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Single cycle supply chain coordination model for fuzzy stochastic demand of perishable items

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Abstract

Market demand of the perishable bakery products are volatile. Hence, demand of such items are random variable. The most appealing demand distribution of such items is lognormal demand distribution. Using historical data, the parameters of lognormal distribution are obtained. But, there is a problem of overestimation or underestimation of the parameters. Fuzzy triangular numbers are used to overcome such problems. Single period (newsboy) inventory model is used to obtain an optimal order quantity. The retailers and manufacturers profit is obtained by using the supply chain coordination model. At the end of day, all the remaining items rotted. Hence, there is no gain from unsold items. Also, the some items are defective at the time of purchase. Thus, loss sharing strategy is proposed between manufacturers and retailers. All of these aspects are incorporated in the supply chain coordination model. The data is collected from Koregaon Park, Pune market, India to illustrate the methodology.

Keywords: Single cycle inventory model, fuzzy random variable, co-ordination method, supply chain, perishable items.

1 Introduction

Many researchers have studied single period inventory model by assuming demand as fuzzy random variable. To develop the single period inventory model, authors considered well known newsboy problem. In real situation, demand of the bakery products vary substantially. Whenever the historical information is not available, demand of the bakery products are fuzzy random variable. We assumed that the demand of the bakery product is lognormal distributed. Petrovic et al. [10] developed a model for the news vendor problem where the demand is considered discrete fuzzy. Single inventory problem is also known as newsboy problem. This problem is very important for theoretical and practical consideration. In real life situation, many products have a limited selling period. Hence, a newsboy problem is useful for decision making. The purpose of this model is to improve the performance of the supply chain by scheduling the plans and objectives of individual enterprises. Co-ordination always focuses on ordering decision and inventory management in distributed inter-company setting. Supply chain co-ordination can be defined as the identifying the separately and independently supply chain activities among supply chain members. Every supply chain involves the several members such as manufacturers, suppliers, distribution centers (warehouses) and costumers etc. In supply chain, members of supply chain works independently. Each member of supply chain tries to improve their performance. If members of the supply chain improved their performance then the performance of supply chain will be improved automatically. The supply chain plans how to choose the entities and distribute goods to customers and minimize the total cost. Supply chain network is the optimal platform to manage supply chain effectively and play an important role in management. The aim of supply chain coordination is to improve the performance of supply chain. The supply chain coordination usually focuses on inventory management and decision. The majority of researcher studied the supply chain design problem with deterministic parameters.
Inventory represents an important assets to any business organization. In real world inventory system there exist parameters and constants which are uncertain and characterized by fuzzy numbers. Kwakernaak [5] defined fuzzy

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Table 1: Classification of Research Papers in different features

Author	Random fuzzy demand	Fuzzy demand	Triangular fuzzy number	Trapezoidal fuzzy number	Single stage	double stage	alpha cut	Perishable items	Sharing Contract
Chang [1]		*	*	*	*				
Hu et al.[4]	*	*	*	*	*				
Li et al.[7]	*		*		*	*			
Petrovic et al.[8]		*					*		
Pertrovic et al.[9]		*	*		*				
Petrovic et al.[10]		*	*		*				
Walker [13]	*		*	*					
Wang et al.[14]		*	*		*				
Wang et al. [15]		*	*		*				*
Xu et al. [16]		*	*		*	*			
Xu et al.[17]		*	*	*		*			
Xu et al.[18]		*							
Zhang et al.[20]		*	*		*		*		

random variable, expectation and conditional expectation of fuzzy random variable and properties of fuzzy random variable. He also introduced fuzzy probability and fuzzy events. Puri et al. [11] defined the concept of fuzzy random variable and expectation of fuzzy random variable. They discussed the properties of integral set-valued function. Zadeh [19] introduced a fuzzy set theory. Feng et al. [3] explained the concept of variance and covariance of fuzzy random variable and discussed the application of variance and covariance. They have also given their further applications in the correlation function and the criterion's of mean-squares calculus for fuzzy stochastic process. They extended the results on fuzzy sets and systems and discussed independency of fuzzy random variable by means of a convergence criterion in uniform Hausdorf metric. Dutta et al.[2] proposed methods for construction of membership function without using alpha-cut. They discussed the arithmetic operation of fuzzy numbers using α-cut. They also made the comparison between fuzzy arithmetic with and without using α-cut. In real situation, the decision maker focuses on different points to configure their supply chain network. Few of the decision maker wants to minimize the expected cost, some decision maker concern on the chance with which the total cost is less than a given cost, and others set some confidence levels as an appropriate safety marking to constrain the total cost of supply chain network.

In the proposed model, we consider the demand of bakery products,which follows lognormal distribution because the support of lognormal distribution and demand should be the same. The exact distribution is unknown. Hence, we assume the fuzzy random variable. Also, the sharing of loss due to perishable items are considered in supply chain coordination model. The defective items purchased by retailers are disposed and same amount is debited to manufacturer. All of these constraints are incorporated in the model. The rest of paper includes preliminaries and mathematical model are presented in Section 2. Numerical case study is presented in Section 3. The results and discussion are incorporated in Section 4. Section 5 gives the concluding remarks.

2 Preliminary definitions and mathematical model

Following preliminary definitions are used for the single cycle supply chain model.

2.1 Fuzzy random variable and its expectation

Definition 2.1. (Fuzzy set)[2]

A fuzzy set $\widetilde{A}$:R→ [0,1] is said to be a fuzzy number if it satisfies,

(i) $\widetilde{A}$ is normal,

(ii) $\widetilde{A}$ is fuzzy convex,

(iii) $\widetilde{A}$ is upper semi continuous,

(iv) The support of $\widetilde{A}$ is compact.

Definition 2.2. (Triangular fuzzy number)[2]
It is a fuzzy number represented with three points by $\widetilde{A} = (a, b, c)$ *and its membership function is:*

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & \text{if } x < a \\ \frac{x-a}{b-a} & \text{if } a \le x \le b \\ \frac{c-x}{c-b} & \text{if } b \le x \le c \\ 0 & \text{if } x > c \end{cases}$$

The precise appearance of the function is determined by the choice of parameters a, b *and* c.

Definition 2.3. (Alpha-cut)[2]
Let $\widetilde{A}$ *be a fuzzy set defined on universal set* X *and any number* α , $\alpha \in [0, 1]$. *The* α*-cut is denoted by* A_α *and is given by,* $A_\alpha = \{x| \;\; \mu_{\widetilde{A}}(x) \ge \alpha\}$, *Alpha cut contains all the elements of the universal set* X *whose, membership grade in* A *are greater than or equal to the specified value* α.

Definition 2.4. (Fuzzy random variable)[5] *Let* $(\Re,\mu,P)$ *be the probability space, and* X *be a random variable on* $(\Re, \mu, P)$ *with a probability density function* $f(x)$. *Fuzzy random variable* $\widetilde{X}$ *is a mapping from* R *to a family of fuzzy numbers,i.e.,* $\widetilde{X}$: $x \to \widetilde{X}_{(x)} \in F$, *where* F *denotes fuzzy set. Fuzzy random variables are the random variables that are valued as fuzzy numbers.*

For given $\alpha \in (0, 1]$, *suppose that the* α*-cut* $\widetilde{X}(x)_\alpha$ *of a number* $\widetilde{X}(x)$ *is* $\widetilde{X}(x)_\alpha = \left[\widetilde{X}(x)^-_\alpha, \widetilde{X}(x)^+_\alpha\right]$. *Let* $\widetilde{X}(x)^-_\alpha, \widetilde{X}(x)^+_\alpha$ *denote the left end point and right end point of the* α*-cut* $\widetilde{X}(x)_\alpha$ *of* $\widetilde{X}(x)$, *where* $\widetilde{X}(x)^-_\alpha$, $\widetilde{X}(x)^+_\alpha$ *are real valued random variables.*

Definition 2.5. (Fuzzy expectation)[5] *The fuzzy expectation of the fuzzy random variable* $\widetilde{X}$ *is defined as:*

$$E(\widetilde{X}) = \bigcup_{\alpha \in [0,1]} \alpha \left[\int_R \widetilde{X}^-_\alpha dp, \int_R \widetilde{X}^+_\alpha dp\right],$$

By using the definition of alpha-cut, then definition of fuzzy expectation becomes:

$$\left[E(\widetilde{X})\right] = \bigcup_{\alpha \in [0,1]} \alpha \left[E(\widetilde{X})\right] = \bigcup_{\alpha \in [0,1]} \alpha \left[\left[E(\widetilde{X})\right]^-_\alpha, \left[E(\widetilde{X})\right]^+_\alpha\right].$$

Due to the $\widetilde{X}^-_\alpha$ *and* $\widetilde{X}^+_\alpha$ *are real valued random variable,their respective expectations are:*

$$E(\widetilde{X}) = \bigcup_{\alpha \in [0,1]} \alpha \left[\left[E(\widetilde{X})\right]^-_\alpha, \left[E(\widetilde{X})\right]^+_\alpha\right]. \tag{1}$$

Definition 2.6. (Signed distance of fuzzy expectation)[2] *Let* $\widetilde{A}$ *be a fuzzy number with* α*-cut* $\widetilde{A}_\alpha = \left[\widetilde{A}^-_\alpha, \widetilde{A}^+_\alpha\right]$, *then signed distance of fuzzy number* $\widetilde{A}$ *is* $d(\widetilde{A}, 0) = \frac{1}{2}\int_0^1 (\widetilde{A}^-_\alpha + \widetilde{A}^+_\alpha) d\alpha$. *Then the signed distance of fuzzy expectation* $E(\widetilde{X})$ *is*

$$d(E(\widetilde{X}), 0) = \frac{1}{2}\int_0^1 \left[E(\widetilde{X}^-_\alpha) + E(\widetilde{X}^+_\alpha)\right] d\alpha \tag{2}$$

2.2 Notations and assumptions

Following notations and assumptions are used for model formulation.
Notations:
s - manufacturers wholesale price per unit.
p - manufacturers production cost per unit.
n - retailers holding cost per unit.
m - retailers shortage cost per unit.
q - retail price per unit.
r - defective rate.
f - retailers inspecting cost per unit.

e - reverse cost of defective products per unit which includes manufacturer price and inspection cost.
Q - retailers order quantity.
Q^* - retailers optimal order quantity.
X-random external demand with probability distribution $f(x)$.
$\widetilde{X}$ - fuzzy random external demand corresponding to X and expressed as the triangular fuzzy number.
$\widetilde{X}$ = (X-δ_1, X, X+δ_2), where δ_1 and δ_2 are determined by managers depending on their experiences and reflect a kind of fuzzy appreciation from their intrinsic understanding.

Assumptions:
1. Demand follows lognormal distribution .
2. Only one item is considered.
3. Life time of items is very short.

2.3 Supply chain coordination model

Every supply chain contains various members like manufacturer, suppliers, retailers and customers. Manufacturer and retailer cooperate to each other and maximize profit of the whole supply chain. Retailers order bakery products to manufacturer. Manufacturer fulfill the order (Q) of retailers. As soon as the retailer inspect all the items. The total rQe cost of defective items is debited to manufacturer. While obtaining an optimal order quantity, the loss incurred due to perishable items are shared by retailers and manufacturers. If manufacturer bears the loss then wholesale price becomes $s(\lambda)$. Where, $s(\lambda) = s + b\lambda$, λ is percentage of loss sharing between retailer and manufacturer. b is a constant to be determined according to market situation. If the loss incurred due to perishable items, manufacturer will bear λ percent loss and retailer will bear $(1-\lambda)$ percent loss. Total supply chain profit becomes;

$$\widetilde{T}(Q) = q\ min(\widetilde{X}, Q(1-r)) - (n+p)(Q(1-r) - \widetilde{X})^+ - m(\widetilde{X} - Q(1-r))^+ - pQ - fQ,$$

where, $min\{\widetilde{X}, Q(1-r)\} = \widetilde{X} - (\widetilde{X} - Q(1-r))^+$, $(Q(1-r) - \widetilde{X})^+ = Q(1-r) - \widetilde{X} + (\widetilde{X} - Q(1-r))^+$, and $(\widetilde{X} - Q(1-r))^+ = Max\big(0, (\widetilde{X} - Q(1-r))\big)$. By using above results the total supply chain fuzzy random profit becomes,

$$\widetilde{T}(Q) = (q+n+p)\widetilde{X} - (q+n+m+p)(\widetilde{X} - Q(1-r))^+ - (n+p)Q(1-r) - pQ - fQ.$$

Fuzzy expectation of whole supply chain profit becomes:

$$E(\widetilde{T}(Q)) = (q+n+p)E(\widetilde{X}) - (q+n+m+p)E(\widetilde{X} - Q(1-r))^+) - (n+p)Q(1-r) - pQ - fQ.$$

By applying defuzzification method of signed distance, the total supply chain profit of centralized system is as follows.

$$d(E(\widetilde{T}(Q)), 0) = (q+n+p)d(E(\widetilde{X}, 0)) - (q+n+m+p)d(E(\widetilde{X} - Q(1-r))^+, 0))) - (n+p)Q(1-r) - pQ - fQ$$

We denote $T(Q) = d(E(\widetilde{T}(Q)), 0)$, where,

$$d(E(\widetilde{X}), 0) = \frac{1}{2}\int_0^1 \left[E(\widetilde{X}_\alpha^-) + E(\widetilde{X}_\alpha^+)\right] d\alpha, \qquad d(E(\widetilde{X}), 0) = E(X) + \frac{\delta_2 - \delta_1}{4},$$

$$d(E(\widetilde{X} - Q(1-r))^+, 0) = \frac{1}{2}\int_0^1 \left[E\left[((\widetilde{X} - Q(1-r))^+, 0)_\alpha^-\right] + E\left[((\widetilde{X} - Q(1-r))^+, 0)_\alpha^+\right]\right] d\alpha.$$

Using these values in the above equation, we get:

$$d(E(\widetilde{T}(Q)), 0) = (q+n+p)\left[E(X) + \frac{\delta_2 - \delta_1}{4}\right] - \frac{q+n+m+p}{2}\int_0^1 \Big[E((\widetilde{X} - Q(1-r))^+)_\alpha^-)$$
$$+ [E((X - Q(1-r))^+)_\alpha^+\Big] d\alpha - (n+p)Q(1-r) - pQ - fQ.$$

For fuzzy random demand can be written as triangular fuzzy numbers, $(\widetilde{X} - \delta_1,\ \widetilde{X},\ \widetilde{X} + \delta_2)$.

The shortage can also expressed as triangular fuzzy numbers.

$$\left[(\widetilde{X} - Q(1-r))^+ - \delta_1, (\widetilde{X} - Q(1-r))^+, (\widetilde{X} - Q(1-r))^+ + \delta_2\right].$$

Using α-cuts, the triangular fuzzy number can be expressed as,

$$\left[(\widetilde{X}-Q(1-r))^{+}-\delta_1+\alpha\delta 1,\;\; (\widetilde{X}-Q(1-r))^{+}+\delta_2-\alpha\delta_2\right].$$

The range of fuzzy demand with shortages for left α-cut of the above interval is $(Q(1-r)+\delta_1-\alpha\delta_1)$ to ∞. The range of fuzzy demand with shortages for right α-cut of the above interval is $(Q(1-r)-\delta_2+\alpha\delta_2)$ to ∞. The total supply chain profit becomes:

$$\begin{aligned} d(E(\widetilde{T}(Q),0)) &= (q+n+p)\left[E(X)+\frac{\delta_2-\delta_1}{4}\right]-(n+p)Q(1-r)-pQ-fQ \\ &-\frac{q+n+m+p}{2}\left[\int_0^1\int_{Q(1-r)+\delta_1-\alpha\delta_1}^{\infty}(x-\delta_1+\alpha\delta_1-Q(1-r))f(x)dxd\alpha\right. \\ &\left.+\int_0^1\int_{Q(1-r)-\delta_2+\alpha\delta_2}^{\infty}(x+\delta_2-\alpha\delta_2-Q(1-r))f(x)dxd\alpha\right] \end{aligned} \tag{3}$$

Differentiate equation (3) with respect to Q and equate to zero to evaluate the optimal order quantity Q.

$T(Q)$ is strictly concave function with respect to Q^* and the optimal order quantity satisfies:

$$\frac{1}{2}\int_0^1\left[F(Q^*(1-r)-\delta_2+\alpha\delta_2)+F(Q^*(1-r)+\delta_1-\alpha\delta_1)d\alpha\right]=\frac{(q+m)(1-r)-p-f}{(q+n+m+p)(1-r)} \tag{4}$$

We can find the optimal order quantity Q^* in the supply chain system from the above equation.

Again, differentiate equation (3) with respect to Q. That is, the term $d(E(\widetilde{T}(Q),0))$ is strictly concave function with respect to Q.

After simplifying equation(3) and using integration by parts rule (Rudin [12]) and Leibnitz Theorem (Murray[6]).

$$\begin{aligned} d(E(\widetilde{T}(Q)),0) &= (q+n+p)\left[e^{\mu+\frac{\sigma^2}{2}}+\frac{\delta_2-\delta_1}{4}\right]-(n+p)Q(1-r)-pQ-fQ \\ &-\frac{q+n+m+p}{2}\left[e^{\mu+\frac{\sigma^2}{2}}\int_0^1\left(\left[2-\phi\left[\frac{(ln(Q(1-b)+\delta_1-\alpha\delta_1)-\mu}{\sigma}-\sigma\right]\right]\right.\right. \\ &\left.-\phi\left[\frac{ln(Q(1-r))-\delta_2+\alpha\delta_2)-\mu}{\sigma}-\sigma\right]\right)d\alpha \\ &-\int_0^1\left[(Q(1-r)-\alpha\delta_1+\delta_1)\left[1-\phi\left[\frac{ln(Q(1-r)+\delta_1-\alpha\delta_1)-\mu}{\sigma}\right]\right]\right]d\alpha \\ &\left.+\int_0^1\left[(\delta_2-\alpha\delta_2-Q(1-r))\left[1-\phi\left[\frac{ln(Q(1-r)-\delta_2+\alpha\delta_2)-\mu}{\sigma}\right]\right]\right]d\alpha\right]. \end{aligned} \tag{5}$$

The optimal order quantity Q^* can be obtained using equation (4) and integration by parts (Rudin [13]).

$$\begin{aligned} &2\phi\left[\frac{ln(Q(1-r)-\mu}{\sigma}\right]+\left[\frac{\sigma}{\delta_1^2}-\frac{\sigma}{\delta_2^2}\right]e^{2\mu+2\sigma^2}\phi\left[-2\sigma+\frac{ln(Q(1-r))-\mu}{\sigma}\right] \\ &+\sigma e^{2\mu+2\sigma^2}\left[\frac{1}{\delta_2^2}\phi\left[-2\sigma+\frac{ln(Q(1-r)-\delta_2)-\mu}{\sigma}\right]-\frac{1}{\delta_1^2}\phi\left[-2\sigma+\frac{ln(Q(1-r)+\delta_1)-\mu}{\sigma}\right]\right] \\ &+\sigma e^{\mu+\frac{\sigma^2}{2}}\left[\frac{Q(1-r)-\delta_2}{\delta_2^2}-\frac{Q(1-r)+\delta_1}{\delta_1^2}\right]\phi\left[-\sigma+\frac{ln(Q(1-r))-\mu}{\sigma}\right] \\ &+\sigma e^{\mu+\frac{\sigma^2}{2}}\left[\frac{Q(1-r)+\delta_1}{\delta_1^2}\phi\left[-\sigma+\frac{ln(Q(1-r)+\delta_1)-\mu}{\sigma}\right]\right] \\ &-\sigma e^{\mu+\frac{\sigma^2}{2}}\left[\frac{Q(1-r)-\delta_2}{\delta_2^2}\phi\left[-\sigma+\frac{ln(Q(1-r)-\delta_2)-\mu}{\sigma}\right]\right] \\ &=\frac{(q+m)(1-r)-p-f}{(q+n+m+p)(1-r)}. \end{aligned} \tag{6}$$

Where ϕ is the cumulative distribution function of the standard normal distribution. We can find the optimal order quantity Q^* for supply chain system by using equation(6) and Newton -Raphson method. Profit for the manufacturer is:

$$M(Q^*)=(s-p)Q^*-rQ^*e-\lambda p\left(Q^*(1-r)-\widetilde{X}+(\widetilde{X}-Q^*(1-r))^{+}\right). \tag{7}$$

Table 2: Different loss sharing strategy of manufacturer for supply chain coordination model

			$\lambda = 0.4$			$\lambda = 0.5$			$\lambda = 0.6$		
δ_1	δ_2	r	Q^*	$M(Q^*)$	$T(Q^*)$	Q^*	$M(Q^*)$	$T(Q^*)$	Q^*	$M(Q^*)$	$T(Q^*)$
160	40	0.02	468.7462	36927.93	50501.57	468.7462	40710.73	50501.57	468.7462	44493.54	50501.57
160	40	0.04	478.0443	36061.1	48993.91	478.0443	39937.4	48993.91	478.0443	43813.7	48993.91
160	40	0.06	487.7088	35158.16	47424.9	487.7088	39131.58	47424.9	487.7088	43105	47424.9
160	40	0.08	497.7604	34216.8	45790.79	497.7604	38291.16	45790.79	497.7604	42365.52	45790.79
120	80	0.02	491.3725	39025.84	53722.53	491.3725	43070.09	53722.53	491.3725	47114.34	53722.53
120	80	0.04	500.6782	38078.8	52163.29	500.6782	42216.21	52163.29	500.6782	46353.61	52163.29
120	80	0.06	510.3278	37093.35	50542.29	510.3278	41327.21	50542.29	510.3278	45561.06	50542.29
120	80	0.08	520.339	36067.15	48855.88	520.339	40400.89	48855.88	520.339	44734.63	48855.88
80	120	0.02	489.6925	38676.04	57017.56	489.6925	42652.38	57017.56	489.6925	46628.72	57017.56
80	120	0.04	498.8901	37716.43	55428.73	498.8901	41782.47	55428.73	498.8901	45848.51	55428.73
80	120	0.06	508.4313	36718.7	53776.63	508.4313	40877.61	53776.63	508.4313	45036.53	53776.63
80	120	0.08	518.3348	35680.51	52057.42	518.3348	39935.62	52057.42	518.3348	44190.74	52057.42
40	160	0.02	487.8608	38074.28	59317.01	488.3928	41780.7	56611.8	487.8608	45768.66	59317.01
40	160	0.04	497.1005	37116.32	57694.24	497.5808	40912.6	55019.91	497.1005	44986.82	57694.24
40	160	0.06	506.6907	36120.18	56006.37	507.113	507.113	53364.71	506.6907	44173.14	56006.37
40	160	0.08	516.6507	35083.53	54249.42	517.008	39069.7	51642.38	516.6507	43325.59	54249.42

3 Numerical case study

The bakery product cake has short life time. It is produced from Millennium Bakery Koregaon Park Pune, India. The data collected from Koregaon Park area store at Pune, India. The daily sales and demand are recorded for the whole year 2017. The demand distribution is fitted. It follows lognormal distribution with parameter μ=6.214 and $\sigma = 0.09631$. Manufacturers wholesale price is $s = Rs.200$ per kg. The retail price per unit $q = Rs.300$ per kg, retailers holding cost per unit $n = Rs.5$, retailers shortage cost per unit $m = Rs.50$. The value of defective products per unit is $e = Rs.175$, the production cost is $p = Rs.150$ and inspection cost is $f = Rs.5$. The defective products are disposed and its value is debited to manufacturer. The defective rate changes from $r = 0.02$ to $r = 0.08$.
The fuzzy random demand is expressed as triangular numbers $\left[\widetilde{X} - \delta_1,\ \widetilde{X},\ \widetilde{X} + \delta_2\right]$. The aspiration level values are δ_1 and δ_2. In supply chain coordination model, the retailer and manufacturer has to bear the loss due to unsold items. In this case, they were jointly taking the decision. Here, manufacturer bears the 50 percent loss incurred due to unsold items without increasing the wholesale price. Using above model parameters, the optimal order quantity Q^* is obtained by using equation (6). The $T_2(Q^*)$ and $M_2(Q^*)$ are obtained by using equation (5) and (7) respectively. These results are presented in Table 2.
In real world situation, if manufacturer bears the risk then manufacturer will increase the wholesale price under the risk. It can be expressed in terms of linear equation with usual wholesale price. $s(\lambda) = s + b\lambda$. Here, we consider $b = 102$. Then $s(\lambda)$ is obtained at different percentages of loss. If $\lambda = 0.4$ then $s(\lambda) = 240.8$. If $\lambda = 0.5$ then $s(\lambda) = 251$. If $\lambda = 0.6$ then $s(\lambda) = 261.2$. Using these parameters and optimal order quantity Q^* is obtained. Using Q^*, the total supply chain profit is evaluated and presented in Table 2.

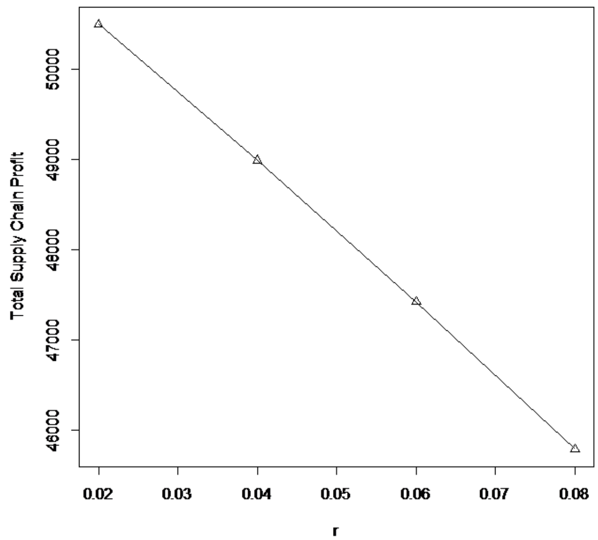

Figure 1: The graph of defective rate vs total supply chain profit

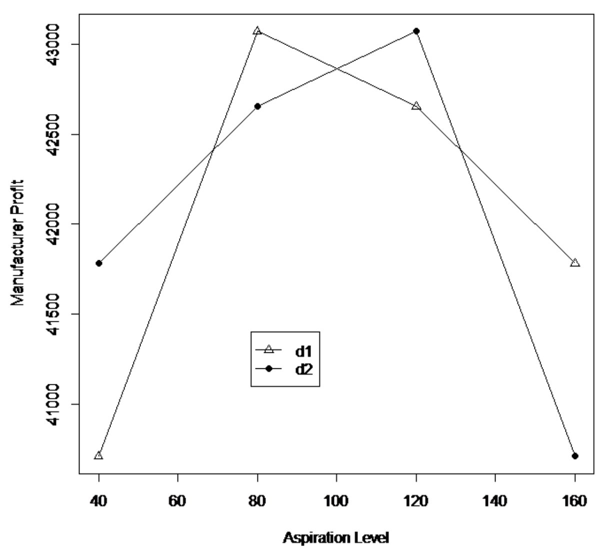

Figure 2: The graph of aspiration level vs manufacturer profit

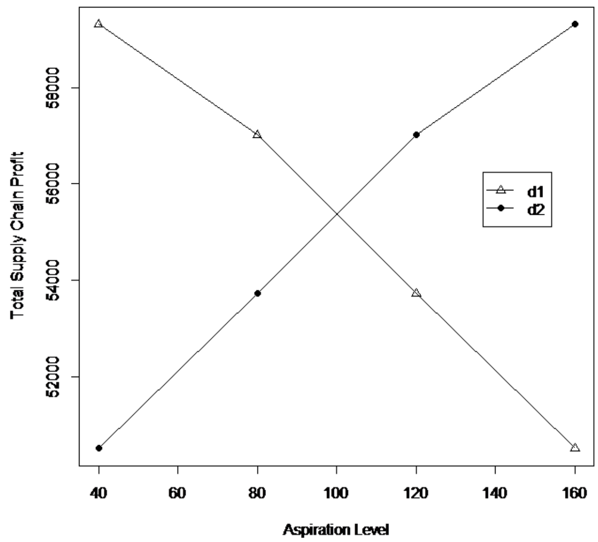

Figure 3: The graph of aspiration level vs total supply chain profit

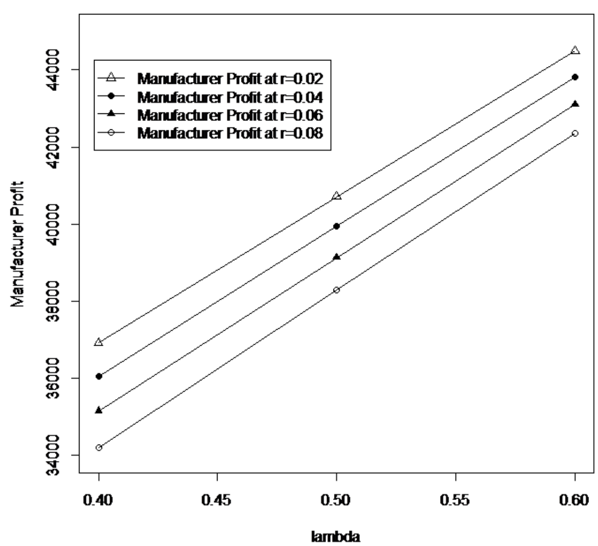

Figure 4: The graph of percentage of loss sharing λ vs manufacturer profit

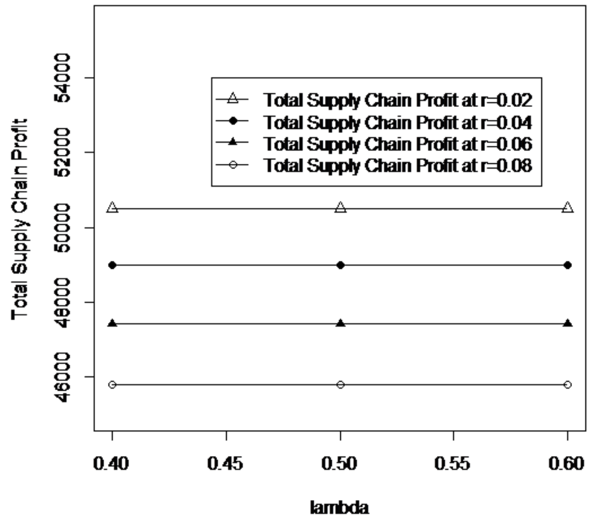

Figure 5: The graph of percentage of loss sharing λ vs total supply chain profit

4 Results and discussion

The sensitivity analysis is carried out by varying the different parameters. As defective rate increased the total supply chain profit decreased as shown in Figure 1. Since the number of defective items are large. It leads to the loss of total inventory. As defective rate increased the manufacturers profit decreased as shown in Table 2. Because, the manufacture has to bear the whole loss incurred due to defective production. Also, as defective rate increased the optimal order quantity increased as shown in Table 2. Whenever, there is large number of defective items in consignment. Retailer has no option to increase the order quantity to meet the market demand. The aspiration level δ_1 increases the manufacturing profit, initially it increases very rapidly and then decreases gradually. The aspiration level δ_2 increases the manufacturing profit, initially it increases gradually and then decreases very rapidly as shown in Figure 2. The aspiration level δ_1 increases the optimal order quantity and total supply chain profit decreases and the aspiration level δ_2 increases with the total supply chain profit as shown in Figure 3. It is obvious that the impact of δ_1 causes least optimal order quantity. Hence there will be less manufacturer profit and total supply chain profit.
In supply chain coordination system, manufacturer has to bear some risk. In such a case, manufacturer will increase the wholesale price of an item. In this case, manufacturer bears λ percent loss incurred due to unsold items. All unsold items perished at the end of day. The percentage of loss sharing and manufacturer profit is shown in Figure 4. As manufacturer take more risks. The manufacturers gain increased. There is no impact of loss sharing strategy on overall supply chain profit is shown in Figure 5.

5 Conclusions

This paper dealt with single cycle (newsboy vendor) perishable items. The daily demand of perishable items is uncertain. That's why fuzzy random demand is considered. We assumed lognormal demand distribution to accommodate the fluctuation of the demand and the support of the distribution. As defective rate increases the optimal order quantity increases, manufacturer and total supply chain profit decreases. Whenever, there is loss due to unsold items, the loss sharing strategy should employed. This model is useful when the risk is high due to perishable items.
There is a huge scope of further research for agricultural products. Majority of products are utilized daily. For example milk, fruits and vegetables. Also, this model is useful for industrial supply chain. It has many applications in logistics and management science. This problem can be extended for multi-product items with multivariate demand distribution.

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