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A new method for fuzzification of nested dummy variables by fuzzy clustering membership functions and its application in financial economy

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Abstract

In this study, the aim is to propose a new method for fuzzification of nested dummy variables. The fuzzification idea of dummy variables has been acquired from non-linear part of regime switching models in econometrics. In these models, the concept of transfer functions is like the notion of fuzzy membership functions, but no principle or linguistic sentence have been used for inputs. Consequently, for the non-linear part including transfer function, there is no reason why the different types of functions such as logistic are used. Therefore, in order to solve the aforementioned problem like the regime switching models, the transfer functions are considered for dummy variables. However, the presented transfer functions are proposed by fuzzy clustering membership function. Finally, using fuzzy logic, the membership functions of clusters are combined with each other and constitute the fuzzy nested regimes. The suggested model has been used in financial data of Iran's stock in order to examine the equity premium Puzzle. The results of using above model helped in modeling appropriate second-order moments in consumption capital asset pricing model.

Keywords: Finance, fuzzy, CCAPM-F, nested dummy variables.

1 Introduction

In time series, there are some models under the title of regime switching which examines the effects of shocks, structural failure due to the political and social events such as war, revolution, strike, riot and sanction and also economic decisions and policies by dummy variables of zero- one based on result. Among these models, the TAR model by Tong [26], Tong and Lim [27] can be mentioned. The existence of dummy binary variable of zero- one in TAR model causes to obtaining switch between two regimes from the specified value of threshold variable. However, as a result of existing uncertainty in these shocks and regimes, their measurement by binary dummy variables is not accurate. For this reason, some models under the title of STAR were introduced by Franses and Dijk [14]. In STAR model, different transfer functions like logistic were substituted for zero- one dummy variables. Considering the increase of threshold variable, these functions gradually propose the transition among regimes between zero and one. However, the idea of gradual transition among regimes had been indicated previously by Bacon and Watts [5], but they entered into the literature of non-linear time series by Chan and Tong [12]. Finally, Granger and Trasvirta [17] and Trasvirta [25] developed it.

By reviewing these transfer functions, it can be found that its concept is too similar to fuzzy membership functions. Therefore, considering transfer function for binary dummy variables of zero and one by researchers may be one of the first steps of dummy variables fuzzificaion, though the researchers were ignorant of this matter. The word fuzzy means ambiguity [31]. This ambiguity is resulted from uncertainty and this uncertainty can be due to some factors such as weakness of human knowledge, weakness in measurement, lack of clearness for subject (being from judgment and value type). In this case, the attributes like warm, cold, good, bad, low and high are along with ambiguity. Similarly, the influence of political, economic, social and cultural shocks is not measurable accurately. Consequently, we can look at their effects on economic and financial variables with a fuzzy attitude.

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Baliamoune [7], in a regression model, used logistic membership function for dummy variables implicitly. However, the subject and aim of that article were not concerning the fuzzification of dummy variables and their comparison with classic dummy variables. Giovanis [15], in his article used fuzzy dummy variables to examine the effects of weekdays on stock returns. The membership functions of these variables were considered as triangular functions. By fuzzification of binary dummy variables for weekdays, he showed that fuzzy dummy variables have better results as compared with classic dummy variables (binary zero and one) and so the classification of zero and one for weekdays contains weak point. Giovanis's paper was the first research regarding to the fuzzification of dummy variables. However, in this paper, the question which is posed is that why the triangular membership function has been used?

Jafari Samimi et al. [23] applied fuzzy qualitative explanatory variables with triangular membership functions in econometric patterns and they have concluded that using these variables leads to better results rather than encoded dummy variables. In another article, Giovanis [16] proposed the similarity of transfer functions in STAR model and fuzzy membership functions. According to this researcher, the transfer functions in STAR model are too similar to the membership functions of fuzzy sets. From this point of view, the transfer functions are not complete. Considering the STAR model in two parts, one part acts as a linear autoregressive (AR) and another part as a non- linear consisting transfer function, therefore, the non- linear part can be criticized. The transfer function is the same as fuzzy membership function but no principle or linguistic sentence has been used for inputs. This researcher compared different transfer functions with different fuzzy membership functions in STAR model and concluded that fuzzy membership functions cause better performance of model.

In a study by Aznarte et al. [3, 4], the similarity between STAR model and modeling of a time series AR within a fuzzy system has been stated well. These researchers modeled the nested STAR model by fuzzy system and showed the weak point of this model in forecasting stage. Following Giovanis's study [16], Abounoori and Shahriyar [1], made fuzzy transfer function by using stability rules of dependent variable in STAR model and then tested that in the money demand function of Iran. The results indicated the higher accuracy of the model within the proposed method as compared with STAR model.

Following the mentioned studies, the present paper has been done. In other words, in this research, the main objective is the fuzzification of nested dummy variables inspired by transfer function of STAR model. The innovation of present paper is observable from two aspects. On the one hand, the use of membership function of fuzzy clustering method has been suggested which is based on data attribute. In clustering method, the data are grouped by regarding their distance from center and their attributes. Since, in STAR model, the data attributes is various in different regimes, then it seems that using fuzzy clustering method is appropriate for determining the membership values. In fuzzy clustering method, a data does not belong to a cluster definitely and it can belong to several clusters simultaneously with different membership degrees. Therefore, in current paper, the reason for using clustering membership function instead of transfer functions is based on this fact that the membership value of each data to regime or cluster is not based on a predetermined function which has been used in STAR model, but each data decide to have what membership regarding to its attribute. On the other hand, using fuzzy rules, the membership functions of clusters are combined with each other, and then fuzzy nested dummy variables are made which are discussed in none of aforementioned studies (the influence of two dummy variables on each other is called nested dummy variable).

In the following, using fuzzy nested dummy variables proposed in C-CAPM model, the equity premium puzzle has been examined in financial data of Iran stock exchange empirically. Mehra and Prescott [21] for the first time published an article under the title of "equity premium: a puzzle" on which remarkably regarded the difference between risky and non- risky asset. The reason why they have regarded this difference between these two rates as a considerable factor and called it puzzle, is that the historical data related to the equity premium in America is significantly larger than that amount which economists can calculate in financial area by conventional and neoclassical standard model (or the same C-CAPM model). These two researchers concluded that the whole of this observed distance between risky and non- risky asset returns is not only because of risk factor. They showed that if we replace the amount of risky and non- risky asset returns considering the historical data in C-CAPM model, the risk aversion coefficient calculated for individuals would be much more than usual. While, many studies have shown that this coefficient is about the number of 2 or certainly is less than 10.

After Mehra and Prescott, many researchers are trying to solve the equity premium puzzle by some modifications in C-CAPM model. However, until now, none of them have successfully proposed a convincing solution. Campbell and Cochrane [11], presented another approach to explain the equity premium puzzle. They incorporates the possibility of recession—that is, a major economic downturn—as a state variable. In this model, the risk aversion of investors rises dramatically when the chances of a recession increase; thus, the model can generate a high equity premium. Since risk aversion increases precisely when consumption is low, it generates a precautionary demand for bonds that helps lower the risk-free rate. This model is consistent with both consumption and asset market data. However, concerning this approach, Mehra [20] stated that: "Whether investors actually display the huge, time varying, countercyclical variations

in risk aversion postulated in this model is, however, open to question". Another research in this case done recently is a paper by Xei and Florackis [30]. They represented disappointment aversion (avoiding conclusions which may be worse than average expectations). They used it in the portfolio selection model which an investor chooses among risky and non- risky asset, and concluded that disappointment aversion plays an important role in explaining the equity premium puzzle in 19 countries under review. Since there are a lot of researches in this case, in the following, only all studies concerning equity premium puzzle which have been done in Iran are summarized in table 1

Name of researcher	**Year of research**	**Result**
Keshavarz Haddad and Esfahani [18]	2013	Non- existence of equity premium puzzle by random mastery tests
Mohamadzadeh et al [22]	2016	Existence of equity premium puzzle by examination of C-CAPM model and modification of model with savings (SC-CAPM)
Shahiki Tash et al [24]	2017	Existence of equity premium puzzle by examination of C-CAPM model and H- CCAPM

Table 1: studies of examining the Equity Premium Puzzle in IRAN

In the present study, according to data provided by Securities and Exchange Organization and Central bank of Islamic Republic of Iran, a gap is observed between risky asset return (stocks) and non- risky asset return (bank deposits). Statistical calculations show that during the years of 1993 – 2016, this gap (equity premium) is 3.1 %. Considering this amount of difference, it seems that more people were motivated enough to drive their capital towards the capital market, and then have decreased the volume of their deposits with banks. However, does it happen actually? Whether the standards provided by C-CAPM for measuring the motivation of individuals to transfer their capital from one market to another one are standards with an appropriate assumptions and correct applications?

One of the assumptions of C-CAPM model is the stability of risk aversion coefficient which seems that is not possible in real world. In current study, considering unacceptable result of fixed risk aversion with negative value obtained from C-CAPM model, in fact, it is assumed that relative risk -averse coefficient is not fixed in C-CAPM model and it changes in financial and economic combined regimes. This theory is considered in C-CAPM by binary nested dummy variables. As discussed before, accurate investigation of influence of regimes by binary dummy variables is not possible. Therefore, in this study, the nested dummy variables have become fuzzy and are inserted in C-CAPM model for the first time. We call this model as CCAPM-F. Finally, three models of C-CAPM, C-CAPM with binary nested dummy variables, and CCAPM-F were compared. The results show that the assumption of stability of relative risk- averse coefficient is not correct in C-CAPM model, and this coefficient changes between the nested regimes of economy and market, so that using nested dummy variables cause to improve the performance and assessment of C-CAPM model. The more important issue is that suggested model of present paper CCAPM-F contains better results rather than those two models. In other words, using fuzzy nested dummy variables in C-CAPM led to better outcomes rather than binary nested dummy variables. However, the research objective is not proposing another solution for equity premium puzzle. The modeling of second order moments by fuzzy logic has been indicated in this research.

To achieve the above objectives, this paper is divided into 6 parts. After introduction, a review on regime switching models is presented. In section 3, the proposed method of fuzzy nested dummy variables is offered. In this section, the relation of transfer function in regime switching models with fuzzy membership functions is described. In the sections 4 and 5, the application of suggested method in financial data and obtained results are shown respectively. The conclusion is in the last part of article.

2 A review on regime switching models[1]

In TAR model, it is assumed that the regimes are determined by q_t and the threshold value by C. The specific time mode occurs so that the regime value is specified with the lagged values of dependent variable. This model is known as SETAR model. For example, for a model with two regimes, when the lagged value of regime is equal to one, (a model of AR (1) in each regime) the SETAR model is indicated as follows:

$$y_t = \left\{ \begin{array}{ll} \varphi_{0,1}+\varphi_{1,1}y_{t-1}+\varepsilon_t & if \quad y_{t-1} \leq c \\ \varphi_{0,1}+\varphi_{1,1}y_{t-1}+\varepsilon_t & if \quad y_{t-1} > c \end{array} \right\} \tag{1}$$

[1]The contents of this section are all from source [14].

where, it is mutually shown as follows:

$$y_t = (\varphi_{0,1} + \varphi_{1,1} y_{t-1})\,(1 - I\,[y_{t-1} > C]) + (\varphi_{0,2} + \varphi_{1,2} y_{t-1})\,I\,[y_{t-1} > C] + \varepsilon_t \tag{2}$$
$$I\,[y_{t-1}\,, C] = \left\{ \begin{array}{ll} 1 & if \quad y_{t-1} > C \\ 0 & o.w \end{array} \right\}$$

The $I\,[A]$ is indicator function, and if the A event takes place, it will equal to 1, otherwise it is zero (the variable is dummy). In SETAR model, it is assumed that the border between two regimes is determined by a certain value of threshold variable of y_{t-1}. By substituting continuous function of $G\,[y_{t-1}\,; \gamma; C]$ instead of indicator function of $I\,[y_{t-1} > C]$, a more gentle transition between different regimes is obtained. The value of this function varies from 0 to 1 as long as y_{t-1} rises. The resultant model is called a Smooth Transition AR (STAR) model and is given by:

$$y_t = (\varphi_{0,1} + \varphi_{1,1} y_{t-1})(1 - G\,(y_{t-1}; \gamma; C)) + (\varphi_{0,2} + \varphi_{1,2} y_{t-1})\,G\,(y_{t-1}; \gamma; C) + \varepsilon_t \tag{3}$$

A popular choice for the so-called transition function is the logistic function:

$$G\,(y_{t-1}; \gamma; C) = \frac{1}{1 + \exp(-\gamma\,[y_{t-1} - C])}$$

and the resultant model is then called a Logistic STAR (LSTAR) model. G is the monotonic and increasing function from transfer function of y_{t-1} and it is limited to a value between 0 and 1. Moreover, there is $G\,(y_{t-1}; C; C) = 0.5$. Therefore, it can be stated that threshold parameter of C presents the transfer point between two final regimes by $lim_{Y_{t-1} \to -\infty}\, G = 0$ and $lim_{Y_{t-1} \to \infty}\, G = 1$. The parameter of γ shows the transfer speed between two regimes. Sometimes, the researchers tend to examine the model under the regimes more than two modes. In this case, the SETAR model is referred to the nested model. The nested TAR model (NeTAR) was suggested by Astatkie et al [2] . as follows:

$$y_t = \left\{ \begin{array}{lll} \varphi_{0,1} + \varphi_{1,1} y_{t-1} + \varepsilon_t & \text{if} & y_{t-1} \le c_1 \text{ and } y_{t-2} \le c_2 \\ \varphi_{0,2} + \varphi_{1,2} y_{t-1} + \varepsilon_t & \text{if} & y_{t-1} \le c_1 \text{ and } y_{t-2} > c_2 \\ \varphi_{0,3} + \varphi_{1,3} y_{t-1} + \varepsilon_t & \text{if} & y_{t-1} > c_1 \text{ and } y_{t-2} \le c_{\,2} \\ \varphi_{0,4} + \varphi_{1,4} y_{t-1} + \varepsilon_t & \text{if} & y_{t-1} > c_1 \text{ and } y_{t-2} > c_{\,2} \end{array} \right\}$$

In the above model, the time series of y_t to y_{t-1} is dependent upon the threshold value of C_1 and y_{t-2} depends on C_2. Totally, this issue presents 4 regimes. Thus, the nested STAR model (NeSTAR) is proposed by [14] as follows:

$$y_t = [\,\acute{\varphi}_1 x_t\,(1 - G_1\,(y_{t-1})) + \acute{\varphi}_2 x_t G_1\,(y_{t-1})]\,[1 - G_2\,(y_{t-2})] + [\acute{\varphi}_3 x_t\,(1 - G_1\,(y_{t-1})) + \acute{\varphi}_4 x_t G_1\,(y_{t-1})]\,G_2\,(y_{t-2}) + \varepsilon_t \tag{4}$$

3 Introducing proposed model

3.1 Fuzzy sets

Suppose that, the set of A can be shown by the function of $I_A\,(x)$ as follows:

$$I_A\,(x)\ : U \to \{0, 1\} \qquad I_A\,(x)\ = \left\{ \begin{array}{l} 1 \text{ if } x \in A \\ 0 \text{ if } x \notin A \end{array} \right\} \tag{5}$$

These sets are referred to the Crisp sets. The dummy variables are also in the crisp set. Therefore, if A indicates a set of periods in which the qualitative event has happened, according to dummy variables, the equation (5) is established. If the range of $\{0, 1\}$ is converted to the closed interval of $[0, 1]$, the Crisp set is changed to the fuzzy set. In other words, assuming the universal set of U, the fuzzy set of A in U is defined as follows:

$$A : U \to [0, 1] \qquad A(u) \in [0, 1]$$

$A\,(u)$ is called membership function which measures the rate of membership of u to A. A may show a concept like "length" and $A(x)$ measures the compatibility (adjustment) degree of X value with the concept of "length" and practically, A function takes the U elements to the [0,1] distance. The fuzziness takes place when the interval does not have a sharp border, but it is a fuzzy set. For example, the definitions of boom and recession do not have sharp border. In other words, for the variable of stock return, the positive values close to zero and negative ones close to zero can be considered as belonging to the recession set and partly to the boom. However, to the extent these values go up from zero to the positive values, its membership to the boom set grows and recession becomes less and vice versa.

With this definition of fuzzy membership function, it is clear that transfer function (2) is also like fuzzy membership functions. They can be defined once more as follows [13]:

$$I\left[y_{t-1}\ ,C\right]=\left\{\begin{array}{ll} 1-G\left(y_{t-1};\gamma;C\right) & \text{if}\ \ y_{t-1}<C \\ 0.5 & \text{if}\ \ y_{t-1}=C \\ G\left(y_{t-1};\gamma;C\right) & \text{if}\ \ y_{t-1}>C \end{array}\right\} \tag{6}$$

Considering the equation (6), it is clear that transfer functions of regime switching models like fuzzy membership functions adopt values between zero and one. The difference is that the regime switching models have no criterion for using available transfer functions unless some standard criterion like Schwarz can be used after estimating model.
However, the raised question is: what is the criterion for selecting membership function? We believe that it is associated with data feature. Indeed, there are some groups or regimes in a sort of data which the data feature will change in that group or regime. Therefore, a transfer function is needed which considers the data feature. In the next sections, this transfer function is presented.

3.2 Fuzzy clustering

In statistics science, different kinds of clustering methods are used for grouping or isolating data. One of these methods is fuzzy clustering. In the certain clustering, a data belongs to a cluster or does not. In other words, the range of data is a two member series of 0 and 1. However, in a fuzzy clustering, a data can belong to several clusters with different membership functions between 0 and 1. The logic of this approach led the researchers to use membership functions of fuzzy clustering as an appropriate substitution for transfer functions in regime switching models. In this study, the method of "fuzzy c-means clustering" (FCM) has been applied to separate the data in fuzzy cluster C. Other fuzzy clustering methods can be compared mutually.
In the "fuzzy c-means clustering", the number of clusters (C) has been already identified. The objective function defined for this algorithm is as follows:

$$J=\sum_{i=1}^{c}\sum_{k=1}^{n}u_{ik}^{m}d_{ik}^{2}=\sum_{i=1}^{c}\sum_{k=1}^{n}u_{ik}^{m}\|x_k-v_i\|^2$$

In the above formula, m is the real number larger than 1 which most of the time the number of 2 is selected for m. The X_k is the sample of k th and v_i is representative or the center of i th cluster. U_{ik} shows the rate of membership of i th in the cluster of k th. The sign of $\|*\|$ is the distance of sample from the center of cluster. A U matrix can be defined from U_{ik} which contains c rows and n columns and its components can select any value between 0 and 1. Although, the components of U matrix can choose any value between 0 and 1, the total number of components of each of the columns should be equal to 1 and we have:

$$\sum_{i=1}^{c}\left(u_{ik}\right)=1 \qquad \forall\ k=1,2,3,\ldots,n$$

This condition means that the total dependency of each sample to the cluster c should be equal to 1. Using above mentioned condition and minimizing the objective function, we have:

$$u_{ik}=\frac{1}{\sum_{j=1}^{c}\left(\frac{d_{ik}}{d_{jk}}\right)^{\frac{2}{m-1}}} \qquad , \qquad v_i=\frac{\sum_{k=1}^{n}u_{ik}^{m}x_k}{\sum_{k=1}^{n}u_{ik}^{m}}$$

3.3 Fuzzy nested dummy variables:

Considering the criticism to the transfer functions which is mentioned in the previous sections, the membership functions which are made based on fuzzy clustering (u_{ik}) have been used for transfer functions or membership functions of fuzzy dummy variables as substitution, so that membership function is estimated out of model and then substituted in model like a variable. For example, suppose that it is intended to estimate the following equation [14]:

$$\begin{aligned} y_t=\ & \varphi_{0,1}+\left(\left(\varphi_{1,1}x_t\right)*\left(1-I\left[z_{t1}>C\right]\right)+\ \left(\varphi_{1,2}x_t\right)*I\left[z_{t1}\ \ >C\right]\right)*I\left[z_{t2}\ \ >R\right] \\ & +\left(\left(\varphi_{1,1}x_t\right)*\left(1-I\left[z_{t1}>C\right]\right)+\ \left(\varphi_{1,2}x_t\right)*I\left[z_{t1}\ \ >C\right]\right)*\left(1-I\left[z_{t2}>R\right]\right) \\ & +\varepsilon_t \end{aligned} \tag{7}$$

The above model is a regression model of nested regime switching. The variables y_t and x_t are dependent and independent variables respectively. $I[z_{ti}]$, $i = 1, 2$ shows the indicator function of model whose output is actually the same binary dummy variable of zero and one and it is the same as z_{ti} based on each desirable variable. C and R are threshold value of model. In this model, the nested dummy variables are made due to the effect feedback of dummy variables on each other. Moreover, in current model is estimated by non-linear methods. According to the previous discussion, if the variable of z_t does not have sharp borders, thus binary range of 0 and 1 from I function can be expanded to the continuous range of 0 to 1. To reach this aim, each of the two transfer variables of z_{t1} and z_{t2} are divided into two clusters separately by fuzzy clustering method. Hence, totally 4 clusters are made. If membership values for time series clusters of z_{t1} are shown by U_{1t} and U_{2t} , and also for time series clusters of $z_{t2}$by $U_{1t}^{'}$ and $U_{2t}^{'}$, then according to the following rules we have:

- If z_{t1} is in cluster 1, then $I[z_{t1} > C] = U_{1t}$.
- If z_{t1} is in cluster 2, then $1 - I[z_{t1} > C] = U_{2t}$.
- If z_{t2} is in cluster 1, then $I[z_{t2} > R] = U_{1t}^{'}$.
- If z_{t2} is in cluster 2, then $1 - I[z_{t2} > R] = U_{2t}^{'}$.

Finally, by combining above rules and using fuzzy logic, four new rules will be made as follows:

- If z_{t1} is in cluster 1and z_{t2} is in cluster 1, then $I[z_{t1} > C] * I[z_{t2} > R] = U_{1t} * U_{1t}^{'}$.
- If z_{t1} is in cluster 1 and z_{t2} is in cluster 2, then $I[z_{t1} > C] * 1 - I[z_{t2} > R] = U_{1t} * U_{2t}^{'}$.
- If z_{t1} is in cluster 2 and z_{t2} is in cluster 1, then $1 - I[z_{t1} > C] * I[z_{t2} > R] = U_{2t} * U_{1t}^{'}$.
- If z_{t1} is in cluster 2 and z_{t2} is in cluster 2, then$1 - I[z_{t1} > C] * 1 - I[z_{t2} > R] = U_{2t} * U_{2t}^{'}$.

By substituting fuzzy nested dummy variables obtained from the above rules in equation (7):

$$y_t = \varphi_{0,1} + (\varphi_{1,1}x_t) * (U_{1t} * U_{1t}^{'}) + (\varphi_{1,2}x_t) * (U_{1t} * U_{2t}^{'}) + (\varphi_{1,3}x_t) * (U_{2t} * U_{1t}^{'}) + (\varphi_{1,4}x_t) * (U_{2t} * U_{2t}^{'}) + \varepsilon_t \quad (7\text{-a})$$

This type of dummy variable modeling can be used in each type of econometric model. The method of estimation and testing is also based on the same applying principles of econometric methods. Although, in some cases, the equations are estimated linearly, they are non- linear inherently. Just for this reason that the fuzzy dummy variables are estimated and optimized out of model and within the FCM algorithm, they seem linear. In diagram (1), the fuzzification of nested dummy variables proposed in present study has been depicted conceptually.

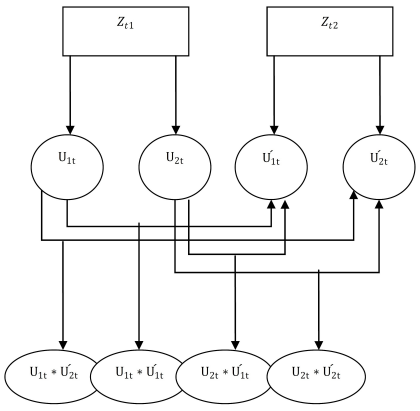


Figure 1: Fuzzification of nested dummy variables using membership functions of FCM

In the following, the application of (7-a) model in financial data is presented to examine the equity premium Puzzle.

4 Application in financial data of stock exchange of Iran

The neoclassical growth model and its random types comprise a central structure in modern financial affairs, public finance, and business cycles theory. This model has been widely used by researchers like Lucas [19]. In fact, the most insights of economy have been taken from the class of this model. The main concept in the framework of mentioned model renders that the present consumption and some coming periods are accomplished by different products. The relative prices of these different products are equal to people's desire to substitute these goods and business abilities for converting goods to each other. When this model is faced with the empirical data of macroeconomics and especially business cycle theory will achieve a considerable success. Unfortunately, this model is rejected in dealing with financial data of stock market. Perhaps, Mehra and Prescott's paper [21] and equity premium puzzle can be the best example for this test. In the following, the concept of equity premium puzzle has been described by a theoretical and empirical model.

4.1 Empirical and theoretical model of equity premium puzzle

The equity premium puzzle is indicated in different ways. In this study, we adopt the approach of Campbell [9, 10] but allow both stock market returns and consumption growth to follow conditionally heteroskedastic processes. We start with a representative agent who maximizes a time-separable utility function:

$$MaxE_t \sum_{j=0}^{\infty} \delta^j U\left(C_{t+j}\right), \tag{8}$$

Where δ is a discount factor, C_{t+j} is the investor's future consumption stream and $U\left(C_{t+j}\right)$ is the period utility derived from such consumption. Equation (9) shows the budget constraint:

$$\text{st.} \qquad \delta W_{t+1} = \left(W_t - C_t\right)\left(1 + R_{p,t}\right) \tag{9}$$

Where W_{t+1} is the wealth and $R_{p,t}$ is the financial asset returns. This problem yields the following Euler equation to describe the optimal consumption and investment path of the representative agent:

$$U(C_t) = \delta E_t[(1 + R_{i,t+1})\, U(\acute{C}_{t+1}) \tag{10}$$

With $1 + R_{i,t+1}$ representing the gross rate of return available on asset i. The investor equates the loss in current consumption with the expected gain in discounted consumption next period. We employ a time-separable power utility function:

$$MaxE_t \sum_{j=0}^{\infty} \delta^j \frac{C_{t+j}^{1-\gamma}}{1-\gamma} \tag{11}$$

Where γ is the coefficient of relative risk aversion. Combining equations (10) and (11), we get the familiar expression,

$$1 = E_t \left[\left(1 + R_{i,t+1}\right) \delta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \right] \tag{12}$$

Following Campbell [9], we assume that the joint conditional distribution of asset returns and consumption is lognormal, with time -varying volatility. Taking logs of equation (12), we get:

$$0 = E_t r_{i,t+1} + \log \delta - \gamma E_t \left[\Delta c_{t+1}\right] + 0.5 \left(h_{rt} + \gamma^2 h_{ct} - 2\gamma h_{r,c}\right) \tag{13}$$

Where $r_{i,t+1} = \log\left(1 + R_{i,t}\right)$, $c_t = \log(C_t)$, h_{rt} and h_{rt} denote the conditional variance of log returns and log consumption growth respectively, and $h_{r,c}$ represents their conditional covariance. The log equity premium is:

$$E_t \left[r_{i,t+1} - r_{f,t+1}\right] + \frac{h_{rt}}{2} = \gamma h_{r,c} \tag{14}$$

The variance term on the left-hand side of equation (14) is Jensen's Inequality adjustment term due to using expectations of log returns. Therefore, the equity premium is equal to the difference in risk- free asset returns from stock return and the log equity premium equivalent with relative risk aversion coefficient multiplied by covariance of stock returns with consumption growth rate.

According to mentioned theory, the relative risk aversion coefficient should stay in the range of 2 to 10. Because the high risk aversion renders the individual's fleeing from instability and volatility in consumption path. However, the consumption is growing over the time. Therefore, in order to modify the current consumption, people should borrow from the future. This propensity for borrowing would lead to increase the rate of real interest of risk- free asset. However, real interest rates of risk- free asset are rarely positive over the time. Consequently, we encounter with risk-free asset rate puzzle proposed by [29]. Since, in empirical studies of equity premium, the relative risk aversion coefficient is unjustifiably obtained in a large value, it is stated that there is a puzzle.
In order to test the equity premium puzzle, the C-CAPM is used with some alteration as follows:

$$E_t\left[r_{i,t+1} - r_{f,t+1}\right] = \gamma_0 + \gamma_1 h_{r,c} + \varepsilon_t \tag{15}$$

The second part in the left side of equation (14) is the Jensen inequality correction factor which is omitted as a result of having little effect and simplifying. Moreover, the intercept is added to model. The equation (18) is estimated after substituting $h_{r,c}$through extraction of conditional covariance from fuzzy bivariate Garch system within stock return (r), and growth of household marginal consumption (c) by least square error method. The following assumes that the investor risk aversion coefficient is variable in different regimes of financial and macroeconomic. In order to examine this hypothesis, the dummy variables suggested in model (15) are used. By these explanations, the equation C-CAPM is offered:

$$\begin{aligned} E\left[r_{i,t+1} - r_{f,t+1}\right]_t = {} & \gamma_0 + \gamma_1 \left(h_{cr,t}\right) * \left(I\left[R_t \le r\right]\right) * \left(I\left[C_t \ \le c\right]\right) \\ & + \gamma_2 \left(h_{cr,t}\right) * \left(I\left[R_t > r\right]\right) * \left(I\left[C_t \ \le c\right]\right) \\ & + \ \gamma_3 \left(h_{cr,t}\right) * \left(I\left[R_t \le r\right]\right) * \left(I\left[C_t \, c\right]\right) \\ & + \gamma_4 \left(h_{cr,t}\right) * \left(I\left[R_t > r\right]\right) * \left(I\left[C_t \ > c\right]\right) + \varepsilon_t \end{aligned} \tag{16}$$

The equation (16) is the regression of nested regime switching, and the functions $I\left[R_t\right]$ * $I\left[C_t\ \right]$ are nested binary dummy variables which have been presented in equation (18). As discussed in introduction, these functions can be considered as fuzzy functions. Therefore, proposing nested fuzzy dummy variables, the C-CAPM F model has been presented in equation (17):

$$E[r_{i,t+1} - r_{f,t+1}]_t = \gamma_0 + \gamma_1(LL_{Cr,t} * h_{cr,t}) + \gamma_2(LH_{Cr,t} * h_{cr,t}) + \gamma_3(HL_{Cr,t} * h_{cr,t}) + \gamma_4\left(HH_{Cr,t} * h_{cr,t}\right) + \varepsilon_t \tag{17}$$

where γ_1 to γ_4 reflect the variable risk aversion of investor respectively in fuzzy regimes (fuzzy nested dummy variables) of economic recession and bear market ($LL_{Cr,t}$), economic recession and bull market ($LH_{Cr,t}$), the economic boom and bear market ($HL_{Cr,t}$), and the economic boom and bull market ($HH_{Cr,t}$) . The equation (17) is estimated after substituting $h_{r,c}$ resulted from fuzzy bivariate Garch system by least square error method. The equation (17) is the application of equation (7-a) in empirical examination of equity premium puzzle. In the following, the manner of fuzzification in nested fuzzy regimes of economy and market ($LL_{Cr,t}$, $LH_{Cr,t}$, $HL_{Cr,t}$, $HH_{Cr,t}$) has been stated.

4.2 Fuzzification of nested dummy variables in C-CAPM (CCAPM-F):

Following these explanations, considering Consumption Capital Asset Pricing Model, respectively, the growth rate variable of household marginal consumption and stock returns have been considered as economy and financial market. Two dummy variables of boom and recession and also bull and bear market have been regarded for macroeconomics and financial market separately. Since the dummy variables of recession and boom in economy and also bear and bull market are respectively presented as follows:

$$\begin{aligned} I_{CL} &= \left\{ \begin{array}{ll} 1 & \text{IF } c_t \le c \\ 0 & \text{o.w} \end{array} \right\} \quad I_{CH} = \left\{ \begin{array}{ll} 1 & \text{IF } c_t > c \\ 0 & \text{o.w} \end{array} \right\} \\ I_{rL} &= \left\{ \begin{array}{ll} 1 & \text{IF } r_t \le r \\ 0 & \text{o.w} \end{array} \right\} \quad I_{rH} = \left\{ \begin{array}{ll} 1 & \text{IF } r_t > r \\ 0 & \text{o.w} \end{array} \right\} \end{aligned} \tag{18}$$

The proposed sets of (18) are definitive and classic with binary range including zero and one values. It can be argued that boom and recession in economy and or bull and bear market are fuzzy concepts and cannot be measured precisely. This concept causes to expand the sets range of (18) from binary set of zero and one to the continuous range between zero and one.
If u_{CL}, u_{CH}, u_{rL}, u_{rH} imply respectively the membership values of recession and boom of time series consumption growth rate (macro economy) and bull and bear stock market, we have:

- If the macro economy is in recession, then $L_{Ct} = u_{CL}$.
- If the macro economy is in boom, then $H_{Ct} = u_{CH}$.
- If the stock market (financial market) is bear, then $L_{rt} = u_{rL}$.
- If the stock market (financial market) is bull, then $H_{rt} = u_{rH}$.

Since there are two dummy variables for each time series, therefore, using fuzzy rules and multiplication, 4 dummy variables have been extracted as follow:

- If the macro economy is in recession and the stock market is bear, then $LL_{Cr,t} = L_{Ct} * L_{rt} = u_{CL} * u_{rL}$.
- If the macro economy is in recession and the stock market is bull, then $LH_{Cr,t} = L_{Ct} * H_{rt} = u_{CL} * u_{rH}$.
- If the macro economy is in boom and the stock market is bear, then $HL_{Cr,t} = H_{Ct} * L_{rt} = u_{CH} * u_{rL}$.
- If the macro economy is in boom and the stock market is bull, then $HH_{Cr,t} = H_{Ct} * H_{rt} = u_{CH} * u_{rH}$.

4.3 Introduction of fuzzy bi-variate garch model:

In order to obtain the conditional covariance between stock return and household marginal consumption rate (h_{rc}) and using that in models (15) to (17), the fuzzy bivariate Garch equations system of VECH- type is proposed as follows [8]:

$$\left\{ \begin{array}{l} c_t = \tau_1 L_{ct} + \tau_2 H_{ct} + \tau_3 c_{t-1} + \varepsilon_{ct} \\ h_{ct} = \omega_1 L_{ct} + \omega_2 H_{ct} + \omega_3 h_{c,t-1} + \omega_4 \varepsilon^2_{c,t-1} \\ r_t = \partial_1 L_{rt} + \partial_2 H_{rt} + \partial_3 r_{t-1} + \varepsilon_{rt} \\ h_{rt} = \phi_1 L_{rt} + \phi_2 H_{rt} + \phi_3 h_{r,t-1} + \phi_4 \varepsilon^2_{r,t-1} \\ h_{cr,t} = \rho_0 + \rho_5 h_{cr,t-1} + \rho_6 \varepsilon_{r,t-1} \varepsilon_{c,t-1} \end{array} \right\} \tag{19}$$

In equations system of (19), the effect of fuzzy regimes of recession and economy boom on consumption growth rate and its volatility, and also the effect of bear and bull market on stock return and its volatilities are investigated.
All in all, in this study the empirical examination method of equity premium puzzle is summarized in the following steps:

1. The main objective is estimating equation (17). Therefore, at first, fuzzy dummy variables of $LL_{Cr,t}$, $LH_{Cr,t}$, $HL_{Cr,t}$ and $HH_{Cr,t}$ should be made. To do this, the rate of household marginal consumption has been considered as economy representative and market return as a representative of financial market.

2. Then using C-Mean fuzzy clustering method, each of time series of consumption and market return are divided into two clusters. Hence, totally, 4 fuzzy clusters with the names of economy recession, economy boom, bull market and bear market are made. Each data in these clusters has membership function. Therefore, membership functions are extracted for each 4 clusters. Now, there are 4 fuzzy dummy variables. In this phase, variables have membership values between zero and one but in binary state, they are just zero and one.

3. In this step, fuzzy nested dummy variables are made through combining fuzzy dummy variables of step 2 and fuzzy rules, as presented in equations of section 4.2.

4. In the next step, conditional covariance between market return and consumption ($h_{cr,t}$) is obtained via equations of bivariate Garch (19). It should be noted that in equations of bivariate Garch system, fuzzy dummy variables of economic recession, economic boom, bull market and bear market are used. Based on the empirical estimation, it was revealed that inserting these variables in equation systems increases the prediction ability of model.

5. After extracting fuzzy nested dummy variables, conditional covariance and their substitution, the equation of regression (17) is estimated through least square error. It is noteworthy that although the equation (17) seems linear apparently, it is non-linear intrinsically. It looks linear just for this reason that dummy variables are estimated out of equation.

4.4 Empirical findings

4.4.1 Introducing and analyzing data.

In this study, Tehran Price Index (TEPIX) based on quarterly frequency and over the period 1993-2016 is used quarterly. Quarterly returns of the stock market based on seasonal price index is computed as $r_{i,t+1} = \frac{p_{t+1}-p_t+d_{t+1}}{p_t}$. P_t is the seasonal price index of stock and d_{t+1} is the value of dividends in market of Tehran. Since, there was no reliable data relating to the dividends for mentioned period, the value was disregarded (with dividends, the value of equity premium will be higher). The time series of household marginal consumption (C_t) has been adopted quarterly from Central bank of Iran in the period of 1993-2016 and smoothed by inflation index in 2013. The third required variable is providing appropriate means for risk-free asset rate. It is largely accepted in literature that the use of the rate that prevails on the money market is a feasible alternative and a suitable compromise for economies where a long-term treasury is not liquid or may not even exist.
In this paper, along with many studies on the average rate of one-year and five-year bank deposits announced by Central bank of Iran in the period of 1993- 2016 quarterly, it is used as a substitute for risk-free assets (for instance refer to [13]). The equity premium is obtained from difference between risk- free asset return rate and stock returns rate as follows: $r_{i,t+1} - r_{f,t+1}$. where, $r_{f,t+1}$ is the seasonal average rate of real bank deposits.
In the following, some distribution characteristics of time series are indicated in table (2). The results of table (2) show that except equity premium, the time series of real growth rate of household marginal consumption and real stock returns have kurtosis and skewness more than normal distribution. Therefore, their normality hypothesis is rejected by Jarque–Bera test.

Name of variable	Average	Standard deviation	Skewness	Kurtosis	P- value of Jarque-Bera tes
Real stock return	0.016	0.017	0.699	3.58	0.01*
Stock Premium	0.031	0.122	0.52	3.09	0.12
Real growth rate of household marginal consumption	0.01	0.066	0.71	3.9	0.03*
Average real return of one- year and five-year bank deposit	-0.014	0.028	-1.7	8.07	0.00*

Source: Authors findings. * indicate significance of test.

Table 2: Descriptive statistics of time series

4.4.2 Results of estimating fuzzy bi-variate garch model.

Before, estimating model (19), the state of autocorrelations with different levels has been investigated in time series and time series square of real growth rate of household marginal consumption and real stock return. The outcomes of this study are reported in table (3).

Autocorrelation of real stock return and its square						
Autocorrelation	$\gamma(1)$ (p-value)	$\gamma(2)$ (p-value)	$\gamma(3)$ (p-value)	$\gamma(4)$ (p-value)	$\gamma(5)$ (p-value)	$\gamma(6)$ (p-value)
r_t	0.366	0.212	0.042*	0.176	0.150	0.207
r_t^2	0.0101*	0.0107*	0.0181*	0.0436*	0.0329*	0.0778
Autocorrelation of real growth rate of household marginal consumption and its square						
Autocorrelation	$\gamma(1)$ (p-value)	$\gamma(2)$ (p-value)	$\gamma(3)$ (p-value)	$\gamma(4)$ (p-value)	$\gamma(5)$ (p-value)	$\gamma(6)$ (p-value)
c_t	0.082	0.033*	0.034*	0.0529	0.064	0.072
c_t^2	0.144	0.088	0.174	0.000*	0.000*	0.000*

Source: Authors findings. * indicate significance of test.

Table 3: Autocorrelation of time series

The results of table 3 indicate that autocorrelation for square of these time series is significant in different lags. Hence, using B-ivariate Garch model is appropriate for modeling two time series with t-distribution. The results of equations system of (19) are reported in table (4). The optimal lags of model are selected based on SBC (Schwarz's Bayesian Criteria), because it loses lesser freedom degrees as compared to other standards.

Name of coefficient	Value of coefficient	Standard error
τ_1	-0.029*	0.005
τ_2	0.08*	0.005
ω_1	0.000268*	0.00007
ω_2	0.0001*	0.00002
∂_1	-0.06*	0.0022
∂_2	0.0139*	0.0099
ϕ_1	0.0098*	0.001
ϕ_2	0.0019*	0.0009
Portmanteau test		
The P- value of adjusted statistic Q for lag =1, 8, 12 are respectively 0.39, 0.48, 0.74		

Source: Authors findings. * indicate significance of test.

Table 4: Estimated results of fuzzy Bi-variate Garch of equations system (19) [2]

The results of assessing model (19) present that the average of stock returns in bull market is significantly determined by quarterly value of 13.9 percent and lower volatility towards bear market with the average of seasonal -6 percent. These observations are compatible with leverage effects in market. It means that with a fall in stock price, debt ratio of firms increase and then it leads to increase the asset risk [6]. Therefore, bear market creates negative returns and high uncertainty for investor.

The household real marginal consumption rate has similar pattern in macro economy. The average of consumption growth rate in boom period is specified by seasonal value of 8% with lower volatility and negative value of 2.9% in recession period with higher volatility. This outcome shows that as a result of uncertainty in the consumption future growth path, the recession periods have presented higher volatility rather than boom periods. Portmanteau test[3] which is stated in table (4) represents lack of autocorrelation in residuals and appropriateness of model.

4.5 Results of examining equity premium puzzle.

Before evaluation of Models (15) to (17), using Phillips-Perron test, the stationary status of equity premium time series data and conditional covariance resulted from system (18) are checked. (To ensure that there is no spurious regression, the stationary test is done). The results of this test in variable levels have been shown in table (5). According to this table, the variables are in stationary level and fixed.

Phillips and Perron (PP) test			
Variable		Value of statistic	P-Value
Equity premium		-12.159	0.0001
Conditional covariance ($h_{r,c}$)		-5.9	0.000
Critical values			
Variable	1%	5%	10%
Equity premium	-3.5	-2.9	-2.58
Conditional covariance ($h_{r,c}$)	-3.5	-2.9	-2.58

Source: Authors findings.

Table 5: Results of unit root test for model variables

The results of models assessed are presented in table (6). The results of C-CAPM model without dummy variables (equation 15) in table (6) show that relative risk averse coefficient $\gamma_1$with negative value of 3.9 is statistically significant in 5% level. Explaining the negative risk averse coefficient in a model on which the economy representative is so careful about his consumption flow is difficult and unreasonable. Therefore, the amount of -3.9% obtained from C-CAPM model for risk aversion in Iran is not acceptable theoretically and empirically. Additionally, the intercept which had been added to the model as test was statistically significant, while it should be zero.

In the following, table (6) shows the results of estimating C-CAPM model with binary nested dummy variables (equation (16) with zero threshold values, $(c = 0, r = 0)$) where all the four relative risk-averse coefficients are statistically

[2]In table 4 only coefficients have been reported which are important in terms of interpretation.

[3]The statistic of Ljung- Box Hasking which is the multivariate form of Portmanteau test is used for checking the autocorrelation effects in residuals of multivariate model.

Parameter					
Model	γ_0	γ_1	γ_2	γ_3	γ_4
CCAPM	0.029 (0.012)*	-3.9 (1.8)*			
CCAPM with binary nested dummy variables	0.009 (0.009)	16.12 (6.19)*	13.3 (5.7)*	10.71(3.69)*	10.3 (1.27)*
CCAPM-F	0.0023 (0.1)	7.9(2.3)*	5.8(1.6)*	2.9(0.2)*	1.8(0.5)*

Source: Authors findings. Standard error of coefficients has been presented in (). * indicate significance of test

Table 6: Estimated results of models

significant in combined regimes of economy and market. However, their numerical values are almost large and out of the range of 2 to 10. The intercept which was added as a test for correctness of this model is not significant. The assessment results of equation coefficients in CCAPM-F (equation (17)) except intercept are all significant statistically. Moreover, all the values of these coefficients are theoretically in acceptable range. This result indicates that the amount of 3.1% (regarding table 2) of extra return (equity premium) in Tehran market is explained by risk factor. In other words, following its excessive risk, extra return which stock index has relative to the investment deposit, is just tolerating more risk. The risk aversion amount of investors changes towards different combined regimes of economy and market. For this reason, the investor's risk aversion is maximum in economy recession regime and bear market and it is minimum in economy boom regime and bull market.

All in all, the results show that regardless to the market conditions, the risk aversion level is higher in recession regime of economy. In contrast, the risk aversion level is lower in boom regime of economy.

Considering the coefficient value of (w1) in equations of bivariate garch system (18) reported in table (2), uncertainty of consumption is higher in recession. Since people have less money and low consumption in bad economic conditions (recession), under this circumstance, they prefer to invest in bank deposits with very little risk. While, in good economic conditions enjoying less consumption fluctuation (coefficient value w1 in table 4), this inclination will be less than that amount and people tend to the rival market, namely stock exchange.

Model	Statistic value of F	Durbin Watson	Schwarz criterion	Adjusted R	Autocorrelation (Ljung Box test)	Theil's inequality coefficient(TIC)
CCAPM	9.14 {0.04}	1.71	-1.3	0.34	Lag of 1={0.044}, Lag of 10={0.089}	0.64
C-CAPM with binary dummy variables	39.95{0.00}	1.22	-1.2	0.49	Lag of 1={0.0}, Lag of 10={0.0}	0.31
CCAPM-F	42.9{0.00}	1.97	-2.24	0.67	Lag of 1={0.2010}, Lag of 10={0.447}	0.011

Source: Authors findings. Standard error of coefficients has been presented in (). {} shows the P-Value and * indicate significance of test

Table 7: Diagnostic Tests and comparison of models

The quality of estimation and comparison of models by diagnostic statistics is presented in table (7). Both Durbin Watson and Ljung Box statistics test the serial autocorrelation in regression residuals. The difference is that Ljung Box statistic examines the autocorrelation to the higher order of lag too. In a standard manner, Durbin Watson statistic should be about 2 in order to non- existence of first- order autocorrelation in model residuals. The assumption of zero in Ljung Box test also indicates non-existence of autocorrelation in residuals. Furthermore, as the Schwarz's criterion is lower, the model is better. According to this issue, the results of table (7) show that in residuals of regression model, there is autocorrelation in both C-CAPM and C-CAPM with binary dummy variables. The status of Durbin Watson, Ljung Box, Statistic value of F and Adjusted R for CCAPM-F are better than other models.

The amount of Schwarz's criterion in CCAPM-F model is less than other models. The Theil's inequality coefficient (TIC) in Table (7) evaluates the model's simulation performance. It will always fall between 0 and 1. If TIC=0, there is a perfect fit. If TIC=1, the simulation performance of the model is as bad as it possibly could be. The value of this coefficient is closer to zero for the model CCAPM-F than for other models. Diagrams 2 through 4 illustrate the simulation of the equity premium time series by three models.

All in all, the results indicate that using fuzzy nested dummy variables leads to improvement of CCAPM model performance.

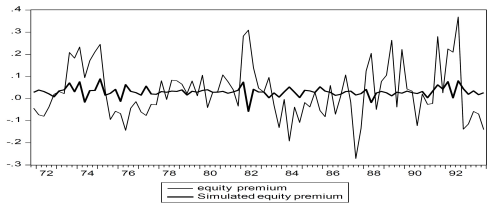


Figure 2: Simulation of equity premium time series by the C-CAPM

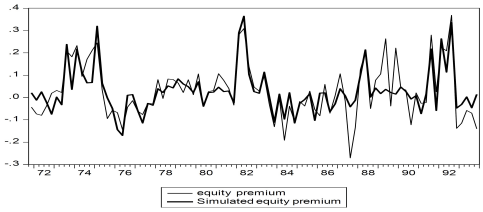


Figure 3: Simulation of equity premium time series by the C-CAPM with binary nested dummy variables

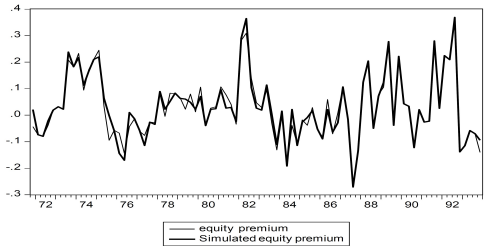


Figure 4: Simulation of equity premium time series by the CCAPM-F

5 Conclusions

The purpose of the present paper was the modeling of fuzzy nested dummy variables. The fuzzification idea of nested dummy variables is extracted from non-linear part of regime switching model of STAR. In the non-linear part of STAR model, different transfer functions like logistic are used. These transfer functions are like membership functions of fuzzy sets. From this point of view, there is a criticism to the non- linear part of model so that there is no reason or linguistic principle (such as fuzzy membership functions) for using these transfer functions. Therefore, in the current study, via correcting this case by fuzzy clustering membership functions, it was used for fuzzification of nested dummy variables. In the following, the proposed method was applied for empirical testing of equity premium puzzle of Iran for the quarterly period of 1993 to 2016. In order to examine the equity premium puzzle empirically, 3 models of C-CAPM, C-CAPM with binary nested dummy variables, and C-CAPM with fuzzy nested dummy variables (CCAPM-F) were used.

The results of investigating C-CAPM model showed that the risk aversion coefficient resulted from this model in Tehran market is -3.9 percent. On the one hand, in the event of C-CAPM, it is claimed that the investor was risk averse and demanded to smooth the consumption during his life. On the other hand, this finding in Iran indicates that the investor is looking for risk. Therefore, this outcome is not justifiable within a theoretical framework. This result was compatible with the results of researches by [22, 24]. In both studies, they reported a negative estimation and out of acceptable value for relative risk averse coefficient in C-CAPM model in data of Iran. Donadelli and Prosperi [13], emphasized that inaccessibility to suitable substitute for risk-free capital asset, inappropriate assumptions of consumption capital asset pricing model and lack of data lead to the abnormal estimation (negative or larger than 10) of relative risk averse coefficient both in developed and newfangled counties.

Then the variability assumption of risk-averse coefficient were inserted into C-CAPM model by nested dummy variables. In both models of C-CAPM with nested dummy variables and CCAPM-F, the results showed that relative risk averse coefficient is statistically significant in combined regimes of macroeconomics-market. However, the values of this coefficient are acceptable and in an appropriate range just for CCAPM-F model. Furthermore, from the viewpoint of diagnostic statistics, CCAPM-F model showed better performance. According to the findings of this model, the investor

is risk- averse in Iran reasonably.
The value of risk aversion for the recession regime of macroeconomics-bear market and boom regime of macroeconomics-bull market was higher and lower respectively. This matter showed that besides, the investor by such treatments is trying to smooth his consumption within these regimes and demanding extra consumption is just for tolerating risk. This result was compatible with the study of Keshavarz Haddad and Esfahani [18]. in stock market of Iran. They examinedthe equity premium puzzle by stochastic dominance approach and out of examination of C-CAPM model.
Moreover, the results indicate that risk aversion moves against recession cycles and macroeconomics boom in Iran data. It means that the risk aversion increases in macroeconomics recession and it will decrease in macroeconomics boom. This result is compatible with the article of Campbell and Cochrane [11]. These two researchers showed that the risk aversion increases in recession by making state variable for recession within CCAPM and inserting habits. Indeed, the state variable obtained from this paper can be interpreted as a dummy variable but with membership values. The membership values of this state variable have been made based on the consumption surplus ratio.
In the current study, the empirical results on one hand showed the appropriate application of second-order moment in C-CAPM model and also it emphasized that using fuzzy nested dummy variables with membership function obtained from fuzzy clustering method can lead to better performance in econometric models (It is important that researches take into dummy variables as fuzzy sets, because measurement of these is ambiguous). On the other hand, it can be a guideline for investors to choose their appropriate combined portfolio between two stock market and bank deposits in different conditions of macroeconomics and market. Moreover, the following cases are proposed as researches for the next studies: i) It is suggested that researchers use new approaches of data mining like fuzzy neural networks in order to fuzzification of nested dummy variables and compare the results of considered method in C-CAPM model with CCAPM-F model of current paper. ii) In this article, the aim of empirical work was not proposing another solution for equity premium puzzle but the main objective was presenting the preference and superiority of fuzzification of fuzzy nested dummy variables rather than binary nested dummy variables in econometric models. However, the researchers in financial area can use appropriate modeling of second-order moments by fuzzy nested dummy variables in all types of capital asset pricing models such as H-CCAPM. The model of H-CCAPM is one of the special statuses of C-CAPM model, with this explanation that consumption expenditures are divided into expenditures on housing section and household consumption expenditures except housing.

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