A fuzzy capacitated facility location-network design model: A hybrid firefly and invasive weed optimization (FIWO) solution

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Abstract

Facility location-network design (FLND) problem, which determines the location of facilities and also communication links between the demand and facility nodes, is arisen from the combination of the facility location and network design problems. This paper deals with fuzzy capacitated facility location-network design model which aims to select the facilities and candidate links in a way that yield to minimize the total costs, containing: costs of opening facilities, link construction and transportation costs. In this study, several types of links have been considered for connecting two nodes in which their capacity, transportation and construction costs are distinct. Besides, the possibility of multiple link selection between two nodes and also the necessity of the existence of a facility in some nodes are considered. There are also several types of facilities which the budget for each type is unique. Moreover, to open the facilities and to construct the links, limited budgets are considered. As a result of the uncertainty in customers demand, an interactive fuzzy solution approach is applied. Because this problem is classified as an NP-hard class, a hybrid meta-heuristic algorithm based on Firefly and Invasive Weed Optimization (FIWO) is developed. Finally, the behavior of the model is tested, and the proposed algorithms performance is compared with some other solving methods.

Keywords: Facility location, network design, interactive fuzzy resolution, firefly and invasive weed optimization.

1 Introduction

In recent years, facility location models are widely used for real applications such as transportation, emergency and distribution networks. Attractiveness of this type of problems has convinced the researchers to turn more and more to location problems. Facility location is determining the location of some facilities so that create maximum coverage for demands or the most workflow be established among the facilities [5]. Optimization of transportation costs or other objectives like the cost of facility locating along with maximizing demand covering are the most common goals for the facility location problems [7].

Network design is a noticeable problem seeks to reduce the cost of transportation and to increase demand covering by creating infrastructure networks [12]. On the other side this problem aims to minimize the maximum traveling time. In this problem a network is represented by a set of nodes and connection links and the objective is to satisfy the demands [13].

Facility location-network design (FLND) problems are arisen from combination of aforementioned problems to determine the location of facilities along with communication links between the demand and facility nodes. In recent years, these problems have been applied to a variety of areas such as distribution and transportation systems. In FLND problems, it is possible that a demand point served by several facilities. In general, these problems look for network designing and facilities locating to minimize the total costs of transportation, facility locating and link construction [30].

In the last decades, many research studies are conducted on facility location-network design models. Daskin et al. (1993) introduced an FLND problem for the first time by proposing an uncapacitated transportation network model

[9]. Along the same line, Melkote (1996) investigated three models for FLND problem [22]. Melkote and Daskin (2001) introduced a capacitated FLND problem. Most basic models of FLND in scientific articles are taken from the model described in this paper [24]. Also, in another paper, a problem is investigated to simultaneously find the optimal solution for the facility locations and the network design [23]. Cocking (2008) considered uncapacitated FLND problem. Additionally, to obtain a more suitable lower bound and upper bound for the optimal solution, several algorithms were studied [5]. Cocking et al. (2012) examined the influence of road network configuration in addition to facility locations on availability of healthcare facilities [6].

In other related works, Drezner and Wesolowsky (2003) conducted a study to minimize the total construction and transportation costs. In their study, a network is considered which its link can be either constructed as one-way or two-way link at a given cost. They also solved this new network design problems by descent algorithm, simulated annealing, tabu search, and genetic algorithm [13]. Kaun et al. (2006) presented the analysis of genetic and ant colony optimization algorithms in solving the feeder bus network design problem [19].

In continued researches and studies on context of the network, Hewitt et al. (2010) proposed a combination approach of exact and heuristic methods to solve network flow with a fixed cost [15]. In order to improve the accessibility of all facility classes, Bigotte et al. (2010) presented an optimization model simultaneously decides about promotion in urban centers and network links [2]. The abovementioned research studies are taken into account the costs as their objective in spite of the fact that some other factors such as travel time, which is a vital ingredient in the logistics and supply chain, should be considered. Contreras et al. (2012) presented an FLND problem to minimize the maximum customer-facility travel time [8]. In some research studies, the objective function is defined as two or more objective. Carrano et al. (2007) presented a multi-criterion algorithm to deal with FLND problems by formulating them as bi-objective problems [3]. Mortezaei and JabalAmeli (2011) introduced a multi-objective capacitated FLND problem and solved it with a new hybrid algorithm [26]. Davari et al. (2015) addressed the budget-constrained preventive health care network design problems aim to maximize the participation level along with the equity among the populations living in different neighborhoods. In addition, to solve these models, fuzzy goal programming and fuzzy chance constrained optimization methodologies are proposed [10].

In scientific studies, uncertainty is an integral part of many real-world phenomena and can be seen in various industries. To precisely mention some of these applications, industries such as, airline and railway networks, fire stations, (Shishebori et al., 2018), bread production (Mirakhorli, 2014), alcohol free beverage production (Gumus et al., 2009), vegetable oils manufacturing (Paksoy et al., 2012) along with hydrocarbon biofuel (Tong et al., 2014) can be mentioned [35, 25, 14, 27, 38].

As a representative of relevant research studies which took uncertainty into consideration, Pishvaee and torabi (2010) proposed a multi-objective possibilistic mixed integer programming model to cope with the issue of uncertainty in closed-loop supply chain network design [29]. Shafia et al. (2012) studied FLND problem under uncertainty. In their paper, a stochastic p-robust optimizing approach is applied to face the uncertainty [33]. Davari et al. (2013) conducted a research to design a hub covering network that the demand positions are unknown and are estimated by means of fuzzy variables [11]. Rahmaniani and Shafia (2013) considered the maximum covering facility location and network design problems with uncertainty intends to locate a predefined number of facilities and optimize the corresponding network to maximize the total covered demand points [31]. In another paper, Shishebori et al. (2013) presented a mixed integer nonlinear programming model to formulate reliable budget-constrained FLND problem [34]. Jafarnejad et al. (2014) considered the system reliability in FLND problem and proposed a mixed integer nonlinear programming formulation [16].

Rahmaniani and Ghaderi (2013) proposed an FLND problem considers different types of link with their corresponding and capacity and transportation and construction costs to minimize the total transportation and operating costs. Besides, this study assumed that only one link can be chosen. Based on the fire-fly algorithm, they proposed a heuristic algorithm to solve the model [30]. Sadat Asl et al. (2016) proposed a complex integer number programming model with fuzzy demands considers backup facilities and multiple links between nodes [32].

In above studies, no one has considered multi-type of facilities and essential facilities. However, in reality, there may exist many different types of facilities to satisfy costumers demand depending on the situation. For the first time, this study considers a capacitated facility location-network design model with multi-type of facilities, multi-type of links and limited budget for opening and construction of the facilities and links. Also, there is no limitation to select more than one link between two nodes. Furthermore, we added the essential facilities assumption which means that facility existence is essential in some nodes. In real-world FLND problems, environmental factors like customers demand are imprecise because some information is incomplete or unavailable over the planning. Besides, when there is not enough information about uncertain parameters, probabilistic approaches lose their efficacy. In these cases, fuzzy set theory can provide us a suitable framework to deal with the uncertainty. It helps researchers to consider real-world uncertainties in the process of optimization. In this paper, we use an interactive fuzzy solution approach to cope with the uncertainty

in customers demand.

The model selects a set of nodes to locate the facilities and determines the way which nodes are allocated to facilities. It also selects a set of links to connect nodes to their allocated facilities so that the total costs of transportation, facility locating and link construction should be minimized. Additionally, to solve this problem a hybrid metaheuristic method is developed.

This paper is organized as follows: Section 2 elaborates the problem definition and illustrates the assumptions and notations along with presenting the crisp model formulation. Then the fuzzy model is shown and its equivalent auxiliary crisp mixed-integer linear programming model along with hybrid interactive solution method is presented in Section 3. In Section 4, after presenting the standard algorithms, our proposed method is described. Besides reporting the computational results in Section 5, a comprehensive comparison is presented which demonstrates the efficiency of the developed algorithm, and finally in Section 6, conclusions are considered.

2   Problem definition and model formulation

2.1   Definition and assumptions

Facility location-network design problem is a network with a set of customers and a set of candidate links. In this problem, by a set of candidate links, nodes are connected to each other and construct the network. In addition to that, each node can be a point of demand or a potential facility site. Since in reality there may be different type of facility, multi-type of facilities are considered to locate in network. Additionally, for moving between two nodes, there are multi-type of links with different quality and transportation costs, and there is no limitation to select more than one link between two nodes. In other words, it is possible to select one or more than one link between two characterized nodes. Each facility and link has its capacity which restricts the amount of satisfied demand and transportation. In the real world, it is obvious that facility existence is essential in some nodes. So, in this paper, we define a set of nodes as essential facilities such that facilities should be located in these nodes. The location of facilit and selection of candidate links should be in a way that yield to minimize the total costs. Objective function contains costs of opening facilities, link construction and transportation costs.

This problem includes the following assumptions: (1) a demand point is shown by a node, (2) the network is a customer-to-server system means that demands move to facilities, (3) there is no link at the first, (4) there are several types of links with different qualities for possible connections, (5) All links are directed, (6) there may be several links between each pair of nodes and constructed links just can be one-sided, (7) a customer may be satisfied by several facilities, (8) there are several types of facilities that each type has its own budget, (9) the budget for facility opening and construction of links is restricted, (10) facilities can be located just on the nodes and each node only accept one facility, (11) if a facility located in a node, the demand of that node will be completely satisfied, (12) some nodes need essential facilities which means in these nodes essential facilities must be located, (13) both facilities and links are capacitated, (14) demands are faced with the uncertainty, and (15) other parameters are certain and static.

2.2   Model formulation

Sets, parameters, and decision variables are defined in Table 1 as follows:

To model our problem, we develop the Rahmaniani and Ghaderi [5] model and consider multi type of facilities. This assumption affects the objective function and all constraints which are related to the facilities. Therefore, the fixed cost of opening a facility, capacity of facility, maximum available budget for opening facility and decision variable Z_j^s depend on the type of facility. Also, we suppose that the budget for construction of links is limited but the surplus budget of opening facility can be used for construction of links. This assumption is shown in Eq. (12).

Furthermore, there is no limitation to select more than one link between two nodes but links should be constructed only in one direction (See Eq. (9)). Also, we add the essential facilities assumption means that facility existence is essential in some nodes. These nodes are defined in *Jessential* set.

Table 1: Sets, parameters, and decision variables used in the proposed model.

Symbol	Description
Sets	
N	Set of nodes $i, j \in N$ and sets of costumers, $p \in N$
T	Set of different type of links $t \in T$
S	Set of different type of facilities $s \in S$
Jessential	Set of essential facilities that must be located; Jessential $\{j|Z_j^s = 1;$ at least for one s $\}$
Parameters	
f_j^s	Fixed cost of opening a facility of type s on node j
$\tilde{d}^p$	Uncertain fuzzy demand of costumer p
c_{ij}^t	Fixed cost of constructing link (i, j) of type t
tr_{ij}^t	Cost of transportation on link (i, j) of type t
V_{ij}^t	Capacity of link (i, j) of type t
Ca_i^s	Capacity of facility of type s in node i
B^s	Maximum available budget for opening facility of type s
Q	Maximum available budget for link construction
Decision Variables	
Z_j^s	If facility at node j of type s be opened 1, otherwise 0
X_{ij}^t	If link (i, j) of type t be constructed 1, otherwise 0
Y_{ij}^{pt}	Fraction of customers demand p traveling on the link (i, j) of type t
W_i^p	Fraction of customers demand p served by facility on node i

Consequently, the developed model, based on the abovementioned symbols and assumption, is as follows:

$$\min\left\{\sum_{s \in S}\sum_{j \in N} f_j^s Z_j^s + \sum_{t \in T}(\sum_{i \in N}\sum_{j \in N}\sum_{p \in N} \tilde{d}^p tr_{ij}^t Y_{ij}^{pt} + \sum_{i \in N}\sum_{j \in N} c_{ij}^t X_{ij}^t)\right\} \tag{1}$$

Subject to:

$$\sum_{s \in S} Z_i^s + \sum_{t \in T}\sum_{j \in N} X_{ij}^t \geq 1; \quad \forall i \in N, \tag{2}$$

$$\sum_{s \in S} Z_i^s + \sum_{t \in T}\sum_{j \in N} Y_{ij}^{it} = 1; \quad \forall i \in N, \tag{3}$$

$$\sum_{t \in T}\sum_{j \in N} Y_{ij}^{pt} = \sum_{t \in T}\sum_{j \in N} Y_{ij}^{it} + W_i^p; \quad \forall i, p \in N, \quad i \neq p \tag{4}$$

$$\sum_{s \in S} Z_p^s + \sum_{i \in N} W_i^p = 1; \quad \forall p \in N, \tag{5}$$

$$\sum_{p \in N} \tilde{d}^p W_i^p \leq Ca_i^s; \quad \forall i \in N, \ s \in S, \tag{6}$$

$$W_i^p \leq \sum_{s \in S} Z_i^s; \quad \forall i, p \in N \tag{7}$$

$$\sum_{s \in S} Z_i^s \leq 1; \quad \forall i \in N, \tag{8}$$

$$X_{ij}^t + X_{ji}^t \leq 1; \quad \forall i, j \in N, \quad t \in T, \tag{9}$$

$$\sum_{p \in N} \tilde{d}^p Y_{ij}^{pt} \leq V_{ij}^t X_{ij}^t; \quad \forall i, j \in N, \ t \in T \tag{10}$$

$$\sum_{j \in N} f_j^s Z_j^s \leq B^s; \quad \forall s \in S, \tag{11}$$

$$\sum_{s \in S}\sum_{j \in N} f_j^s Z_j^s + \sum_{t \in T}\sum_{i \in N}\sum_{j \in N} c_{ij}^t X_{ij}^t \leq \sum_{s \in S} B^s + Q; \tag{12}$$

$$\sum_{s \in S} Z_j^s = 1; \quad \forall j \in \text{Jessential,} \tag{13}$$

$$0 \leq Y_{ij}^{pt} \leq 1; \quad \forall i, j, p \in N, \quad t \in T, \tag{14}$$

$$0 \leq W_i^p \leq 1; \quad \forall i, p \in N, \tag{15}$$

$$Z_j^s \in \{0, 1\}; \quad \forall j \in N, \quad s \in S, \tag{16}$$

$$X_{ij}^t \in \{0, 1\}; \quad \forall i, j \in N, \quad t \in T, \tag{17}$$

In the above model, the objective function (1) minimizes the total costs required for the opening of facilities, transportation and link construction. Constraints (2) imply that demands in each node are satisfied by itself or are transported to another nodes through capacitated links in which several link will be used in case the capacity of link is not enough. Constraints (3) state if node i is a facility node, it should not construct any flow on the links of the network; otherwise, it should find a route to transfer its demand. Constraints (4) express that the costumer ps flow which comes to node i should be equivalent to satisfied demand in node i and demand which traverses it. Constraints (5) declare each costumers demand should be satisfied totally. In addition, node p can satisfy its demand if it is a facility node. Constraints (6) say that the capacity of facilities is limited; in other words, the total demands of all the customers that should be satisfied in node i, should not exceed the capacity of the facility in node i, in case that node i is a facility node. Constraints (7) guarantee that a node can satisfy customers demand in case that it is a facility node. Constraints (8) imply that for each facility node just one type of facilities can be chosen. Constraints (9) state that links should be constructed only in one direction. Constraints (10) imply the limitation of the capacity of links by expressing that if link (i, j) of type t is created, total demand of the customers passes through should be less than its capacity. Constraints (11) and (12) deal with the budget. Constraints (11) represent the limitation of budget on the opening facilities. Constraint (12) states the limited budget for opening and construction of the facilities and links. Constraints (13) are related to the construction of essential facilities. Finally, Constraints (14) (17) state the condition of decision variables of problem.

3 The proposed solution method

3.1 The equal auxiliary crisp model

The proposed model assumes certainty in all aspects of the problem. However, in real-world problems, we encounter the uncertainty of information regarding the parameters. For instance, customer demand has uncertain nature in the proposed model. It can change for a specific customer over an intermediate time horizon. If we consider it deterministic, this assumption can lead to an inappropriate results. Stochastic and fuzzy set theory approaches are the first options in the literature to cope with this issue. Lodwick et al. compared some fuzzy, stochastic and deterministic optimization methods for solving linear programming problems. Their work shows the ability of fuzzy approaches to deal with these problems [20].

In the present paper, we use an interactive fuzzy resolution (IFR) method proposed by Jimnez et al. for solving linear programming problems such that all the coefficients are fuzzy numbers. They used a fuzzy ranking method to rank the fuzzy objective values and also to cope with the inequality relation on constraints [17]. This method can keep its linearity and do not increase the number of objective functions and inequality constraints.

The Jimnez et al. method is based on the definition of the expected interval and the expected value of a fuzzy number [17]. Assume that $\tilde{d}$ is a fuzzy number. The following equation can be defined as the membership function of d.

$$\mu_{\tilde{d}}(x) = \begin{cases} 0 & \forall x \in (-\infty, d_1] \\ f_{\tilde{d}}(x) & \text{increasing on} & [d_1, d_2] \\ 1 & \forall x \in [d_2, d_3] \\ g_{\tilde{d}}(x) & \text{decreasing on} & [d_3, d_4] \\ 0 & \forall x \in [d_4, +\infty) \end{cases} \tag{18}$$

where functions $f_{\tilde{d}}$ and $g_{\tilde{d}}$ represent a continuous and monotonically increasing function on the left-hand side of $\tilde{d}$ and a continuous and monotonically decreasing function on the right-hand side of $\tilde{d}$, respectively. If $f_{\tilde{d}}$ and $g_{\tilde{d}}$ are linear functions, a trapezoidal fuzzy number can be denoted by $\tilde{d} = (d_1, d_2, d_3, d_4)$. If $d_2 = d_3$, the fuzzy set becomes

triangular. According to Jimnez et al. (2007) the expected interval of a fuzzy set $\tilde{d}$ can be defined as follows:

$$EI(\tilde{d}) = [E_1^{\tilde{d}}, E_2^{\tilde{d}}] = \left[\int_0^1 f_{\tilde{d}}^{-1}(\alpha)d\alpha, \int_0^1 g_{\tilde{d}}^{-1}(\alpha)d\alpha \right]. \tag{19}$$

The expected value of a fuzzy set $\tilde{d}$ is the mean point of its expected interval:

$$EV(\tilde{d}) = \frac{E_1^{\tilde{d}} + E_2^{\tilde{d}}}{2}. \tag{20}$$

The uncertain and/or imprecise nature of many parameters leads to concerns of system feasibility and optimality. In this way, Jimnez et al. (2007) conducted a fuzzy relationship analysis to compare fuzzy sets. According to the ranking method of Jimnez, for any pair of fuzzy numbers $\tilde{a}$ and $\tilde{b}$, the degree in which $\tilde{a}$ is bigger than $\tilde{b}$ can be defined as follows:

$$\mu_M(\tilde{a}, \tilde{b}) = \begin{cases} 0 & \text{if } E_2^{\tilde{a}} - E_1^{\tilde{b}} < 0 \\ \dfrac{E_2^{\tilde{a}} - E_1^{\tilde{b}}}{E_2^{\tilde{a}} - E_1^{\tilde{b}} - (E_1^{\tilde{a}} - E_2^{\tilde{b}})} & \text{if } 0 \in [E_1^{\tilde{a}} - E_2^{\tilde{b}}, E_2^{\tilde{a}} - E_1^{\tilde{b}}] \\ 1 & \text{if } E_1^{\tilde{a}} - E_2^{\tilde{b}} > 0 \end{cases} \tag{21}$$

where $[E_1^{\tilde{a}}, E_2^{\tilde{a}}]$ and $[E_1^{\tilde{b}}, E_2^{\tilde{b}}]$ are the expected intervals of $\tilde{a}$ and $\tilde{a}$. When $\mu_M(\tilde{a}, \tilde{b}) = 0.5$, we will say that $\tilde{a}$ and $\tilde{b}$ are indifferent. When $\mu_M(\tilde{a}, \tilde{b}) \geq \alpha$ we will say that $\tilde{a}$ is bigger than, or equal, to $\tilde{b}$ at least in a degree α and we will represent it by $\tilde{a} \geq_\alpha \tilde{b}$.

Now, consider the following fuzzy mathematical programming model in which all parameters are defined as fuzzy numbers.

$$\begin{aligned} & \min z = \tilde{c}^t x \\ & s.t. \\ & \tilde{a}_i x \geq \tilde{b}_i, \quad i = 1, ..., l \\ & \tilde{a}_i x \leq \tilde{b}_i, \quad i = l+1, ..., m \\ & x \geq 0. \end{aligned} \tag{22}$$

Based on Jimnez et al., a decision vector $x \in \mathcal{R}^n$ is feasible in degree of α if $\min_{i=1,...,m}\{\mu_M(\tilde{a}_i x, \tilde{b}_i)\} = \alpha$. According to (21), equation $\tilde{a}_i x \geq \tilde{b}_i$ and $\tilde{a}_i x \leq \tilde{b}_i$ are equivalent to the following equations, respectively:

$$\frac{E_2^{\tilde{a}_i x} - E_1^{\tilde{b}_i}}{E_2^{\tilde{a}_i x} - E_1^{\tilde{a}_i x} + E_2^{\tilde{b}_i} - E_1^{\tilde{b}_i}} \geq \alpha, \quad i = 1, ..., l \tag{23}$$

$$\frac{E_2^{\tilde{a}_i x} - E_1^{\tilde{b}_i}}{E_2^{\tilde{a}_i x} - E_1^{\tilde{a}_i x} + E_2^{\tilde{b}_i} - E_1^{\tilde{b}_i}} \leq 1 - \alpha, \quad i = l+1, ..., m \tag{24}$$

or can be rewritten as follows:

$$[(1-\alpha)E_2^{\tilde{a}_i} + \alpha E_1^{\tilde{a}_i}]x \geq \alpha E_2^{\tilde{b}_i} + (1-\alpha)E_1^{\tilde{b}_i}, \quad i = 1, ..., l \tag{25}$$

$$[\alpha E_2^{\tilde{a}_i} + (1-\alpha)E_1^{\tilde{a}_i}]x \leq (1-\alpha)E_2^{\tilde{b}_i} + \alpha E_1^{\tilde{b}_i}, \quad i = l+1, ..., m \tag{26}$$

Also, Jimnez et al. showed that a feasible solution like x^0 is an α -acceptable optimal solution of the model (22) if and only if for all feasible decision vectors say x such that $\tilde{a}_i x \geq_\alpha \tilde{b}_i = 1, ..., l$ and $\tilde{a}_i x \leq_\alpha \tilde{b}_i, i = l+1, ..., m, x \geq 0$, the following equation holds:

$$\tilde{c}_x^t \geq_{1/2} \tilde{c}^t x^0$$

Therefore, with the objective of minimizing, x^0 is a better choice at least in degree $1/2$ as opposed to the other feasible vectors. The above equation can be rewritten as follows;

$$\frac{E_2^{\tilde{c}^t x} + E_1^{\tilde{c}^t x}}{2} \geq \frac{E_2^{\tilde{c}^t x^0} + E_1^{\tilde{c}^t x^0}}{2} \tag{27}$$

Finally, the equivalent crisp α -parametric model of the model (22) can be written as follows by using the definition of expected interval and expected value of a fuzzy number:

$$
\begin{aligned}
&\min EV(\tilde{c})x \\
&s.t. \\
&[(1-\alpha)E_2^{\tilde{a}_i} + \alpha E_1^{\tilde{a}_i}]x \geq \alpha E_2^{\tilde{b}_i} + (1-\alpha)E_1^{\tilde{b}_i}, \quad i = 1, ..., l \\
&[\alpha E_2^{\tilde{a}_i} + (1-\alpha)E_1^{\tilde{a}_i}]x \leq (1-\alpha)E_2^{\tilde{b}_i} + \alpha E_1^{\tilde{b}_i}, \quad i = l+1, ..., m \\
&x \geq 0.
\end{aligned}
\tag{28}
$$

where $EV(\tilde{c}) = (EV(\tilde{c}_1), EV(\tilde{c}_2), ..., EV(\tilde{c}_n))$ represents the expected value of the fuzzy vector $\tilde{c}$.

According to Jimenez et al. method, the equal auxiliary crisp model of the proposed model can be formulated as follows:

$$
\min \left\{ \sum_{s \in S} \sum_{j \in N} f_j^s Z_j^s + \sum_{t \in T}(\sum_{i \in N} \sum_{j \in N} \sum_{p \in N} (\frac{E_1^{\tilde{d}} + E_2^{\tilde{d}}}{2})^p tr_{ij}^t Y_{ij}^{pt} + \sum_{i \in N} \sum_{j \in N} c_{ij}^t X_{ij}^t) \right\}
\tag{29}
$$

Subject to:

$$
\sum_{s \in S} Z_i^s + \sum_{t \in T} \sum_{j \in N} X_{ij}^t \geq 1; \quad \forall i \in N,
\tag{30}
$$

$$
\sum_{s \in S} Z_i^s + \sum_{t \in T} \sum_{j \in N} Y_{ij}^{it} = 1; \quad \forall i \in N,
\tag{31}
$$

$$
\sum_{t \in T} \sum_{j \in N} Y_{ij}^{pt} = \sum_{t \in T} \sum_{j \in N} Y_{ij}^{pt} + W_i^p; \quad \forall i, p \in N, \quad i \neq p,
\tag{32}
$$

$$
\sum_{s \in S} Z_p^s + \sum_{i \in N} W_i^p = 1; \quad \forall p \in N,
\tag{33}
$$

$$
\sum_{p \in N} (\alpha(E_2^{\tilde{d}})^p + (1-\alpha)(E_1^{\tilde{d}})^p) W_i^p \leq Ca_i^s; \quad \forall i \in N, \ s \in S,
\tag{34}
$$

$$
W_i^p \leq \sum_{s \in S} Z_i^s; \quad \forall i, p \in N
\tag{35}
$$

$$
\sum_{s \in S} Z_i^s \leq 1; \quad \forall i \in N,
\tag{36}
$$

$$
X_{ij}^t + X_{ji}^t \leq 1; \quad \forall i, j \in N, \quad t \in T,
\tag{37}
$$

$$
\sum_{p \in N} (\alpha(E_2^{\tilde{d}})^p + (1-\alpha)(E_1^{\tilde{d}})^p) Y_{ij}^{pt} \leq V_{ij}^t X_{ij}^t; \quad \forall i, j \in N, \ t \in T
\tag{38}
$$

$$
\sum_{j \in N} f_j^s Z_j^s \leq B^s; \quad \forall s \in S,
\tag{39}
$$

$$
\sum_{s \in S} \sum_{j \in N} f_j^s Z_j^s + \sum_{t \in T} \sum_{i \in N} \sum_{j \in N} c_{ij}^t X_{ij}^t \leq \sum_{s \in S} B^s + Q;
\tag{40}
$$

$$
\sum_{s \in S} Z_j^s = 1; \quad \forall j \in Jessential,
\tag{41}
$$

$$
0 \leq Y_{ij}^{pt} \leq 1; \quad \forall i, j, p \in N, \ t \in T,
\tag{42}
$$

$$
0 \leq W_i^p \leq 1; \quad \forall i, p \in N,
\tag{43}
$$

$$
Z_j^s \in \{0, 1\}; \quad \forall j \in N, \ s \in S,
\tag{44}
$$

$$
X_{ij}^t \in \{0, 1\}; \quad \forall i, j \in N, \ t \in T,
\tag{45}
$$

3.2   Interactive resolution method

Jimnez et al. (2007) proposed an interactive method to solve the α-parametric linear programming problem. The interactive fuzzy resolution (IFR) method allows decision makers (DMs) to consider feasibility of the constraints and

satisfaction degree of the goal in an interactive way. The DMs have to acquire a balanced solution between improving the objective-function value and improving the constraint feasibility.

The best way to reflect the DMs preferences is to use natural language, establishing a semantic correspondence for different degrees of feasibility; therefore, Jimnez et al. (2007) established 11 scales, which allowed a sufficient distinction between different levels without being excessive. The 11 scales are as follows [17]:

0.0 - Unacceptable solution

0.1 - Practically unacceptable solution

0.2 - Almost unacceptable solution

0.3 - Very unacceptable solution

0.4 - Quite unacceptable solution

0.5 - Neither acceptable nor unacceptable solution

0.6 - Quite acceptable solution

0.7 - Very acceptable solution

0.8 - Almost acceptable solution

0.9 - Practically acceptable solution

1.0 - Completely acceptable solution

Obviously, depending of wishes of DM, other scales can be used. If α_0 is the minimum constraint feasibility degree that the DM is willing to admit, the feasibility interval of α is reduced to $\alpha_0 \leq \alpha \leq 1$ and, according to the semantic scale, we will work with discrete values of α [17]:

$$M = \{\alpha_k = \alpha_0 + 0.1k | k = 0, 1, ..., \frac{1 - \alpha_0}{0.1}\} \subset [0, 1] \tag{46}$$

In the first step of their method, they solved the corresponding ordinary linear program for each α_k . Jimnez et al. obtained the space $O = \{x^0(\alpha_k), \alpha_k \in M\}$ of the α_k -acceptable optimal solution of the original problem, and the corresponding possibility distribution of the objective value: $\tilde{z}^0(\alpha_k) = \tilde{c}x^0(\alpha_k)$ (see Fig. 1).
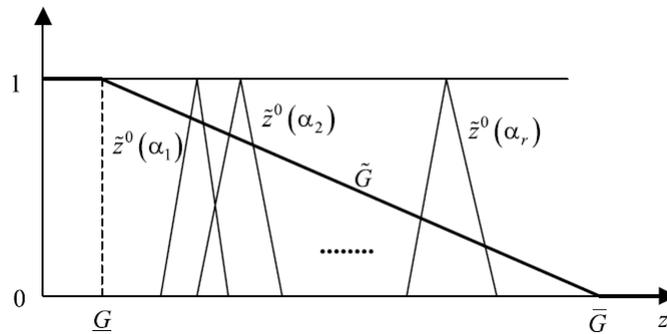

Figure 1: Possibility distribution of the objective values and the fuzzy goal provided by the DM [17].

In order to obtain a decision vector that complies with the expectations of the DMs, two conflicting factors should be evaluated: feasibility of the constraints and acceptability of the objective-function value. After obtaining the results under different $\tilde{z}^0(\alpha_k)$, the DMs are asked to specify a goal $\underline{G}$ and its tolerance threshold $\overline{G}$. The goal is expressed by means of a fuzzy set $\tilde{G}$ whose membership function is:

$$\mu_{\tilde{G}}(z) = \begin{cases} 1 & \text{if } z \leq \underline{G} \\ \lambda \in [0, 1] & \text{decreasing on } \underline{G} \leq z \leq \overline{G} \\ 0 & \text{if } z \geq \overline{G}. \end{cases} \tag{47}$$

Fig. 1 shows the possibilistic distributions of the objective function values under different feasibility degrees and the fuzzy goal provided by the DMs. In the second step of the IFR method, we have to compute the degree of satisfaction of the fuzzy goal $\tilde{G}$ by each α-acceptable optimal solution, that is to say the membership degree of each fuzzy number

$\tilde{z}^0(\alpha_k)$ to the fuzzy set $\tilde{G}$. In this way, Jimnez et al. suggested using an index proposed by Yager (1979):

$$K_{\tilde{G}}(\tilde{z}^0(\alpha)) = \frac{\int_{-\infty}^{+\infty} \mu_{\tilde{z}^0(\alpha)}(z).\mu_{\tilde{G}}(z)dz}{\int_{-\infty}^{+\infty} \mu_{\tilde{z}^0(\alpha)}(z)dz}, \tag{48}$$

where the denominator is the area under $\mu_{\tilde{z}^0(\alpha)}$ and, in the numerator, the possibility of occurrence $\mu_{\tilde{z}^0(\alpha)}z$ of each crisp value z is weighted by its satisfaction degree $\mu_{\tilde{G}}(z)$ of the goal $\tilde{G}$ (see Fig. 2).
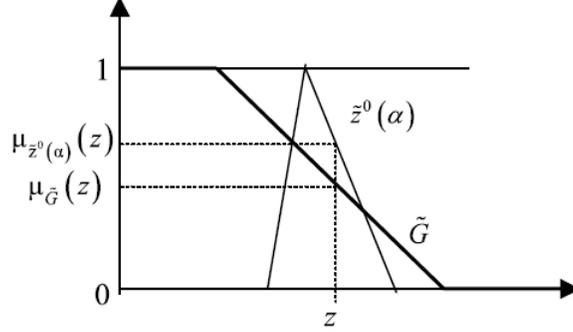

Figure 2: Occurrence possibility of a crisp objective value z and its goal satisfaction degree [17].

In the third step of the IFR method, in an attempt to identify a balance between feasibility of the constraints and satisfaction degree of the goal in the space of α_μ -acceptable solutions (O), Jimnez et al. (2007) proposed to build two fuzzy sets: $\tilde{F}$ and $\tilde{S}$ with the following membership functions: $\mu_{\tilde{F}}(x^0(\alpha_k)) = \alpha_k$ and $\mu_{\tilde{S}}(x^0(\alpha_k)) = K_{\tilde{G}}(\tilde{z}^0(\alpha_k))$, respectively. Then, a fuzzy decision is defined by aggregating the two aforementioned fuzzy sets (i.e., $\tilde{D} = \tilde{F} \cap \tilde{S}$) [17]:

$$\mu_{\tilde{D}}(x^0(\alpha_k)) = \alpha_k * K_{\tilde{G}}(\tilde{z}^0(\alpha_k)), \tag{49}$$

where * represents a t-norm which can be the minimum, the algebraic product, etc. The solution with the highest membership grade will be the final decision for the fuzzy linear programming problem [17]:

$$\mu_{\tilde{D}}(x^*) = \max_{\alpha_k \in M} \{\alpha_k * K_{\tilde{G}}(\tilde{z}^0(\alpha_k))\}. \tag{50}$$

4   Solution methodology

Nowadays, due to the extent of science and engineering applications, different businesses face enormous challenges. Majority of them, either constrained or unconstrained, can be considered as optimization problems. As solving those optimization problems is challenging, many algorithms are proposed in the past decades [40].

Rahmaniani and Ghaderi (2013) express that facility location network design problems belong to NP-hard class of combinatorial optimization problems. Since solving these problems is hard, we utilize a meta-heuristic algorithm, Firefly-Invasive Weed Optimization (FIWO), to struggle with this dilemma.

4.1   Firefly algorithm

Firefly Algorithm (FA), first time proposed by Yang (2008), is a nature-inspired metaheuristic algorithm that is capable of computing a solution to an optimization problem. It mimics by the flashing behavior of fireflies [39].

The three main rules to designing the standard algorithm are the following: 1. All fireflies are unisex, in the other words, any Firefly can be attracted to any other brighter one; 2. The attractiveness of any Firefly is measured by its brightness which is associated with the encoded objective function; 3. The attractiveness of a Firefly depends on the brightness of it. The brightness will be inversely proportional to the distance. Besides, it will move randomly if there is no brighter Firefly than a specific one [1].

The attractiveness of each Firefly determines by its brightness. As the distance between fireflies increases, the attractiveness decreases exponentially because of the light absorption of medium. Hence, the main form of attraction is measured by a decreasing function, which is proportional to the light intensity seen by adjacent fireflies [4].

$$\beta(r) = \beta_0 exp[-\gamma r^2] \tag{51}$$

In the above formulation, β_0 is the attractiveness at $r = 0$ and γ is an absorption coefficient. The distance between any two fireflies calculated by Cartesian distance as follows [4]:

$$r_{pq} = \sqrt{\sum_{s=1}^{d}(x_p^s - x_q^s)^2} \tag{52}$$

where x_p^s is the sth component of the spatial coordinate of the pth Firefly and d is the is the number of dimensions.

The movement of Firefly p when is attracted to Firefly q, which is a brighter Firefly, calculates by [4]:

$$x_p^{t+1} = x_p^t + \beta(r)(x_q^t - x_p^t) + \alpha\varepsilon_p \tag{53}$$

where x_p^t is the current position of a Firefly and x_p^{t+1} is the next generation Firefly position. The Fireflys attractiveness is represented by the second term. In case that there is not any brighter Firefly, Firefly p will move randomly. The last term denotes this random movement where α represents the randomness parameter and ε_p is a random number generated uniformly distributed between 0 and 1.

4.2   Invasive weed optimization

Invasive Weed Optimization (IWO) algorithm is a novel ecologically inspired metaheuristic based on the characteristics of the invasive weeds. As weeds can be classified as robust plants which are adaptive to environments change, they can be a suitable role model to design a powerful optimization algorithm. In IWO the solutions are represented by weeds which grow and disperse seeds randomly over the search area [21].

Firstly, an initial population is generated by generating a finite number of solutions which are randomly dispread in the search area. After seeds grow to flowering plants in each iteration, all plants are evaluated and ranked based on their fitness value. Then, to determine the number of seeds based on the maximum number ($s_{\max}$) and minimum numbers ($s_{\min}$) of seeds, the below Equation is utilized, where *popcost_i* is fitness value of weed ith and *bestcost* and *worstcost* are respectively best and worst fitness value within the set of weeds.

$$seed_i = \frac{floor(s_{\min} + (s_{\max} - s_{\min}) \times (popcost_i - wortcost)}{bestcost - worstcost} \tag{54}$$

To ensure that generated seeds are around their parent plant, they are normally distributed with different decreasingly standard deviations (*sigma*) in each iteration. sigma is calculated for each iteration ($sigma_{iter}$) according to:

$$sigma_{iter} = \left(\frac{iter_{\max} - iter}{iter_{\max}}\right)^{exponent}(sigma_{initial} - sigma_{final}) + sigma_{final} \tag{55}$$

where exponent, $sigma_{initial}$, $sigma_{final}$, $iter_{\max}$ demonstrate the pre-specified nonlinear modulation index, initial and final standard deviations, and the upper limit for the number of iterations, respectively ($sigma_{final} < sigma_{initial}$) [36].

Then, newly-generated seeds are aggregated to colonies. If population size exceeds the upper limit of the population size, IWO ranks all the weeds and eliminates the surplus according to the defined number of colony population.

4.3   Hybridization of firefly algorithm and invasive weed optimization

To improve the potentials and ameliorate the deficiencies of aforementioned algorithms, some utilized the hybridization methods. Kasdirin et al. (2015) discussed that FA has shortcoming on getting trapped at local optimum and, on the other hand, IWO is effective with precise global search ability. Therefore, they hybridize IWO and FA (HIWFO) to achieve a more powerful algorithm. Authors, also compared this algorithm with other algorithms (co-evolutionary PSO, PSO-DE, hybrid charged system with PSO, HGSO, FA and IWO and showed that this hybrid algorithm is far better than others [18].

Moreover, Panda et al. (2017) introduced invasive weed-Firefly optimization (IWFO) algorithm and compared it with IWO and FA algorithms for multi-robot motion planning problem [28]. In this paper, we introduce a new hybrid metaheuristic algorithm base on FA and IWO to make use of the capabilities of the two algorithms to solve the proposed model.

4.4 Firefly-invasive weed optimization (FIWO) algorithm

During our examination we faced that the standard algorithms have some deficiencies in solving this model. On one hand, FA reached near the desirable solution in the first iterations, yet couldnt manage to improve it, and as mentioned in the literature it trapped in local optimum. On the other hand, IWO had a good search capability and improve the solution in every iteration though it couldnt reach a solution as good as FA. Such deficiencies were also mentioned in Kasdirin et al. (2015) and Panda et al. (2017). In order to overcome these deficiencies, we hybridize these algorithms with high-level relay hybridization method. In this method, the self-contained metaheuristic algorithms (two or multi) are executed in a sequence [37]. So, the problem will be solved by FA in the first place. After that, the best solution will be improved by IWO. The pseudo code of FIWO is illustrated in Fig. 3. The Difference between the new algorithm and (HIWFO and IWFO) is that, HIWFO solves the problem with IWO and in every iteration of IWO the solution will be improved by FA as an inner part. But, IWFO has a general procedure which in every iteration, first the problem will be solved by IWO. Then, the population will be improved by FA.

Firefly-Invasive Weed Optimization (FIWO) algorithm

Initialization. Random initial algorithm population
$x_i, (i = 1, \cdots, n)$;
Determine the initial parameters and evaluate the population produced;
while $t_{FA} \leq$ maximum iteration **do**
    for $i=1$ to n **do**
        for $j=1$ to n **do**
        Evaluate r between two units (x_i, x_j) and their attractiveness via $e^{-\gamma r^2}$;
            if $(I_j > I_i)$, move i towards j **then**
            Evaluate new solution x_{i+1};
            end
        end
    end
    Update light intensity, $I(x_d)$ based on the update location;
end
Improve the locations by using IWO localization;
while $t_{IWO} \leq$ maximum iteration **do**
Update sigma and generate seeds over the search space;
    if The number of weeds and seeds $>$ maximum population **then**
    Eliminate unit with lower fitness value;
    end
end

Figure 3: The proposed Firefly-Invasive Weed Optimization (FIWO) algorithm pseudo-code.

To analyze the FIWO algorithm, we compare it with some other metaheuristic algorithms: Genetic algorithm, FA, IWO, HIWFO and Shuffled complex evolution (SCE). The results, which are demonstrated in next section, show that FIWO solve the proposed model, effectively.

5 Computational experience

This section presents the results of the test experiments. In this way, to show the performance of the proposed algorithm and also to investigate the behavior of the model, we generated various test networks. To solve the test problems, the GAMS 24.8.3 software is utilized. Moreover, the algorithm has been coded in MATLAB 7.6 and run under personal computer with 3.1 GHz processor and 10 GB of RAM. In the proposed algorithm, we have adjusted the parameters by trial and error. Table 2 shows the parameters along with their values and descriptions.

Table 2: FIWO parameters.

Parameter	Description	Value
IWO		
Smin_IWO	Minimum number of seeds	0
Smax_IWO	Maximum number of seeds	5
Exponent_IWO	Variance reduction exponent	8
sigma_initial_IWO	Initial value of standard deviation	1
sigma_final_IWO	Final value of standard deviation	0.0001
Firefly		
gamma	Light absorption coefficient	3.5
beta0	Attraction coefficient base value	2
alpha	Mutation coefficient	0.2
alpha_damp	Mutation coefficient damping ratio	0.6

We solve our problem for different test networks with a number of nodes $n \in \{10, 20, 40, 60, 80, 100\}$ to show the model sensitivity. The required parameters for each test network like fixed costs and demand averages are randomly generated from uniform distributions.

For each link, we suppose three different levels of quality. The link of type 1 has high quality in which construction and transportation costs are high and low, respectively. The link of type 3 has low quality such that construction and transportation costs are low and high, respectively. The link of type 2 is between two others.

The values of parameter c_{ij}^1 (construction cost of link of type 1) is achieved from uniform distribution in (100,400) and construction costs of links of type 2 and type 3 are obtained by 10 and 25 percent reduction in construction costs of type 1, respectively. The values of parameter t_{ij}^1 (transportation cost on link (i, j) of type 1) are achieved from uniform distribution (40, 80). To obtain transportation costs on links of type 2 and type 3, we increase transportation costs on links type 1 as much as 15 and 25 percent, respectively.

Because of the importance of the existence of a facility in some nodes, two, four, six, eight and ten nodes are randomly selected as essential facilities for scales 10, 20, 40, 60, 80 and 100 nodes respectively.

In this work, we assume two types of facilities such that the budget for opening of facility is limited. The fixed costs of opening facility for each type are uniformly obtained from interval (1800, 2300). For the sake of the uncertainty in costumers demand, we consider Gaussian fuzzy number for them. The mean parameter of costumers demand is randomly generated from uniform distribution (10, 40) and the left and right variances obtained from interval (1.2,6) randomly. Facilities capacity and links capacity are supposed constantly and are equal to 60, 15 respectively. Budget needed for opening the facilities and also for construction of links for each scale are presented in table below:

Table 3: Budget needed for each test problems (according to monetary unit).

Scale	Budget needed for facility type 1	Budget needed for facility type 2	Budget needed for construction of links
100	50000	60000	20000
80	40000	48000	16000
60	30000	36000	12000
40	20000	24000	8000
20	10000	12000	4000
10	5000	6000	2000

Model can be solved through the solution method as described in previous section under a set of feasibility degrees (α). Assume that the decision makers would not admit high risks of constraint violation. Thus, the discrete levels of α were 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1.

We solve our model in each scale (10, 20, 40, 60, 80 and 100 nodes) under the set of feasibility degrees and compare the results of proposed algorithm and standard mathematical programming software GAMS for the highest membership degree in the fuzzy set decision.

In the case of 10 nodes, as shown in Table 4, the system costs gradually increase when α increases and objective function would be (15153.10, 16985.33, 18850.56) under $\alpha = 0.4$ and (15424.76, 17305.19, 19218.38) under $\alpha = 1$.

Table 4: The α-acceptable solutions obtained from the proposed model in scale 10 nodes.

Feasibility degree, α	Facility locating costs	Possibility distribution of transportation costs	Link construction costs	Possibility distribution of objective value, $\tilde{Z}^0(\alpha)$
0.4	9780	(3858.25,5690.48,7555.71)	1514.85	(15153.10,16985.33,18850.56)
0.5	9780	(3868.14,5703.94,7573.11)	1514.85	(15162.99,16998.79,18867.96)
0.6	9780	(3877.59,5716.79,7589.72)	1514.85	(15172.44,17011.64,18884.57)
0.7	9780	(3886.89,5730.16,7607.34)	1609.35	(15276.24,17119.51,18996.69)
0.8	9780	(3897.44,5749.62,7634.82)	1609.35	(15286.79,17138.97,19024.17)
0.9	9780	(3911.40,5773.63,7668.27)	1700.85	(15392.25,17254.48,19149.12)
1	9780	(3943.91,5824.34,7737.53)	1700.85	(15424.76,17305.19,19218.38)

Then, the DMs would be asked to establish a goal $\underline{G}$ and its tolerance threshold $\overline{G}$. We will suppose that the DM is fully satisfied with an objective value lower than 15153.1 and that he will not be able to assume a cost of more than 19218.3. Then, the goal is expressed by means of a fuzzy set $\tilde{G}$ whose membership function is:

$$
\mu_{\tilde{G}}(z) = \begin{cases} 1 & if \ z \leq 15153.1 \\ \dfrac{19218.3 - z}{19218.3 - 15153.1} & if \ 15153.1 \leq z \leq 19218.3 \\ 0 & if \ z \geq 19218.3 \end{cases} \tag{56}
$$

Now, the compatibility index for each solution is determined by DMs aspirations. After that, a fuzzy decision ($\tilde{D}$) is employed to aggregate feasibility and satisfaction degrees under each α-acceptable solution. According to (Jimnez et al., 2007), the t-norm minimum is utilized to determine the membership degree of each α-acceptable optimal solution to ($\tilde{D}$) , which would be the solution with the highest mean of membership degrees in the fuzzy decision. Table 5 shows the satisfaction degree of the fuzzy goal $K_{\tilde{G}}(\tilde{z}^0(\alpha))$ and membership degree of fuzzy decision $\mu_{\tilde{D}}(x^0(\alpha_k))$ under each α-acceptable system costs.

Table 5: Satisfaction degree of the fuzzy goal and membership degrees of the fuzzy decision in scale 10 nodes.

Feasibility degree, α	Satisfaction degree of fuzzy goal, $K_{\tilde{G}}(\tilde{z}^0(\alpha))$	membership degree of fuzzy decision, $\mu_{\tilde{D}}(x^0(\alpha_k))$
0.4	0.549	0.4
0.5	0.545	0.5
0.6	0.542	0.542
0.7	0.516	0.516
0.8	0.511	0.511
0.9	0.483	0.483
1	0.470	0.470

As it is obvious in Table 5, the 0.6-feasibility optimal solution would be the best choice. Fig. 4 shows the obtained network under 0.6- feasibility optimal solutions.
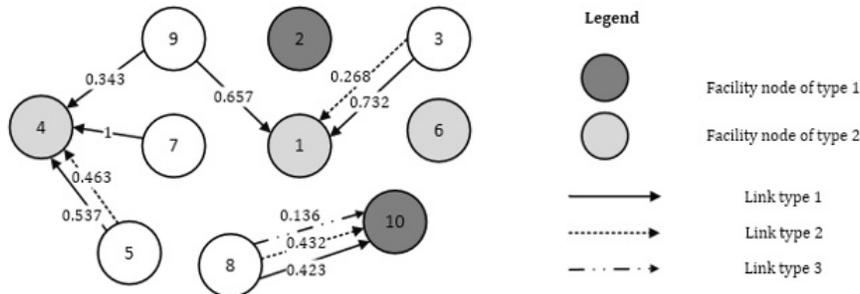

Figure 4: Obtained network under 0.6- feasibility optimal solutions

According to Fig. 4, facilities opened in nodes 1,2,4,6 and 10 which the other demand nodes satisfied by them. In this network, because of the importance of facility existence in some nodes, we considered nodes 2 and 6 as essential set members. Also, obviously, there is no limitation to select more than one link between two nodes. The values for each link represent Y_{ij}^{pt}. For example, according to the network, $Y_{31}^{32} = 0.268$. It means that fraction of customers demand 3 traveling on the link (3,1) of type 2 is 0.268. Also, customers demand 3 completely satisfied by node 1, so $W_1^3 = 1$. Other values of W_i^p for each i and p have been reported in the following table:

Table 6: Values of W_i^p for 10 nodes under 0.6- feasibility optimal solutions.

ip	3	5	7	8	9
1	1				0.658
4		1	1		0.342
10				1	

To show the effectiveness of the proposed algorithm, we compared FIWO performance with some other metaheuristic algorithms. The results shown in Table 7 justify the efficiency of FIWO. Comparison of the algorithms in average time and average error show that FIWO has a better performance than the others (see Fig. 5).

Table 7: Comparison of metaheuristics performance.

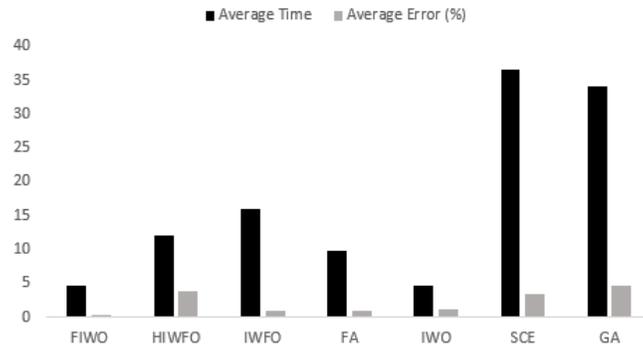|	FIWO	HIWFO	IWFO	FA	IWO	SCE	GA
Average time	4.59	12.17	16.04	9.72	4.68	36.48	34.01
Average error (%)	0.32	3.80	0.96	1.02	1.25	3.38	4.58

Figure 5: Comparison of the algorithms in average time and average error.

We solved different test problems to show the proposed algorithms performance and also the behavior of the model. The solution procedure for different tests is similar to case 10 nodes which described in previous. For each case we solved the model and obtained α- feasibility optimal solutions and then, compared the results of FIWO algorithm and standard mathematical programming software to show the appropriate performance of the proposed algorithm. Table 8 presents the computational results for test instances.

Table 8: Computational results for test instances.

Test problem	Optimal α-feasibility	GAMS		FIWO	
		Optimal Total costs	Time (s)	Average error (%)	Average time (s)
100	0.5	147367.30	4387.09	2.46	3222.31
80	0.6	121993.60	2410.20	2.13	1776.84
60	0.6	93799.57	370.65	1.09	682.45
40	0.6	62650.17	61.46	0.40	187.07
20	0.6	34246.37	3.34	0.54	21.89
10	0.6	17018.61	0.51	0.32	4.59
Average	-	-	1205.54	1.15	982.52

As shown in Table 8, GAMS can solve the test problems exactly in appropriate time in short scales but with the increasing size of the problem, because of the high complexity, the performance of GAMS reduces and it solved the test problems in 1205 seconds on average. Furthermore, FIWO solved the problem with almost one percent error on average with a suitable time (982 seconds on average). In comparison with GAMS and by relying on results, FIWO algorithm can solve the problem efficiently. Fig. 6 demonstrates performance comparison for the test instances.
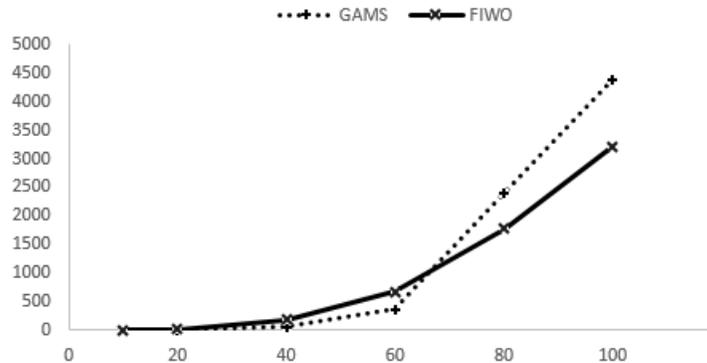

Figure 6: FIWO in comparison with GAMS in time for the test instances.

6   Conclusions

Facility location and network design problems have attracted a lot of attention in past decades and have widely used for real applications such as transportation, emergency and distribution networks. Broadly speaking, FLND problems look for designing the network and locating some facilities.

This study, a fuzzy capacitated facility location-network design model was presented. In this model, different types of links with their corresponding capacity can be constructed to connect each two nodes. Besides, there was no limitation to select just one link between two nodes. Also, we considered several types of facilities which the budget for each type is different. In addition, the budget for opening of facility and construction of links was restricted. Since opening some facilities in some nodes was necessary, we defined some nodes as essential set in advance. To cope with the uncertainty in customers demand, an interactive fuzzy solution approach was applied. We also proposed an efficient hybrid Firefly and Invasive Weed Optimization (FIWO) algorithm to solve the presented model. To validate the model and analyze the algorithms performance, various instances were randomly generated. The computational results have demonstrated the efficiency of proposed algorithm in comparison with GAMS in large scale.

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