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Extensions of ELECTRE-I and TOPSIS methods for group decision-making under complex Pythagorean fuzzy environment

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Abstract

Multi-criteria group decision-making is a process in which decision makers assess the performance of alternatives on the basis of conflicting criteria to opt the most worthy alternative as solution. TOPSIS and ELECTRE are effective and commonly used methods to solve multiple criteria decision-making problems. The aim of this study is to propose two new models, namely, complex Pythagorean fuzzy TOPSIS (CPF-TOPSIS) method and complex Pythagorean fuzzy ELECTRE I (CPF-ELECTRE I) method, to tackle multiple criteria group decision-making problems comprising complex Pythagorean fuzzy data. In these methods, we compare complex Pythagorean fuzzy numbers on the basis of their score functions. We use revised closeness index for the ranking of alternatives in CPF-TOPSIS method. In complex Pythagorean fuzzy concordance and discordance sets, we compare the alternatives on being superior and inferior to other alternatives on the basis of score degree, accuracy degree and indeterminacy. In CPF-ELECTRE I method, we use outranking decision graph to obtain the best alternative. We illustrate the structure of both methods with the help of flow charts. To verify the accuracy of proposed methods, we present an explanatory example for selection of best interior designer for a hotel renovation. We authenticate the proposed techniques by providing a brief comparative analysis of these methods with existing methods.

Keywords: Complex Pythagorean fuzzy set, TOPSIS, ELECTRE I, normalized Euclidean distance.

1 Introduction

Decision-making is a process of solving problems for choosing the best alternative. Multi-criteria decision-making (MCDM) can be explicated as a discipline of operations research in which the satisfactory solution is opted regarding some significant factors which can be considered as conflicting criteria for the decision-making problems. MCDM is widely used in medical, economics, engineering and social sciences. MCDM can be classified according to the number of decision makers (DMs), i.e., individual decision-making and group decision-making (GDM). Multi-criteria group decision-making (MCGDM) is a procedure to deal with the problems to find the most suitable alternative relative to some criteria by a cooperative group of decision makers (DMs). In MCGDM the individual decision matrices of all DMs are merged to get a group satisfactory solution. To deal with MCDM problems, sundry techniques involving "TOPSIS" [17], "ELECTRE" [8], etc., have been proposed.

The technique of order preferences by similarity to ideal solution method (TOPSIS) is used to solve MCDM and MCGDM problems. The main idea behind the TOPSIS method is to opt the most suitable solution which is nearest to positive ideal solution (PIS) and farthest to negative ideal solution (NIS). Hwang and Yoon [17] established the TOPSIS method in 1981. The classical TOPSIS method deals with crisp data for the evaluation of performance of alternatives and relative importance of the criteria. In real life decision-making problem, it is very rare to find such crisp and precise data because of complexity and uncertainty of practical decision-making and vagueness of human

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judgements. Therefore, to handle such imprecise and vague data, we seek help from the fuzzy set (FS) theory initiated by Zadeh [30] in 1965. Firstly, Chen [11] extended the TOPSIS method in fuzzy environment in 2000. Shen et al. [23] and Amiri [6] employed fuzzy TOPSIS method for supplier selection and project selection in oil-fields development, respectively. Later, Atanassov [7] generalized the concept of fuzzy sets and established intuitionistic fuzzy (IF) sets by adding non-membership ν and hesitancy π to the fuzzy set, along with the membership μ, subjected to the condition $\mu+\nu \leq 1$. Boran et al. [9] combined the TOPSIS method with the notion of intuitionistic fuzzy set (IFS) to select most worthy supplier for an automobile company. Aloini [5] proposed a peer based modification of IF-TOPSIS method to select the best packaging machine. Zhang et al. [32] extended the TOPSIS method for covering-based variable precision intuitionistic fuzzy rough set model. In 2013, Yager [28, 29] presented the concept of Pythagorean fuzzy sets (PFSs) with membership μ, non-membership ν, and hesitancy $\pi = \sqrt{1-\mu^2-\nu^2}$ which relaxes the condition of IFSs $\mu+\nu \leq 1$ by $\mu^2+\nu^2 \leq 1$. Xu and Zhang [33] introduced Pythagorean fuzzy numbers (PFNs) and extended the TOPSIS method for PFN in 2014. Later, Akram et al. [1] proposed the PF-TOPSIS method for group decision making and illustrated the method with solid examples. Garg [14] extended the PFS as linguistic Pythagorean fuzzy set . Recently, Zhan et al. [31] proposed a new version of PF-TOPSIS method for Covering based Pythagorean fuzzy rough set model. Feng et al. [12] presented the generalized intuitionistic fuzzy soft sets.

The FS, IFS and PFS are not capable enough to deal with ambiguous and inconsistent data of periodic nature. These models effectively deal with one dimensional phenomena, but there is a major deficiency in these models, that is, lack of capability to model two dimensional phenomena. To overcome this problem, Ramot et al. [19, 18] introduced complex fuzzy set (CFS) in which range of membership function was extended from $[0,1]$ to unit circle. In CFS, membership $\mu = ue^{i\alpha}$ consists of two terms, i.e., amplitude term u which lies in the unit interval $[0,1]$ and phase term (periodic term) α which lies in the interval $[0,2\pi]$. The phase term is of vital importance in defining the complex fuzzy model which makes complex fuzzy model distinct and superior to all previous models in the literature. CFSs are proficient in modeling the two dimensional phenomena, such as signal processing. In 2012, Alkouri and Salleh [4] presented the complex intuitionistic fuzzy sets (CIFSs) by adding non-membership $\nu = ve^{i\beta}$ to the CFS with the conditions $u+v \leq 1$ and $\alpha+\beta \leq 2\pi$. Rani and Garg [20] presented a decision-making strategy on the basis of distance measure for complex intuitionistic fuzzy model. In 2019, Ullah et al. [24] put forward the notion of complex Pythagorean fuzzy set (CPFS) and defined some distance measures for CPFSs. In CPFS membership $\mu = ue^{i\alpha}$ and non-membership $\nu = ve^{i\beta}$ can take values in the unit circle with the conditions $\mu^2+\nu^2 \leq 1$ and $(\frac{\alpha}{2\pi})^2 + (\frac{\beta}{2\pi})^2 \leq 1$. CPFS, containing the phase term, is a more effective tool to capture the vague and uncertain data of periodic nature than the PFS. For example, a clothing brand considers five locations to open new outlet regarding some particular criteria. If an expert assigns membership 0.8 and non-membership 0.6 to a location with respect to a criteria then IFS fails to deal this problem because $0.8+0.6 \geq 1$, but this problem can be effectively handled by PFS as $0.8^2+0.6^2 \leq 1$. On the other hand, if we consider maximum number of people visiting the outlet at a particular time then PFS also fails because to handle time we have to introduce the periodic term. Now expert assigns membership $0.8e^{i(1.4\pi)}$ and non-membership $0.6e^{i(1.1\pi)}$ which satisfy the conditions of CPFS as $0.8^2+0.6^2 \leq 1$ and $(0.7)^2+(0.55)^2 \leq 1$. Therefore, CPFSs are proficient to deal with data of periodic nature due to complex membership and non-membership grades.

On the other hand, ELimination and Choice Expressing REality (ELECTRE) method was developed by Benayoun et al. [8] which was later named as ELECTRE I method. Figueira et al. [13] discussed the ELECTRE family of methods. Hatami-Marbini and Tavana [16] presented the fuzzy ELECTRE I method and Rouyendegh et al. [22] utilized the fuzzy ELECTRE method for staff selection. Chen and Wu [27] extended the ELECTRE I method in intuitionistic fuzzy environment and applied it to select the project manager. Rouyendegh [21] and Vahdani et al. [25] used the intuitionistic fuzzy ELECTRE method for selection of best location and best flexible manufacturing system, respectively. Çali and Balaman [10] presented a comparative analysis of VIKOR and ELECTRE methods under IF environment. In 2019, Akram et al. [2] introduced PF-ELECTRE I method and demonstrated it with examples. In this research article, we present two methods including CPF-TOPSIS and CPF-ELECTRE I as well as their flow charts to deal with the MCGDM problems comprising the complex Pythagorean fuzzy (CPF) data. We use Euclidean distance to assess the distance among alternatives and PIS or NIS in CPF-TOPSIS method. We rank the alternatives in descending order of the revised closeness index. In CPF-ELECTRE I method, we compare complex Pythagorean fuzzy numbers (CPFNs) on the basis of score function, accuracy function and indeterminacy. We obtain the CPF concordance and discordance sets by pairwise comparison of alternatives with respect to each criteria which are employed to construct the CPF concordance and discordance matrices. We obtain CPF effective concordance or discordance matrices by comparing concordance or discordance level with the CPF concordance or discordance indices which leads to the construction of aggregated outranking matrix. Finally, we choose the best alternative with the help of outranking graph.

This research article consists of 8 sections. Section 1 provides the literature review of the proposed methods. Section 2 comprises preliminary concepts related to CPFS including aggregation operators, score function, accuracy function and distance measure. Section 3 presents algorithm and flow chart of CPF-TOPSIS method. Section 4 consists of the numerical example to select the best interior designer using CPF-TOPSIS method. Section 5 presents algorithm and flow chart of CPF-ELECTRE I method. Section 6 consists of the numerical example to select the best interior designer using CPF-ELECTRE I method. Section 7 provides the comparative analysis of both proposed methods and section 8 consists of concluding remarks.

2 Preliminaries

Definition 2.1. [24] *Let X be the universe of discourse. A complex Pythagorean fuzzy set (CPFS) $\mathbb{A}$ defined on X is an object of the form*

$$\mathbb{A} = \{(x, u_{\mathbb{A}}(x)e^{i\alpha_{\mathbb{A}}(x)}, v_{\mathbb{A}}(x)e^{i\beta_{\mathbb{A}}(x)}) : x \in X\},$$

where $i = \sqrt{-1}$, $u_{\mathbb{A}}(x), v_{\mathbb{A}}(x) \in [0,1]$, $\alpha_{\mathbb{A}}(x),\ \beta_{\mathbb{A}}(x) \in [0, 2\pi]$, $0 \leq u^2_{\mathbb{A}}(x) + v^2_{\mathbb{A}}(x) \leq 1$ and $0 \leq (\frac{\alpha_{\mathbb{A}}(x)}{2\pi})^2 + (\frac{\beta_{\mathbb{A}}(x)}{2\pi})^2 \leq 1$. For simplicity, the pair $(ue^{i\alpha}, ve^{i\beta})$ is called the complex Pythagorean fuzzy number (CPFN).

Definition 2.2. *Let $\psi = (u_\psi e^{i\alpha_\psi}, v_\psi e^{i\beta_\psi})$, $\psi_1 = (u_{\psi_1} e^{i\alpha_{\psi_1}}, v_{\psi_1} e^{i\beta_{\psi_1}})$ and $\psi_2 = (u_{\psi_2} e^{i\alpha_{\psi_2}}, v_{\psi_2} e^{i\beta_{\psi_1}})$ be three CPFNs. The operations corresponding to these three CPFNs can be defined as follows:*

1. $\rho\psi = \left(\sqrt{1-(1-u^2_\psi)^\rho}\, e^{i2\pi\sqrt{1-(1-(\frac{\alpha_\psi}{2\pi})^2)^\rho}}, v^\rho_\psi e^{i2\pi(\frac{\beta_\psi}{2\pi})^\rho}\right), (\rho > 0);$
2. $\psi^\rho = \left(u^\rho_\psi e^{i2\pi(\frac{\alpha_\psi}{2\pi})^\rho}, \sqrt{1-(1-v^2_\psi)^\rho}\, e^{i2\pi\sqrt{1-(1-(\frac{\beta_\psi}{2\pi})^2)^\rho}}\right), (\rho > 0);$
3. $\psi_1 \oplus \psi_2 = \left(\sqrt{u^2_{\psi_1} + u^2_{\psi_2} - u^2_{\psi_1}u^2_{\psi_2}}\, e^{i2\pi\sqrt{(\frac{\alpha_{\psi_1}}{2\pi})^2 + (\frac{\alpha_{\psi_2}}{2\pi})^2 - (\frac{\alpha_{\psi_1}}{2\pi})^2(\frac{\alpha_{\psi_2}}{2\pi})^2}}, v_{\psi_1}v_{\psi_2} e^{i2\pi(\frac{\beta_{\psi_1}}{2\pi})(\frac{\beta_{\psi_2}}{2\pi})}\right);$
4. $\psi_1 \otimes \psi_2 = \left(u_{\psi_1}u_{\psi_2} e^{i2\pi(\frac{\alpha_{\psi_1}}{2\pi})(\frac{\alpha_{\psi_2}}{2\pi})}, \sqrt{v^2_{\psi_1} + v^2_{\psi_2} - v^2_{\psi_1}v^2_{\psi_2}}\, e^{i2\pi\sqrt{(\frac{\beta_{\psi_1}}{2\pi})^2 + (\frac{\beta_{\psi_2}}{2\pi})^2 - (\frac{\beta_{\psi_1}}{2\pi})^2(\frac{\beta_{\psi_2}}{2\pi})^2}}\right).$

Definition 2.3. [3] *Let $\psi_i = (u_i e^{i\alpha_i}, v_i e^{i\beta_i})(i = 1, 2, ..., n)$ be a collection of CPFNs. Let $q = (q_1, q_2, ..., q_n)^T$ be the weight vector of ψ_i such that $q_i \in [0,1]$ and $\sum\limits_{i=1}^{n} q_i = 1$. The complex Pythagorean fuzzy weighted averaging (CPFWA) operator can be defined as:*

$$\begin{aligned} CPFWA_q(\psi_1, \psi_2, \ldots, \psi_n) &= q_1\psi_1 \oplus q_2\psi_2 \oplus \ldots \oplus q_n\psi_n \\ &= \left(\sqrt{1-\prod_{i=1}^{n}(1-(u_{\psi_i})^2)^{q_i}}\, e^{i2\pi\sqrt{1-\prod_{i=1}^{n}\left(1-\left(\frac{\alpha_{\psi_i}}{2\pi}\right)^2\right)^{q_i}}}, \prod_{i=1}^{n}(v_{\psi_i})^{q_i} e^{i2\pi\prod_{i=1}^{n}\left(\frac{\beta_{\psi_i}}{2\pi}\right)^{q_i}}\right). \end{aligned} \tag{1}$$

Definition 2.4. *Let $\psi_1 = (u_{\psi_1} e^{i\alpha_{\psi_1}}, v_{\psi_1} e^{i\beta_{\psi_1}})$ and $\psi_2 = (u_{\psi_2} e^{i\alpha_{\psi_2}}, v_{\psi_2} e^{i\beta_{\psi_2}})$ be two CPFNs. The normalized Euclidean distance between ψ_1 and ψ_2 can be defined as:*

$$d(\psi_1, \psi_2) = \sqrt{\frac{(u^2_{\psi_1} - u^2_{\psi_2})^2 + (v^2_{\psi_1} - v^2_{\psi_2})^2 + (w^2_{\psi_1} - w^2_{\psi_2})^2 + \frac{1}{16\pi^4}\left((\alpha^2_{\psi_1} - \alpha^2_{\psi_2})^2 + (\beta^2_{\psi_1} - \beta^2_{\psi_2})^2 + (\gamma^2_{\psi_1} - \gamma^2_{\psi_2})^2\right)}{2}}. \tag{2}$$

Definition 2.5. [3] *Let $\psi = (ue^{i\alpha}, ve^{i\beta})$ be a CPFN. The score of CPFN can be defined as:*

$$s(\psi) = (u^2 - v^2) + \frac{1}{4\pi^2}(\alpha^2 - \beta^2), \tag{3}$$

where $s(\psi) \in [-2, 2]$ and s is the score function of ψ.

Definition 2.6. [3] *Let $\psi = (ue^{i\alpha}, ve^{i\beta})$ be a CPFN. Then accuracy of CPFN is*

$$h(\psi) = (u^2 + v^2) + \frac{1}{4\pi^2}(\alpha^2 + \beta^2), \tag{4}$$

where $h(\psi) \in [0, 2]$ and h is the accuracy function of ψ.

Definition 2.7. [3] *For the comparison of any two CPFNs $\psi_1 = (u_1 e^{i\alpha_1}, v_1 e^{i\beta_1})$ and $\psi_2 = (u_2 e^{i\alpha_2}, v_2 e^{i\beta_2})$*

1. *If $s(\psi_1) > s(\psi_2)$, then $\psi_1 \succ \psi_2$ (ψ_1 is superior to ψ_2);*
2. *If $s(\psi_1) = s(\psi_2)$, then*
   - *If $h(\psi_1) > h(\psi_2)$, then $\psi_1 \succ \psi_2$ (ψ_1 is superior to ψ_2);*
   - *If $h(\psi_1) = h(\psi_2)$, then $\psi_1 \sim \psi_2$ (ψ_1 is equivalent to ψ_2).*

3 CPF-TOPSIS method for MCGDM

In this section, we introduce a mathematical technique, namely, CPF-TOPSIS, to solve MCGDM problems comprising CPF data. The proposed CPF-TOPSIS method tries to find the most suitable alternative which is nearest to the positive ideal solution (PIS) and farthest from negative ideal solution (NIS) as the solution of MCGDM problem.

In a MCGDM problem, where we have to deal with CPF data, let $\mathbb{A} = \{\mathbb{A}_1, \mathbb{A}_2, ..., \mathbb{A}_l\}$ be the set of l alternatives from which the best alternative is to be chosen on the basis of some criteria. Let $\mathfrak{C} = \{\mathfrak{c}_1, \mathfrak{c}_2, ..., \mathfrak{c}_m\}$ be the set of decision criteria. Let $\mathbb{E} = \{\mathbb{E}_1, \mathbb{E}_2, ..., \mathbb{E}_n\}$ be the set of experts hired for decision-making. Each expert $\mathbb{E}_r$ assigns a CPFN $s^{(r)}_{pq}$ to alternative $\mathbb{A}_p$ with respect to criteria $\mathfrak{c}_q$ by carefully observing the influence of $\mathfrak{c}_q$ on the alternative $\mathbb{A}_p$ and provide the result in a complex Pythagorean fuzzy decision matrix (CPFDM) $\mathbb{S}^{(r)} = (s^{(r)}_{pq})_{l\times m}$. $s^{(r)}_{pq} = (\mu^{(r)}_{pq}, \nu^{(r)}_{pq}, \pi^{(r)}_{pq}) = (u^{(r)}_{pq} e^{i\alpha^{(r)}_{pq}}, v^{(r)}_{pq} e^{i\beta^{(r)}_{pq}}, w^{(r)}_{pq} e^{i\gamma^{(r)}_{pq}})$ represents the CPFN assigned by the expert $\mathbb{E}_r$ after judging the alternative $\mathbb{A}_p$ on the criteria $\mathfrak{c}_q$.

The hesitancy can be evaluated as:

$$\pi^{(r)}_{pq} = \sqrt{1 - (u^{(r)}_{pq})^2 - (v^{(r)}_{pq})^2} e^{i2\pi\sqrt{1-(\frac{\alpha^{(r)}_{pq}}{2\pi})^2-(\frac{\beta^{(r)}_{pq}}{2\pi})^2}}.$$

The CPFDM of expert $\mathbb{E}_r$, denoted by $\mathbb{S}^{(r)}$, is represented as follows:

$$\mathbb{S}^{(r)} = \begin{pmatrix} (\mu^{(r)}_{11}, \nu^{(r)}_{11}, \pi^{(r)}_{11}) & (\mu^{(r)}_{12}, \nu^{(r)}_{12}, \pi^{(r)}_{12}) & \dots & (\mu^{(r)}_{1m}, \nu^{(r)}_{1m}, \pi^{(r)}_{1m}) \\ (\mu^{(r)}_{21}, \nu^{(r)}_{21}, \pi^{(r)}_{21}) & (\mu^{(r)}_{22}, \nu^{(r)}_{22}, \pi^{(r)}_{22}) & \dots & (\mu^{(r)}_{2m}, \nu^{(r)}_{2m}, \pi^{(r)}_{2m}) \\ \vdots & \vdots & \ddots & \vdots \\ (\mu^{(r)}_{l1}, \nu^{(r)}_{l1}, \pi^{(r)}_{l1}) & (\mu^{(r)}_{l2}, \nu^{(r)}_{l2}, \pi^{(r)}_{l3}) & \dots & (\mu^{(r)}_{lm}, \nu^{(r)}_{lm}, \pi^{(r)}_{lm}) \end{pmatrix}.$$

The target is to find the most suitable alternative as the solution of this MCGDM problem.

We describe our CPF-TOPSIS method in the following algorithm.

Step 1: The importance of experts can be considered as linguistic terms expressed in CPFNs. Let $S_r = (\mu_r, \nu_r, \pi_r) = (u_r e^{i\alpha_r}, v_r e^{i\beta_r}, w_r e^{i\gamma_r})$ be the credibility of each expert. Then weight of rth expert can be calculated as:

$$\xi_r = \frac{u_r + w_r(\frac{u_r}{u_r+v_r}) + \frac{\alpha_r}{2\pi} + \frac{\gamma_r}{2\pi}(\frac{\frac{\alpha_r}{2\pi}}{\frac{\alpha_r}{2\pi}+\frac{\beta_r}{2\pi}})}{\sum\limits_{r=1}^{n}\Big(u_r + w_r(\frac{u_r}{u_r+v_r}) + \frac{\alpha_r}{2\pi} + \frac{\gamma_r}{2\pi}(\frac{\frac{\alpha_r}{2\pi}}{\frac{\alpha_r}{2\pi}+\frac{\beta_r}{2\pi}})\Big)}, \tag{5}$$

where $\xi_r \in [0,1]$ and satisfy the condition $\sum\limits_{r=1}^{n} \xi_r = 1$.

Step 2: $\mathbb{S}^{(r)} = (s^{(r)}_{pq})_{l\times m}$ denotes the CPFDM of each expert $\mathbb{E}_r$ with weight ξ_r. For MCGDM, the individual perspectives of the experts are assembled to have a collective opinion of all expert about an alternative with respect to a particular criteria. This leads to the construction of aggregated complex Pythagorean fuzzy decision matrix (ACPFDM) $\mathbb{S} = (s_{pq})_{l\times m}$ using CPFWA operator.

$$\begin{aligned} s_{pq} &= CPFWA_\xi(s^{(1)}_{pq}, s^{(2)}_{pq}, ..., s^{(n)}_{pq}) \\ &= \xi_1 s^{(1)}_{pq} \oplus \xi_2 s^{(2)}_{pq} \oplus ... \oplus \xi_n s^{(n)}_{pq} \\ &= \left(\sqrt{1 - \prod_{r=1}^{n}\Big(1-(u^{(r)}_{pq})^2\Big)^{\xi_r}} e^{i2\pi\sqrt{1-\prod\limits_{r=1}^{n}\Big(1-\Big(\frac{\alpha^{(r)}_{pq}}{2\pi}\Big)^2\Big)^{\xi_r}}}, \prod_{r=1}^{n}(v^{(r)}_{pq})^{\xi_r} e^{i2\pi\prod\limits_{r=1}^{n}\Big(\frac{\beta^{(r)}_{pq}}{2\pi}\Big)^{\xi_r}}\right), \end{aligned} \tag{6}$$

$$\pi_{\mathbb{A}_p}(\mathfrak{c}_q) = \sqrt{\prod_{r=1}^{n}\Big(1-(u^{(r)}_{pq})^2\Big)^{\xi_r} - (\prod_{r=1}^{n}(v^{(r)}_{pq})^{\xi_r})^2} e^{i2\pi\sqrt{\prod\limits_{r=1}^{n}\Big(1-(\frac{\alpha^{(r)}_{pq}}{2\pi})^2\Big)^{\xi_r} - \Big(\prod\limits_{r=1}^{n}(\frac{\beta^{(r)}_{pq}}{2\pi})^{\xi_r}\Big)^2}},$$

where $s_{pq} = (\mu_{\mathbb{A}_p}(\mathfrak{c}_q), \nu_{\mathbb{A}_p}(\mathfrak{c}_q), \pi_{\mathbb{A}_p}(\mathfrak{c}_q))= (u_{\mathbb{A}_p}(\mathfrak{c}_q)e^{i\alpha_{\mathbb{A}_p}(\mathfrak{c}_q)}, v_{\mathbb{A}_p}(\mathfrak{c}_q)e^{i\beta_{\mathbb{A}_p}(\mathfrak{c}_q)}, w_{\mathbb{A}_p}(\mathfrak{c}_q)e^{i\gamma_{\mathbb{A}_p}(\mathfrak{c}_q)})$, $p = 1,2,...,l$ and $q = 1,2,3,...,m$. Correspondingly, $\mathbb{S} = (s_{pq})_{l\times m}$ is constructed as:

$$\mathbb{S} = \begin{pmatrix} (\mu_{\mathbb{A}_1}(\mathfrak{c}_1), \nu_{\mathbb{A}_1}(\mathfrak{c}_1), \pi_{\mathbb{A}_1}(\mathfrak{c}_1)) & (\mu_{\mathbb{A}_1}(\mathfrak{c}_2), \nu_{\mathbb{A}_1}(\mathfrak{c}_2), \pi_{\mathbb{A}_1}(\mathfrak{c}_2)) & \ldots & (\mu_{\mathbb{A}_1}(\mathfrak{c}_m), \nu_{\mathbb{A}_1}(\mathfrak{c}_m), \pi_{\mathbb{A}_1}(\mathfrak{c}_m)) \\ (\mu_{\mathbb{A}_2}(\mathfrak{c}_1), \nu_{\mathbb{A}_2}(\mathfrak{c}_1), \pi_{\mathbb{A}_2}(\mathfrak{c}_1)) & (\mu_{\mathbb{A}_2}(\mathfrak{c}_2), \nu_{\mathbb{A}_2}(\mathfrak{c}_2), \pi_{\mathbb{A}_2}(\mathfrak{c}_2)) & \ldots & (\mu_{\mathbb{A}_2}(\mathfrak{c}_m), \nu_{\mathbb{A}_2}(\mathfrak{c}_m), \pi_{\mathbb{A}_2}(\mathfrak{c}_m)) \\ \vdots & \vdots & \ddots & \vdots \\ (\mu_{\mathbb{A}_l}(\mathfrak{c}_1), \nu_{\mathbb{A}_l}(\mathfrak{c}_1), \pi_{\mathbb{A}_l}(\mathfrak{c}_1)) & (\mu_{\mathbb{A}_l}(\mathfrak{c}_2), \nu_{\mathbb{A}_l}(\mathfrak{c}_2), \pi_{\mathbb{A}_l}(\mathfrak{c}_2)) & \ldots & (\mu_{\mathbb{A}_l}(\mathfrak{c}_m), \nu_{\mathbb{A}_l}(\mathfrak{c}_m), \pi_{\mathbb{A}_l}(\mathfrak{c}_m)) \end{pmatrix}.$$

Step 3: All criteria may not be equally valueable. Each expert $\mathbb{E}_r$ rate these criteria and assigns a CPF weightage to each criteria. Let $w_q^{(r)} = (\mu_q^{(r)}, \nu_q^{(r)}, \pi_q^{(r)}) = (u_q^{(r)}e^{i\alpha_q^{(r)}}, v_q^{(r)}e^{i\beta_q^{(r)}}, w_q^{(r)}e^{i\gamma_q^{(r)}})$ be the CPF weight assigned to the criteria $\mathfrak{c}_q$ by expert $\mathbb{E}_r$. The individual perspectives of the experts are assembled to construct the weightage matrix $\mathbb{W}$ using CPFWA operator.

$$\begin{aligned} w_q &= CPFWA_\xi(w_q^{(1)}, w_q^{(2)}, ..., w_q^{(n)}) \\ &= = \xi_1 w_q^{(1)} \oplus \xi_2 w_q^{(2)} \oplus ... \oplus \xi_n w_q^{(n)} \\ &= \left(\sqrt{1 - \prod_{r=1}^{n}\Big(1-(u_q^{(r)})^2\Big)^{\xi_r}}\, e^{i2\pi\sqrt{1-\prod_{r=1}^{n}\Big(1-\Big(\frac{\alpha_q^{(r)}}{2\pi}\Big)^2\Big)^{\xi_r}}}, \prod_{r=1}^{n}(v_q^{(r)})^{\xi_r} e^{i2\pi\prod_{r=1}^{n}(\frac{\beta_q^{(r)}}{2\pi})^{\xi_r}} \right), \qquad (7) \\ \pi_w(\mathfrak{c}_q) &= \sqrt{\prod_{r=1}^{n}\Big(1-(u_q^{(r)})^2\Big)^{\xi_r} - (\prod_{r=1}^{n}(v_q^{(r)})^{\xi_r})^2}\, e^{i2\pi\sqrt{\prod_{r=1}^{n}\Big(1-\Big(\frac{\alpha_q^{(r)}}{2\pi}\Big)^2\Big)^{\xi_r} - (\prod_{r=1}^{n}(\frac{\beta_q^{(r)}}{2\pi})^{\xi_r})^2}}. \end{aligned}$$

$\mathbb{W} = [w_1, w_2, ..., w_m]^T$ and $w_q = (\mu_w(\mathfrak{c}_q), \nu_w(\mathfrak{c}_q), \pi_w(\mathfrak{c}_q))= (u_w(\mathfrak{c}_q)e^{i\alpha_w(\mathfrak{c}_q)}, v_w(\mathfrak{c}_q)e^{i\beta_w(\mathfrak{c}_q)}, w_w(\mathfrak{c}_q)e^{i\gamma_w(\mathfrak{c}_q)})$, $q = 1,2,...,m$.

Step 4: The aggregated weighted complex Pythagorean fuzzy decision matrix (AWCPFDM) $\mathbb{S}' = (s'_{pq})_{l\times m}$ can be evaluated with the help of ACPFDM and weight matrix $\mathbb{W}$. The entries of AWCPFDM $s'_{pq} = s_{pq} \otimes w_q$ can be calculated as:

$$s'_{pq} = \left(u_{\mathbb{A}_p}(\mathfrak{c}_q)u_w(\mathfrak{c}_q)e^{i2\pi(\frac{\alpha_{\mathbb{A}_p}(\mathfrak{c}_q)}{2\pi})(\frac{\alpha_w(\mathfrak{c}_q)}{2\pi})}, \sqrt{v^2_{\mathbb{A}_p}(\mathfrak{c}_q) + v^2_w(\mathfrak{c}_q) - v^2_{\mathbb{A}_p}(\mathfrak{c}_q)v^2_w(\mathfrak{c}_q)}\, e^{i2\pi\sqrt{(\frac{\beta_{\mathbb{A}_p}(\mathfrak{c}_q)}{2\pi})^2 + (\frac{\beta_w(\mathfrak{c}_q)}{2\pi})^2 - (\frac{\beta_{\mathbb{A}_p}(\mathfrak{c}_q)}{2\pi})^2(\frac{\beta_w(\mathfrak{c}_q)}{2\pi})^2}} \right). \qquad (8)$$

The AWCPFDM can be constructed as:

$$\mathbb{S}' = \begin{pmatrix} (\mu'_{\mathbb{A}_1}(\mathfrak{c}_1), \nu'_{\mathbb{A}_1}(\mathfrak{c}_1), \pi'_{\mathbb{A}_1}(\mathfrak{c}_1)) & (\mu'_{\mathbb{A}_1}(\mathfrak{c}_2), \nu'_{\mathbb{A}_1}(\mathfrak{c}_2), \pi'_{\mathbb{A}_1}(\mathfrak{c}_2)) & \ldots & (\mu'_{\mathbb{A}_1}(\mathfrak{c}_m), \nu'_{\mathbb{A}_1}(\mathfrak{c}_m), \pi'_{\mathbb{A}_1}(\mathfrak{c}_m)) \\ (\mu'_{\mathbb{A}_2}(\mathfrak{c}_1), \nu'_{\mathbb{A}_2}(\mathfrak{c}_1), \pi'_{\mathbb{A}_2}(\mathfrak{c}_1)) & (\mu'_{\mathbb{A}_2}(\mathfrak{c}_2), \nu'_{\mathbb{A}_2}(\mathfrak{c}_2), \pi'_{\mathbb{A}_2}(\mathfrak{c}_2)) & \ldots & (\mu'_{\mathbb{A}_2}(\mathfrak{c}_m), \nu'_{\mathbb{A}_2}(\mathfrak{c}_m), \pi'_{\mathbb{A}_2}(\mathfrak{c}_m)) \\ \vdots & \vdots & \ddots & \vdots \\ (\mu'_{\mathbb{A}_l}(\mathfrak{c}_1), \nu'_{\mathbb{A}_l}(\mathfrak{c}_1), \pi'_{\mathbb{A}_l}(\mathfrak{c}_1)) & (\mu'_{\mathbb{A}_l}(\mathfrak{c}_2), \nu'_{\mathbb{A}_l}(\mathfrak{c}_2), \pi'_{\mathbb{A}_l}(\mathfrak{c}_2)) & \ldots & (\mu'_{\mathbb{A}_l}(\mathfrak{c}_m), \nu'_{\mathbb{A}_l}(\mathfrak{c}_m), \pi'_{\mathbb{A}_l}(\mathfrak{c}_m)) \end{pmatrix},$$

where $s'_{pq} = (\mu'_{\mathbb{A}_p}(\mathfrak{c}_q), \nu'_{\mathbb{A}_p}(\mathfrak{c}_q), \pi'_{\mathbb{A}_p}(\mathfrak{c}_q)) = (u'_{\mathbb{A}_p}(\mathfrak{c}_q)e^{i\alpha'_{\mathbb{A}_p}(\mathfrak{c}_q)}, v'_{\mathbb{A}_p}(\mathfrak{c}_q)e^{i\beta'_{\mathbb{A}_p}(\mathfrak{c}_q)}, w'_{\mathbb{A}_p}(\mathfrak{c}_q)e^{i\gamma'_{\mathbb{A}_p}(\mathfrak{c}_q)})$ and

$$\pi'_{\mathbb{A}_p}(\mathfrak{c}_q) = \sqrt{1 - u'^2_{\mathbb{A}_p}(\mathfrak{c}_q) - v'^2_{\mathbb{A}_p}(\mathfrak{c}_q)}\, e^{i2\pi\sqrt{1-(\frac{\alpha'_{\mathbb{A}_p}(\mathfrak{c}_q)}{2\pi})^2 - (\frac{\beta'_{\mathbb{A}_p}(\mathfrak{c}_q)}{2\pi})^2}}.$$

Step 5: It is not possible to obtain the positive and negative ideal solution from the AWCPFDM $\mathbb{S}'$ because each CPFN has complex membership and non-membership grades and we cannot compare two complex number. To tackle this problem, we seek help from the score function of CPFNs, as defined in Eq. (3). The score matrix $\mathbb{S}^* = (s^*_{pq})_{l\times m}$ can be obtained by calculating the score function of each entry in $\mathbb{S}'$.

$$s^*_{pq} = u'^2_{\mathbb{A}_p}(\mathfrak{c}_q) - v'^2_{\mathbb{A}_p}(\mathfrak{c}_q) + (\frac{\alpha'_{\mathbb{A}_p}(\mathfrak{c}_q)}{2\pi})^2 - (\frac{\beta'_{\mathbb{A}_p}(\mathfrak{c}_q)}{2\pi})^2, \qquad (9)$$

$$\mathbb{S}^* = \begin{pmatrix} s^*_{11} & s^*_{12} & \cdots & s^*_{1m} \\ s^*_{21} & s^*_{22} & \cdots & s^*_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ s^*_{l1} & s^*_{l2} & \cdots & s^*_{lm} \end{pmatrix}.$$

Step 6: Let $\mathfrak{C}_B$ and $\mathfrak{C}_Y$ represent the set of benefit type and cost type criteria. The PIS $\mathbb{A}^+$ and NIS $\mathbb{A}^-$ can be obtained as:

$$\begin{aligned}\mathbb{A}^+ &= \{\mathbb{A}^+(\mathfrak{c}_1), \mathbb{A}^+(\mathfrak{c}_2), ..., \mathbb{A}^+(\mathfrak{c}_m)\},\\ \mathbb{A}^- &= \{\mathbb{A}^-(\mathfrak{c}_1), \mathbb{A}^-(\mathfrak{c}_2), ..., \mathbb{A}^-(\mathfrak{c}_m)\},\end{aligned}$$

where the PIS and NIS according to the criteria $\mathfrak{c}_q$ is evaluated as:

$$\mathbb{A}^+(\mathfrak{c}_q) = \{(\max_p s^*_{pq}|\mathfrak{c}_q \in \mathfrak{C}_B), (\min_p s^*_{pq}|\mathfrak{c}_q \in \mathfrak{C}_Y)|p = 1, 2, 3, ..., l\}, \tag{10}$$

$$\mathbb{A}^-(\mathfrak{c}_q) = \{(\min_p s^*_{pq}|\mathfrak{c}_q \in \mathfrak{C}_B), (\max_p s^*_{pq}|\mathfrak{c}_q \in \mathfrak{C}_Y)|p = 1, 2, 3, ..., l\}. \tag{11}$$

Step 7: It is not possible to find an exact PIS while working on real life decision-making problems. Similarly, NIS, being the worst choice as a solution, is difficult to find. To tackle this problem, we define distance measure and compute the distance of PIS and NIS from each alternative $\mathbb{A}_p$. For this purpose, we use Euclidean distance between two sets, defined as follows:

$$d(\mathbb{A}_p, \mathbb{A}^+) = \sqrt{\sum_{q=1}^{m} (\mathbb{A}_p(\mathfrak{c}_q) - \mathbb{A}^+(\mathfrak{c}_q))^2}, \tag{12}$$

$$d(\mathbb{A}_p, \mathbb{A}^-) = \sqrt{\sum_{q=1}^{m} (\mathbb{A}_p(\mathfrak{c}_q) - \mathbb{A}^-(\mathfrak{c}_q))^2}. \tag{13}$$

Step 8: The relative closeness index of an alternative $\mathbb{A}_p$ can be computed employing the formula:

$$R_{p^+} = \frac{d(\mathbb{A}_p, \mathbb{A}^-)}{d(\mathbb{A}_p, \mathbb{A}^+) + d(\mathbb{A}_p, \mathbb{A}^-)}, \tag{14}$$

$p = 1, 2, ..., l.$
Later, Hadi-Vencheh and Mirjaberi [26] proved that under some particular conditions the closeness index fails to provide proper result. To deal such problem, they introduced the revised closeness index ζ as follows:

$$\zeta(\mathbb{A}_p) = \frac{d(\mathbb{A}_p, \mathbb{A}^-)}{d_{\max}(\mathbb{A}_p, \mathbb{A}^-)} - \frac{d(\mathbb{A}_p, \mathbb{A}^+)}{d_{\min}(\mathbb{A}_p, \mathbb{A}^+)}, \tag{15}$$

$p = 1, 2, ..., l.$
The revised closeness index of an alternative measures the closeness to the PIS and separation from NIS. The alternative with largest revised closeness index is most suitable one.

Step 9: After the computing revised closeness index, the alternatives are compared by ranking them in descending order of revised closeness index $\zeta(\mathbb{A}_p)$. We find the solution by choosing the alternative $\check{\mathbb{A}}$ with highest value of revised closeness index.

$$\check{\mathbb{A}} = \{\mathbb{A}_p : (p, \zeta(\mathbb{A}_p) = \max_{1\leq k\leq l} \zeta(\mathbb{A}_k))\}.$$

The structure of CPF-TOPSIS method is explained with the help of flow chart (Figure 2(a)) as follows:

4 Numerical example

In this section, we apply CPF-TOPSIS method to select the best interior designer for the renovation of a hotel to effectively demonstrate the proposed method.

4.1 Selection of best interior designer for hotel renovation

Interior designing is an art to boost the interior of a building to create an attractive environment for the visitors. Interior design is an effective tool to satisfy customer and bring in revenue. Effective interior design of a hotel balances the operational functionality (guest's ease, security and food quality) and the physical environmental elements (hotel furniture, lights, wall paints, flowers, art and designing) to develop an attractive environment. An interior designer decorates interior spaces with perfect combination of lights, furniture, paint, flowers and curtains which creates the attractive ambience for visitors. A hotel owner wants to hire an interior designer for renovation of the hotel interior design to improve the guest experience. A group of three experts is hired to assess the hotel exigencies and select the most suitable interior designer. After preliminary screening, a list of five candidates $\mathbb{A}_1, \mathbb{A}_2, \mathbb{A}_3, \mathbb{A}_4, \mathbb{A}_5$ is shortlisted for further evaluation. The experts judge the candidates on the basis of four criteria, namely, creativity and artistic sense ($\mathfrak{c}_1$), educational background ($\mathfrak{c}_2$), planning and communication skills ($\mathfrak{c}_3$) and experience ($\mathfrak{c}_4$).

Step 1: The linguistic terms for rating the experts relative to their importance are tabulated in Table 1. The weights of experts can be evaluated by assigning a linguistic term to each expert and using the Eq. (5), as described in Table 2.

Table 1: Linguistic terms to rate experts and criteria relative to their importance

Linguistic terms	CPFNs
Very important($\mathbb{VI}$)	$[0.90e^{i2\pi(0.87)}, 0.15e^{i2\pi(0.10)}, 0.41e^{i2\pi(0.48)}]$
Important($\mathbb{I}$)	$[0.83e^{i2\pi(0.80)}, 0.20e^{i2\pi(0.21)}, 0.52e^{i2\pi(0.56)}]$
Medium($\mathbb{M}$)	$[0.75e^{i2\pi(0.72)}, 0.45e^{i2\pi(0.50)}, 0.48e^{i2\pi(0.48)}]$
Unimportant($\mathbb{UI}$)	$[0.45e^{i2\pi(0.50)}, 0.70e^{i2\pi(0.73)}, 0.55e^{i2\pi(0.47)}]$
Very unimportant($\mathbb{VUI}$)	$[0.20e^{i2\pi(0.25)}, 0.85e^{i2\pi(0.87)}, 0.49e^{i2\pi(0.42)}]$

Table 2: The importance of experts and their weights

Expert	Linguistic terms	CPFNs	$weights(\xi_r)$
$\mathbb{E}_1$	Very important	$[0.90e^{i2\pi(0.87)}, 0.15e^{i2\pi(0.10)}, 0.41e^{i2\pi(0.48)}]$	0.3595
$\mathbb{E}_2$	Important	$[0.83e^{i2\pi(0.80)}, 0.20e^{i2\pi(0.21)}, 0.52e^{i2\pi(0.56)}]$	0.3512
$\mathbb{E}_3$	Medium	$[0.75e^{i2\pi(0.72)}, 0.45e^{i2\pi(0.50)}, 0.48e^{i2\pi(0.48)}]$	0.2893

Step 2: The linguistic terms to rate the candidates relative to their performance are tabulated in Table 3. Each expert assesses the performance of the candidates on the basis of a particular criteria and deduce the result, as tabulated in Table 4. The individual opinions of expert $\mathbb{E}_1$, $\mathbb{E}_2$ and $\mathbb{E}_3$ are arranged to form the CPFDMs, as shown in Table 6, 7 and 8, repectively. The ACPFDM, shown in Table 9, is constructed by averaging the decisions of experts with the help of CPFWA operator, as defined in Eq. (6).

Table 3: Linguistic terms to rate the interior designers

Linguistic terms	CPFNs
Extremely good($\mathbb{EG}$)	$[1e^{i2\pi}, 0, 0]$
Very very good($\mathbb{VVG}$)	$[0.90e^{i2\pi(0.92)}, 0.10e^{i2\pi(0.07)}, 0.42e^{i2\pi(0.39)}]$
Very good($\mathbb{VG}$)	$[0.85e^{i2\pi(0.83)}, 0.30e^{i2\pi(0.25)}, 0.43e^{i2\pi(0.50)}]$
Good($\mathbb{G}$)	$[0.80e^{i2\pi(0.78)}, 0.35e^{i2\pi(0.37)}, 0.49e^{i2\pi(0.50)}]$
Medium good($\mathbb{MG}$)	$[0.75e^{i2\pi(0.73)}, 0.47e^{i2\pi(0.45)}, 0.47e^{i2\pi(0.51)}]$
Medium($\mathbb{M}$)	$[0.65e^{i2\pi(0.67)}, 0.55e^{i2\pi(0.52)}, 0.52e^{i2\pi(0.53)}]$
Medium bad($\mathbb{MB}$)	$[0.50e^{i2\pi(0.53)}, 0.70e^{i2\pi(0.68)}, 0.51e^{i2\pi(0.51)}]$
Bad($\mathbb{B}$)	$[0.40e^{i2\pi(0.45)}, 0.75e^{i2\pi(0.72)}, 0.53e^{i2\pi(0.53)}]$
Very bad($\mathbb{VB}$)	$[0.30e^{i2\pi(0.28)}, 0.85e^{i2\pi(0.80)}, 0.43e^{i2\pi(0.53)}]$
Very very bad($\mathbb{VVB}$)	$[0.10e^{i2\pi(0.15)}, 0.90e^{i2\pi(0.92)}, 0.42e^{i2\pi(0.36)}]$

Table 4: The rating of interior designers with respect to criteria by experts

	$\mathfrak{c}_1$					$\mathfrak{c}_2$					$\mathfrak{c}_3$					$\mathfrak{c}_4$				
	$\mathbb{A}_1$	$\mathbb{A}_2$	$\mathbb{A}_3$	$\mathbb{A}_4$	$\mathbb{A}_5$	$\mathbb{A}_1$	$\mathbb{A}_2$	$\mathbb{A}_3$	$\mathbb{A}_4$	$\mathbb{A}_5$	$\mathbb{A}_1$	$\mathbb{A}_2$	$\mathbb{A}_3$	$\mathbb{A}_4$	$\mathbb{A}_5$	$\mathbb{A}_1$	$\mathbb{A}_2$	$\mathbb{A}_3$	$\mathbb{A}_4$	$\mathbb{A}_5$
$\mathbb{E}_1$	$\mathbb{G}$	$\mathbb{MG}$	$\mathbb{VVG}$	$\mathbb{MG}$	$\mathbb{M}$	$\mathbb{MG}$	$\mathbb{M}$	$\mathbb{VG}$	$\mathbb{M}$	$\mathbb{MB}$	$\mathbb{VG}$	$\mathbb{G}$	$\mathbb{VG}$	$\mathbb{VG}$	$\mathbb{G}$	$\mathbb{G}$	$\mathbb{MG}$	$\mathbb{VG}$	$\mathbb{G}$	$\mathbb{M}$
$\mathbb{E}_2$	$\mathbb{G}$	$\mathbb{M}$	$\mathbb{VG}$	$\mathbb{G}$	$\mathbb{MG}$	$\mathbb{MG}$	$\mathbb{G}$	$\mathbb{VG}$	$\mathbb{MG}$	$\mathbb{M}$	$\mathbb{VG}$	$\mathbb{MG}$	$\mathbb{G}$	$\mathbb{G}$	$\mathbb{MG}$	$\mathbb{G}$	$\mathbb{MG}$	$\mathbb{G}$	$\mathbb{MG}$	$\mathbb{M}$
$\mathbb{E}_3$	$\mathbb{VG}$	$\mathbb{G}$	$\mathbb{VG}$	$\mathbb{G}$	$\mathbb{MG}$	$\mathbb{G}$	$\mathbb{MG}$	$\mathbb{G}$	$\mathbb{M}$	$\mathbb{M}$	$\mathbb{G}$	$\mathbb{MG}$	$\mathbb{VG}$	$\mathbb{G}$	$\mathbb{G}$	$\mathbb{G}$	$\mathbb{M}$	$\mathbb{VG}$	$\mathbb{MG}$	$\mathbb{MG}$

Step 3: Each expert assesses the significance of each criteria and the decision is aggregated in Table 5. The individual perspectives of the experts about a criteria are assembled to construct the weightage matrix $\mathbb{W}$ with the help of CPFWA operator, as defined in Eq. (7).

Step 4: The AWCPFDM $\mathbb{S}'$ can be obtained by Eq. (8) and is shown in Table 10.

Step 5: The score of each CPFN can be computed by using Eq. (9) and the score matrix $\mathbb{S}^*$ is shown in Table 11.

Table 5: Importance of criteria in linguistic terms and their weights

Criteria	$\mathbb{E}_1$	$\mathbb{E}_2$	$\mathbb{E}_3$	CPF weights
$\mathfrak{c}_1$	$\mathbb{VI}$	$\mathbb{I}$	$\mathbb{VI}$	$[0.8798e^{i2\pi(0.8491)}, 0.1659e^{i2\pi(0.1298)}, 0.4455e^{i2\pi(0.5120)}]$
$\mathfrak{c}_2$	$\mathbb{I}$	$\mathbb{I}$	$\mathbb{I}$	$[0.8300e^{i2\pi(0.8000)}, 0.2000e^{i2\pi(0.2100)}, 0.5207e^{i2\pi(0.5620)}]$
$\mathfrak{c}_3$	$\mathbb{I}$	$\mathbb{M}$	$\mathbb{I}$	$[0.8058e^{i2\pi(0.7754)}, 0.2659e^{i2\pi(0.2848)}, 0.5291e^{i2\pi(0.5636)}]$
$\mathfrak{c}_4$	$\mathbb{M}$	$\mathbb{M}$	$\mathbb{I}$	$[0.7769e^{i2\pi(0.7465)}, 0.3559e^{i2\pi(0.3890)}, 0.5194e^{i2\pi(0.5398)}]$

Step 6: The criteria artistic sense and creativity, educational background , planning and communication skill and experience are benefit criteria. The PIS and NIS can be obtained by using Eqs. (10)-(11).

$$\mathbb{A}^+ = \{1.0249, 0.6564, 0.5411, 0.3539\},$$

$$\mathbb{A}^- = \{0.2575, -0.2391, 0.2427, -0.1839\}.$$

Step 7: The distance of each alternative from PIS and NIS can be evaluated by employing Euclidean distance defined in Eq. (12). The revised closeness index can be computed using Eq. (15). The assessment information is summarized in Table 12.

Step 8: Corresponding to the revised closeness indices, we infer that the alternatives can be ranked in the following order:

$$\mathbb{A}_3 \succ \mathbb{A}_1 \succ \mathbb{A}_4 \succ \mathbb{A}_2 \succ \mathbb{A}_5.$$

Thus, we conclude that $\mathbb{A}_3$ is best interior designer.

5 The CPF-ELECTRE I method

In this section, we will try to tackle the MCGDM problems comprising CPF data by developing a new method, namely, complex Pythagorean fuzzy ELECTRE I (CPF-ELECTRE I) method. We present the flow chart to illustrate the method effectively. Let $\mathbb{A} = \{\mathbb{A}_1, \mathbb{A}_2, ..., \mathbb{A}_l\}$ be the set of l alternatives from which the best alternative is to be choosen on the basis of some criteria. Let $\mathfrak{C} = \{\mathfrak{c}_1, \mathfrak{c}_2, ..., \mathfrak{c}_m\}$ be the set of decision criteria. Let $\mathbb{E} = \{\mathbb{E}_1, \mathbb{E}_2, ..., \mathbb{E}_n\}$ be the set of experts hired for decision-making. Each expert $\mathbb{E}_r$ assigns a CPFN $s_{pq}^{(r)}$ to alternative $\mathbb{A}_p$ with respect to criteria $\mathfrak{c}_q$ and provide the result in a complex Pythagorean fuzzy decision matrix (CPFDM) $\mathbb{S}^{(r)} = (s_{pq}^{(r)})_{l\times m}$. The target is to find the most suitable alternative as the solution of this MCGDM problem. We describe our proposed method step by step as follows:
In CPF-ELECTRE I method, **step 1-step 4** are same as in CPF-TOPSIS method.

5.1 Determination of complex Pythagorean fuzzy concordance and discordance sets

The main idea behind the CPF-ELECTRE I method is to compare each pair of alternatives on some specific criteria. In CPF-ELECTRE I method, the comparison between CPFNs is made on the basis of score degree, accuracy degree and hesitancy degree. The alternatives with higher score degree are preferred over other alternatives. In case of same score degree, the alternative with higher accuracy degree is preferred over the other alternatives. The CPF concordance and discordance sets are classified as strong, midrange and weak concordance and discordance sets depending on the score function, accuracy function and hesitancy. For each pair of alternatives $(\mathbb{A}_\psi, \mathbb{A}_\chi)$, $(\psi, \chi = 1, 2, ..., l, \psi \neq \chi)$, the CPF concordance sets comprise the subscripts of all those criteria for which $\mathbb{A}_\psi$ is preferable to $\mathbb{A}_\chi$.

(i) The complex Pythagorean fuzzy strong concordance set $\mathfrak{J}_{\mathfrak{C}_{\psi\chi}}$ can be defined as:

$$\mathfrak{J}_{\mathfrak{C}_{\psi\chi}} = \{q : u'_\psi(\mathfrak{c}_q) \geq u'_\chi(\mathfrak{c}_q), \alpha'_\psi(\mathfrak{c}_q) \geq \alpha'_\chi(\mathfrak{c}_q), v'_\psi(\mathfrak{c}_q) < v'_\chi(\mathfrak{c}_q), \beta'_\psi(\mathfrak{c}_q) < \beta'_\chi(\mathfrak{c}_q), w'_\psi(\mathfrak{c}_q) < w'_\chi(\mathfrak{c}_q), \gamma'_\psi(\mathfrak{c}_q) < \gamma'_\chi(\mathfrak{c}_q)\}. \quad (16)$$

(ii) The complex Pythagorean fuzzy midrange concordance set $\mathfrak{J}_{\mathfrak{C}^*_{\psi\chi}}$ can be defined as:

$$\mathfrak{J}_{\mathfrak{C}^*_{\psi\chi}} = \{q : u'_\psi(\mathfrak{c}_q) \geq u'_\chi(\mathfrak{c}_q), \alpha'_\psi(\mathfrak{c}_q) \geq \alpha'_\chi(\mathfrak{c}_q), v'_\psi(\mathfrak{c}_q) < v'_\chi(\mathfrak{c}_q), \beta'_\psi(\mathfrak{c}_q) < \beta'_\chi(\mathfrak{c}_q), w'_\psi(\mathfrak{c}_q) \geq w'_\chi(\mathfrak{c}_q), \gamma'_\psi(\mathfrak{c}_q) \geq \gamma'_\chi(\mathfrak{c}_q)\}. \quad (17)$$

(iii) The complex Pythagorean fuzzy weak concordance set $\mathfrak{J}_{\mathfrak{C}^{**}_{\psi\chi}}$ can be defined as:

$$\mathfrak{J}_{\mathfrak{C}^{**}_{\psi\chi}} = \{q : u'_\psi(\mathfrak{c}_q) \geq u'_\chi(\mathfrak{c}_q), \alpha'_\psi(\mathfrak{c}_q) \geq \alpha'_\chi(\mathfrak{c}_q), v'_\psi(\mathfrak{c}_q) \geq v'_\chi(\mathfrak{c}_q), \beta'_\psi(\mathfrak{c}_q) \geq \beta'_\chi(\mathfrak{c}_q)\}. \quad (18)$$

Table 6: CPFDM of the expert $\mathbb{E}_1$

$\mathbb{S}^{(1)}$	$\mathfrak{c}_1$	$\mathfrak{c}_2$	$\mathfrak{c}_3$	$\mathfrak{c}_4$
$\mathbb{A}_1$	$[0.80e^{i2\pi(0.78)}, 0.35e^{i2\pi(0.37)}, 0.49e^{i2\pi(0.50)}]$	$[0.75e^{i2\pi(0.73)}, 0.47e^{i2\pi(0.45)}, 0.47e^{i2\pi(0.51)}]$	$[0.85e^{i2\pi(0.83)}, 0.30e^{i2\pi(0.25)}, 0.43e^{i2\pi(0.50)}]$	$[0.80e^{i2\pi(0.78)}, 0.35e^{i2\pi(0.37)}, 0.49e^{i2\pi(0.50)}]$
$\mathbb{A}_2$	$[0.75e^{i2\pi(0.73)}, 0.47e^{i2\pi(0.45)}, 0.47e^{i2\pi(0.51)}]$	$[0.65e^{i2\pi(0.67)}, 0.55e^{i2\pi(0.52)}, 0.52e^{i2\pi(0.53)}]$	$[0.80e^{i2\pi(0.78)}, 0.35e^{i2\pi(0.37)}, 0.49e^{i2\pi(0.50)}]$	$[0.75e^{i2\pi(0.73)}, 0.47e^{i2\pi(0.45)}, 0.47e^{i2\pi(0.51)}]$
$\mathbb{A}_3$	$[0.90e^{i2\pi(0.92)}, 0.10e^{i2\pi(0.07)}, 0.42e^{i2\pi(0.39)}]$	$[0.85e^{i2\pi(0.83)}, 0.30e^{i2\pi(0.25)}, 0.43e^{i2\pi(0.50)}]$	$[0.85e^{i2\pi(0.83)}, 0.30e^{i2\pi(0.25)}, 0.43e^{i2\pi(0.50)}]$	$[0.85e^{i2\pi(0.83)}, 0.30e^{i2\pi(0.25)}, 0.43e^{i2\pi(0.50)}]$
$\mathbb{A}_4$	$[0.75e^{i2\pi(0.73)}, 0.47e^{i2\pi(0.45)}, 0.47e^{i2\pi(0.51)}]$	$[0.65e^{i2\pi(0.67)}, 0.55e^{i2\pi(0.52)}, 0.52e^{i2\pi(0.53)}]$	$[0.85e^{i2\pi(0.83)}, 0.30e^{i2\pi(0.25)}, 0.43e^{i2\pi(0.50)}]$	$[0.80e^{i2\pi(0.78)}, 0.35e^{i2\pi(0.37)}, 0.49e^{i2\pi(0.50)}]$
$\mathbb{A}_5$	$[0.65e^{i2\pi(0.67)}, 0.55e^{i2\pi(0.52)}, 0.52e^{i2\pi(0.53)}]$	$[0.50e^{i2\pi(0.53)}, 0.70e^{i2\pi(0.68)}, 0.51e^{i2\pi(0.51)}]$	$[0.80e^{i2\pi(0.78)}, 0.35e^{i2\pi(0.37)}, 0.49e^{i2\pi(0.50)}]$	$[0.65e^{i2\pi(0.67)}, 0.55e^{i2\pi(0.52)}, 0.52e^{i2\pi(0.53)}]$

Table 7: CPFDM of the expert $\mathbb{E}_2$

$\mathbb{S}^{(2)}$	$\mathfrak{c}_1$	$\mathfrak{c}_2$	$\mathfrak{c}_3$	$\mathfrak{c}_4$
$\mathbb{A}_1$	$[0.80e^{i2\pi(0.78)}, 0.35e^{i2\pi(0.37)}, 0.49e^{i2\pi(0.50)}]$	$[0.75e^{i2\pi(0.73)}, 0.47e^{i2\pi(0.45)}, 0.47e^{i2\pi(0.51)}]$	$[0.85e^{i2\pi(0.83)}, 0.30e^{i2\pi(0.25)}, 0.43e^{i2\pi(0.50)}]$	$[0.80e^{i2\pi(0.78)}, 0.35e^{i2\pi(0.37)}, 0.49e^{i2\pi(0.50)}]$
$\mathbb{A}_2$	$[0.65e^{i2\pi(0.67)}, 0.55e^{i2\pi(0.52)}, 0.52e^{i2\pi(0.53)}]$	$[0.80e^{i2\pi(0.78)}, 0.35e^{i2\pi(0.37)}, 0.49e^{i2\pi(0.50)}]$	$[0.75e^{i2\pi(0.73)}, 0.47e^{i2\pi(0.45)}, 0.47e^{i2\pi(0.51)}]$	$[0.75e^{i2\pi(0.73)}, 0.47e^{i2\pi(0.45)}, 0.47e^{i2\pi(0.51)}]$
$\mathbb{A}_3$	$[0.85e^{i2\pi(0.83)}, 0.30e^{i2\pi(0.25)}, 0.43e^{i2\pi(0.50)}]$	$[0.85e^{i2\pi(0.83)}, 0.30e^{i2\pi(0.25)}, 0.43e^{i2\pi(0.50)}]$	$[0.80e^{i2\pi(0.78)}, 0.35e^{i2\pi(0.37)}, 0.49e^{i2\pi(0.50)}]$	$[0.80e^{i2\pi(0.78)}, 0.35e^{i2\pi(0.37)}, 0.49e^{i2\pi(0.50)}]$
$\mathbb{A}_4$	$[0.80e^{i2\pi(0.78)}, 0.35e^{i2\pi(0.37)}, 0.49e^{i2\pi(0.50)}]$	$[0.75e^{i2\pi(0.73)}, 0.47e^{i2\pi(0.45)}, 0.47e^{i2\pi(0.51)}]$	$[0.80e^{i2\pi(0.78)}, 0.35e^{i2\pi(0.37)}, 0.49e^{i2\pi(0.50)}]$	$[0.75e^{i2\pi(0.73)}, 0.47e^{i2\pi(0.45)}, 0.47e^{i2\pi(0.51)}]$
$\mathbb{A}_5$	$[0.75e^{i2\pi(0.73)}, 0.47e^{i2\pi(0.45)}, 0.47e^{i2\pi(0.51)}]$	$[0.65e^{i2\pi(0.67)}, 0.55e^{i2\pi(0.52)}, 0.52e^{i2\pi(0.53)}]$	$[0.75e^{i2\pi(0.73)}, 0.47e^{i2\pi(0.45)}, 0.47e^{i2\pi(0.51)}]$	$[0.65e^{i2\pi(0.67)}, 0.55e^{i2\pi(0.52)}, 0.52e^{i2\pi(0.53)}]$

Table 8: CPFDM of the expert $\mathbb{E}_3$

$\mathbb{S}^{(3)}$	$\mathfrak{c}_1$	$\mathfrak{c}_2$	$\mathfrak{c}_3$	$\mathfrak{c}_4$
$\mathbb{A}_1$	$[0.85e^{i2\pi(0.83)}, 0.30e^{i2\pi(0.25)}, 0.43e^{i2\pi(0.50)}]$	$[0.80e^{i2\pi(0.78)}, 0.35e^{i2\pi(0.37)}, 0.49e^{i2\pi(0.50)}]$	$[0.80e^{i2\pi(0.78)}, 0.35e^{i2\pi(0.37)}, 0.49e^{i2\pi(0.50)}]$	$[0.80e^{i2\pi(0.78)}, 0.35e^{i2\pi(0.37)}, 0.49e^{i2\pi(0.50)}]$
$\mathbb{A}_2$	$[0.80e^{i2\pi(0.78)}, 0.35e^{i2\pi(0.37)}, 0.49e^{i2\pi(0.50)}]$	$[0.75e^{i2\pi(0.73)}, 0.47e^{i2\pi(0.45)}, 0.47e^{i2\pi(0.51)}]$	$[0.75e^{i2\pi(0.73)}, 0.47e^{i2\pi(0.45)}, 0.47e^{i2\pi(0.51)}]$	$[0.65e^{i2\pi(0.67)}, 0.55e^{i2\pi(0.52)}, 0.52e^{i2\pi(0.53)}]$
$\mathbb{A}_3$	$[0.85e^{i2\pi(0.83)}, 0.30e^{i2\pi(0.25)}, 0.43e^{i2\pi(0.50)}]$	$[0.80e^{i2\pi(0.78)}, 0.35e^{i2\pi(0.37)}, 0.49e^{i2\pi(0.50)}]$	$[0.85e^{i2\pi(0.83)}, 0.30e^{i2\pi(0.25)}, 0.43e^{i2\pi(0.50)}]$	$[0.85e^{i2\pi(0.83)}, 0.30e^{i2\pi(0.25)}, 0.43e^{i2\pi(0.50)}]$
$\mathbb{A}_4$	$[0.80e^{i2\pi(0.78)}, 0.35e^{i2\pi(0.37)}, 0.49e^{i2\pi(0.50)}]$	$[0.65e^{i2\pi(0.67)}, 0.55e^{i2\pi(0.52)}, 0.52e^{i2\pi(0.53)}]$	$[0.80e^{i2\pi(0.78)}, 0.35e^{i2\pi(0.37)}, 0.49e^{i2\pi(0.50)}]$	$[0.75e^{i2\pi(0.73)}, 0.47e^{i2\pi(0.45)}, 0.47e^{i2\pi(0.51)}]$
$\mathbb{A}_5$	$[0.75e^{i2\pi(0.73)}, 0.47e^{i2\pi(0.45)}, 0.47e^{i2\pi(0.51)}]$	$[0.65e^{i2\pi(0.67)}, 0.55e^{i2\pi(0.52)}, 0.52e^{i2\pi(0.53)}]$	$[0.80e^{i2\pi(0.78)}, 0.35e^{i2\pi(0.37)}, 0.49e^{i2\pi(0.50)}]$	$[0.75e^{i2\pi(0.73)}, 0.47e^{i2\pi(0.45)}, 0.47e^{i2\pi(0.51)}]$

Table 9: Aggregated complex Pythagorean fuzzy decision matrix

$\mathbb{S}$	$\mathfrak{c}_1$	$\mathfrak{c}_2$	$\mathfrak{c}_3$	$\mathfrak{c}_4$
$\mathbb{A}_1$	$[0.8162e^{i2\pi(0.7960)}, 0.3347e^{i2\pi(0.3303)}, 0.4709e^{i2\pi(0.5072)}]$	$[0.7658e^{i2\pi(0.7457)}, 0.4316e^{i2\pi(0.4252)}, 0.4767e^{i2\pi(0.5130)}]$	$[0.8371e^{i2\pi(0.8170)}, 0.3136e^{i2\pi(0.2800)}, 0.4482e^{i2\pi(0.5041)}]$	$[0.8000e^{i2\pi(0.7800)}, 0.3500e^{i2\pi(0.3700)}, 0.4873e^{i2\pi(0.5047)}]$
$\mathbb{A}_2$	$[0.7377e^{i2\pi(0.7277)}, 0.4561e^{i2\pi(0.4474)}, 0.4978e^{i2\pi(0.5199)}]$	$[0.7407e^{i2\pi(0.7308)}, 0.4484e^{i2\pi(0.4425)}, 0.5003e^{i2\pi(0.5197)}]$	$[0.7695e^{i2\pi(0.7494)}, 0.4227e^{i2\pi(0.4194)}, 0.4787e^{i2\pi(0.5124)}]$	$[0.7252e^{i2\pi(0.7141)}, 0.4919e^{i2\pi(0.4692)}, 0.4818e^{i2\pi(0.5195)}]$
$\mathbb{A}_3$	$[0.8705e^{i2\pi(0.8710)}, 0.2021e^{i2\pi(0.1582)}, 0.4488e^{i2\pi(0.4651)}]$	$[0.8371e^{i2\pi(0.8170)}, 0.3137e^{i2\pi(0.2800)}, 0.4482e^{i2\pi(0.5041)}]$	$[0.8342e^{i2\pi(0.8141)}, 0.3167e^{i2\pi(0.2869)}, 0.4515e^{i2\pi(0.5049)}]$	$[0.8342e^{i2\pi(0.8141)}, 0.3167e^{i2\pi(0.2869)}, 0.4515e^{i2\pi(0.5049)}]$
$\mathbb{A}_4$	$[0.7835e^{i2\pi(0.7634)}, 0.3891e^{i2\pi(0.3970)}, 0.4845e^{i2\pi(0.5095)}]$	$[0.6900e^{i2\pi(0.6928)}, 0.5205e^{i2\pi(0.4943)}, 0.5030e^{i2\pi(0.5251)}]$	$[0.8199e^{i2\pi(0.7997)}, 0.3311e^{i2\pi(0.3214)}, 0.4671e^{i2\pi(0.5071)}]$	$[0.7695e^{i2\pi(0.7494)}, 0.4227e^{i2\pi(0.4194)}, 0.4787e^{i2\pi(0.5124)}]$
$\mathbb{A}_5$	$[0.7187e^{i2\pi(0.7101)}, 0.4973e^{i2\pi(0.4740)}, 0.4860e^{i2\pi(0.5207)}]$	$[0.6047e^{i2\pi(0.6274)}, 0.5998e^{i2\pi(0.5726)}, 0.5240e^{i2\pi(0.5277)}]$	$[0.7839e^{i2\pi(0.7638)}, 0.3882e^{i2\pi(0.3963)}, 0.4846e^{i2\pi(0.5095)}]$	$[0.6834e^{i2\pi(0.6889)}, 0.5255e^{i2\pi(0.4987)}, 0.5068e^{i2\pi(0.5260)}]$

Table 10: Aggregated weighted complex Pythagorean fuzzy decision matrix

$\mathbb{S}'$	$\mathfrak{c}_1$	$\mathfrak{c}_2$	$\mathfrak{c}_3$	$\mathfrak{c}_4$
$\mathbb{A}_1$	$[0.7181e^{i2\pi(0.6759)}, 0.3694e^{i2\pi(0.3522)}, 0.5898e^{i2\pi(0.6474)}]$	$[0.6356e^{i2\pi(0.5966)}, 0.4678e^{i2\pi(0.4657)}, 0.6141e^{i2\pi(0.6536)}]$	$[0.6745e^{i2\pi(0.6335)}, 0.4026e^{i2\pi(0.3913)}, 0.6188e^{i2\pi(0.6675)}]$	$[0.6215e^{i2\pi(0.5823)}, 0.4834e^{i2\pi(0.5172)}, 0.6165e^{i2\pi(0.6272)}]$
$\mathbb{A}_2$	$[0.6490e^{i2\pi(0.6179)}, 0.4794e^{i2\pi(0.4622)}, 0.5907e^{i2\pi(0.6361)}]$	$[0.6148e^{i2\pi(0.5846)}, 0.4827e^{i2\pi(0.4809)}, 0.6237e^{i2\pi(0.6534)}]$	$[0.6201e^{i2\pi(0.5811)}, 0.4866e^{i2\pi(0.4927)}, 0.6154e^{i2\pi(0.6477)}]$	$[0.5634e^{i2\pi(0.5331)}, 0.5814e^{i2\pi(0.5815)}, 0.5870e^{i2\pi(0.6145)}]$
$\mathbb{A}_3$	$[0.7659e^{i2\pi(0.7396)}, 0.2593e^{i2\pi(0.2036)}, 0.5884e^{i2\pi(0.6415)}]$	$[0.6948e^{i2\pi(0.6536)}, 0.3667e^{i2\pi(0.3450)}, 0.6187e^{i2\pi(0.6736)}]$	$[0.6722e^{i2\pi(0.6313)}, 0.4049e^{i2\pi(0.3959)}, 0.6198e^{i2\pi(0.6669)}]$	$[0.6481e^{i2\pi(0.6077)}, 0.4629e^{i2\pi(0.4703)}, 0.6047e^{i2\pi(0.6399)}]$
$\mathbb{A}_4$	$[0.6893e^{i2\pi(0.6482)}, 0.4180e^{i2\pi(0.4145)}, 0.5917e^{i2\pi(0.6388)}]$	$[0.5727e^{i2\pi(0.5542)}, 0.5478e^{i2\pi(0.5269)}, 0.6099e^{i2\pi(0.6444)}]$	$[0.6607e^{i2\pi(0.6201)}, 0.4154e^{i2\pi(0.4196)}, 0.6252e^{i2\pi(0.6629)}]$	$[0.5978e^{i2\pi(0.5594)}, 0.5317e^{i2\pi(0.5483)}, 0.5999e^{i2\pi(0.6216)}]$
$\mathbb{A}_5$	$[0.6323e^{i2\pi(0.6029)}, 0.5177e^{i2\pi(0.4876)}, 0.5764e^{i2\pi(0.6315)}]$	$[0.5019e^{i2\pi(0.5019)}, 0.6208e^{i2\pi(0.5979)}, 0.6022e^{i2\pi(0.6250)}]$	$[0.6317e^{i2\pi(0.5923)}, 0.4591e^{i2\pi(0.4748)}, 0.6246e^{i2\pi(0.6510)}]$	$[0.5309e^{i2\pi(0.5143)}, 0.6065e^{i2\pi(0.6020)}, 0.5919e^{i2\pi(0.6108)}]$

Table 11: Score matrix $\mathbb{S}^*$

$\mathbb{S}^*$	$\mathfrak{c}_1$	$\mathfrak{c}_2$	$\mathfrak{c}_3$	$\mathfrak{c}_4$
$\mathbb{A}_1$	0.7120	0.3242	0.5411	0.2242
$\mathbb{A}_2$	0.3595	0.2555	0.2427	-0.0746
$\mathbb{A}_3$	1.0249	0.6564	0.5297	0.3539
$\mathbb{A}_4$	0.5488	0.0574	0.4724	0.0870
$\mathbb{A}_5$	0.2575	-0.2391	0.3135	-0.1839

Table 12: Distance measure of alternative from PIS and NIS and their ranking

Alternatives	$d(\mathbb{A}_p, \mathbb{A}^-)$	$d(\mathbb{A}_p, \mathbb{A}^+)$	Revised closeness index	Ranking
$\mathbb{A}_1$	0.8829	0.4744	-40.9949	2
$\mathbb{A}_2$	0.5167	0.9360	-81.7161	4
$\mathbb{A}_3$	1.3276	0.0114	0	1
$\mathbb{A}_4$	0.5467	0.8133	-70.9303	3
$\mathbb{A}_5$	0.0708	1.3160	-115.3853	5

Where $\mathfrak{J} = \{q : q = 1, 2, ..., m\}$ denotes the set of subscripts of all criteria. These CPF concordance sets illustrate the degree to which $\mathbb{A}_\psi$ is preferable over $\mathbb{A}_\chi$. In CPF strong concordance set, $\mathbb{A}_\psi$ contains all three properties upon which the alternatives are judged, i.e., higher score function, higher accuracy function and least hesitancy as compared to $\mathbb{A}_\chi$. Therefore, CPF strong concordance set is most concordant among concordant sets, whereas in CPF midrange concordance set, $\mathbb{A}_\psi$ has larger hesitancy as compared to $\mathbb{A}_\chi$. Therefore, $\mathfrak{J}_{\mathfrak{C}^*_{\psi\chi}}$ is less concordant than $\mathfrak{J}_{\mathfrak{C}_{\psi\chi}}$. The CPF weak concordance set is least concordant due to larger amplitude and phase term of the non-membership grade than membership grade.

The CPF discordance sets are basically complementary subsets of CPF concordance sets. The CPF discordance sets comprise the subscripts of all those criteria for which $\mathbb{A}_\psi$ is not preferable over $\mathbb{A}_\chi$. The CPF discordance sets can be classified into strong, midrange and weak discordance sets.

(i) The complex Pythagorean fuzzy strong discordance set $\mathfrak{J}_{\mathfrak{D}_{\psi\chi}}$ can be defined as:

$$\mathfrak{J}_{\mathfrak{D}_{\psi\chi}} = \{q : u'_\psi(\mathfrak{c}_q) < u'_\chi(\mathfrak{c}_q), \alpha'_\psi(\mathfrak{c}_q) < \alpha'_\chi(\mathfrak{c}_q), v'_\psi(\mathfrak{c}_q) \geq v'_\chi(\mathfrak{c}_q), \beta'_\psi(\mathfrak{c}_q) \geq \beta'_\chi(\mathfrak{c}_q), w'_\psi(\mathfrak{c}_q) \geq w'_\chi(\mathfrak{c}_q), \gamma'_\psi(\mathfrak{c}_q) \geq \gamma'_\chi(\mathfrak{c}_q)\}. \quad (19)$$

(ii) The complex Pythagorean fuzzy midrange discordance set $\mathfrak{J}_{\mathfrak{D}^*_{\psi\chi}}$ can be defined as:

$$\mathfrak{J}_{\mathfrak{D}^*_{\psi\chi}} = \{q : u'_\psi(\mathfrak{c}_q) < u'_\chi(\mathfrak{c}_q), \alpha'_\psi(\mathfrak{c}_q) < \alpha'_\chi(\mathfrak{c}_q), v'_\psi(\mathfrak{c}_q) \geq v'_\chi(\mathfrak{c}_q), \beta'_\psi(\mathfrak{c}_q) \geq \beta'_\chi(\mathfrak{c}_q), w'_\psi(\mathfrak{c}_q) < w'_\chi(\mathfrak{c}_q), \gamma'_\psi(\mathfrak{c}_q) < \gamma'_\chi(\mathfrak{c}_q)\}. \quad (20)$$

(iii) The complex Pythagorean fuzzy weak discordance set $\mathfrak{J}_{\mathfrak{D}^{**}_{\psi\chi}}$ can be defined as:

$$\mathfrak{J}_{\mathfrak{D}^{**}_{\psi\chi}} = \{q : u'_\psi(\mathfrak{c}_q) < u'_\chi(\mathfrak{c}_q), \alpha'_\psi(\mathfrak{c}_q) < \alpha'_\chi(\mathfrak{c}_q), v'_\psi(\mathfrak{c}_q < v'_\chi(\mathfrak{c}_q), \beta'_\psi(\mathfrak{c}_q) < \beta'_\chi(\mathfrak{c}_q)\}. \quad (21)$$

$\mathfrak{J}_{\mathfrak{D}_{\psi\chi}}$ and $\mathfrak{J}_{\mathfrak{D}^{**}_{\psi\chi}}$ represent the most discordant and least discordant sets, respectively.

5.2 Evaluation of complex Pythagorean fuzzy concordance index and matrix

A CPF concordance index indicates the extent to which one alternative is superior to other. For example, $k_{\psi\chi}$ $(\psi, \chi = 1, 2, 3, ..., l, \psi \neq \chi)$ denotes the superiority of alternative $\mathbb{A}_\psi$ to alternative $\mathbb{A}_\chi$. The concordance indices can be calculated by assigning weights to concordance indicators in concordance set. The CPF concordance index $k_{\psi\chi} = (\mu^{(q)}_{k_{\psi\chi}}, \nu^{(q)}_{k_{\psi\chi}}, \pi^{(q)}_{k_{\psi\chi}})= (u^{(q)}_{k_{\psi\chi}} e^{i\alpha^{(q)}_{k_{\psi\chi}}}, v^{(q)}_{k_{\psi\chi}} e^{i\beta^{(q)}_{k_{\psi\chi}}}, w^{(q)}_{k_{\psi\chi}} e^{i\gamma^{(q)}_{k_{\psi\chi}}})$, $(\psi, \chi = 1, 2, 3, ..., l, \psi \neq \chi)$ in CPF-ELECTRE I method is defined as:

$$k_{\psi\chi} = w_{\mathfrak{J}_\mathfrak{C}} \otimes \sum_{q\in\mathfrak{J}_\mathfrak{C}} w_q \oplus w_{\mathfrak{J}_{\mathfrak{C}^*}} \otimes \sum_{q\in\mathfrak{J}_{\mathfrak{C}^*}} w_q \otimes w_{\mathfrak{J}_{\mathfrak{C}^{**}}} \otimes \sum_{q\in\mathfrak{J}_{\mathfrak{C}^{**}}} w_q, \quad (22)$$

where $w_{\mathfrak{J}_\mathfrak{C}}$, $w_{\mathfrak{J}_{\mathfrak{C}^*}}$ and $w_{\mathfrak{J}_{\mathfrak{C}^{**}}}$ denote the weights assigned by decision makers to the CPF strong, midrange and weak concordance sets. $w_q = (\mu_w(\mathfrak{c}_q), \nu_w(\mathfrak{c}_q), \pi_w(\mathfrak{c}_q))= (u_w(\mathfrak{c}_q)e^{i\alpha_w(\mathfrak{c}_q)}, v_w(\mathfrak{c}_q)e^{i\beta_w(\mathfrak{c}_q)}, w_w(\mathfrak{c}_q)e^{i\gamma_w(\mathfrak{c}_q)})$ denotes the CPF weight associated with qth criteria. The CPF concordance matrix $\mathfrak{K} = (k_{\psi\chi})_{l\times l}$ can be constructed by arranging all the CPF concordance indices as follows:

$$\mathfrak{K}=\begin{pmatrix} - & (\mu^{(q)}_{k_{12}},\nu^{(q)}_{k_{12}},\pi^{(q)}_{k_{12}}) & \cdots & (\mu^{(q)}_{k_{1(l-1)}},\nu^{(q)}_{k_{1(l-1)}},\pi^{(q)}_{k_{1(l-1)}}) & (\mu^{(q)}_{k_{1l}},\nu^{(q)}_{k_{1l}},\pi^{(q)}_{k_{1l}}) \\ (\mu^{(q)}_{k_{21}},\nu^{(q)}_{k_{21}},\pi^{(q)}_{k_{21}}) & - & \cdots & (\mu^{(q)}_{k_{2(l-1)}},\nu^{(q)}_{k_{2(l-1)}},\pi^{(q)}_{k_{2(l-1)}}) & (\mu^{(q)}_{k_{2l}},\nu^{(q)}_{k_{2l}},\pi^{(q)}_{k_{2l}}) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ (\mu^{(q)}_{k_{l1}},\nu^{(q)}_{k_{l1}},\pi^{(q)}_{k_{l1}}) & (\mu^{(q)}_{k_{l2}},\nu^{(q)}_{k_{l2}},\pi^{(q)}_{k_{l2}}) & \cdots & (\mu^{(q)}_{k_{l(l-1)}},\nu^{(q)}_{k_{l(l-1)}},\pi^{(q)}_{k_{l(l-1)}}) & - \end{pmatrix}.$$

5.3 Evaluation of complex Pythagorean fuzzy discordance index and matrix

On contrary to the CPF concordance index, the CPF discordance index denotes the extent to which one alternative is inferior to other. For example, $h_{\psi\chi}$ $(\psi,\chi=1,2,3,...,l,\psi\neq\chi)$ denotes the degree to which alternative $\mathbb{A}_\psi$ is not preferable to alternative $\mathbb{A}_\chi$. The CPF discordance indices $h_{\psi\chi}$, $(\psi,\chi=1,2,3,...,l,\psi\neq\chi)$ can be defined using distance measure and weights assigned to the discordance indicators in discordance sets as follows:

$$h_{\psi\chi}=\frac{\max\limits_{q\in\mathfrak{J}_{\mathfrak{D}}\cup\mathfrak{J}_{\mathfrak{D}^*}\cup\mathfrak{J}_{\mathfrak{D}^{**}}}\{w_{\mathfrak{J}_{\mathfrak{D}}}\times d(s'_{\psi q},s'_{\chi q}),w_{\mathfrak{J}_{\mathfrak{D}^*}}\times d(s'_{\psi q},s'_{\chi q}),w_{\mathfrak{J}_{\mathfrak{D}^{**}}}\times d(s'_{\psi q},s'_{\chi q})\}}{\max\limits_{q\in\mathfrak{J}} d(s'_{\psi q},s'_{\chi q})}, \tag{23}$$

where $s'_{\psi q}$ and $s'_{\chi q}$ denote the entries of AWCPFDM ($\mathbb{S}'$). The distance measure $d(s'_{\psi q},s'_{\chi q})$ between the ψth and χth alternative on the criteria $\mathfrak{c}_q$ can be calculated using Eq. (2). Further, $w_{\mathfrak{J}_{\mathfrak{D}}}$, $w_{\mathfrak{J}_{\mathfrak{D}^*}}$ and $w_{\mathfrak{J}_{\mathfrak{D}^{**}}}$ denote the weight assigned by decision makers to the CPF strong, midrange and weak discordance sets, respectively. The CPF discordance index $h_{\psi\chi}$ indicates the comparison of performances of alternative $\mathbb{A}_\psi$ and alternative $\mathbb{A}_\chi$ with respect to the discordant criteria $\mathfrak{c}_q$.The CPF discordance matrix can be constructed by arranging all the CPF discordance indices as follows:

$$\mathfrak{H}=\begin{pmatrix} - & h_{12} & \cdots & h_{1(l-1)} & h_{1l} \\ h_{21} & - & \cdots & h_{2(l-1)} & h_{2l} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ h_{(l-1)1} & h_{(l-1)2} & \cdots & - & h_{(l-1)l} \\ h_{l1} & h_{l2} & \cdots & h_{l(l-1)} & - \end{pmatrix}.$$

5.4 Construction of complex Pythagorean fuzzy effective concordance matrix

To effectively observe the dominance of an alternative to the other alternatives, the CPF concordance indices are compared by a threshold value. The threshold value is called concordance level, denoted by $\tilde{k}$. The concordance level can be calculated as:

$$\tilde{k}=(\mu_{\tilde{k}_{\psi\chi}},\nu_{\tilde{k}_{\psi\chi}},\pi_{\tilde{k}_{\psi\chi}})=\frac{1}{l(l-1)}\max_{\psi\chi}(\mu^{(q)}_{k_{\psi\chi}},\nu^{(q)}_{k_{\psi\chi}},\pi^{(q)}_{k_{\psi\chi}}), \tag{24}$$

$(\psi,\chi=1,2,3,...,l,\psi\neq\chi)$.

The comparison between two CPFNs is made on basis of their score function defined in Eq. (3). For concordance level, we choose the CPF concordance index with maximum score function and proceed further. During the comparison of CPF concordance indices and the concordance level, two situation arises. If $k_{\psi\chi}$ exceeds $\tilde{k}$ then there are more chances that the alternative $\mathbb{A}_\psi$ is preferable over the $\mathbb{A}_\chi$ and if $\tilde{k}$ exceeds $k_{\psi\chi}$ then chance of superiority of the alternative $\mathbb{A}_\psi$ over the alternative $\mathbb{A}_\chi$ decreases. The CPF effective concordance matrix $\Phi=(\phi_{\psi\chi})_{l\times l}$ can be constructed as:

$$\Phi=\begin{pmatrix} - & \phi_{12} & \cdots & \phi_{1(l-1)} & \phi_{1l} \\ \phi_{21} & - & \cdots & \phi_{2(l-1)} & \phi_{2l} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \phi_{(l-1)1} & \phi_{(l-1)2} & \cdots & - & \phi_{(l-1)l} \\ \phi_{l1} & \phi_{l2} & \cdots & \phi_{l(l-1)} & - \end{pmatrix},$$

where

$$\Phi_{\psi\chi}=\begin{cases} 1, & \text{if } k_{\psi\chi}\geq\tilde{k}; \\ 0, & \text{if } k_{\psi\chi}<\tilde{k}. \end{cases} \tag{25}$$

5.5 Construction of complex Pythagorean fuzzy effective discordance matrix

In order to observe the dominance of alternatives in the discordance sense, we seek help from the CPF discordance matrix where each entry (discordance index) $h_{\psi\chi}$ represents the inferiority of alternative $\mathbb{A}_\psi$ to the alternative $\mathbb{A}_\chi$. These CPF discordance indices are compared by discordance level to construct the CPF effective discordance matrix. The discordance level $\tilde{h}$ can be evaluated by averaging all the entries of CPF discordance matrix $\mathfrak{H}$ as follows:

$$\tilde{h} = \frac{1}{l(l-1)} \sum_{\psi,\psi\neq\chi} \sum_{\chi,\psi\neq\chi} h_{\psi\chi}. \tag{26}$$

If $h_{\psi\chi}$ exceeds discordance level $\tilde{h}$, it verifies that alternative $\mathbb{A}_\psi$ is a worse choice than alternative $\mathbb{A}_\chi$. The discordant Boolean matrix (CPF effective discordance matrix) can be constructed as:

$$\eta = \begin{pmatrix} - & \eta_{12} & \cdots & \eta_{1(l-1)} & \eta_{1l} \\ \eta_{21} & - & \cdots & \eta_{2(l-1)} & \eta_{2l} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \eta_{(l-1)1} & \eta_{(l-1)2} & \cdots & - & \eta_{(l-1)l} \\ \eta_{l1} & \eta_{l2} & \cdots & \eta_{l(l-1)} & - \end{pmatrix},$$

where

$$\eta_{\psi\chi} = \begin{cases} 1, & \text{if } h_{\psi\chi} \leq \tilde{h}; \\ 0, & \text{if } h_{\psi\chi} > \tilde{h}. \end{cases} \tag{27}$$

5.6 Construction of aggregated outranking Boolean matrix

The CPF effective concordance and discordance matrices are merged to get aggregated outranking Boolean matrix which provides more authentic information about the outranking relation and dominance of an alternative over the other. The aggregated outranking Boolean matrix can be constructed as follows:

$$\Lambda = \begin{pmatrix} - & \Lambda_{12} & \cdots & \Lambda_{1(l-1)} & \Lambda_{1l} \\ \Lambda_{21} & - & \cdots & \Lambda_{2(l-1)} & \Lambda_{2l} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \Lambda_{(l-1)1} & \Lambda_{(l-1)2} & \cdots & - & \Lambda_{(l-1)l} \\ \Lambda_{l1} & \Lambda_{l2} & \cdots & \Lambda_{l(l-1)} & - \end{pmatrix},$$

where

$$\Lambda_{\psi\chi} = \phi_{\psi\chi} \times \eta_{\psi\chi}, (\psi, \chi = 1, 2, 3, ..., l, \psi \neq \chi). \tag{28}$$

A non-zero entry $\Lambda_{\psi\chi}$ of aggregated outranking Boolean matrix proposes that the alternative $\mathbb{A}_\psi$ is strictly preferable over the alternative $\mathbb{A}_\chi$, i.e.,$\mathbb{A}_\chi$ is outperforming $\mathbb{A}_\psi$.

5.7 Sketching of outranking graph

Now, our next target is to use the information of aggregated outranking Boolean matrix to find the most satisfactory solution of MCGDM problem. Outranking relations are visualized with the help of a directed graph as graphs are most convenient tool to represent relations. Thus, a directed graph $\mathbb{G} = (\mathbb{V}, \mathbb{E})$ is constructed, where $\mathbb{V}$ (vertex set) denotes the set of alternatives and each edge in $\mathbb{E}$ (edge set) represents the relationship between the corresponding alternatives. For any pair of alternatives $\mathbb{A}_\psi$ and $\mathbb{A}_\chi$, the following cases arise, as shown in Figure 1.

1. If $\Lambda_{\psi\chi} = 1$; an arc is drawn from $\mathbb{A}_\psi$ pointing toward $\mathbb{A}_\chi$ which depicts that $\mathbb{A}_\psi$ is outperforming $\mathbb{A}_\chi$, i.e., $(\mathbb{A}_\psi \succ \mathbb{A}_\chi)$.
2. If $\Lambda_{\psi\chi} = 1$ and $\Lambda_{\chi\psi} = 1$; a two sided arrowed arc is drawn which illustrate the equivalent relation between the alternatives $\mathbb{A}_\psi$ and $\mathbb{A}_\chi$, i.e., $(\mathbb{A}_\psi \approx \mathbb{A}_\chi)$.
3. If $\Lambda_{\psi\chi} = 0$; no edge will be drawn between the corresponding alternative which shows that $\mathbb{A}_\psi$ and $\mathbb{A}_\chi$ are incomparable, i.e., $(\mathbb{A}_\psi ? \mathbb{A}_\chi)$.

The structure of CPF-ELECTRE I method is explained with the help of flow chart (Figure 2(b)) as follows:

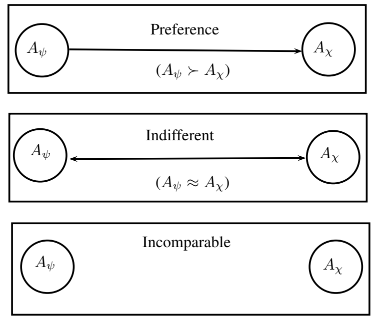


Figure 1: Graphical representation of relations in outranking decision graph

6 Numerical example

In this section, we apply our proposed CPF-ELECTRE I method to the same MCGDM problem "selection of best interior designer for hotel renovation", elaborated in subsection 4.1.
In CPF-ELECTRE I method, **step 1-step 4** are the same as in CPF-TOPSIS method.

Step 5: The CPF strong concordance set $\mathfrak{J}_{\mathfrak{C}}$, CPF midrange concordance set $\mathfrak{J}_{\mathfrak{C}^*}$ and CPF weak concordance set $\mathfrak{J}_{\mathfrak{C}^{**}}$ are given by:

$$\mathfrak{J}_{\mathfrak{C}} = \begin{pmatrix} - & \{\} & \{\} & \{\} & \{\} \\ \{\} & - & \{\} & \{\} & \{\} \\ \{1\} & \{\} & - & \{\} & \{\} \\ \{\} & \{\} & \{\} & - & \{\} \\ \{\} & \{\} & \{\} & \{\} & - \end{pmatrix}, \quad \mathfrak{J}_{\mathfrak{C}^*} = \begin{pmatrix} - & \{3,4\} & \{\} & \{2,4\} & \{1,2,4\} \\ \{\} & - & \{\} & \{2\} & \{1,2\} \\ \{2\} & \{3,4\} & - & \{2,4\} & \{1,2,4\} \\ \{\} & \{1,3,4\} & \{\} & - & \{1,2,3,4\} \\ \{\} & \{3\} & \{\} & \{\} & - \end{pmatrix}, \mathfrak{J}_{\mathfrak{C}^{**}} = \begin{pmatrix} - & \{\} & \{\} & \{\} & \{\} \\ \{\} & - & \{\} & \{\} & \{\} \\ \{\} & \{\} & - & \{\} & \{\} \\ \{\} & \{\} & \{\} & - & \{\} \\ \{\} & \{\} & \{\} & \{\} & - \end{pmatrix}.$$

The CPF strong discordance set $\mathfrak{J}_{\mathfrak{D}}$, CPF midrange discordance set $\mathfrak{J}_{\mathfrak{D}^*}$ and CPF weak discordance set $\mathfrak{J}_{\mathfrak{D}^{**}}$ are given by:

$$\mathfrak{J}_{\mathfrak{D}} = \begin{pmatrix} - & \{\} & \{1\} & \{\} & \{\} \\ \{\} & - & \{\} & \{\} & \{\} \\ \{\} & \{\} & - & \{\} & \{\} \\ \{\} & \{\} & \{\} & - & \{\} \\ \{\} & \{\} & \{\} & \{\} & - \end{pmatrix}, \quad \mathfrak{J}_{\mathfrak{D}^*} = \begin{pmatrix} - & \{\} & \{2\} & \{\} & \{\} \\ \{3,4\} & - & \{3,4\} & \{1,3,4\} & \{3\} \\ \{\} & \{\} & - & \{\} & \{\} \\ \{2,4\} & \{2\} & \{2,4\} & - & \{\} \\ \{1,2,4\} & \{1,2\} & \{1,2,4\} & \{1,2,3,4\} & - \end{pmatrix}, \quad \mathfrak{J}_{\mathfrak{D}^{**}} = \begin{pmatrix} - & \{\} & \{\} & \{\} & \{\} \\ \{\} & - & \{\} & \{\} & \{\} \\ \{\} & \{\} & - & \{\} & \{\} \\ \{\} & \{\} & \{\} & - & \{\} \\ \{\} & \{\} & \{\} & \{\} & - \end{pmatrix}.$$

Step 6: The weightage of CPF strong, midrange and weak concordance sets is decided by experts, as given in Eq. (29). The CPF concordance matrix can be evaluated by assembling all concordance indices $k_{\psi\chi}$ and the concordance indices can be evaluated by using Eq. (22).

$$(w_{\mathfrak{J}_{\mathfrak{C}}}, w_{\mathfrak{J}_{\mathfrak{C}^*}}, w_{\mathfrak{J}_{\mathfrak{C}^{**}}}) = \left(1, \frac{2}{3}, \frac{1}{3}\right). \tag{29}$$

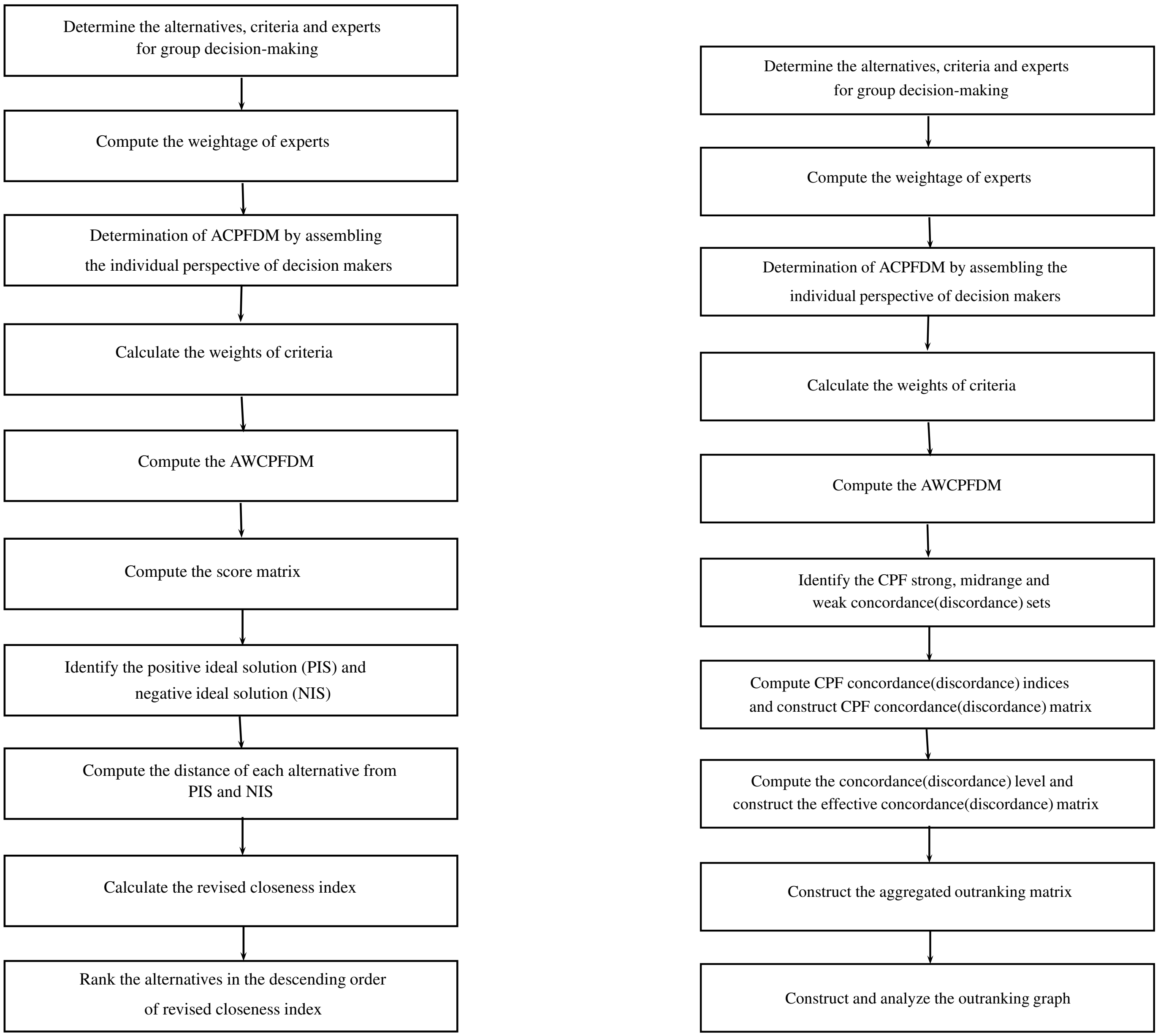
The CPF concordance matrix $\mathfrak{K}$ is computed in Table 13.

Table 13: Concordance matrix

	$\mathbb{A}_1$	$\mathbb{A}_2$	$\mathbb{A}_3$
$\mathbb{A}_1$	-	$(0.8554e^{i2\pi(0.8278)}, 0.2076e^{i2\pi(0.2307)}, 0.4745e^{i2\pi(0.5114)})$	0
$\mathbb{A}_2$	0	-	0
$\mathbb{A}_3$	$(0.9467e^{i2\pi(0.9267)}, 0.0567e^{i2\pi(0.0459)}, 0.3171e^{i2\pi(0.3730)})$	$(0.8554e^{i2\pi(0.8278)}, 0.2076e^{i2\pi(0.2307)}, 0.4745e^{i2\pi(0.5114)})$	-
$\mathbb{A}_4$	0	$(0.9488e^{i2\pi(0.9305)}, 0.0627e^{i2\pi(0.0592)}, 0.3096e^{i2\pi(0.3615)})$	0
$\mathbb{A}_5$	0	$(0.7090e^{i2\pi(0.6769)}, 0.4135e^{i2\pi(0.4329)}, 0.5713e^{i2\pi(0.5953)})$	0

	$\mathbb{A}_4$	$\mathbb{A}_5$
$\mathbb{A}_1$	$(0.8673e^{i2\pi(0.8403)}, 0.1718e^{i2\pi(0.1883)}, 0.4672e^{i2\pi(0.5084)})$	$(0.9530e^{i2\pi(0.9351)}, 0.0518e^{i2\pi(0.0483)}, 0.2985e^{i2\pi(0.3511)})$
$\mathbb{A}_2$	$(0.7354e^{i2\pi(0.7028)}, 0.3420e^{i2\pi(0.3533)}, 0.5850e^{i2\pi(0.6175)})$	$(0.9108e^{i2\pi(0.8853)}, 0.1033e^{i2\pi(0.0907)}, 0.3997e^{i2\pi(0.4561)})$
$\mathbb{A}_3$	$(0.8673e^{i2\pi(0.8403)}, 0.1718e^{i2\pi(0.1883)}, 0.4672e^{i2\pi(0.5084)})$	$(0.9530e^{i2\pi(0.9351)}, 0.5180e^{i2\pi(0.0483)}, 0.2985e^{i2\pi(0.3511)})$
$\mathbb{A}_4$	-	$(0.9769e^{i2\pi(0.9654)}, 0.0213e^{i2\pi(0.0208)}, 0.2126e^{i2\pi(0.2173)})$
$\mathbb{A}_5$	0	-

Step 7: The distance measures, used in computing the CPF discordance indices, can be calculated by employing normalized Euclidean distance which is defined in Eq. (2) and are arranged in Table 14. The weightage of CPF strong,



(a) Flow chart of CPF-TOPSIS method

(b) Flow chart of CPF-ELECTRE I method

Figure 2: Flow charts of proposed methods

midrange and weak discordance sets is decided by experts, as shown in Eq. (30). The CPF discordance matrix can be evaluated by assembling all discordance indices $h_{\psi\chi}$ and the discordance indices can be evaluated by using Eq. (23).

$$(w_{\mathfrak{I}_{\mathfrak{D}}}, w_{\mathfrak{I}_{\mathfrak{D}^*}}, w_{\mathfrak{I}_{\mathfrak{D}^{**}}}) = \left(1, \frac{2}{3}, \frac{1}{3}\right). \tag{30}$$

The CPF discordance matrix is represented as:

$$\mathfrak{H} = \begin{pmatrix} - & 0 & 0.9299 & 0 & 0 \\ 0.5952 & - & 0.4397 & 0.6667 & 0.1014 \\ 0 & 0 & - & 0 & 0 \\ 0.6663 & 0.5842 & 0.6667 & - & 0 \\ 0.8342 & 0.6664 & 0.6666 & 0.6667 & - \end{pmatrix}.$$

Step 8: The concordance level $\tilde{k}$, calculated by Eq. (24), is $(0.3782e^{i2\pi(0.3546)}, 0.8249e^{i2\pi(0.8240)}, 0.4201e^{i2\pi(0.4419)})$. The effective concordance matrix, constructed by the comparison of concordance level and CPF concordance indices on

Table 14: Complex Pythagorean fuzzy distances $d(s'_{\psi q}, s'_{\xi q})$

	s'_{11}	s'_{21}	s'_{31}	s'_{41}	s'_{51}		s'_{12}	s'_{22}	s'_{32}	s'_{42}	s'_{52}
s'_{11}	-	0.1255	0.1114	0.0586	0.1628	s'_{12}	-	0.0267	0.1198	0.0965	0.2037
s'_{21}	-	-	0.2354	0.0677	0.0397	s'_{22}	-	-	0.1438	0.0734	0.1805
s'_{31}	-	-	-	0.1690	0.2718	s'_{32}	-	-	-	0.2151	0.3227
s'_{41}	-	-	-	-	0.1057	s'_{42}	-	-	-	-	0.1077
s'_{51}	-	-	-	-	-	s'_{52}	-	-	-	-	-
	s'_{13}	s'_{23}	s'_{33}	s'_{43}	s'_{53}		s'_{14}	s'_{24}	s'_{34}	s'_{44}	s'_{54}
s'_{13}	-	0.1080	0.0042	0.0261	0.0831	s'_{14}	-	0.1121	0.0502	0.0523	0.1496
s'_{23}	-	-	0.1039	0.0837	0.0274	s'_{24}	-	-	0.1553	0.0600	0.0399
s'_{33}	-	-	-	0.0218	0.0789	s'_{34}	-	-	-	0.0966	0.1939
s'_{43}	-	-	-	-	0.0578	s'_{44}	-	-	-	-	0.0984
s'_{53}	-	-	-	-	-	s'_{54}	-	-	-	-	-

the basis of score function, is given by:

$$\Phi = \begin{pmatrix} - & 1 & 1 & 1 & 1 \\ 1 & - & 1 & 1 & 1 \\ 1 & 1 & - & 1 & 1 \\ 1 & 1 & 1 & - & 1 \\ 1 & 1 & 1 & 1 & - \end{pmatrix}.$$

Step 9: The discordance level $\tilde{h}$, evaluated by Eq. (26), is 0.3742. The effective discordance matrix (boolean discordant matrix) η, constructed by the comparison of the discordance level and CPF discordance indices, is :

$$\eta = \begin{pmatrix} - & 1 & 0 & 1 & 1 \\ 0 & - & 0 & 0 & 1 \\ 1 & 1 & - & 1 & 1 \\ 0 & 0 & 0 & - & 1 \\ 0 & 0 & 0 & 0 & - \end{pmatrix}.$$

Step 10: The aggregated outranking matrix Λ, obtained by multiplying the corresponding entries of CPF effective concordance and CPF effective discordance matrices, is given by:

$$\Lambda = \begin{pmatrix} - & 1 & 0 & 1 & 1 \\ 0 & - & 0 & 0 & 1 \\ 1 & 1 & - & 1 & 1 \\ 0 & 0 & 0 & - & 1 \\ 0 & 0 & 0 & 0 & - \end{pmatrix}.$$

Step 11: The outranking graph is shown in Figure 3. The complete analysis of outranking graph (Figure 3) is

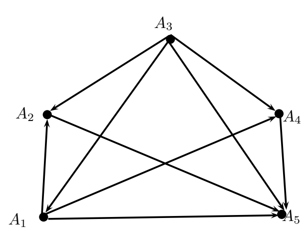


Figure 3: Outranking decision graph

presented in Table 15. By analyzing the graph, we conclude that $\mathbb{A}_3$ is outranking all other alternatives. Thus, $\mathbb{A}_3$ is

Table 15: Analysis of outranking graph

Alternatives	Submissive alternatives	Incomparable alternatives
$\mathbb{A}_1$	$\mathbb{A}_2$, $\mathbb{A}_4$, $\mathbb{A}_5$	-
$\mathbb{A}_2$	$\mathbb{A}_5$	$\mathbb{A}_4$
$\mathbb{A}_3$	$\mathbb{A}_1$, $\mathbb{A}_2$, $\mathbb{A}_4$, $\mathbb{A}_5$	-
$\mathbb{A}_4$	$\mathbb{A}_5$	$\mathbb{A}_2$
$\mathbb{A}_5$	-	-

best interior designer for this project.

7 Comparative analysis

In this section, we provide a comparison between the proposed techniques, namely, CPF-TOPSIS method and CPF-ELECTRE I method. We also compare the proposed methods with existing techniques to highlight their significance.

7.1 Comparison between CPF-TOPSIS and CPF-ELECTRE I

TOPSIS and ELECTRE family of methods are widely used to tackle the MCGDM problems. Both of these methods have completely different working principle. CPF-TOPSIS method basis upon the principle to find an alternative which is closest to PIS and farthest from NIS. Whereas, CPF-ELECTRE I is an outranking method which basis upon the pairwise comparison of alternatives with respect to decision criteria. CPF-ELECTRE I method may not give complete ranking of the alternatives. On the other hand, CPF-TOPSIS method always provides the ranking of alternatives on the basis of revised closeness index. In some situations, CPF-ELECTRE I method may not conclude a single optimal solution, but provides a solution set. On the other hand, CPF-TOPSIS provides a single alternative as solution. Since we deal two types of criteria in MCGDM, i.e., benefit criteria and cost criteria. In CPF-TOPSIS method, during the selection of PIS and NIS we identify both types of criteria and proceed further according to the nature of criteria. In CPF-ELECTRE I method, type of criteria is not identified. The alternatives are rated differently in CPF-TOPSIS method and CPF-ELECTRE I method. In CPF-TOPSIS method, alternatives are assigned linguistic terms according to being good or bad in the criteria quantitatively, but in CPF-ELECTRE I method, linguistic terms are assigned on the basis how preferable the alternative is according to that criteria. For example, an alternative according to benefit criteria "quality" will be rated as good (larger membership and smaller non-membership) in CPF-TOPSIS method as well as CPF-ELECTRE I because of being preferable. Therefore, when we deal with a numerical problem comprising all benefit criteria, both methods provide the same solution as for the selection of interior designer. Comparison of CPF-TOPSIS method and CPF-ELECTRE I method is shown in Table 16.

Table 16: Comparative analysis of CPF-TOPSIS method and CPF ELECTRE I method

Methods	Ranking of alternatives	Best alternative
CPF-TOPSIS method	$\mathbb{A}_3 \succ \mathbb{A}_1 \succ \mathbb{A}_4 \succ \mathbb{A}_2 \succ \mathbb{A}_5$	$\mathbb{A}_3$
CPF-ELECTRE I method	$\mathbb{A}_3 \succ \mathbb{A}_1 \succ \mathbb{A}_5$	$\mathbb{A}_3$

The problem arises with the appearance of cost criteria because the alternatives with respect to cost criteria are rated differently. For example, an alternative according to cost criteria "pollution" will be rated as bad (smaller membership and larger non-membership) in CPF-TOPSIS method, but in CPF-ELECTRE I, it will be rated as good (larger membership and smaller non-membership), being preferable due to least cost. Both of these methods effectively handle the MCGDM problems comprising conflicting criteria. For the comparison of CPF-TOPSIS and CPF-ELECTRE I in the MCGDM problem containing conflicting criteria, following the rules of rating the alternatives of a single method, this issue can be resolved by the interchanging the membership and non-memberships of the alternatives only according to cost criteria in ACPFDM.

7.2 Comparison with existing techniques

- The complex Pythagorean fuzzy sets are capable to handle both aspects of two dimensional information efficiently because of the phase term which enables them to deal with periodic phenomena, accommodate more amount of vagueness and provide more accurate results in decision-making environments. The proposed methods have an edge over the existing MCDM and MCGDM approaches as the combination of the decision-making skills of TOPSIS and ELECTRE methods with flexibility of CPFS make them eminent among the existing decision-making strategies.

- The existing techniques [1, 2, 9, 11, 15, 16], designed to tackle the MCGDM problems, are limited to deal with the one dimensional information and using these methods for two dimensional information cause the loss of significant information which lead to an inappropriate and incongruent decision. Therefore, the proposed methods are perfectly suitable for those MCGDM problems which contain inexact data of periodic nature.

- The proposed methods not only perform for CPF data rather they can be successfully applied to one dimensional phenomena for Pythagorean fuzzy and intuitionistic fuzzy information by taking their phase terms equal to zero. So, the proposed methods are more flexible to overcome the limitations of the existing methods.

8 Conclusions

Uncertainty and vagueness of human decisions can be effectively handled by CPFSs. The CPFSs are highly capable to represent two dimensional phenomena because of the phase term which makes it superior to FS, IFS and PFS. The conditions on the amplitude and phase terms increase flexibility of CPFSs than CFSs and CIFSs. In this research article, two MCGDM methods are presented, along with their flow charts, to deal with two dimensional data. The Euclidean distance is used to assess the distance among alternatives and PIS or NIS in CPF-TOPSIS method. In CPF-ELECTRE I method, the performance of the each pair of alternatives is compared relative to each criteria on the basis of score function, accuracy function and indeterminacy. This comparison is employed to construct CPF concordance and discordance sets which are further utilized in the evaluation of CPF concordance and discordance indices. The CPF effective concordance or discordance matrices are obtained by comparison of concordance or discordance level with the CPF concordance or discordance indices. The results of CPF effective concordance and discordance matrices are merged in aggregated outranking matrix to identify the optimal alternative with the help of outranking relations. The proposed methods can be applied to tackle all MCGDM problems comprised of CPFN in different fields such as medical, engineering, social science, business and automobile industry. In future, we plan to extend our research work to CPF-ELECTRE II method, CPF-ELECTRE III method, CPF-ELECTRE IV method and CPF-VIKOR method.
Conflict of interest: The authors declare no conflicts of interest.

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