Stability and stabilization of network T-S fuzzy systems with random packet-loss for Synchronous machine

M. S. Aslam¹, X. Dai² and T. Zhao³

¹,²,³School of Electrical and Information Engineering, Guangxi University of Science and Technology, Liuzhou 545006, China
shamroz.aslam@yahoo.com, mathdxs@163.com, zhaott1993@163.com

Abstract

This article deals with the problem $H_\infty$ control under network T-S fuzzy system with random packet dropouts for the synchronous machine. In the control design, the intermittent data loss existing in the closed-loop system is taken into account. Two main approaches adopted in this paper to design the fuzzy controller. First, a new version of the T-S fuzzy state-feedback controller is constructed to guarantee the resulting closed-loop system to be stochastically stable. Furthermore, in this regard, by taking into account a new version of fuzzy Lyapunov-Krasovskii functional (LKF) in conjunction with free weighting matrices, containing mode-dependent non-integral term such that the resulting system is stochastically stable with the desired performance. Second, to establish the pole place method (PPM) with the membership function. Furthermore, to make the controller design more convenient, the designed controller does not need to share the number of fuzzy rules and membership functions with the model. Synchronous machine parameters are used for the simulation results which are provided to show the usefulness of the proposed algorithm.

Keywords: Networked systems (NSs), packet-loss phenomena, T-S fuzzy system, synchronous machine (SM).

1 Introduction

In the industrial sector, the improvement in control theory and power systems have been empowered AC generators and motor drives to handle the challenging problems like high-performance and high-efficiency. In the type of AC drives as induction motors and synchronous motors related to others of the electric machine, one has been attained significant popularity due to cost-effective, high robustness and excellent reliability. Therefore, related to DC motors, synchronous motors (SM) are more complex to handle the speed control and not good enough for high-performance practical applications due to their natural nonlinear behavior [1, 16, 25, 27, 29, 42]. So SM is frequently used at a constant speed, for this DC motors were favored for variable-speed drives (VSD) [16] and the advances in latest microcomputers and power electronics [5]. In this regard, the role of synchronous machines had played a significant role in the analysis of power system’s stability [24, 32, 38]. In the view of the above discussion, it’s not easy to find out the optimal controller in the field of power systems, which remain the system at their best performance. To get this goal, to resolve the SM stability and relay on the system variables/parameters is critical to avoid unnecessary circumstances.

With the latest development in the field of the power system, researchers have been achieved a lot of results and the methods in the stability and stabilization problem [14, 21, 22, 23, 31, 39]. For power systems, the output-based controller was investigated in [21], while the exponential stability was also considered in [19] with the same problem. This problem is further investigated in [18, 20], with robust fuzzy control. While on the other side, the authors put a lot of efforts on the Induction motors with different methodology, e.g. Sliding Mode Control (SMC) [13], Model Predictive Control (MPC) [4, 10], Model Reference Adaptive System (MRAS) [12]. In [13], researchers proposed a control method considers the disturbance inputs representing the system nonlinearity or the unmodeled uncertainty with sufficient LMIs. In [4, 10], authors developed for a permanent-magnet synchronous motor drive, while speed and current controllers are combined...
together, including all of the state variables of the system. A considerable research effort has been made with high-precision, efficient control of sensor less IM drives throughout its operating range demands an exact knowledge of a few of the IM parameters in [4, 10, 12, 13]. Although there are some shortcomings, these methods can achieve anomaly performance. However, to overcome this problem of power control networks in the network control system. The above-mentioned methods generally have some drawback in dealing with overshoots, rise-time, accuracy, and reliability. For this reason, it is a great interest to design the generalize fuzzy Lyapunov Krasovskii Function (LKF) which overcome these issues.

The T-S Fuzzy system is introduced to analyzing and designing the nonlinear control system, which proposed firstly by Takagi and Sugeno [36]. After this, it becomes a very popular approach. Many researchers have paid great attention in recent years because T-S fuzzy system provides various advantages such as simple in the calculation, extreme flexibility and highly reliable. In recent years, a huge number of noteworthy results and methods for nonlinear systems based on the T-S fuzzy model approach have been investigated. For example, the problems of feedback power control system [11, 15, 17], fuzzy controller design [30], LMI approaches [9, 10, 41] and fuzzy filtering problems [6]. In above-mentioned work, mostly researchers investigated different problem based on T–S fuzzy systems with common Lyapunov-Krasovskii functional (LKF). This may yield more conservatism. In this regards, we have to design a new version of associated LKF for SM, which gives better results to the existing one. This motivates the current research.

This paper contains two studies: First, we design the $H_\infty$ control problem for SM, which based on the T-S fuzzy system. Secondly, we follow the standard step to design the pole-place method to achieve the fuzzy gain for our problem. The main contribution and novelties in this article are summarized as:

1) A general kind of model for T–S fuzzy model is established with the network control system with synchronous premises induced by the networks is considered.
2) The notion of $H_\infty$ control is presented, which has the ability to analyze the performances of $H_\infty$ index in a unified framework.
3) On the account of LMIs, a novel fuzzy-Lyapunov functional is presented to achieve appropriate condition and makes the system stochastically stable, which gives less conservative delay-dependent conditions with the fuzzy pole-place method. Our proposed methodology provides an efficient from the others. It is verified by parameters of the synchronous machine. Notations: Most of the notation deployed in this paper are standard and well found in the existing literature. Specifically, $X$ and $Y$ used for real symmetric matrices, particularly in this paper, the $X \geq Y$ and $X > Y$ represents the positive definite and semi-matrices, respectively. The superscript for the transpose of the matrix is ‘$T$’. $I$ denotes an identity matrix of suitable dimension. The notation ‘$\cdot$’ indicates as an ellipsis in terms that are brought by symmetry. Block diagonal matrix is represents by $\text{diag}(...)$. $L_2[0, \infty)$ shows the space of vector functions over $[0, \infty)$.

Figure 1: Structure of T–S fuzzy system under Network Control system for Synchronous machine.

2 Problem formulation and system descriptions

In the continuous-time domain for the conventional NCS’s comprises of a nonlinear plant, its configuration includes sensors, samplers, fuzzy controller and a network channel as illustrated in Fig. 1. A class of wireless sensor system can be illustrated by the aforementioned configuration, where the controller receives the measurements from the sensor. An
assumption is made here that the network channel signal is communicated in the form of a solitary packet.

\[
\begin{align*}
\frac{dx(t)}{dt} &= g_1(x(t)) + g_{u_1}(u(t)) + f_1(x(t))\omega(t) \\
y(t) &= g_3(x(t)).
\end{align*}
\]

Where \(x(t) \in \mathbb{R}^n\), \(y(t) \in \mathbb{R}^m\), \(u(t) \in \mathbb{R}^p\) and \(\omega(t) \in \mathbb{R}^{n_u}\) are the state, the measurement, control input signal, and the disturbance signal \(\omega(t)\), which belongs to \(L_2[0, \infty)\) respectively.

As shown in Fig. 1, a nonlinear system could be approximated by a T-S fuzzy model with plant rules [?]:

**Plant Rule i:** IF \(g_1(x(t))\) is \(M_{i1}\) and \(\cdots\) and \(g_p(x(t))\) is \(M_{i_p}\), THEN

\[
\begin{align*}
\dot{x}(t) &= A_ix(t) + B_iu(t) + D_i\omega(t) \\
y(t) &= C_ix(t).
\end{align*}
\]

where \(i = 1, 2, \cdots, r\); \(r\) represent the fuzzy rules (IF-THEN) \(x(t) \in \mathbb{R}^{n_x}\), \(y(t) \in \mathbb{R}^{n_y}\), and \(u(t) \in \mathbb{R}^{n_u}\) are the state, the measured output signal, and the control input, respectively and \(\omega(t) \in \mathbb{R}^{n_u}\) disturbance input signal which belongs to \(L_2[0, \infty)\). Premise variables are denoted by \(g_i(x(t))(i = 1, 2, \cdots, p)\), which are function of output; \(A_i, B_i, D_i, C_i\) are constant matrices with proper dimensions. For the system (1), initial conditions are defined as \(x(t_0) = x_0\). To avoid the complexity, \(g_j(x(t))\) denotes as \(g_j(x)\).

By using center-average for de-fuzzifier, product fuzzy interface and a singleton fuzzifier, the global dynamic of the fuzzy system (2) is inferred as:

\[
\begin{align*}
\dot{x}(t) &= \sum_{i=1}^r \mu_i(g(x))[A_ix(t) + B_iu(t) + D_i\omega(t)] \\
y(t) &= \sum_{i=1}^r \mu_i(g(x))C_ix(t).
\end{align*}
\]

where \(\mu_i(g(x)) = \frac{v_i(g(x))}{\sum_{j=1}^r v_j(g(x))} \geq 0\), \(v_i(g(x)) = \prod_{j=1}^p M_{ij}^j(g_j(x))\). \(M_{ij}^j(g_j(x))\) is the membership function of the fuzzy set \(M_{i1}^1 \times \cdots \times M_{ir}^r\), \(v_i(g(x))\) is the normalized membership function. It is obvious that weight dependent functions \(\mu_i(g(x))\) satisfy:

\[
\mu_i(g(x)) \geq 0, \quad \sum_{i=1}^r \mu_i(g(x)) = 1.
\]

### 2.1 Event-triggered Scheme

In this paper, the problem of event-triggered controller design with random packet loss via T-S fuzzy model is explored. The structure of the T-S fuzzy control system with event-triggered scheme is shown in Fig. 2.

In the existence of the network, the premise variable \(\mu_i(\tilde{\phi}(x))\) of system (2) is not the same as the one in fuzzy controller, then, consider the rule of \(j\)th controller model:

**Controller Rule j** IF \(g_1(\hat{x}(t))\) is \(W_{1j}\) and \(\cdots\) and \(g_p(\hat{x}(t))\) is \(W_{pj}\), THEN

\[
u(t) = K_j\hat{x}(t),
\]

where \(K_j\), \((j = 1, 2, \cdots, p)\) are controller parameters to be determined later. \(\hat{x}(t)\) is the control signal, which is transmitted through the channel. \(W_{jj}\), \((j = 1, 2, \cdots, r\); \(g = 1, 2, \cdots, p)\) denotes the fuzzy sets, \(g_w(\hat{x})\) presents the premise variables via channel. For simplicity, \(g_w(\hat{x}(t))\) is used to present the \(g_w(\hat{x})\) and \(g_w(\hat{x}) = [g_1(\hat{x}), g_2(\hat{x}), \cdots, g_p(\hat{x})]\).
Then, fuzzy controller can be written as:

\[ u(t) = \sum_{j=1}^{r} h_j(g(\hat{x}))K_j \hat{x}(t), \]

where \( h_i(g(\hat{x})) = \frac{\varsigma_i(g(\hat{x}))}{\sum_{i=1}^{n} \varsigma_i(g(\hat{x}))} \), \( \varsigma_i(g(\hat{x})) = \prod_{j=1}^{n} W_{ij}(g_j(\hat{x})) \). \( W_{ij}(g_j(\hat{x})) \) is the membership function of the fuzzy set \( W_1^i \times \cdots \times W_n^i \), \( \mu_i(g(x)) \) satisfy:

\[ h_i(g(\hat{x})) \geq 0, \quad \sum_{i=1}^{r} h_i(g(\hat{x})) = 1. \]

As shown in Fig. 2, considering the limited bandwidth of the communication channels, ETCs is inserted between the sampled-data and the latest transmitted sampled-data, our transmission scheme able to determine whether or not the error between the current sampled-data and the latest transmitted sampled-data, our transmission scheme able to determine whether or not the sampled-data should be transmitted. More-ever, once \( x_t(i_k \ h) \) is transmitted, the next triggered instant is determined by the following logic:

\[ i_{k+1}h = i_kh + \min_{j \geq 1} \{ jh \mid e_k^T(t^nh_k)\alpha e_k(t^nh_k) \geq \kappa x^T(t_kh)\alpha x(i_kh) \}, \]

where \( \kappa(0 \leq \kappa < 1) \) and \( \alpha(\alpha > 0) \) are triggering parameters. The threshold error between the current sampling output and the latest transmission one is defined as

\[ e_k(t^nh_k) = x(t^nh_k) - x(i_kh), \]

where \( t^nh_k = i_kh + jh, \ n \in \mathbb{N}. \)

To analyze more easily, similar to [3] and [2], the interval \([t_kh + \tau_k, t_{k+1}h + \tau_{k+1})\) can be divided into several subintervals. Suppose that there exists a constant \( \varsigma \) which satisfies \([t_kh + \tau_k, t_{k+1}h + \tau_{k+1}) = \bigcup_{j=1}^{\rho} \Pi_j \), where \( \Pi_j = [t_kh + jh + \tau_{k+j}, t_{k+1}h + jh + \tau_{k+j+1}), j = \{1, 2, \ldots, \rho\}, \rho = t_{k+1} - t_k - 1 \) Define \( \tau(t) = t - t_kh - jh, 0 \leq \tau_k \leq \tau(t) \leq h + \tau_{k+j+1} \leq \tau_M \). The sampled signals via event-triggered scheme can be described as follows:

\[ \hat{x}(t) = x(t - \tau(t)) + e_k(t). \]

Now, we will explain how the unreliable channel affect the communication through packet-loss.

### 2.2 Communication links

Plant model with the communication channel is shown in Fig. 2 In this paper, the concept of intermittent data missing through the channel should be considered. First of all, we considered states of the plant and controller are not same e.g. \( x(t) \neq \hat{x}(t) \). A stochastic method is implemented to explain them and it can be modeled as:

\[ \check{x}(t) = \beta(t) \hat{x}(t), \]

where \( \beta(t) \) fulfills the bernoulli process. The procedure of \( \beta(t) \) describe the

\[
\text{Prob}\{\beta(t)\} = \begin{cases} \mathbb{E}\{\beta(t)\} = \bar{\beta}, & \beta(t) = 1 \\ 1 - \bar{\beta}, & \beta(t) = 0. \end{cases}
\]

According to this definition, we have

\[ u(t) = \sum_{j=1}^{r} h_j(g(\hat{x}))\beta(t)K_j[x(t - \tau(t)) + e_k(t)]. \]

\( g_i(x) \) is abbreviated as \( g_i \) and \( h_j(\hat{x}) \) is abbreviated as \( h_j \), then, the closed-loop T-S fuzzy system (2) can be described as

\[
\begin{align*}
\dot{x}(t) &= \sum_{i=1}^{r} \sum_{j=1}^{n} g_i h_j [A_{ij} x(t) + B_{ij} x(t - \tau(t)) + \bar{B}_{ij} e_k(t) + D_{ij} u(t)] \\
y(t) &= \sum_{i=1}^{r} g_i C_i x(t),
\end{align*}
\]

where \( \bar{B}_{ij} = \bar{\sigma} B_{ij} K_j + \bar{\sigma}(t) B_{ij} K_j \), \( \bar{\sigma}(t) = \sigma(t) - \check{e} \). It is noted that

\[
E\{\bar{\sigma}(t)\} = \begin{cases} \check{e}(1 - \check{e}), & \bar{\sigma}(t) = \bar{\sigma}(t) \check{e} \bar{\sigma}(t) \\ 0, & \bar{\sigma}(t) = \bar{\sigma}(t). \end{cases}
\]

Now, the problem addressed is formulated as follows design networked T-S fuzzy controller in the form of (12) with random packet-loss phenomena.
3 Stability analysis

In this segment, we will show the stochastically stability for the nonlinear system network systems. As mentioned in [37], we suppose the fuzzy weight dependent function $\mu_i(\phi(t))$ are $C^1$ functions.

**Theorem 3.1.** Suppose that

$$| \mu_i(\phi(t)) | \leq \rho_i, \quad i = 1, 2, \cdots, r, \quad (13)$$

where $\rho_i \geq 0$. The $H_\infty$ control design is solved if there exist matrix $P_j > 0, j = 1, 2, \cdots, r$, such that the following inequalities hold:

$$P_k < P_i, \quad k = 1, 2, \cdots, r - 1, \quad (14)$$

$$\theta_{ij} + \theta_{ji} < 0, \quad i \leq j, \quad (15)$$

where

$$\theta_{ij} = \begin{bmatrix} \Gamma_{ij} & A_i^T P_1 \\ \circ & -\tau_M R \end{bmatrix}$$

$$\Gamma_{ij} = \begin{bmatrix} \Gamma_{ij}^1 & P_i \Xi_{ij} & 0 & P_i \Xi_{ij} P_i D_i \\ \circ & \Gamma_{ij}^2 & 0 & -\kappa C_i^T \alpha & 0 \\ \circ & \circ & -Q_2 - R & 0 & 0 \\ \circ & \circ & \circ & (1 - \kappa) \alpha & 0 \end{bmatrix}.$$  

$$\Gamma_{ij}^1 = \sum_{k=1}^{r-1} \rho_k (P_k - P_r) + P_r A_i + A_i^T P_j + \sum_{l=1}^{2} Q_l + C_i^T C_i,$$

$$\Gamma_{ij}^2 = -Q_2 + \kappa C_i^T \alpha C_i,$$

$$\Lambda_{ij} = \begin{bmatrix} A_i & \Xi_{ij} & 0 & \Xi_{ij} & D_i \end{bmatrix}.$$

**Proof.** At this point, we implement the Lyapunov-Krasovskii function candidate for system [12],

$$J(x_t, t) = x(t)^T P(h) x(t) + \int_{t-\tau(t)}^{t} x(s)^T Q_1 x(s) ds + \int_{t-\tau_M}^{t} x(s)^T Q_2 x(s) ds + \tau_M \int_{t-\tau_M}^{t} \int_{s}^{t} \dot{x}(v)^T R \dot{x}(v) dv, \quad (16)$$

where

$$P(h) = \sum_{i=1}^{r} \mu_i(g(t)) P_i. \quad (17)$$

Further computing

$$\dot{J}(x_t, t) = x(t)^T (\dot{P}(h) + \sum_{l=1}^{2} Q_l) x(t) + 2x(t)^T P(h) \dot{x}(t) - x(t - \tau_M)^T Q_2 x(t - \tau_M) + \dot{x}(t)^T (\tau_M R) \dot{x}(t)$$

$$- \tau_M \int_{t-\tau_M}^{t} \dot{x}(s)^T R \dot{x}(s) ds + e_k^T (t) \alpha e_k(t) - e_k^T (t) \alpha e_k(t). \quad (18)$$

To deal the above inequality

$$\tau_M \int_{t-\tau_M}^{t} \dot{x}(s)^T R \dot{x}(s) ds \leq \begin{bmatrix} x(t) \\ x(t - \tau_M) \end{bmatrix}^T \begin{bmatrix} -R & R \\ R & -R \end{bmatrix} \begin{bmatrix} x(t) \\ x(t - \tau_M) \end{bmatrix}. \quad (19)$$

Based on event-triggered condition [8], \(t \in [i_k h + \tau_{i_k}, i_k+1 h + \tau_{i_{k+1}}]\) we have:

$$e_k^T (t) \alpha e_k(t) \leq \kappa x(i_k h)^T \alpha x(i_k h),$$

which is equivalent to

$$[x^T (t - \tau(t)) \quad e_k^T (t)] \begin{bmatrix} \kappa C_i^T \alpha C_i & -\kappa C_i^T \alpha \\ \circ & \kappa \alpha \end{bmatrix} \begin{bmatrix} x(t - \tau(t)) \\ e_k(t) \end{bmatrix} \leq 0. \quad (20)$$
In the same way
\[ \dot{J}(x_i, t) + y(t)^T y(t) - \gamma^2 \omega(t)^T \omega(t) \leq \eta(t)^T \theta_{ij} \eta(t), \] (21)
where
\[ \eta(t) = [x(t)^T, x(t - \tau(t))^T, x(t - \tau_M)^T, \epsilon_k(t)^T, \omega(t)^T]. \]
Differentiating the yields that
\[ \dot{\mu}_r(g(t)) = - \sum_{k=1}^{r-1} \dot{\mu}_k(g(t)), \]
we have:
\[ \dot{P}(h) = \sum_{k=1}^{r} \dot{\mu}_k(g(t))P_k = \sum_{k=1}^{r-1} \mu_k(g(t))P_k + \dot{\mu}_r(g(t))P_r = \sum_{k=1}^{r-1} \rho_k(P_k - P_r). \] (22)
According (13), (14), (15) and (22) with Schur complement, we have:
\[ \theta_{ij} = \begin{bmatrix} \Gamma_{ij} & \Lambda_{ij}^T P_i \\ \alpha & -\tau_M^2 R \end{bmatrix}. \]
This, together with (19), implies that
\[ \dot{J}(x_i, t) + y(t)^T y(t) - \gamma^2 \omega(t)^T \omega(t) \leq 0. \] (23)

According to the Theorem 3.1, we are in position to design the fuzzy controller in the form (12) in the next theorem.

3.1 Fuzzy controller designing

In this section, we develop the main result for the solvable of the fuzzy control problem for system (12).

**Theorem 3.2.** Suppose the NCSs with given \( \gamma \) with the close-loop system (12). If matrices \( X_j > 0 \), \( \bar{P}_r \), and \( F_j \) with appropriate dimensions, then the succeeding LMIs hold for \( i, j = 1, 2, \ldots, r \):
\[ \bar{P}_k - P_i < 0, \quad k = 1, 2, \ldots, r-1, \] (24)
\[ \bar{\theta}_{ij} + \bar{\theta}_{ji} < 0, \quad i \leq j, \] (25)
where
\[ \bar{\theta}_{ij} = \begin{bmatrix} \bar{\Gamma}_{ij} & X_j \Lambda_{ij}^T \\ \alpha & -\tau_M^2 R \end{bmatrix} \quad \text{and} \quad \bar{\Gamma}_{ij} = \begin{bmatrix} \bar{\Gamma}_{ij}^{11} & \bar{b}_{ij} & 0 & \bar{b}_{ij} & D_i X_j \\ \alpha & \bar{\Gamma}_{ij}^{22} & 0 & -\kappa C_i^T \alpha & 0 \\ \alpha & \alpha & -\bar{Q}_2 - \bar{R} & 0 & 0 \\ \alpha & \alpha & \alpha & (1 - \kappa) \alpha & 0 \\ \alpha & \alpha & \alpha & \alpha & -\gamma^2 I \end{bmatrix}, \]
where
\[ \bar{b}_{ij} = \bar{\sigma} B_i Y_j + \bar{\sigma}(t) B_i Y_j \quad \text{and} \quad \bar{Q}_\ell = X_j Q_\ell X_j^T, \quad \ell = 1, 2 \quad \bar{R} = X_j R X_j^T. \]
The controller gain is given as follow:
\[ K_j = F_j X_j^{-1}, \] (26)

**Proof.** Define \( X_j = P_j^{-1} \) Pre and post multiply to (15) by \( \{X_j, X_j, X_j, X_j, I, X_j\} \) and its transpose, which yields to (25). Therefore, when the given LMIs in (24-25) are feasible, all the conditions are satisfied. The proof can be finished consequently. \( \square \)

**Remark 1.** The obtained conditions in Theorem 3.2 are established on the equation (13), where the membership functions (MFs) and their rates are assumed to be differentiable and bounded. In particular, equation (13) is important for a fuzzy Lyapunov candidate, which grants us to take more significant information on MFs into the \( H_\infty \) controller design conditions. However, some of the practical MFs have not differentiation, therefore, it should also be mentioned that equation (13) may be bounded to some extent. Moreover, when the membership functions are not differentiable, it may not be practicable to employ the fuzzy matrices in the L-K functional. For that reason, we can only exploit the quadratic L-K functionals (i.e., Lyapunov matrices are not dependent upon the membership functions). In this scenario, the obtained conditions in Theorem 3.2 are valid by setting \( \rho_i \) to be appropriately small and by limiting matrices variables \( \bar{P}_k > 0 \) to \( \bar{P} > 0 \). However, this may lead to some limitation, when these variables are delimited to special cases.
4 Fuzzy pole placement method (FPPM)

In this section, we will design the controller gain via Fuzzy Pole Placement Method (FPPM) due to its simple controller. In this paper, we proposed the FPPM for a power system. Many researchers put a lot effects on designing the pole-placement method for different systems, e.g. power system [1, 29, 42], nonlinear systems [31] and dynamical systems [22]. We considered the same the system (2) for FPPM of Synchronous machine. But in control designing, our control input:

\[ u(t) = -\sum_{j=1}^{r} K_j x(t), \quad i = 1, 2, \cdots, r \]  

(27)

In the above control gain \( K_j \) determined through FPPM. In this method our desired pole denoted \( \{P_{d_1}, P_{d_2}, \cdots, P_{d_N}\} \), \((N=\text{No. of system states})\). First, we determine the controller gain \( K_j \) using Algorithm 1, then convert it into the T-S fuzzy system using the membership function. At the last, plot the figures through standard software MATLAB.

5 Simulation examples

In this section, we provide the most sensible practical example of the non-PDC design process for a synchronous machine connected to the infinite bus system with packet-loss phenomena. The system mathematical model of the d-q reference frame is described by the following \([18, 19, 20, 21]\).

\[
\begin{align*}
\frac{d(x(t))}{dt} &= g_1(x(t)) + g_{u_1}(u(t)) + f_1(x(t))\omega(t) \\
y(t) &= g_{\delta}(x(t)).
\end{align*}
\]  

(28)
Table 1: Algorithm for FPPM design procedure.

\[
\begin{align*}
\text{Table 1: Algorithm for FPPM design procedure.} \\
\end{align*}
\]

\[
\begin{align*}
g_1((x(t)) &= \left[ \begin{array}{c} x_2(t) \\ -a_1 \sin(2x_1(t)) - a_2 x_3(t) \sin(x_1(t)) + a_3 x_5(t) \cos(x_1(t)) + a_4 u_1(t) \\ b_1 \cos(x_1(t)) + b_2 x_3(t) \\ -c_1 \sin(x_1(t)) - c_2 x_4(t) + c_3 x_5(t) + c_4 u_2(t) \\ -d_1 \sin(x_1(t)) + d_2 x_4(t) + d_3 x_5(t) + d_4 u_2(t) \end{array} \right], \\
g_{u_1} &= \left[ \begin{array}{cccc} 0 & a_4 & 0 & 0 \end{array} \right],
\end{align*}
\]

where state vector
\[
x(t) = \left[ \begin{array}{c} \theta_d(t) \\ \omega_d(t) \\ E''_d(t) \\ E''_q(t) \\ E''_q(t) \end{array} \right],
\]

Figure 5: Output response of the \( g(t) \) for Case-I.

In Table 3, coefficients of the synchronous machine system are given. Before going to further detail of the synchronous machine. First, we talk about the premise variables which is independent of the input and often used as part of
the state vector. Now we define the premise variable \( \phi(t) = [\phi_1(x_1(t)) \ \phi_2(x_3(t))] \) with \( \phi_1(x_1(t)) = \theta_d(t) \) and \( \phi_2(x_3(t)) = x_3(t) \).

For the sake of premise variable terms, we define, \( \phi_i(t) = x_i(t), \ i = 1, 2 \). In the same way, we calculate the the maximum and minimum values of \( \phi_i(t) \) under all \( x(t) \in [-a, a], \ a > 0 \), where \( a \) and \( -a \) presents the maximum and minimum values of \( \phi_i(t) \), respectively. So maximum and minimum values of \( \phi_i(t) \) can be written as:

\[
\phi_i(t) = M_1^i(\phi_i(t)).a + M_2^i(\phi_i(t)).(-a) \]

\[
\therefore M_1^i(\phi_i(t)) + M_2^i(\phi_i(t)) = 1.
\]

According to the above analysis, our membership function becomes:

\[
M_1^i(\phi_i(t)) = \frac{\phi_i(t) + a}{2a}, \quad M_2^i(\phi_i(t)) = \frac{a - \phi_i(t)}{2a}.
\]

Finally, our T–S fuzzy system consist of four fuzzy rules, which is given below:

Table 2: Algorithm for fuzzy controller design procedure.

\[
\begin{align*}
&\text{Determine the synchronous machine parameters.} \\
&\text{Extract the matrices } A_1, A_2, B, C, \text{ and } D_i \\
&\text{Solve the LMI of Theorem 2 using MATLAB.} \\
&\text{Obtain the solution of LMI } X_i \text{ and } F_i \\
&\text{Calculate the fuzzy controller gain using the formula (26).}
\end{align*}
\]

Before going to detail analysis, we define two cases for our simulation. In the first case, our synchronous machine contains the damper effect, while in the second case we ignore the effect of damper phenomena. According to the eq. (8), our systems matrices with suitable dimensions under the damping effect are:

\[
\begin{bmatrix}
A_1 & A_2
\end{bmatrix} = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
2a_1 + a_2.a & 1 & 0 & 0 & a_3 & 2a_1 - a_2.a & 1 & 0 & 0 & a_3 \\
b_1 & b_2 & 0 & 0 & c_1 & b_1 & 0 & b_2 & 0 & 0 \\
c_1 & 0 & c_2 & c_3 & d_1 & c_1 & 0 & c_2 & c_3 \\
d_1 & 0 & d_2 & d_3 & d_1 & 0 & d_2 & d_3
\end{bmatrix}.
\]
with

\[ K_1 = \sin(\frac{\theta_{10}}{\theta_{10}})[2a_2\cos(\theta_{10}) + a_2a], \quad K_2 = \sin(\frac{\theta_{10}}{\theta_{10}})[2a_2\cos(\theta_{10}) - a_2a], \]

\[ B = B_1 = [0 \ a_3 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T, \quad C = C_1 = [0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T, \]

In the same, we define the systems matrices with proper dimensions under the no damping effect are

\[ A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 2\ a_1 + a & 0 & 0 \\ b_1 & 0 & b_2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 2\ a_1 - a & 0 & 0 \\ b_1 & 0 & b_2 \end{bmatrix}, \]

\[ A_3 = \begin{bmatrix} 0 & 1 & 0 \\ K_1 & 0 & a_2\cos(\theta_{10}) \\ b_1\sin(\theta_{10})/\theta_{10} & 0 & b_2 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 0 & 1 & 0 \\ K_2 & 0 & a_2\cos(\theta_{10}) \\ b_1\sin(\theta_{10})/\theta_{10} & 0 & b_2 \end{bmatrix}, \]

with

\[ K_1 = \sin(\frac{\theta_{10}}{\theta_{10}})[2a_2\cos(\theta_{10}) + a_2a], \quad \text{and} \quad K_2 = \sin(\frac{\theta_{10}}{\theta_{10}})[2a_2\cos(\theta_{10}) - a_2a], \]

\[ B = B_1 = [0 \ a_3 \ 0 \ 0 \ b_3]^T, \quad C = C_1 = [0 \ 1 \ 0 \ 0]^T, \]

Furthermore, the disturbance could be expressed as follows:

\[ \omega(t) = \begin{cases} t, & 1 \leq t \leq 2 \\ \sin(t), & 2 \leq t \leq 3 \\ 0, & \text{elsewhere} \end{cases} \quad (29) \]

We select the initial condition as \( x_0 = [0.5; 0.75; -5; 0.9; 0.85]^T \). In the following scenario, we will consider the design of \( H_\infty \) fuzzy controller for the synchronous controller.

**Case-I: \( H_\infty \) Controller with damping effect:** Let us choose \((\rho_1, \rho_2, \rho_3, \rho_4) = 100\) and \(\gamma = 2.5\). Then it is noticed that the LMI's \[24]-[25] are feasible, and the feasible solutions of the controller gains to those LMI's are obtained as follows:

\[
\begin{bmatrix}
X_1 & F_{1}^T \\
X_2 & F_{2}^T \\
X_3 & F_{3}^T \\
X_4 & F_{4}^T
\end{bmatrix} =
\begin{bmatrix}
0.0301 & -0.1188 & -0.0859 & 0.0660 & 0.0179 & 0.0012 \\
-0.1188 & 3.2724 & 2.6771 & 0.0228 & -0.0121 & 9.2307 \\
-0.0859 & 2.6771 & 2.9067 & 0.1250 & 0.0133 & -0.0546 \\
0.0660 & 0.0228 & 0.1250 & 0.1903 & 0.0483 & -0.0033 \\
0.0179 & -0.0121 & 0.0133 & 0.0483 & 0.0125 & -0.0005 \\
0.0301 & -0.1173 & -0.0859 & 0.0660 & 0.0179 & 0.0024 \\
-0.1173 & 3.1986 & 2.6742 & 0.0245 & -0.0115 & 9.1764 \\
-0.0859 & 2.6742 & 2.9078 & 0.1251 & 0.0133 & -0.0744 \\
0.0660 & 0.0245 & 0.1251 & 0.1903 & 0.0483 & -0.0033 \\
0.0179 & -0.0115 & 0.0133 & 0.0483 & 0.0125 & -0.0003 \\
1.0210 & -0.6576 & 0.6303 & 2.5552 & 0.6723 & -0.8380 \\
-0.6576 & 0.9264 & 0.0763 & -1.5886 & -0.4212 & 9.6966 \\
0.6303 & 0.0763 & 0.9493 & 1.6430 & 0.4285 & -0.6476 \\
2.5552 & -1.5886 & 1.6430 & 6.4041 & 1.6844 & -2.1128 \\
0.6723 & -0.4212 & 0.4285 & 1.6844 & 0.4431 & -0.5550 \\
0.8798 & -0.5732 & 0.5332 & 2.1783 & 0.5714 & -0.7026 \\
-0.5732 & 0.8370 & 0.0927 & -1.3671 & -0.3616 & 9.6092 \\
0.5332 & 0.0927 & 0.8306 & 1.3796 & 0.3585 & -0.5431 \\
2.1783 & -1.3671 & 1.3796 & 5.4019 & 1.4166 & -1.7538 \\
0.5714 & -0.3616 & 0.3585 & 1.4166 & 0.3715 & -0.4593
\end{bmatrix}
\]
Then, the $H_\infty$ control constraints can be attained in the form of (26). Implemented the Algorithm (see Table 2), the estimated states of the synchronous machine under damping effect with open-loop and closed-loop are demonstrated in Fig. 3 & 4 respectively. Fig. 5 shows the output response curve. One can see from these figures that the proposed fuzzy dynamic output feedback controller is effective to the synchronous machine. In Fig 6 shows the control input and Fig. 7 displays the packet-loss phenomena from the sensor node to the controller.

**Figure 6:** Random data loss for Case-I.

**Figure 7:** State response of the $x(t)$ for Case-II.

**Remark 2.** Note that the obtained results of sufficient conditions are derived from the free-weighting matrix approach, which is known as the less conservative method. In (19) in Theorem 1, the membership function is assumed to be differentiable and the rate is assumed to be bounded. This assumption is necessary because we want to use the fuzzy Lyapunov functional method, which allows us to bring more information on membership functions into the design conditions. This method is helpful to reduce the conservatism of the derived results especially when the variation occurs in the system model.

**Case-II: $H_\infty$ Controller without Damping Effect:** Let we choose $(\rho_1, \rho_2, \rho_3, \rho_4) = 100$ and $\gamma = 2.5$. Then it is noticed that the LMIs (24)–(25) are feasible, and the feasible solutions of the controller gains to those LMIs are
obtained as follows:

$$\begin{bmatrix}
X_1 & F_1^T \\
X_2 & F_2^T \\
X_3 & F_3^T \\
X_4 & F_4^T \\
\end{bmatrix} =
\begin{bmatrix}
40.7257 & -39.3788 & 21.6897 & 66.0817 \\
-39.3788 & 69.6735 & -54.6436 & 835.5597 \\
21.6897 & -54.6436 & 77.5549 & -146.4439 \\
-40.0218 & 50.4318 & -22.7853 & 835.6783 \\
22.2152 & -22.7853 & 75.9384 & -195.0689 \\
61.9303 & -47.4752 & 37.5306 & -0.3288 \\
-47.4752 & 56.0930 & -14.8474 & 1.1202 \\
37.5306 & -14.8474 & 86.7355 & -0.5585 \\
63.8505 & -45.3531 & 39.0351 & -0.3674 \\
-45.3531 & 62.8563 & -2.8415 & 1.0877 \\
39.0351 & -2.8415 & 87.6566 & -0.6534
\end{bmatrix}$$

On the other side, we also implement the FPPM for the synchronous machine. We follow the Algorithm 1 (see Table 1). For the calculation of controller gain, first, we choose the desired pole location for our system. For this, our desire pole locations for case-II are:

$$\begin{bmatrix}
P_{d1} \\
P_{d2} \\
P_{d3} \\
P_{d4} \\
\end{bmatrix} =
\begin{bmatrix}
-10, 25 + i, -25 - i \\
-15, 25 + i, -25 - i \\
-12, 28 + i, -28 - i \\
-18, 28 + i, -28 - i
\end{bmatrix}$$

According to the desired pole location, our controller gains are:

$$\begin{bmatrix}
K_1 \\
K_2 \\
K_3 \\
K_4 \\
\end{bmatrix} =
\begin{bmatrix}
5.5924 & 0.1155 & -2.4491 \\
6.7454 & 0.0598 & -1.2683 \\
10.6673 & 0.0933 & -2.3081 \\
10.1644 & 0.0264 & -0.0623
\end{bmatrix}$$

Figure 8: Comparison of the control input for Case-II.
Figure 9: Output trajectory of $y(t)$ for Case-II.

Then, the desired fuzzy controller parameters can be calculated in the form of (27). By selecting the initial condition of the control system as $x_0 = [0.5\pi; 0.75\pi; -5]^T$, the state behavior of the control system are presented in Fig. 7 with two different control strategies. While in Fig. 8 illustrates the behavior of output signal $y(t)$, it observes that finally, the output trajectory reaches to zero. $H_\infty$ analysis is important for the physical system and it denotes that the estimates the states of the system. From the Electrical power circuit perspective, this factor is necessary for our system. Because of this, we examined this controller for our system.

It is observed that there is no work regarding these issues which we handle in our paper. But in [20], it is noted that the authors investigated this problem. So we compare our results to show the less conservative results. In Fig. 9, we make a comparison of the control input $u(t)$, which shows that our proposed method is less conservative. In Fig. 10 presents the output response for case-II. While in Fig. 10. shows the random packet loss of the synchronous machine.

Remark 3.: To elaborate on the feasibility of design controller, we demonstrate the synchronous machine example. Different from the [18, 19] and LMI approach with the same problem in [20], design algorithm studied the T-S fuzzy system in this paper. It is noted that attained results with sufficient conditions are derived from the weight-dependent matric approach, which is well known as the less conservative method. For a fuzzy controller gain $K_i$ with associated $H_\infty$ performance index $\gamma$ is calculated from the derived results, which can be quickly solved by the standard numerical software (MATLAB). For the time being, if the performance level $\gamma$ is guaranteed with the fewer numbers of packets dropouts need to be transmitted.

Remark 4. In the future, extending the proposed algorithm into T-S fuzzy systems with event-triggered [7, 28, 35], switched systems [26, 33, 34] and Markov jump systems [8] needs more investigation.

6 Conclusions

In view of the characteristics of above $H_\infty$ fuzzy control system for a synchronous machine, a feasible solution is proposed. This paper first establishes a new version of Lyapunov krassovskii function (LKF) in the account of LMIs which makes the stochastically stable with random path loss data. On the other side, we establish the controller gain pole-placement method with fuzzy membership function. However, packet losses are a significant issue appearing in networked systems, which of course affect the nonlinear systems. Finally, the synchronous machine example has been delivered to demonstrate the efficiency of our design method. When we consider consideration of both time delays and event-triggered in the problem of our paper is of much interest. It is also possible to examine the T-S fuzzy Markovian systems with fault isolation delay and actuator saturation by integrating the methods proposed in this paper.

Appendix I

$$a_1 = \frac{-\omega_0}{2T_L} \left( \frac{1}{X_d} - \frac{1}{X_q} \right); \quad a_2 = \frac{-\omega_0}{T_L X_q},$$

$$a_3 = \frac{-\omega_0}{T_L X_d}; \quad a_4 = \frac{\omega_0}{T_L},$$
\[ b_1 = -\frac{1}{T'_{d0}} \left(1 \right) \frac{X_q}{X'_{q}}; \quad b_2 = \frac{-X_q}{T''_{a}X'_{q}}, \]
\[ c_1 = -\frac{1}{T'_{d0}} \left[ \frac{X_d - X_a}{X'_d - X_a} \left(1 \right) - \frac{X_q}{X'_{d}} \right] \quad \left(1 \right) \frac{X'_d}{X''_{q}}, \]
\[ c_2 = \frac{-1}{T'_{d0}} \left( \frac{X_d - X_a}{X'_d - X_a} \right), \]
\[ c_3 = -\frac{1}{T''_{d0}} \left( \frac{X'_d - X_d - X_a}{X'_d - X_a} \right) \quad c_4 = \frac{-1}{T''_{d0}} \left( \frac{X'_d - X_a}{X'_d - X_a} \right) \left(1 \right) \frac{X''_{q}}{X''_{q}}, \]
\[ d_1 = \frac{X'_d - X_a}{X'_d - X_a} c_1 + \frac{1}{T'_{d0}} \left( \frac{X'_d}{X''_{d}} \right), \quad d_2 = \frac{X''_{d} - X_a}{X'_d - X_a} c_2 \quad \frac{1}{T''_{d0}}, \]
\[ d_3 = \frac{X'_d - X_a}{X'_d - X_a} c_3 + \frac{1}{T''_{d0}} \frac{X'_d}{X''_{d}}, \quad d_4 = \frac{X''_{d} - X_a}{X'_d - X_a} c_4, \]

![Graph](image_url)

Figure 10: Random data loss for Case-II.

### Table 3: Data of Synchronous Machine (Capacity Power 200VA)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Numerical value (p.u)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T'_{d0} )</td>
<td>d-axis time constant of transient open circuit</td>
<td>7.00</td>
</tr>
<tr>
<td>( T''_{d0} )</td>
<td>d-axis time constant in sub-transient open circuit</td>
<td>0.07</td>
</tr>
<tr>
<td>( T''_{q} )</td>
<td>q-axis time constant in sub-transient open circuit</td>
<td>0.18</td>
</tr>
<tr>
<td>( X_d )</td>
<td>Magnetic reactance in d-axis</td>
<td>1.10</td>
</tr>
<tr>
<td>( X'_d )</td>
<td>Transient reactance in d-axis</td>
<td>0.50</td>
</tr>
<tr>
<td>( X''_{d} )</td>
<td>Sub-transient reactance in d-axis</td>
<td>0.35</td>
</tr>
<tr>
<td>( X_q )</td>
<td>Magnetic reactance in q-axis</td>
<td>1.10</td>
</tr>
<tr>
<td>( X''_{q} )</td>
<td>Sub-transient reactance in q-axis</td>
<td>0.30</td>
</tr>
<tr>
<td>( X_a )</td>
<td>Field leakage reactance</td>
<td>0.19</td>
</tr>
<tr>
<td>( T_L )</td>
<td>Magnetic dipole moment</td>
<td>10.00</td>
</tr>
<tr>
<td>( \omega_0 )</td>
<td>Synchronous rotor angular</td>
<td>100 ( \pi )</td>
</tr>
</tbody>
</table>
Acknowledgement

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Conflict of interest

The authors declare that they have no conflict of interest.

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