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Robust Takagi-Sugeno sensor fault tolerant control strategy for nonlinear system

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Abstract

This paper presents a robust Fault Tolerant Tracking Control (FTTC) design for nonlinear uncertain systems described by Takagi Sugeno (T-S) fuzzy models with unmeasurable premise variables subject to sensor faults. A Proportional Integral Observer (PIO) is proposed to estimate the faulty states and the time-varying sensor faults. The FTTC is synthesized based on the estimation derived from the PIO with a guaranteed H_∞ performance to minimize the effect of the external disturbance. The trajectory tracking performances and the stability of the closed loop system are analyzed based on the Lyapunov theory and the L_2 optimization. The stability condition are formulated in terms of Linear Matrix Inequality (LMI). The proposed robust FTTC is illustrated using simulation results.

Keywords: T-S fuzzy model, fault tolerant tracking control (FTTC), parametric uncertainties, proportional integral observer (PIO), LMIs contrainst .

1 Introduction

In many industrial application, the control system depends on the sensor measurement. Hence if a fault occurs, it can lead to the closed loop stability if no proper action is performed. To overcome this problem, many researches have been devoted to the study of the Fault Tolerant Controllers (FTC). Indeed, the FTC can be defined as a control technique that combines diagnosis with controls methods to preserve stability and performances even in faulty cases.

In literature, we find two approaches dealing with the FTC problem: Passive Fault Tolerant Control (PFTC) and Active Fault Tolerant Control (AFTC). In the first approach, a priori information about the fault which may affect the system is required. In order to compensate faults, this approach considers the fault as uncertainty or disturbance which are taken into account in the design of the control law [18, 31, 34]. In contrast, the second approach is more important. It has the possibility to change its structure according to the information provided by the fault detection and isolation (FDI) block which is designed to characterize the faults. The success of the aforementioned methods depended on the system model complexity. Indeed, several work have been developed for linear system [12, 13, 17, 20]. In reality, the systems are extremely nonlinear. Moreover, a large class of nonlinear systems can be represented by T-S fuzzy models. The aim is to decompose the model of the nonlinear system into a series of linear models involving nonlinear weighting functions. The equivalent fuzzy model describes the dynamic of behavior of the system [26, 27]. Accordingly, several FTC approach have been developed based on T-S representation [7, 10, 21, 24, 28, 11, 32, 35, 30, 33]. For example, in [22] an active FTC for vehicle lateral dynamics represented by T-S model subject to sensor fault has been proposed using the observer bank scheme. In [25], a T-S fuzzy sensor FTC for wind turbine is proposed using adaptive fault estimator. Lately, few research studies fault-tolerant control problem using descriptor approach. Authors in [3] presented a method for designing robust static output-feedback controller design against sensor failure for vehicle dynamics. Authors in [3, 14, 16] used the descriptor observer approach to design an active FTC against sensor fault for T-S system with unmeasurable premise variable without considering parametric uncertainty.

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Recently, some studies have been done regarding to FTC problem for nonlinear uncertain system. These can be considered through two approaches. The first one uses a fuzzy observer to estimate the actuator fault when the problem of trajectory tracking is formulated using LMI formulations [9]. The second one studied the same problem when the premise variables are non measured and the tracking performances was expressed in terms of bilinear matrix inequality (BMI) [1].

Concerning uncertain fuzzy T-S system affected by sensor fault, a few work have been studied the problem. For example, the author in [2] use the descriptor technique to design a fuzzy fault tolerant control for vehicle dynamics subject to constant sensor fault. Authors in [8] present FTC design for uncertain T-S systems by trajectory tracking using a descriptor approach.

Until now, there is very few works dealing the FTC strategies for T-S fuzzy models subject both to unmeasurable premise variables and sensor time-varying faults. This field remains very important, which motivates our study in this work. In this paper, our objective is to design a FTTC law by using a strategy based on the reference model. The considered T-S fuzzy model is subject both to unmeasurable premise variables and parametric uncertainties. The proposed control scheme use a fuzzy PIO to compensate implicitly the sensor faults effect and handle large classes of sensor faults signals [6, 23, 5]. Our main contribution is to deal with the trajectory tracking problem with unmeasurable premise variables. To guarantee the stability in closed loop with unmeasurable premise variables the sufficient conditions are derived by using the Lyapunov stability theory and L_2 performance, all formulated in terms of LMI constraints.

This paper is organized as follows: In section 2, the problem of FTC design for uncertain T-S models is formulated. A fuzzy observer-based FTTC is considered in section 3. Finally, simulation results are provided to demonstrate the design effectiveness.

Notation: The following notations are considered. X^T and X^{-1} are the transpose and the inverse of matrix X, respectively. I is the identity matrix with appropriate dimension. The star symbol $*$ in a symmetric matrix denotes the transposed block in the symmetric position. $Sym(X)$ denotes the Hermitian of the matrix X, that it $Sym(X) = X + X^T$. The following lemmas are considered.

Lemma 1.1. [19] *Consider two real matrices X and Y with appropriate dimensions, for any positive scalar ε, the following inequality is verified:*

$$X^TY + Y^TX \leq \varepsilon X^TX + \varepsilon^{-1}Y^TY \tag{1}$$

Lemma 1.2. [15] *Let a matrix $\Omega < 0$, a matrix X with appropriate dimension such that $X^T\Omega X \leq 0$, and a scalar α, the following inequality holds:*

$$X^T\Omega X \leq -2\alpha X - \alpha^2\Omega^{-1} \tag{2}$$

2 Problem formulation

Let us consider the nonlinear dynamical model described by the following if then rules and tacking into account the model uncertainties:

$$\begin{array}{l}\textbf{Rule}: if\, z_1(t)\, is\, M_{1i}\, \textbf{and}, ..., \textbf{and}\, z_r(t)\, is\, M_{li},\, \textbf{then} \\ \left\{ \begin{array}{l} \dot{x}(t) = (A_i + \Delta A_i)\, x(t) + (B_i + \Delta B_i)\, u(t) \\ y(t) = Cx(t) \end{array} \right.\end{array} \tag{3}$$

where $x(t) \in \Re^n, u(t) \in \Re^m$ and $y(t) \in \Re^p$ are respectively the state vector, the input vector and the output vector. A_i, B_i and C are matrices with appropriate dimensions. $z_i(i = 1, ..., r)$ are the premises variables. $M_{ij}(i = 1, ..., r; j = 1, ..., l)$ are the fuzzy sets and r is the number of rules. $\Delta A_i, \Delta B_i$ are the uncertain matrices corresponding to the i^{th} subsystem.

The overall fuzzy T-S system affected by sensor fault $f(t)$ and bounded unknown input $w(t)$ is inferred as follow by using the interpolation between the local models:

$$\left\{ \begin{array}{l} \dot{x}(t) = \sum\limits_{i=1}^{r} h_i(z(t))\left((A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u_F(t) + w(t)\right) \\ y(t) = Cx(t) + Df(t) \end{array} \right. \tag{4}$$

where

$$\begin{cases} h_i(z(t)) = \frac{\mu_i(z(t))}{\sum\limits_{i=1}^{r}\mu_i(z(t))}, \mu_i(z(t)) = \prod\limits_{k=1}^{l} M_{ik}(z_k(t)) \\ h_i(z(t)) > 0, \sum\limits_{i=1}^{r} h_i(z(t)) = 1 \end{cases} \tag{5}$$

and $u_F(t)$ represent the input vector.

Proposition 2.1. *The parameter uncertainties considered are norm-bounded, in the form: $\Delta A_i = D_{Ai}F_{Ai}E_{Ai}$ and $\Delta B_i = D_{Bi}F_{Bi}E_{Bi}$, where D_{Ai}, E_{Ai}, D_{Bi}, E_{Bi} are known real constant matrices of appropriate dimension. F_{Ai} and F_{Bi} are unknown Lebesgue measurable matrices satisfy $\forall t$:*

$$\begin{array}{l} (F_{Ai}(t))^T F_{Ai}(t) \leq I \\ (F_{Bi}(t))^T F_{Bi}(t) \leq I \end{array} \tag{6}$$

In the diagnosis problem, we can't consider the premise variable $z(t)$ is measurable because it depend on the input vector $u_F(t)$ or output vector $y(t)$. So that it is interesting to develop T-S model which considers the system state as premise variable. For this, we adding and subtracting the term $\sum\limits_{i=1}^{r} h_i(\hat{z}(t))\,(A_i x(t) + B_i u_F(t))$, then we obtain the following equivalent system with the weighting functions depending on the estimated state

$$\begin{cases} \dot{x}(t) = \sum\limits_{i=1}^{r} h_i(\hat{z}(t))\,((A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u_F(t) + w(t) + \phi(t)) \\ y(t) = Cx(t) + Df(t) \end{cases} \tag{7}$$

where

$$\phi(t) = \sum_{i=1}^{r} (h_i(z(t) - h_i(\hat{z}(t))\,((A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u_F(t))$$

Proposition 2.2. *The sensor fault is unknown but it's q^{th} derivative, that is $f^{(q)}$ is bounded. Let the i^{th} derivatives of the sensor fault f be represented by a state variable formulation as follows:*
$\xi_i = f^{(q-i)} (i = 1, 2, ...q)$
ie
$\dot{\xi}_1 = f^{(q)}, \quad \dot{\xi}_2 = \xi_1, \quad \dot{\xi}_3 = \xi_2, ..., \quad \dot{\xi}_q = \xi_{q-1}.$

3 Fault tolerant tracking control design

In order to take into account both sensor fault, the following augmented system containing the state system and the sensor fault derivatives is constructed:

$$\begin{cases} \dot{\bar{x}}(t) = \sum\limits_{i=1}^{r} h_i(\hat{z}(t))\left((\bar{A}_i + \Delta\bar{A}_i)\bar{x}(t) + (\bar{B}_i + \Delta\bar{B}_i)u_F(t) + \bar{H}w(t) + \bar{G}f^{(q)}(t) + \bar{N}\phi(t)\right) \\ y(t) = \bar{C}\bar{x}(t) \end{cases} \tag{8}$$

where

$$\bar{x} = \begin{bmatrix} x^T & \xi_1^T & \xi_2^T & \xi_3^T & \cdots & \xi_q^T \end{bmatrix}^T \in \Re^{\bar{n}}$$

$$\bar{A}_i = \begin{bmatrix} A_i & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix} \in \Re^{\bar{n}}, \Delta\bar{A}_i = \begin{bmatrix} \Delta A_i & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \in \Re^{\bar{n}}$$

$$\bar{B}_i = \begin{bmatrix} B_i^T & 0 & 0 & \cdots & 0 \end{bmatrix}^T \in \Re^{\bar{n}*m}, \Delta\bar{B}_i = \begin{bmatrix} \Delta B_i^T & 0 & 0 & \cdots & 0 \end{bmatrix}^T \in \Re^{\bar{n}*m}$$

$$\bar{G} = \begin{bmatrix} 0 & I_k & 0 & \cdots & 0 \end{bmatrix}^T \in \Re^{\bar{n}*k}, \bar{C} = \begin{bmatrix} C & 0 & 0 & \cdots & D \end{bmatrix} \in \Re^{p*\bar{n}}$$

$$\bar{H} = \begin{bmatrix} I & 0 & 0 & \cdots & 0 \end{bmatrix}^T \in \Re^{\bar{n}*l}, \bar{N} = \begin{bmatrix} I & 0 & 0 & \cdots & 0 \end{bmatrix}^T, \bar{n} = n + kq$$

The objective is to present a control law $u_F(t)$ which can be used to reconfigure the nominal control in sensor faulty case. The controller strategy is described in figure (1). The FTTC law is given by:

$$u_F(t) = u(t) + \sum_{j=1}^{r} h_j(\hat{z}(t))K_j\big(\hat{x}(t) - x_r(t)\big) \tag{9}$$

where K_j are the gain matrices with appropriate dimension.
In order to derive the FTTC law the following T-S PIO is proposed to estimate simultaneously the system states and the sensor faults:

$$\dot{\hat{\bar{x}}}(t) = \sum_{i=1}^{r} h_i(\hat{z}(t))\left(\bar{A}_i\hat{\bar{x}}(t) + \bar{B}u_F(t) + \bar{L}_i(y - \bar{C}\hat{\bar{x}}(t))\right) \tag{10}$$

where $\hat{\bar{x}}(t) = \begin{bmatrix} \hat{x}^T & \hat{\xi}_1^T & \hat{\xi}_2^T & \cdots & \hat{\xi}_q^T \end{bmatrix}^T$ is the estimate of the augmented state $\bar{x}(t) \in \Re^{\bar{n}}$, $\hat{x}(t) = \begin{bmatrix} I_n & 0_{n*q} \end{bmatrix}\hat{\bar{x}}(t) \in \Re^n$ is the estimate of the original system state $x(t) \in \Re^n$, $\bar{L}_i \in \Re^{\bar{n}*p}$ is the observer gain to be designed.
To specify the desired trajectory, we consider the following reference model [29]:

$$\dot{x}_r(t) = \sum_{i=1}^{r} h_i(z(t))\left(A_i x_r(t) + B_i u(t)\right) \tag{11}$$

with $x_r(t)$ is the reference state.

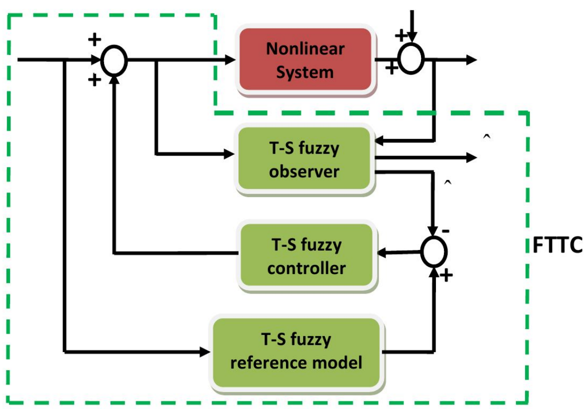

Figure 1: Fault Tolerant Tracking Control (FTTC) strategy

We define the estimation error and the tracking error respectively as follows
$e(t) = \bar{x}(t) - \hat{\bar{x}}(t)$, $e_t(t) = x(t) - x_r(t)$.
The dynamics of $e(t)$ and $e_t(t)$ are given by

$$\begin{aligned}\dot{e}(t) = &\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\hat{z}(t))h_j(\hat{z}(t))((\bar{A}_i - \bar{L}_i\bar{C})e(t) + \Delta\bar{A}_i\bar{x}(t)\\ &+\Delta\bar{B}_i(u(t) + K_j(\hat{x}(t) - x(t) + e_t(t)))) + \bar{H}w(t) + \bar{G}f^{(q)}(t) + \bar{N}\phi(t)\end{aligned} \tag{12}$$

and

$$\dot{e}_t(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\hat{z}(t))h_j(\hat{z}(t))\Big((A_i + \Delta A_i)e_t(t) - \big((B_i + \Delta B_i)K_j(\hat{x} - x_r)\big)e(t) + \Delta A_i x_r(t) + \Delta B_i u(t) + w(t) + \phi(t)\Big) \tag{13}$$

in other side we have

$$\begin{aligned}&\Delta\bar{A}_i\bar{x}(t) = \Delta\tilde{A}_i x(t)\\ &K_j\big(x(t) - \hat{x}(t)\big) = \tilde{K}_j\big(\bar{x}(t) - \hat{\bar{x}}(t)\big) = \tilde{K}_j e(t)\\ &\big(B_i + \Delta B_i\big)K_j\left(\hat{x}(t) - x_r(t)\right) = \big(B_i + \Delta B_i\big)\left(K_j e_t(t) - \tilde{K}_j e(t)\right)\end{aligned}$$

where

$$\Delta\tilde{A}_i = \begin{bmatrix} \Delta A_i \\ 0_{q*n} \end{bmatrix}$$
$$\tilde{K}_j = \begin{bmatrix} K_j & 0_{m*q} \end{bmatrix}$$

Then equation (3) and (3) can be written respectively as follow

$$\dot{e}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\hat{z}(t))h_j(\hat{z}(t))\Big(\big(\bar{A}_i - \bar{L}_i\bar{C} - \Delta\bar{B}_i\tilde{K}_j\big)e(t) + \big(\Delta\bar{B}_iK_j + \Delta\tilde{A}_i\big)e_t(t) + \Delta\tilde{A}_ix_r(t) + \Delta\bar{B}_iu(t) + \bar{H}w(t) + \bar{G}f^{(q)}(t) + \bar{N}\phi(t)\Big) \tag{14}$$

and

$$\dot{e}_t(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\hat{z}(t))h_j(\hat{z}(t))\Big(\big(A_i + \Delta A_i + B_iK_j + \Delta B_iK_j\big)e_t(t) - \big((B_i + \Delta B_i)\tilde{K}_j\big)e(t) + \Delta A_ix_r(t) + \Delta B_iu(t) + w(t) + \phi(t)\Big) \tag{15}$$

After some easy manipulation, the closed-loop system containing the tracking error, the estimation error and the reference state can be expressed as follow:

$$\dot{\tilde{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\hat{z}(t))h_j(\hat{z}(t))\Big(\tilde{G}_{ij}\tilde{x}(t) + \tilde{F}_{ij}\Upsilon(t)\Big) \tag{16}$$

where

$$\tilde{x}(t) = \begin{bmatrix} e_t(t) \\ e(t) \\ x_r(t) \end{bmatrix}, \tilde{F}_{ij} = \begin{bmatrix} I & 0 & \Delta B_i & I \\ \bar{H} & \bar{G} & \Delta\bar{B}_i & \bar{N} \\ 0 & 0 & B_i & 0 \end{bmatrix}, \Upsilon(t) = \begin{bmatrix} w(t) \\ f^{(q)}(t) \\ u(t) \\ \phi(t) \end{bmatrix}$$
$$\tilde{G}_{ij} = \begin{bmatrix} A_i + B_iK_j + \Delta A_i + \Delta B_iK_j & -(B_i\tilde{K}_j + \Delta B_i\tilde{K}_j) & \Delta A_i \\ \Delta\tilde{A}_i + \Delta\bar{B}_iK_j & \bar{A}_i - \bar{L}_i\bar{C} - \Delta\bar{B}_i\tilde{K}_j & \Delta\tilde{A}_i \\ 0 & 0 & A_i \end{bmatrix} \tag{17}$$

Note that the H_∞ tracking performance is defined as follow [29]:

$$\int_0^{t_f} \big(x^T(t)Qx(t)\big)dt \le \gamma^2 \int_0^{t_f} w^T(t)w(t)dt \tag{18}$$

which can be reformulated as:

$$\int_0^{t_f} \Big(\tilde{x}^T(t)\tilde{Q}\tilde{x}(t)\Big)dt \le \gamma^2 \int_0^{t_f} \Upsilon^T(t)\Upsilon(t)dt \tag{19}$$

where

$$\tilde{Q} = diag\begin{bmatrix} Q & 0 & 0 \end{bmatrix} \tag{20}$$

In order to satisfy the stabilization condition derived in LMI terms of system (3) and guarantees robustness against external disturbances, we propose the following theorem:

Theorem 3.1. *For a given positive scalar α, the closed-loop fuzzy system in* (3) *is asymptotically stable and the H_∞ tracking performance is guaranteed with an attenuation level γ, provided that the signal $\Upsilon(t)$ is bounded, if there exist symmetric positive definite matrices X_1, P_{2i}, P_{3i} and matrices J_i, Y_i jointly with positive scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6, \varepsilon_7$, ε_8, ε_9, ε_{10} satisfying the following constraint*

$$\begin{array}{ll} \min\gamma \text{ such that} & \\ \aleph_{ii} < 0 & for \quad i = j = 1, 2, ..., r \\ \frac{2}{r-1}\aleph_{ij} + \aleph_{ij} + \aleph_{ji} < 0 & for \quad i \neq j \end{array} \tag{21}$$

where

$$\aleph_{ij} = \begin{bmatrix} \aleph_{ij}^{11} & \aleph_{ij}^{12} \\ * & \aleph_{ij}^{22} \end{bmatrix}$$

$$\aleph_{ij}^{11} = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & 0 & 0 & I & 0 & 0 & I \\ * & -2\alpha\bar{X}_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -2\alpha I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -2\alpha I & 0 & 0 & 0 & 0 \\ * & * & * & * & -2\alpha I & 0 & 0 & 0 \\ * & * & * & * & * & -2\alpha I & 0 & 0 \\ * & * & * & * & * & * & -2\alpha I & 0 \\ * & * & * & * & * & * & * & -2\alpha I \end{bmatrix}$$

$$\aleph_{ij}^{12} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & X_1 & Y_i^T & 0 \\ \alpha I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \begin{bmatrix} Y_i & 0 \end{bmatrix}^T \\ 0 & \alpha I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha I & 0 & 0 & 0 \end{bmatrix}$$

$$\aleph_{ij}^{22} = \begin{bmatrix} \Sigma_{99} & P_{2i} & 0 & P_{2i}\bar{H} & P_{2i}\bar{G} & 0 & P_{2i}\bar{N} & 0 & 0 & 0 \\ * & \Sigma_{1010} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Sigma_{1111} & 0 & 0 & P_{3i}\bar{B}_i & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Sigma_{1414} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Sigma_{1616} & 0 & 0 \\ * & * & * & * & * & * & * & * & \Sigma_{1717} & 0 \\ * & * & * & * & * & * & * & * & * & \Sigma_{1818} \end{bmatrix}$$

and

$$\begin{aligned}
&\Sigma_{11} = Sym(A_iX_1 + BY_i) + \left(\varepsilon_1^{-1} + \varepsilon_4\right) D_{Ai}D_{Ai}^T + \left(\varepsilon_2 + \varepsilon_3 + \varepsilon_5\right) D_{Bi}D_{Bi}^T \\
&\Sigma_{12} = -B \begin{bmatrix} Y_i & 0 \end{bmatrix} \\
&\Sigma_{99} = Sym\left(P_{2i}\bar{A}_i - \bar{J}_i\bar{C}\right) \\
&\Sigma_{1010} = -\left(\left(\varepsilon_6 + \varepsilon_8 + \varepsilon_{10}^{-1}\right) \bar{D}_{Bi}\bar{D}_{Bi}^T + \left(\varepsilon_7 + \varepsilon_9^{-1}\right) \tilde{D}_{Ai}\tilde{D}_{Ai}^T\right)^{-1} \\
&\Sigma_{1111} = Sym\left(P_{3i}A_i\right) + \varepsilon_4^{-1}E_A^TE_A + \varepsilon_7^{-1}\tilde{E}_A^T\tilde{E}_A \\
&\Sigma_{1414} = -\gamma^2 I + \varepsilon_5^{-1}E_B^TE_B + \varepsilon_8^{-1}\bar{E}_B^T\bar{E}_B \\
&\Sigma_{1616} = -\left(Q + \varepsilon_1E_A^TE_A + \varepsilon_9\tilde{E}_A^T\tilde{E}_A\right)^{-1} \\
&\Sigma_{1717} = -\left(\varepsilon_2^{-1}E_B^TE_B + \varepsilon_{10}\bar{E}_B^T\bar{E}_B\right)^{-1} \\
&\Sigma_{1818} = -\left(\varepsilon_3^{-1}E_B^TE_B + \varepsilon_6^{-1}\bar{E}_B^T\bar{E}_B\right)^{-1} \\
&\bar{X}_1 = diag \begin{bmatrix} X_1 & I_{q*q} \end{bmatrix}, X_1 = P_1^{-1} \\
&K_i = X_1^{-1}Y_i, \bar{L}_i = P_{2i}^{-1}\bar{J}_i
\end{aligned}$$

Remark 3.2. *The presented work illustrates the design of a fault tolerant controller for T-S model with unmeasurable premise variables. It is clear that the discussed problem in this manuscript is very difficult particularly for unmeasurable premise variables case. For less conservativeness, fuzzy Lyapunov functions should be considered in future works.*

Remark 3.3. *The proposed approach concern the uncertain T-S system affected by time varying sensor fault. Contrary to* [2]*, in this approach we can handle a large class of sensors faults by considering that the fault q^{th} derivatives are bounded.*

Proof. Consider the following candidate Lyapunov function for the augmented system (16)

$$V(\tilde{x},(t)) = \tilde{x}^T(t)\tilde{P}\tilde{x}(t) \;\; with \;\; \tilde{P} = \tilde{P}^T > 0 \tag{22}$$

The stability of the closed-loop model (16) is satisfied under the performance (19) with the attenuation level γ if:

$$\dot{V}(\tilde{x},t) + \tilde{x}^T(t)\tilde{Q}\tilde{x}(t) - \gamma^2\Upsilon^T(t)\Upsilon(t) \leq 0 \tag{23}$$

The inequality (23) can be written as:

$$\begin{bmatrix} \tilde{x}(t) \\ \Upsilon(t) \end{bmatrix}^T \begin{bmatrix} \tilde{G}_{ij}^T\tilde{P} + \tilde{P}\tilde{G}_{ij} + \tilde{Q} & \tilde{P}\tilde{F}_{ij} \\ \tilde{F}_{ij}^T\tilde{P} & -\gamma^2 I \end{bmatrix} \begin{bmatrix} \tilde{x}(t) \\ \Upsilon(t) \end{bmatrix} \leq 0 \tag{24}$$

Then the inequality (24) is satisfied if the following condition hold

$$\begin{bmatrix} \tilde{G}_{ij}^T\tilde{P} + \tilde{P}\tilde{G}_{ij} + \tilde{Q} & \tilde{P}\tilde{F}_{ij} \\ \tilde{F}_{ij}^T\tilde{P} & -\gamma^2 I \end{bmatrix} \leq 0 \tag{25}$$

Let us assume that $\tilde{P} = diag\begin{bmatrix} P_1 & P_{2i} & P_{3i} \end{bmatrix}$
Equation (25) can be written as

$$\Pi_{ij} + \Delta\Pi_{ij} \leq 0 \tag{26}$$

with

$$\Pi_{ij} = \begin{bmatrix} Sym\left(P_1A_i + P_1B_iK_i\right) + Q & -P_1B_i\tilde{K}_j & 0 & P_1 & 0 & 0 & P_1 \\ * & Sym\left(P_{2i}\left(\bar{A}_i - \bar{L}_i\bar{C}\right)\right) & 0 & P_{2i}\bar{H} & P_{2i}\bar{G} & 0 & P_{2i}\bar{N} \\ * & * & Sym\left(P_{3i}A_i\right) & 0 & 0 & P_{3i}B_i & 0 \\ * & * & * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & * & -\gamma^2 I \end{bmatrix} \text{ and}$$

$$\Delta\Pi_{ij} = \begin{bmatrix} Sym\left(P_1\Delta A_i + P_1\Delta B_iK_j\right) & -P_1\Delta B_i\tilde{K}_j + \Delta\tilde{A}_i^T P_2 + K_j^T\Delta\bar{B}_iP_2 & P_1\Delta A_i & 0 & 0 & P_1\Delta B_i & 0 \\ * & Sym\left(-P_{2i}\Delta\bar{B}_i\tilde{K}_j\right) & P_{2i}\Delta\tilde{A}_i & 0 & 0 & P_{2i}\Delta\bar{B}_i & 0 \\ * & * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & 0 \end{bmatrix}$$

Then using the uncertainties structure defined in assumption 1 and the well-known property given in proposition 2.1, $\sum\limits_{i=1}^{r}\sum\limits_{j=1}^{r} h_i(\hat{z}(t))h_j(\hat{z}(t))\Delta\Pi_{ij}$ can be bounded as follows:

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\hat{z}(t))h_j(\hat{z}(t))\Delta\Pi_{ij} \leq diag\begin{bmatrix} S_{11ij} & S_{22ij} & S_{33ij} & 0 & 0 & S_{66ij} & 0 \end{bmatrix} \tag{27}$$

with

$$\begin{aligned} S_{11ij} &= P_1\left((\varepsilon_1^{-1} + \varepsilon_4)D_{Ai}D_{Ai}^T + (\varepsilon_2 + \varepsilon_3 + \varepsilon_5)D_{Bi}D_{Bi}^T\right)P_1 + \varepsilon_1E_{Ai}^TE_{Ai} + \varepsilon_9\tilde{E}_{Ai}^T\tilde{E}_{Ai} \\ &\quad + K_j^T\left(\varepsilon_2^{-1}E_{Bi}^TE_{Bi} + \varepsilon_{10}\bar{E}_{Bi}^T\bar{E}_{Bi}\right)K_j \\ S_{22ij} &= P_{2i}\left(\left(\varepsilon_6 + \varepsilon_8 + \varepsilon_{10}^{-1}\right)\bar{D}_{Bi}\bar{D}_{Bi}^T + \left(\varepsilon_7 + \varepsilon_9^{-1}\right)\tilde{D}_{Ai}\tilde{D}_{Ai}^T\right)P_{2i} \\ &\quad + \tilde{K}_j^T\left(\varepsilon_3^{-1}E_{Bi}^TE_{Bi} + \varepsilon_6^{-1}\bar{E}_{Bi}^T\bar{E}_{Bi}\right)\tilde{K}_j \\ S_{33ij} &= \varepsilon_4^{-1}E_{Ai}^TE_{Ai} + \varepsilon_7^{-1}\tilde{E}_{Ai}^T\tilde{E}_{Ai} \\ S_{66ij} &= \varepsilon_5^{-1}E_{Bi}^TE_{Bi} + \varepsilon_8^{-1}\bar{E}_{Bi}^T\bar{E}_{Bi} \end{aligned}$$

Consequently, the inequality (26) hold if

$$\begin{bmatrix} \Theta_{11} + S_{11ij} & -P_1B_i\tilde{K}_j & 0 & P_1 & 0 & 0 & P_1 \\ * & \Theta_{22} + S_{22ij} & 0 & P_{2i}\bar{H} & P_{2i}\bar{G} & 0 & P_{2i}\bar{N} \\ * & * & \Theta_{33} + S_{33ij} & 0 & 0 & P_{3i}B_i & 0 \\ * & * & * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & * & * & -\gamma^2 I + S_{66ij} & 0 \\ * & * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{28}$$

with

$$\Theta_{11} = Sym\left(P_1A_i + P_1B_iK_j\right) + Q$$
$$\Theta_{22} = Sym\left(P_{2i}\left(\bar{A}_i - \bar{L}_j\bar{C}\right)\right)$$
$$\Theta_{33} = Sym\left(P_{3i}A_i\right)$$

The condition (28) contains nonlinear terms. Now, the goals is to formulate it as an LMI problem. To do this end, we partition the inequality (28) as follow:

$$\Gamma_{ij} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ * & \Gamma_{22} \end{bmatrix} \tag{29}$$

where

$$\Gamma_{11} = \Theta_{11} + S_{11ij}$$

$$\Gamma_{12} = \begin{bmatrix} -P_1B_i\tilde{K}_j & 0 & P_1 & 0 & 0 & P_1 \end{bmatrix}$$

$$\Gamma_{22} = \begin{bmatrix} \Theta_{22} + S_{22ij} & 0 & P_{2i}\bar{H} & P_{2i}\bar{G} & 0 & P_{2i}\bar{N} \\ * & \Theta_{33} + S_{33ij} & 0 & 0 & P_{3i}B_i & 0 \\ * & * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & * & -\gamma^2 I + S_{66ij} & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix}$$

Multiplying inequality (29) left and right by $Z = diag\begin{bmatrix} P_1^{-1} & X \end{bmatrix}$, where $X = diag\begin{bmatrix} \bar{X}_1 & I & I & I & I & I \end{bmatrix}$, $\bar{X}_1 = diag\begin{bmatrix} P_1^{-1} & I_{q*q} \end{bmatrix}$, then we obtain

$$\begin{bmatrix} P_1^{-1}\Gamma_{11}P_1^{-1} & P_1^{-1}\Gamma_{12}X \\ * & X\Gamma_{22}X \end{bmatrix} < 0 \tag{30}$$

The block Γ_{22} can be written as

$$\Gamma_{22} = \Gamma_{221} + \Gamma_{222} \tag{31}$$

where

$$\Gamma_{221} = \begin{bmatrix} \Theta_{22} + S_{221ij} & 0 & P_{2i}\bar{H} & P_{2i}\bar{G} & 0 & P_{2i}\bar{N} \\ * & \Theta_{33} + S_{33ij} & 0 & 0 & P_{3i}B_i & 0 \\ * & * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & * & -\gamma^2 I + S_{66ij} & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix}$$

$$S_{221ij} = P_2\left(\left(\varepsilon_6 + \varepsilon_8 + \varepsilon_{10}^{-1}\right)\bar{D}_B\bar{D}_B^T + \left(\varepsilon_7 + \varepsilon_9^{-1}\right)D_AD_A^T\right)P_2$$

and

$$\Gamma_{222} = diag\begin{bmatrix} S_{222ij} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$S_{222ij} = \tilde{K}_j^T\left(\varepsilon_3^{-1}E_B^TE_B + \varepsilon_6^{-1}\bar{E}_B^T\bar{E}_B\right)\tilde{K}_j$$

Inequality (29) implies that $\Gamma_{22} \leq 0$, so using Lemma 2, it yelds

$$X\Gamma_{22}X \leq X\Gamma_{222}X - 2\alpha X - \alpha^2\Gamma_{221}^{-1} \tag{32}$$

By substituting (32) into (30) and using the Schur complement, the inequality (28) can be written in the following form:

$$\begin{bmatrix} P_1^{-1}\Gamma_{11}P_1^{-1} & P_1^{-1}\Gamma_{12}X & 0 \\ * & -2\alpha X + X\Gamma_{222}X & \alpha I \\ * & * & \Gamma_{221} \end{bmatrix} \leq 0 \tag{33}$$

and the LMI condition in theorem 1 can be obtained. □

4 Simulation results

In this section numerical simulations have been performed to validate the developed FTTC scheme. Let us consider the two tanks system [4] with the above analysis technique (fig.2):

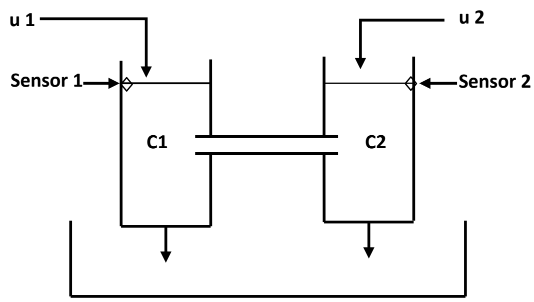

Figure 2: Two tanks system

The system can be expressed in the following form:

$$\begin{cases} \dot{x}(t) = A(x(t)) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{34}$$

where

$$x(t) = \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^T$$
$$A(x(t)) = \begin{bmatrix} \frac{-R_1}{\sqrt{x_1(t)}} - \frac{R_{12}}{\sqrt{|x_1-x_2|}} & \frac{R_{12}}{\sqrt{|x_1-x_2|}} \\ \frac{R_{12}}{\sqrt{|x_1-x_2|}} & \frac{-R_2}{\sqrt{x_2(t)}} - \frac{R_{12}}{\sqrt{|x_1-x_2|}} \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Using the well-known sector nonlinearity approach, a T-S fuzzy model can be constructed where the nonlinear entries of the matrix $A(x(t))$ are considered as premise variables. The system (34) is constituted by the following nonlinearities:

$$z_1(t) = \frac{1}{\sqrt{x_1(t)}}, z_2(t) = \frac{1}{\sqrt{x_2(t)}}$$

Consequently, the global fuzzy system affected by sensor fault $f(t)$ can be written in the following form:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{4} h_i(z(t))\,(A_i x(t)) + Bu(t) \\ y(t) = Cx(t) + Df(t) \end{cases} \tag{35}$$

where

$$A_1 = \begin{bmatrix} -0.00692 & 0.0580 \\ 0.0580 & -0.0682 \end{bmatrix}, A_2 = \begin{bmatrix} -0.0295 & 0.0183 \\ 0.0183 & -0.0248 \end{bmatrix} A_3 = \begin{bmatrix} -0.211 & 0.0155 \\ 0.0155 & -0.0257 \end{bmatrix}, A_4 = \begin{bmatrix} -0.0315 & 0.0259 \\ 0.0259 & -0.0324 \end{bmatrix}$$

The sensor fault can be represented as an additive fault in which the fault signal desponds on the measured state, as illustrated below:

$$y(t) = Cx(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} (0.5 * x_1(t))$$

The system parameters deviate until 10% from their nominal value that $D_A = 0.1$ and $E_A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. An unknown disturbance $w(t)$ with band-limited white noise given by fig.8 is considered. By solving the LMI condition of theorem 1, the designed observer and controller gains are obtained as:

$$L_1 = \begin{bmatrix} 4.9916 \\ 0.1954 \\ 22.495 \\ 25.0178 \end{bmatrix}, L_2 = \begin{bmatrix} 4.9663 \\ 0.0627 \\ 22.6391 \\ 25.2505 \end{bmatrix}, L_3 = \begin{bmatrix} 4.9751 \\ 0.0535 \\ 22.5831 \\ 25.1638 \end{bmatrix}, L_4 = \begin{bmatrix} 4.9643 \\ 0.0880 \\ 22.6531 \\ 25.2712 \end{bmatrix}$$

$$K_1 = \begin{bmatrix} -24.654 & -2.9 \\ -2.9 & -21.59 \end{bmatrix}, K_2 = \begin{bmatrix} -23.525 & -0.915 \\ -0.9150 & -23.76 \end{bmatrix}, \ K_3 = \begin{bmatrix} -23.945 & -0.775 \\ -0.775 & -23.715 \end{bmatrix}, K_4 = \begin{bmatrix} -23.425 & -1.295 \\ -1.295 & -23.38 \end{bmatrix}$$

The simulation results are illustrated by figures(3-8).

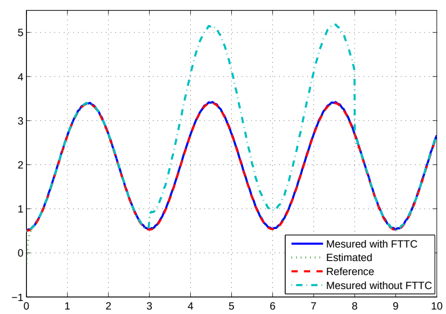

Figure 3: State $x_1(t)$

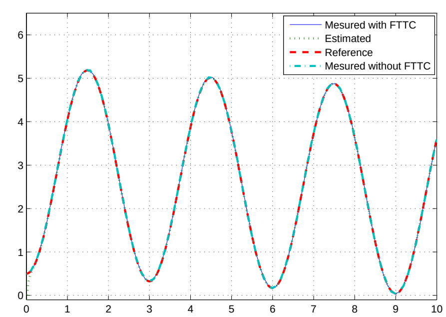

Figure 4: State $x_2(t)$

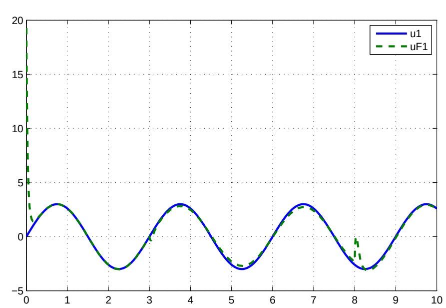

Figure 5: first input control $u_1(t)$

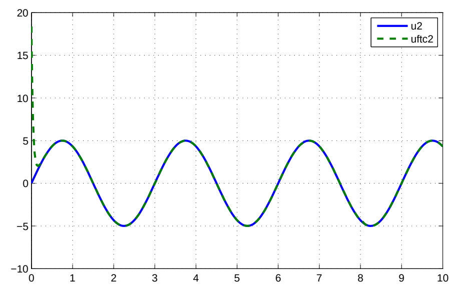

Figure 6: second input control $u_2(t)$

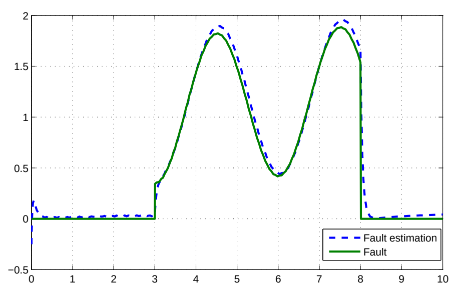

Figure 7: sensor fault estimation

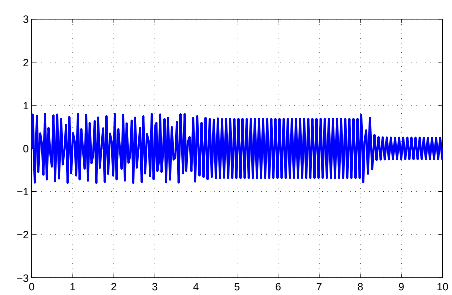

Figure 8: disturbance $w(t)$

Figure (7) illustrates the evolution of the sensor fault with its estimate value. The figures(3-4) show the evolution of the system state together with the reference state and the estimated state. We can see that without FTTC the system state deviate from their reference. However, the proposed fuzzy controllers can preserve the tracking performances. Indeed, a small tracking error is observed in spite of the variation of the parameters. Figures(5,6) show the nominal control input together with FTTC. From this results, we can conclude that the performances of the FTTC approach are satisfactory and can guarantee acceptable functioning in spite the occurrence of sensor fault. This is due to the fact that the fuzzy observer can ensure implicit fault estimation and compensation of the sensor fault.

Remark 4.1. *Note that maintaining state estimation without changes is due to the fact that the PIO perform implicit fault estimation and compensation of sensor fault from the input of PIO. This fact is clearly interpreted from the error signal* $(\bar{y} - \bar{C}\hat{\bar{x}})$ *which can be written as* $Cx + Df - C\hat{x} - D\hat{f}$*, then as long as there are no sensor fault,* $\hat{f} = 0$*. However, once a sensor fault occurs the fault estimation* $\hat{f}$ *compensates the effect of the fault signal* f *and hence the observer always receives a fault free error signal.*

5 Conclusions

In this study, we treat the problem of FTTC design for uncertain nonlinear systems described by T-S models with unmeasurable premise variables. The goal of the FTTC is to keep the state trajectory tracking. In order to reject the sensor fault effect, we use a PIO that can give simultaneously the state and the sensor fault. Based on Lyapunov theory, a strategy dealing with the considered fault is described to ensure the tracking performances. Numerical example have been considered to illustrate the efficiency of the proposed FTTC. It can be remarked that these results will be extended by considering time varying parameters or delay to obtain more accurate results.

References

[1] S. Aouaouda, T. Bouarar, O. Bouhali, *Fault tolerant tracking control using unmeasurable premise variables for vehicle dynamics subject to time varying faults*, Journal of the Franklin Institute, **351** (2014), 4514–4537.

[2] S. Aouaouda, M. Chadli, M. Boukhnifer, H. R. Karimi, *Robust fault tolerant tracking controller design for vehicle dynamics: A descriptor approach*, Mechatronics, **30** (2015), 316–326.

[3] S. Aouaouda, M. Chadli, H. R. Karimi, *Robust static output-feedback controller design against sensor failure for vehicle dynamics*, Journal of Control Theory Applications, IET, **8** (2014), 728–737.

[4] A. Benzaouia, A. Hmamed, A. El Hajjaji, *Stabilization of controlled positive discrete-time T-S fuzzy system by state feedback control*, International Journal of Adaptive Control and Signal processing, **24** (2010), 1091–1106.

[5] H. Ben Zina, M. Allouche, M. Souissi, M. Chaabane,*Takagi-Sugeno fuzzy observer based fault tolerant tracking control for induction motor drive*, 4th International Conference on Systems and Control (ICSC), 2015.

[6] M. Bouattour, M. Chadli, A. El Hajjaji, M. Chaabane, *Estimation of state, actuators and sensors faults for T-S models*, 49th IEEE Conference on Decision and Control CDC'10, Atlanta, Georgia, USA, 2010.

[7] M. Bouattour, M. Chadli, A. El Hajjaji, M. Chaabane, *Design of robust fault detection observer for Takagi-Sugeno models using the descriptor approach*, International Journal of Control, Automation and Systems, **9** (2011), 973–979.

[8] T. Bouarar, B. Marx, D. Maquin, J. Ragot, *Fault-tolerant control design for uncertain Takagi-Sugeno systems by trajectory tracking: A descriptor approach*, Control Theory Applications, IET, **7**, **14** (2013), 1789–1805.

[9] M. Chadli, S. Aouaouda, H. R. Karimi, P. Shi, *Robust fault tolerant tracking controller design for a VTOL aircraft*, Journal of the Franklin Institute, **350** (2013), 2627–2654.

[10] A. Chamsedine, D. Theillol, Y. M. Zhang, C. Join, C. Rabbath, *Active fault tolerant control system design with trajectory re-planing against actuator faults and saturation: Application to a quadratic unmanned arial vehicle*, International Journal of Adaptive Control and Signal processing, **29** (2015), 1–23.

[11] L. Cui, Y. Liu, Y. Wang, D. Duan, *New approach to exponential stability analysis and stabilization for delayed T-S fuzzy markovian jump systems*, Iranian Journal of Fuzzy Systems, **13**(5) (2016), 1–19.

[12] E. Gao, P. Antsaklis, *Stability of the pseudo-inverse method for reconfugurable control system*, International Journal of Control, **53** (1991), 717–729.

[13] Z. Gao, P. Antasklis, *Reconfigurable control system design via perfect model following*, International Journal of Control, **56** (1992), 783–789.

[14] H. Ghorbel, A. El Hajjaji, M. Souissi, M. Chaabane, *Fault-tolerant trajectory tracking control for takagi-sugeno systems with unmeasurable premise variables: Descriptor approach*, Journal of Circuits, Systems and Signal Processing, **33** (2014), 1763–1781.

[15] T. Guerra, A. Kruszewski, L. Vermeiren, H. Tirmant, *Conditions of output stabilization for non linear models in the Takagi-Sugeno form*, Fuzzy Sets and Systeme, **157** (2006), 1248–1295.

[16] I. Haj Brahim, M. Bouattour, D. Mehdi, M. Chaabane, *Robust fault tolerant control for T-S fuzzy system with unmeasurable premise variables: LMI approach*, 12th International Multi-Conference on Systems, Signals and Devices (SSD), 2015.

[17] C. Y. Huang, R. F. stengle, *Restructable control using proportionel integral implicit model following*, Journal of Guidance, Control, and Dynamics, **13** (1990), 303–309.

[18] B. Jiang, M. Starosweiki, V. Cocquempot, *Fault accomodation for a class of nonlinear dynamic systems*, IEEE Transaction on Automatic Control, **51** (2006), 1578–1583.

[19] N. Manamanni, B. Mansouri, A. Hamzaoui, J. Zayton, *Relaxed conditions in tracking control desiyn for T-S fuzzy model*, Journal of Intelligent and Fuzzy Systems, **18** (2007), 1985–2010.

[20] B. Marx, D. Koeing, D. George, *Robust fault tolerant control for descriptor system*, IEEE Transaction on Automatic Control, **49** (2004), 1896–1875.

[21] K. Micheal, B. C. Eugenio, J. S. L. Volter, F. P. S. Louis, *Fuzzy dynamic output feedback control throught nonlinear Takagi-Sugeno models*, Fuzzy Sets and Systems, **263** (2015), 92–111.

[22] M. Oudghiri, M. Chadli, A. E. Hajjaji, *Robust observer-based fault-tolerant control for vehicle lateral dynamics*, International Journal of Vehicle Design, **48** (2008), 173–189.

[23] M. Sami, R. J. Patton, *Wind turbine sensor fault tolerant control via a multiple-model approach*, UKACC International Conference on Control, Cardiff, 2012.

[24] M. Sami Shaker, R. J. Patton, *Active sensor fault tolerant output feedback tracking control for wind turbine systems via T-S model*, Engineering Applications of Artificial Intelligence, **34** (2014), 1–12.

[25] M. S. Shaker, R. J. Patton, *Wind tubine power maximisation based on adaptive sensor fault sliding mode control*, Proceding of the 20th Mediterranean Conference on Control and Automation, 2012.

[26] T. Takagi, M. Sugeno, *Fuzzy identification of system and its applications to modeling and control*, IEEE Transactions on Systems Man and Cybernetics, **15** (1985), 116–132.

[27] K. Tanaka, T. Ikeda, H.O. Wang, *Robust stabilization of a class of uncertain nonlinear systems via fuzzy control: Quadratic stabilizability, H_∞ control theory,and linear matrix inequalities*, IEEE Transactions on Fuzzy Systems, **4**(1) (1996), 1–13.

[28] S. Tong, T. Wang, W. Zhang, *Fault tolerant control for uncertain fuzzy systems with actuators failures*, International Journal of Innovative Computing, Information and Control, **4** (2008), 2461–2474.

[29] C. S. Tseng, B. S. Chen, H. J. Uang, *Fuzzy traking control design for nonlinear dynamic systems via T-S fuzzy model*, IEEE Transactions on Fuzzy Systems, **9** (2001), 381–392.

[30] D. Yang, X. Xie, *Observer-based state estimation of discrete-time nonlinear systems via a novel maximum-priority-based fuzzy observer*, Signal Processing, **137** (2017), 63–68.

[31] X. Yu, Y. Zhang, *Design of passive fault-tolerant flight controller against actuator failures*, Chinese Journal of Aeronautics, **28** (2015), 180–190.

[32] H. Yue, J. Li, *Adaptive fuzzy traking control for a class of nonlinear systems with unknown distributed time-varying delays and unknown control directions*, Iranian Journal of Fuzzy Systems, **11**(1) (2014), 1–25.

[33] H. Zhang, Y. Cui, Y. Wang, *Hybrid fuzzy adaptive fault-tolerant control for a class of uncertain nonlinear systems with unmeasured state*, IEEE Transactions on Fuzzy Systems, **25** (2017), 1041–1051.

[34] K. Zhang, B. Jiang, M. Starosweiki, *Dynamic output feedback fault tolerant controller design for Takagi-Sugeno fuzzy system with actuator fault*, IEEE Transaction on Fuzzy Systems, **18** (2010), 194–201.

[35] H. Zhang, X. Xie, *Relaxed stability conditions for continuous-time T-S fuzzy-control systems via augmented multi-indexed matrix approach*, IEEE Transactions on Fuzzy Systems, **19** (2011), 478–492.