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Original paper

An advanced decision support framework to assess sustainable transport projects using a new uncertainty modeling tool: Interval-valued Pythagorean trapezoidal fuzzy numbers

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Abstract

This paper introduces a new decision support framework for transport projects appraisal, which is based on the multi-criteria decision analysis (MCDA) and the cost-benefit analysis (CBA) to address both monetary and non-monetary characteristics of the problem at the same time. Substantial inaccuracies are present while obtaining non-monetary and monetary inputs to the MCDA and the CBA, respectively. To provide the MCDA part of the framework with uncertainty, an interval-valued Pythagorean fuzzy set (IVPFS) is used as an attractive uncertain modeling tool. Additionally, a new multi-attribute group decision-making (MAGDM) method with partially known attribute weights and unknown expert weights under the IVPFS environment is proposed to find the best sustainable transport project from the non-monetary criteria point of view. Furthermore, a novel uncertainty modeling tool, namely an interval-valued Pythagorean trapezoidal fuzzy set, is introduced to address ill-known quantities and inaccuracies that exist in the estimation of monetary impacts. Finally, this framework is used to handle a real case study of a complex decision process of finding the best transport project to fund several candidate projects. This approach has made it possible to handle the decision process from economic, social, environmental points of view at the same time. The result of this case study shows that the decision-making framework can be used as a valuable decision support tool. Moreover, it is shown that the presented framework can be applied to effectively support appraisals of transport projects.

Keywords: Decision support framework, sustainable transport appraisal, multi-criteria decision analysis, cost-benefit analysis, interval-valued Pythagorean fuzzy set.

1 Introduction

Addressing sustainability has been receiving growing attention in a transport projects appraisal. Discussing the concept of sustainability makes it necessary to consider the revision of traditional decision-making methods, in which often the cost-benefit analysis (CBA) is applied to systematically quantify and compare various benefits and costs made by a project [10, 17]. Decision making by solely focusing on the CBA is proven to be inadequate for problems with often conflicting objectives or criteria. Examples of such situations include problems with environmental or social matters that are mostly intrinsically hard to quantify [5, 6]. Therefore, it is required to further develop the decision-making process to address more than economic factors [4, 31]. Using such a decision-making framework by using multiple criteria needs multi-disciplinary and multi-participatory methods [12, 22, 23].

The MCDA has previously been applied in transport planning to overcome the issue of evaluating effects that cannot (or only with difficulties) be denoted by a monetary value or quantified [2, 3, 11, 14, 16, 18, 19, 21, 24, 25, 27]. However, the CBA is known a fixed section of transport project assessment in most countries [4]. As a result, it is vital to introduce an approach that consists of both the CBA and MCDA parts of an assessment. This approach can

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produce an outcome based on the aforementioned perspectives. Recently, such attempts have been made, and a few of them are relevant to this context. The EUNET method is incorporated the CBA result in terms of a benefit-cost ratio (BCR) or net present value (NPV) as an additional criterion in the MCDA, and then introduced the composite outcome as a relative weight value [13]. Later, the COSIMA approach tries to translate the MCDA outcome into the CBA 'language' by giving shadow prices to the criteria and then denotes the composite outcome as total rates of attractiveness [5, 28]. It should be noted that the COSIMA method has later proved to be hard to use in practical situations due to the trade-off issues that have to be considered between the CBA and MCDA to assess the shadow prices. Recently, several applications with similarities to the EUNET approach have been made to enhance the effective application of a transport policy while prioritizing transport infrastructure alternatives [4, 15, 30] . Salling and Pryn [29], attempted to enable the DMs to make the most informed DM under a highly uncertain situation by providing the outputs of the CBA and MCDA parts separately.

In this paper, a decision support framework is introduced that is based on the approach extended by Salling and Pryn [29]. The framework has two main sections, namely CBA and MCDA, which make two separate sorts of results by addressing non-monetary criteria of a sustainable feature in addition to the conventional monetary effects, respectively.

Because the CBA highly depends on the estimation of the potential benefits and costs of alternatives, vagueness and uncertainty are involved with it. This ultimately causes making infeasible economic outcome. The proposed framework improves the CBA-part by extending interval-valued Pythagorean trapezoidal fuzzy numbers (IVPTrFNs) as a novel practical tool to address ill-known data of the benefit and cost under vague conditions. To handle the CBA when the inputs are in the form of IVPTrFNs, the definition of basic IVPTrF-operations is needed. So, by considering the operational laws of interval-valued Pythagorean fuzzy numbers (IVPFNs), new operations for IVPTrFNs are introduced.. Based on these operations, the IVPTrF net present value (IVPTrF-NPV) is proposed and applied to fund the best sustainable transport project from the CBA part.

The framework enhances the MCDA-part by developing a new multi-attribute group decision-making (MAGDM) method, in which the attribute values are expressed with IVPFNs, and the information on the DMs weights is completely unknown. It is also employed to decide the importance of criteria in situations, in which the weight information is not completely known. The computational process of the MCDA-part takes place in three phases. In the first phase, unknown DM weights are derived by considering the correlation coefficient of the individual decision matrices with the positive and negative ideal decision matrices. In the next phase, incompletely known attribute weights are used, and the attributes weights are derived objectively by a multi-objective optimization model. This model is built on a correlation coefficient concept in an IVPF environment rather than a distance measure. Subsequently, the candidate alternatives are ranked by exploiting a new extension of the MULTIMOORA approach.

In summary, the main features of the proposed decision support framework that separate it from similar models in this area are as follows:

- A novel decision support framework to assess transport projects is introduced that provides the DMs with the power to make a well-informed decision under a highly vague condition by providing separate results from the CBA and the MCDA at the same time.
- The IVPTrFNs as the new extension of IVPFSs are introduced to increase the ability to express and calculate the uncertainty in the CBA part of a decision support framework.
- Operational laws of IVPTrFNs are defined to facilitate decision making in an IVPTrF environment.
- A net present value (NPV) method is extended to the IVPTrF environment and is applied to find the best sustainable transport project from the CBA part.
- Two new division and subtraction operations for IVPFSs are presented by considering the operational laws of IVPFNs.
- New measures for calculating the correlation coefficient and the weighted correlation coefficient between IVPFSs are presented, and their desirable axiomatic properties are proved.
- A new method in an IVPF environment is proposed to objectively compute the weights of the DMs according to the concept of the correlation coefficient between the individual decision matrix and IVPF-ideal decision matrices, unlike the previous studies that were based on the distance measures.
- A multi-objective optimization model is presented to objectively distinguish the incompletely known attribute weights in an IVPS environment.

- A new extension of MULTIMOORA in an IVPF environment is proposed and applied to find the best sustainable transport project from the MCDA part.

The remainder of the paper is presented as follows. The basic concepts, definitions, and notations used throughout the paper are described in Section 2. The principles for the decision support framework are presented in Section 3. The proposed decision support framework is applied in a real case study, and the results are presented and discussed in Section 4. Finally, Section 5 provides concluding remarks.

2 Preliminaries

This section reviews some of the basic concepts, definitions, and operational rules that are in association with IVPFSs. Moreover, two novel operations of division and subtraction along with the correlation and correlation coefficient of IVPFSs are presented. The section ends with a definition of IVPTrFNs and introducing some basic related concepts, which will be used in this paper.

2.1 IVPFSs and basic operational laws for IVPFSs

Definition 2.1. *Let $D([0,1])$ denote the set of all closed subintervals of $[0,1]$. Let X be a universe of discourse* [26]. *An IVPFS $\tilde{P}$ in X is given by:*

$$\tilde{P} = \left\{ \left\langle x,\ \left[\widetilde{\mu}^{l}_{\tilde{P}}(x), \widetilde{\mu}^{u}_{\tilde{P}}(x)\right],\ \left[\widetilde{\nu}^{l}_{\tilde{P}}(x), \widetilde{\nu}^{u}_{\tilde{P}}(x)\right] \right\rangle \mid x \in X \right\}, \tag{1}$$

where $\left[\widetilde{\mu}^{l}_{\tilde{P}}(x), \widetilde{\mu}^{u}_{\tilde{P}}(x)\right] \subseteq [0,1]$, $\left[\widetilde{\nu}^{l}_{\tilde{P}}(x), \widetilde{\nu}^{u}_{\tilde{P}}(x)\right] \subseteq [0,1]$ are interval values presenting the levels of membership and non-membership of element $x \in X$ in $\tilde{P}$, respectively. For every $x \in X$, $\widetilde{\mu}^{l}_{\tilde{P}}(x), \widetilde{\mu}^{u}_{\tilde{P}}(x), \widetilde{\nu}^{l}_{\tilde{P}}(x), \widetilde{\nu}^{u}_{\tilde{P}}(x) \in [0,1]$ and $\left(\widetilde{\mu}^{u}_{\tilde{P}}(x)\right)^2 + \left(\widetilde{\nu}^{u}_{\tilde{P}}(x)\right)^2 \leq 1$. The degree of indeterminacy of the membership function is denoted by:

$$\widetilde{\pi}_{\tilde{P}}(x) = \left[\widetilde{\pi}^{l}_{\tilde{P}}(x), \widetilde{\pi}^{u}_{\tilde{P}}(x)\right] = \left[\sqrt{1-\left(\widetilde{\mu}^{u}_{\tilde{P}}(x)\right)^2-\left(\widetilde{\nu}^{u}_{\tilde{P}}(x)\right)^2}, \sqrt{1-\left(\widetilde{\mu}^{l}_{\tilde{P}}(x)\right)^2-\left(\widetilde{\nu}^{l}_{\tilde{P}}(x)\right)^2}\right]. \tag{2}$$

For convenience, $\tilde{P} = ([\widetilde{\mu}^{l}_{\tilde{P}}, \widetilde{\mu}^{u}_{\tilde{P}}], [\widetilde{\nu}^{l}_{\tilde{P}}, \widetilde{\nu}^{u}_{\tilde{P}}])$ is called an interval-valued Pythagorean fuzzy number (IVPFN)[26].

Definition 2.2. *For any IVPFN $\tilde{P} = ([\widetilde{\mu}^{l}_{\tilde{P}}, \widetilde{\mu}^{u}_{\tilde{P}}], [\widetilde{\nu}^{l}_{\tilde{P}}, \widetilde{\nu}^{u}_{\tilde{P}}])$, the score function of $\tilde{P}$ is presented by* [26]:

$$S\left(\tilde{P}\right) = \frac{\left(\widetilde{\mu}^{l}_{\tilde{P}}\right)^2 + \left(\widetilde{\mu}^{u}_{\tilde{P}}\right)^2 - \left(\widetilde{\nu}^{l}_{\tilde{P}}\right)^2 - \left(\widetilde{\nu}^{u}_{\tilde{P}}\right)^2}{2}, \qquad S\left(\tilde{P}\right) \in [-1,1]. \tag{3}$$

Definition 2.3. *Let $\tilde{P} = ([\widetilde{\mu}^{l}_{\tilde{P}}, \widetilde{\mu}^{u}_{\tilde{P}}], [\widetilde{\nu}^{l}_{\tilde{P}}, \widetilde{\nu}^{u}_{\tilde{P}}])$, $\tilde{P}_1 = ([\widetilde{\mu}^{l}_{\tilde{P}_1}, \widetilde{\mu}^{u}_{\tilde{P}_1}], [\widetilde{\nu}^{l}_{\tilde{P}_1}, \widetilde{\nu}^{u}_{\tilde{P}_1}])$ and $\tilde{P}_2 = ([\widetilde{\mu}^{l}_{\tilde{P}_2}, \widetilde{\mu}^{u}_{\tilde{P}_2}], [\widetilde{\nu}^{l}_{\tilde{P}_2}, \widetilde{\nu}^{u}_{\tilde{P}_2}])$ be three IVPFNs, and $\lambda > 0$, then their operations are defined as follows* [26]:

$$\left(\tilde{P}\right)^{C} = \left(\left[\widetilde{\nu}^{l}_{\tilde{P}}, \widetilde{\nu}^{u}_{\tilde{P}}\right], \left[\widetilde{\mu}^{l}_{\tilde{P}}, \widetilde{\mu}^{u}_{\tilde{P}}\right] \right), \tag{4}$$

$$\tilde{P}_1 \bigoplus \tilde{P}_2 = \left\langle \left[\sqrt{\left(\widetilde{\mu}^{l}_{\tilde{P}_1}\right)^2 + \left(\widetilde{\mu}^{l}_{\tilde{P}_2}\right)^2 - \left(\widetilde{\mu}^{l}_{\tilde{P}_1}\right)^2\left(\widetilde{\mu}^{l}_{\tilde{P}_2}\right)^2}, \sqrt{\left(\widetilde{\mu}^{u}_{\tilde{P}_1}\right)^2 + \left(\widetilde{\mu}^{u}_{\tilde{P}_2}\right)^2 - \left(\widetilde{\mu}^{u}_{\tilde{P}_1}\right)^2\left(\widetilde{\mu}^{u}_{\tilde{P}_2}\right)^2}\right], \left[\widetilde{\nu}^{l}_{\tilde{P}_1}\widetilde{\nu}^{l}_{\tilde{P}_2}, \widetilde{\nu}^{u}_{\tilde{P}_1}\widetilde{\nu}^{u}_{\tilde{P}_2}\right] \right\rangle, \tag{5}$$

$$\tilde{P}_1 \bigotimes \tilde{P}_2 = \left\langle \left[\widetilde{\mu}^{l}_{\tilde{P}_1}\widetilde{\mu}^{l}_{\tilde{P}_2}, \widetilde{\mu}^{u}_{\tilde{P}_1}\widetilde{\mu}^{u}_{\tilde{P}_2}\right], \left[\sqrt{\left(\widetilde{\nu}^{l}_{\tilde{P}_1}\right)^2 + \left(\widetilde{\nu}^{l}_{\tilde{P}_2}\right)^2 - \left(\widetilde{\nu}^{l}_{\tilde{P}_1}\right)^2\left(\widetilde{\nu}^{l}_{\tilde{P}_2}\right)^2}, \sqrt{\left(\widetilde{\nu}^{u}_{\tilde{P}_1}\right)^2 + \left(\widetilde{\nu}^{u}_{\tilde{P}_2}\right)^2 - \left(\widetilde{\nu}^{u}_{\tilde{P}_1}\right)^2\left(\widetilde{\nu}^{u}_{\tilde{P}_2}\right)^2}\right] \right\rangle, \tag{6}$$

$$\lambda\tilde{P} = \left\langle \left[\sqrt{1-\left(1-\left(\widetilde{\mu}^{l}_{\tilde{P}}\right)^2\right)^{\lambda}}, \sqrt{1-\left(1-\left(\widetilde{\mu}^{u}_{\tilde{P}}\right)^2\right)^{\lambda}}\right], \left[\left(\widetilde{\nu}^{l}_{\tilde{P}}\right)^{\lambda}, \left(\widetilde{\nu}^{u}_{\tilde{P}}\right)^{\lambda}\right] \right\rangle, \tag{7}$$

$$\left(\tilde{P}\right)^{\lambda}=\left\langle\left[(\widetilde{\mu}^{l}_{\tilde{P}})^{\lambda},(\widetilde{\mu}^{u}_{\tilde{P}})^{\lambda}\right],\left[\sqrt{1-\left(1-\left(\widetilde{\nu}^{l}_{\tilde{P}}\right)^{2}\right)^{\lambda}},\sqrt{1-\left(1-\left(\widetilde{\nu}^{u}_{\tilde{P}}\right)^{2}\right)^{\lambda}}\right]\right\rangle, \tag{8}$$

Assume $\tilde{P}_j=([\widetilde{\mu}^{l}_{\tilde{P}_j},\widetilde{\mu}^{u}_{\tilde{P}_j}],\ [\widetilde{\nu}^{l}_{\tilde{P}_j},\widetilde{\nu}^{u}_{\tilde{P}_j}])$ $(j=1,2,\ldots,n)$ as a collection of IVPFNs [26]. Then, it is possible to call the function $IPFWA_w:\ \Omega^n\to\Omega$ an interval-valued Pythagorean fuzzy weighted averaging operator. This is defined as follows:

$$IPFWA_w\left(\tilde{P}_1,\tilde{P}_2,\ldots,\tilde{P}_n\right)=w_1\tilde{P}_1\oplus w_2\tilde{P}_2\oplus\cdots\oplus w_n\tilde{P}_n$$
$$=\left\langle\left[\sqrt{1-\prod_{j=1}^{n}\left(1-\left(\widetilde{\mu}^{l}_{\tilde{P}_j}\right)^{2}\right)^{w_j}},\sqrt{1-\prod_{j=1}^{n}\left(1-\left(\widetilde{\mu}^{u}_{\tilde{P}_j}\right)^{2}\right)^{w_j}}\right],\left[\prod_{j=1}^{n}\left(\widetilde{\nu}^{l}_{\tilde{P}_j}\right)^{w_j},\prod_{j=1}^{n}\left(\widetilde{\nu}^{u}_{\tilde{P}_j}\right)^{w_j}\right]\right\rangle, \tag{9}$$

where w_j is the weight of $\tilde{P}_j$ $(j=1,2,\ldots,n)$, $w_j\in[0,1]$ and $\sum_{j=1}^{n}w_j=1$.

2.2 Subtraction and division operations for IVPFNs

Definition 2.4. *Let $\tilde{P}_1=([\widetilde{\mu}^{l}_{\tilde{P}_1},\widetilde{\mu}^{u}_{\tilde{P}_1}],\ [\widetilde{\nu}^{l}_{\tilde{P}_1},\widetilde{\nu}^{u}_{\tilde{P}_1}])$ and $\tilde{P}_2=([\widetilde{\mu}^{l}_{\tilde{P}_2},\widetilde{\mu}^{u}_{\tilde{P}_2}],\ [\widetilde{\nu}^{l}_{\tilde{P}_2},\widetilde{\nu}^{u}_{\tilde{P}_2}])$ be two IVPFNs, then*

$$\tilde{P}_1\ominus\ \tilde{P}_2=\ \left\langle\left[\sqrt{\frac{\left(\widetilde{\mu}^{l}_{\tilde{P}_1}\right)^{2}-\left(\widetilde{\mu}^{l}_{\tilde{P}_2}\right)^{2}}{1-\left(\widetilde{\mu}^{l}_{\tilde{P}_2}\right)^{2}}},\sqrt{\frac{\left(\widetilde{\mu}^{u}_{\tilde{P}_1}\right)^{2}-\left(\widetilde{\mu}^{u}_{\tilde{P}_2}\right)^{2}}{1-\left(\widetilde{\mu}^{u}_{\tilde{P}_2}\right)^{2}}}\right],\left[\frac{\widetilde{\nu}^{l}_{\tilde{P}_1}}{\widetilde{\nu}^{l}_{\tilde{P}_2}},\frac{\widetilde{\nu}^{u}_{\tilde{P}_1}}{\widetilde{\nu}^{u}_{\tilde{P}_2}}\right]\right\rangle,$$
$$\text{if } \tilde{P}_1\geq\ \tilde{P}_2,\ \ \widetilde{\mu}^{u}_{\tilde{P}_2}\neq1,\ \widetilde{\nu}^{l}_{\tilde{P}_2}\neq0,\ \left(\widetilde{\mu}^{u}_{\tilde{P}_1}\right)^{2}\left(\widetilde{\nu}^{u}_{\tilde{P}_2}\right)^{2}-\left(\widetilde{\mu}^{u}_{\tilde{P}_2}\right)^{2}\left(\widetilde{\nu}^{u}_{\tilde{P}_1}\right)^{2}\leq\left(\widetilde{\nu}^{u}_{\tilde{P}_2}\right)^{2}-\left(\widetilde{\nu}^{u}_{\tilde{P}_1}\right)^{2}, \tag{10}$$

$$\tilde{P}_1\oslash\ \tilde{P}_2=\ \left\langle\left[\frac{\left(\widetilde{\mu}^{l}_{\tilde{P}_1}\right)^{2}}{\left(\widetilde{\mu}^{l}_{\tilde{P}_2}\right)^{2}},\frac{\left(\widetilde{\mu}^{u}_{\tilde{P}_1}\right)^{2}}{\left(\widetilde{\mu}^{u}_{\tilde{P}_2}\right)^{2}}\right],\left[\sqrt{\frac{\left(\widetilde{\nu}^{l}_{\tilde{P}_1}\right)^{2}-\left(\widetilde{\nu}^{l}_{\tilde{P}_2}\right)^{2}}{1-\left(\widetilde{\nu}^{l}_{\tilde{P}_2}\right)^{2}}},\sqrt{\frac{\left(\widetilde{\nu}^{u}_{\tilde{P}_1}\right)^{2}-\left(\widetilde{\nu}^{u}_{\tilde{P}_2}\right)^{2}}{1-\left(\widetilde{\nu}^{u}_{\tilde{P}_2}\right)^{2}}}\right]\right\rangle,$$
$$\text{if } \tilde{P}_1\leq\tilde{P}_2,\ \ \widetilde{\mu}^{l}_{\tilde{P}_2}\neq0\ ,\widetilde{\nu}^{u}_{\tilde{P}_2}\neq1,\ \left(\widetilde{\mu}^{u}_{\tilde{P}_1}\right)^{2}\left(\widetilde{\nu}^{u}_{\tilde{P}_2}\right)^{2}-\left(\widetilde{\mu}^{u}_{\tilde{P}_2}\right)^{2}\left(\widetilde{\nu}^{u}_{\tilde{P}_1}\right)^{2}\geq\left(\widetilde{\mu}^{u}_{\tilde{P}_1}\right)^{2}-\left(\widetilde{\mu}^{u}_{\tilde{P}_2}\right)^{2}. \tag{11}$$

Theorem 2.5. *Let $\tilde{P}_1=([\widetilde{\mu}^{l}_{\tilde{P}_1},\widetilde{\mu}^{u}_{\tilde{P}_1}],\ [\widetilde{\nu}^{l}_{\tilde{P}_1},\widetilde{\nu}^{u}_{\tilde{P}_1}])$ and $\tilde{P}_2=([\widetilde{\mu}^{l}_{\tilde{P}_2},\widetilde{\mu}^{u}_{\tilde{P}_2}],\ [\widetilde{\nu}^{l}_{\tilde{P}_2},\widetilde{\nu}^{u}_{\tilde{P}_2}])$ be two IVPFNs satisfying conditions within (10). Let $\tilde{P}=\tilde{P}_1\ominus\tilde{P}_2$, where $\ominus$ is the operation defined by (10). Then it holds $\tilde{P}_1=\tilde{P}\oplus\tilde{P}_2$, where $\oplus$ is the operation defined by (5).*

Proof. Let us consider an equation of type, $\tilde{P}_1=\tilde{P}\oplus\ \tilde{P}_2$, where the IVPFNs $\tilde{P}_2$ and $\tilde{P}_1$ are provided, and the problem is to obtain the unknown IVPN $\tilde{P}$, which satisfies $\left[\widetilde{\mu}^{l}_{\tilde{P}},\widetilde{\mu}^{u}_{\tilde{P}}\right]\subseteq[0,1]$, $\left[\widetilde{\nu}^{l}_{\tilde{P}},\widetilde{\nu}^{u}_{\tilde{P}}(x\right]\subseteq[0,1]$ and $\left(\widetilde{\mu}^{u}_{\tilde{P}}\right)^{2}+\left(\widetilde{\nu}^{u}_{\tilde{P}}\right)^{2}\leq1$.

$$\sqrt{\left(\widetilde{\mu}^{l}_{\tilde{P}}\right)^{2}+\left(\widetilde{\mu}^{l}_{\tilde{P}_2}\right)^{2}-\left(\widetilde{\mu}^{l}_{\tilde{P}}\right)^{2}\left(\widetilde{\mu}^{l}_{\tilde{P}_2}\right)^{2}}=\widetilde{\mu}^{l}_{\tilde{P}_1}\quad,\quad\sqrt{\left(\widetilde{\mu}^{u}_{\tilde{P}}\right)^{2}+\left(\widetilde{\mu}^{u}_{\tilde{P}_2}\right)^{2}-\left(\widetilde{\mu}^{u}_{\tilde{P}}\right)^{2}\left(\widetilde{\mu}^{u}_{\tilde{P}_2}\right)^{2}}=\widetilde{\mu}^{u}_{\tilde{P}_1}$$
$$\widetilde{\nu}^{l}_{\tilde{P}}\,\widetilde{\nu}^{l}_{\tilde{P}_2}=\widetilde{\nu}^{l}_{\tilde{P}_1}\quad,\quad\widetilde{\nu}^{u}_{\tilde{P}}\,\widetilde{\nu}^{u}_{\tilde{P}_2}=\widetilde{\nu}^{u}_{\tilde{P}_1}$$

Then, we have:

$$\widetilde{\mu}^{l}_{\tilde{P}}=\sqrt{\frac{\left(\widetilde{\mu}^{l}_{\tilde{P}_1}\right)^{2}-\left(\widetilde{\mu}^{l}_{\tilde{P}_2}\right)^{2}}{1-\left(\widetilde{\mu}^{l}_{\tilde{P}_2}\right)^{2}}}\quad,\quad\mu^{u}_{\tilde{P}}=\sqrt{\frac{\left(\widetilde{\mu}^{u}_{\tilde{P}_1}\right)^{2}-\left(\widetilde{\mu}^{u}_{\tilde{P}_2}\right)^{2}}{1-\left(\widetilde{\mu}^{u}_{\tilde{P}_2}\right)^{2}}}$$
$$\widetilde{\nu}^{l}_{\tilde{P}}=\frac{\widetilde{\nu}^{l}_{\tilde{P}_1}}{\widetilde{\nu}^{l}_{\tilde{P}_2}}\quad,\quad\widetilde{\nu}^{u}_{\tilde{P}}=\frac{\widetilde{\nu}^{u}_{\tilde{P}_1}}{\widetilde{\nu}^{u}_{\tilde{P}_2}}$$

Unfortunately, $\tilde{P}$ with relations above may not be an IVPFN. The membership degree of $\tilde{P}$ should take values in the interval $[0,1]$, i.e., $0\leq\sqrt{\frac{\left(\widetilde{\mu}^{l}_{\tilde{P}_1}\right)^{2}-\left(\widetilde{\mu}^{l}_{\tilde{P}_2}\right)^{2}}{1-\left(\widetilde{\mu}^{l}_{\tilde{P}_2}\right)^{2}}}\leq1$ and $0\leq\sqrt{\frac{\left(\widetilde{\mu}^{u}_{\tilde{P}_1}\right)^{2}-\left(\widetilde{\mu}^{u}_{\tilde{P}_2}\right)^{2}}{1-\left(\widetilde{\mu}^{u}_{\tilde{P}_2}\right)^{2}}}\leq1$.

The right-hand sides of both conditions are valid because of $\widetilde{\mu}^l_{\tilde{P}_1} \le 1$ and $\widetilde{\mu}^u_{\tilde{P}_1} \le 1$. To proof the left-hand sides, these conditions, $\left(\widetilde{\mu}^l_{\tilde{P}_1}\right)^2 \ge \left(\widetilde{\mu}^l_{\tilde{P}_2}\right)^2$, $\left(\widetilde{\mu}^u_{\tilde{P}_1}\right)^2 \ge \left(\widetilde{\mu}^u_{\tilde{P}_2}\right)^2$, $\left(\widetilde{\mu}^l_{\tilde{P}_2}\right)^2 \ne 1$ and $\left(\widetilde{\mu}^u_{\tilde{P}_2}\right)^2 \ne 1$, which are equal to $\widetilde{\mu}^l_{\tilde{P}_1} \ge \widetilde{\mu}^l_{\tilde{P}_2}$, $\widetilde{\mu}^u_{\tilde{P}_1} \ge \widetilde{\mu}^u_{\tilde{P}_2}$ and $\widetilde{\mu}^u_{\tilde{P}_2} \ne 1$, are needed.

Similarly, non-membership degree of $\tilde{P}$ should take values in the interval $[0,1]$, i.e., $0 \le \frac{\widetilde{\nu}^l_{\tilde{P}_1}}{\widetilde{\nu}^l_{\tilde{P}_2}} \le 1$ and $0 \le \frac{\widetilde{\nu}^u_{\tilde{P}_1}}{\widetilde{\nu}^u_{\tilde{P}_2}} \le 1$.

The left-hand sides of these conditions are correct because $\widetilde{\nu}^l_{\tilde{P}_1} \ge 0$, $\widetilde{\nu}^l_{\tilde{P}_2} \ge 0$, $\widetilde{\nu}^u_{\tilde{P}_1} \ge 0$ and $\widetilde{\nu}^u_{\tilde{P}_2} \ge 0$; however, the right-hand sides of them are correct in the cases that $\widetilde{\nu}^l_{\tilde{P}_1} \le \widetilde{\nu}^l_{\tilde{P}_2}$, $\widetilde{\nu}^u_{\tilde{P}_1} \le \widetilde{\nu}^u_{\tilde{P}_2}$ and $\widetilde{\nu}^l_{\tilde{P}_2} \ne 0$. Since $\widetilde{\nu}^l_{\tilde{P}_2} \le \widetilde{\nu}^u_{\tilde{P}_2}$, it can be concluded that $\widetilde{\nu}^u_{\tilde{P}_2} \ne 0$ is satisfied under the condition that $\widetilde{\nu}^l_{\tilde{P}_2} \ne 0$.

Integrating the conditions $\widetilde{\mu}^l_{\tilde{P}_1} \ge \widetilde{\mu}^l_{\tilde{P}_2}$, $\widetilde{\mu}^u_{\tilde{P}_1} \ge \widetilde{\mu}^u_{\tilde{P}_2}$, $\widetilde{\nu}^l_{\tilde{P}_1} \le \widetilde{\nu}^l_{\tilde{P}_2}$ and $\widetilde{\nu}^u_{\tilde{P}_1} \le \widetilde{\nu}^u_{\tilde{P}_2}$ result in $\tilde{P}_1 \ge \tilde{P}_2$ based on Definition 2.2. Hence, the above-mentioned inequalities hold only if $\tilde{P}_1 \ge \tilde{P}_2$, $\widetilde{\mu}^u_{\tilde{P}_2} \ne 1$ and $\widetilde{\nu}^l_{\tilde{P}_2} \ne 0$.

Furthermore, $\tilde{P}$ is an IVPFN and thus,

$$\left(\widetilde{\mu}^u_{\tilde{P}}\right)^2 + \left(\widetilde{\nu}^u_{\tilde{P}}\right)^2 = \left(\sqrt{\frac{\left(\widetilde{\mu}^u_{\tilde{P}_1}\right)^2 - \left(\widetilde{\mu}^u_{\tilde{P}_2}\right)^2}{1-\left(\widetilde{\mu}^u_{\tilde{P}_2}\right)^2}}\right)^2 + \left(\frac{\widetilde{\nu}^u_{\tilde{P}_1}}{\widetilde{\nu}^u_{\tilde{P}_2}}\right)^2 = \frac{\left(\widetilde{\mu}^u_{\tilde{P}_1}\right)^2\left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2 - \left(\widetilde{\mu}^u_{\tilde{P}_2}\right)^2\left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2 + \left(\widetilde{\nu}^u_{\tilde{P}_1}\right)^2 - \left(\widetilde{\mu}^u_{\tilde{P}_2}\right)^2\left(\widetilde{\nu}^u_{\tilde{P}_1}\right)^2}{\left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2 - \left(\widetilde{\mu}^u_{\tilde{P}_2}\right)^2\left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2} \le 1.$$

Based on the above-mentioned conditions, we know that $\widetilde{\mu}^u_{\tilde{P}_2} < 1$ and $\widetilde{\nu}^u_{\tilde{P}_2} \ne 0$; hence $\left(\widetilde{\mu}^u_{\tilde{P}_2}\right)^2 < 1$ and $\left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2 \ne 0$ then $\left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2 - \left(\widetilde{\mu}^u_{\tilde{P}_2}\right)^2\left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2 = \left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2\left(1-\left(\widetilde{\mu}^u_{\tilde{P}_2}\right)^2\right) > 0$. Then, $\left(\widetilde{\mu}^u_{\tilde{P}_1}\right)^2\left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2 - \left(\widetilde{\mu}^u_{\tilde{P}_2}\right)^2\left(\widetilde{\nu}^u_{\tilde{P}_1}\right)^2 \le \left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2 - \left(\widetilde{\nu}^u_{\tilde{P}_1}\right)^2$, which completes the proof of Theorem 2.5. □

Theorem 2.6. *Let* $\tilde{P}_1 = ([\widetilde{\mu}^l_{\tilde{P}_1}, \widetilde{\mu}^u_{\tilde{P}_1}], [\widetilde{\nu}^l_{\tilde{P}_1}, \widetilde{\nu}^u_{\tilde{P}_1}])$ *and* $\tilde{P}_2 = ([\widetilde{\mu}^l_{\tilde{P}_2}, \widetilde{\mu}^u_{\tilde{P}_2}], [\widetilde{\nu}^l_{\tilde{P}_2}, \widetilde{\nu}^u_{\tilde{P}_2}])$ *be two IVPFNs satisfying conditions within (11). Let* $\tilde{P} = \tilde{P}_1 \oslash \tilde{P}_2$*, where* $\oslash$ *is the operation defined by (11). Then it holds* $\tilde{P}_1 = \tilde{P} \otimes \tilde{P}_2$*, where* $\otimes$ *is the operation defined by (6).*

Proof. Let us consider an equation of type, $\tilde{P}_1 = \tilde{P} \otimes \tilde{P}_2$, where the IVPFNs $\tilde{P}_2$ and $\tilde{P}_1$ are given, and the problem is to find the unknown IVPN $\tilde{P}$, which satisfies $\left[\widetilde{\mu}^l_{\tilde{P}}, \widetilde{\mu}^u_{\tilde{P}}\right] \subseteq [0,1]$, $\left[\widetilde{\nu}^l_{\tilde{P}}, \widetilde{\nu}^u_{\tilde{P}}(x\right] \subseteq [0,1]$ and $\left(\widetilde{\mu}^u_{\tilde{P}}\right)^2 + \left(\widetilde{\nu}^u_{\tilde{P}}\right)^2 \le 1$.

$$\widetilde{\mu}^l_{\tilde{P}} . \widetilde{\mu}^l_{\tilde{P}_2} = \widetilde{\mu}^l_{\tilde{P}_1}, \qquad \widetilde{\mu}^u_{\tilde{P}} . \widetilde{\mu}^u_{\tilde{P}_2} = \widetilde{\mu}^u_{\tilde{P}_1}$$

$$\sqrt{\left(\widetilde{\nu}^l_{\tilde{P}}\right)^2 + \left(\widetilde{\nu}^l_{\tilde{P}_2}\right)^2 - \left(\widetilde{\nu}^l_{\tilde{P}}\right)^2\left(\widetilde{\nu}^l_{\tilde{P}_2}\right)^2} = \widetilde{\nu}^l_{\tilde{P}_1}, \qquad \sqrt{\left(\widetilde{\nu}^u_{\tilde{P}}\right)^2 + \left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2 - \left(\widetilde{\nu}^u_{\tilde{P}}\right)^2\left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2} = \widetilde{\nu}^u_{\tilde{P}_1}$$

Then, we have:

$$\widetilde{\mu}^l_{\tilde{P}} = \frac{\widetilde{\mu}^l_{\tilde{P}_1}}{\widetilde{\mu}^l_{\tilde{P}_2}}, \qquad \widetilde{\mu}^u_{\tilde{P}} = \frac{\widetilde{\mu}^u_{\tilde{P}_1}}{\widetilde{\mu}^u_{\tilde{P}_2}}$$

$$\widetilde{\nu}^l_{\tilde{P}} = \sqrt{\frac{\left(\widetilde{\nu}^l_{\tilde{P}_1}\right)^2 - \left(\widetilde{\nu}^l_{\tilde{P}_2}\right)^2}{1-\left(\widetilde{\nu}^l_{\tilde{P}_2}\right)^2}}, \qquad \widetilde{\nu}^u_{\tilde{P}} = \sqrt{\frac{\left(\widetilde{\nu}^u_{\tilde{P}_1}\right)^2 - \left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2}{1-\left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2}}$$

Unfortunately, $\tilde{P}$ with relations above may not be an IVPFN. The membership degree of $\tilde{P}$ should take values in the interval $[0,1]$, i.e., $0 \le \frac{\widetilde{\mu}^l_{\tilde{P}_1}}{\widetilde{\mu}^l_{\tilde{P}_2}} \le 1$ and $0 \le \frac{\widetilde{\mu}^u_{\tilde{P}_1}}{\widetilde{\mu}^u_{\tilde{P}_2}} \le 1$.

The left-hand sides of both conditions are valid because of $\widetilde{\mu}^l_{\tilde{P}_1} \ge 0$, $\widetilde{\mu}^l_{\tilde{P}_2} \ge 0$, $\widetilde{\mu}^u_{\tilde{P}_1} \ge 0$ and $\widetilde{\mu}^u_{\tilde{P}_2} \ge 0$. The right-hand sides are correct in the case that $\widetilde{\mu}^l_{\tilde{P}_2} \ge \widetilde{\mu}^l_{\tilde{P}_1}$, $\widetilde{\mu}^u_{\tilde{P}_2} \ge \widetilde{\mu}^u_{\tilde{P}_1}$ and $\widetilde{\mu}^l_{\tilde{P}_2} \ne 0$. Since $\widetilde{\mu}^l_{\tilde{P}_2} \le \widetilde{\mu}^u_{\tilde{P}_2}$ then $\widetilde{\mu}^u_{\tilde{P}_2} \ne 0$ is satisfied when $\widetilde{\mu}^l_{\tilde{P}_2} \ne 0$. Similarly, non-membership degree of $\tilde{P}$ should take values in the interval $[0,1]$ (i.e., $0 \le \sqrt{\frac{\left(\widetilde{\nu}^l_{\tilde{P}_1}\right)^2 - \left(\widetilde{\nu}^l_{\tilde{P}_2}\right)^2}{1-\left(\widetilde{\nu}^l_{\tilde{P}_2}\right)^2}} \le 1$ and $0 \le \sqrt{\frac{\left(\widetilde{\nu}^u_{\tilde{P}_1}\right)^2 - \left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2}{1-\left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2}} \le 1$).

The right-hand sides of these conditions are valid because $\widetilde{\nu}^l_{\tilde{P}_1} \le 1$ and $\widetilde{\nu}^u_{\tilde{P}_1} \le 1$ so $\left(\widetilde{\nu}^l_{\tilde{P}_1}\right)^2 \le 1$ and $\left(\widetilde{\nu}^u_{\tilde{P}_1}\right)^2 \le 1$; however, the left-hand sides of them are correct in the cases that $\left(\widetilde{\nu}^l_{\tilde{P}_1}\right)^2 \ge \left(\widetilde{\nu}^l_{\tilde{P}_2}\right)^2$, $\left(\widetilde{\nu}^u_{\tilde{P}_1}\right)^2 \ge \left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2$ and $\left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2 \ne 1$.

Since $\widetilde{\nu}^l_{\tilde{P}_2} \le \widetilde{\nu}^u_{\tilde{P}_2}$ then $\left(\widetilde{\nu}^l_{\tilde{P}_2}\right)^2 \ne 1$ is satisfied when $\widetilde{\nu}^u_{\tilde{P}_2} \ne 1$. Since $\widetilde{\mu}^l_{\tilde{P}_1}, \widetilde{\mu}^l_{\tilde{P}_2}, \widetilde{\mu}^u_{\tilde{P}_1}, \widetilde{\mu}^u_{\tilde{P}_2} \in [0,1]$, then the left-hand sides are correct when $\widetilde{\nu}^l_{\tilde{P}_1} \ge \widetilde{\nu}^l_{\tilde{P}_2}$, $\widetilde{\nu}^u_{\tilde{P}_1} \ge \widetilde{\nu}^u_{\tilde{P}_2}$ and $\widetilde{\nu}^u_{\tilde{P}_2} \ne 1$.

Integrating the conditions $\widetilde{\mu}^l_{\tilde{P}_1} \le \widetilde{\mu}^l_{\tilde{P}_2}$, $\widetilde{\mu}^u_{\tilde{P}_1} \le \widetilde{\mu}^u_{\tilde{P}_2}$, $\widetilde{\nu}^l_{\tilde{P}_1} \ge \widetilde{\nu}^l_{\tilde{P}_2}$ and $\widetilde{\nu}^u_{\tilde{P}_1} \ge \widetilde{\nu}^u_{\tilde{P}_2}$ result in $\tilde{P}_1 \le \tilde{P}_2$ based on Definition 2.2. Hence, the above-mentioned inequalities hold only if $\tilde{P}_1 \le \tilde{P}_2$, $\widetilde{\mu}^l_{\tilde{P}_2} \ne 0$ and $\widetilde{\nu}^u_{\tilde{P}_2} \ne 1$.

Furthermore, $\tilde{P}$ is an IVPFN and thus,

$$\left(\widetilde{\mu}^u_{\tilde{P}}\right)^2 + \left(\widetilde{\nu}^u_{\tilde{P}}\right)^2 = \left(\frac{\widetilde{\mu}^u_{\tilde{P}_1}}{\widetilde{\mu}^u_{\tilde{P}_2}}\right)^2 + \left(\sqrt{\frac{\left(\widetilde{\nu}^u_{\tilde{P}_1}\right)^2 - \left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2}{1-\left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2}}\right)^2 = \frac{\left(\widetilde{\mu}^u_{\tilde{P}_1}\right)^2 - \left(\widetilde{\mu}^u_{\tilde{P}_1}\right)^2\left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2 + \left(\widetilde{\mu}^u_{\tilde{P}_2}\right)^2\left(\widetilde{\nu}^u_{\tilde{P}_1}\right)^2 - \left(\widetilde{\mu}^u_{\tilde{P}_2}\right)^2\left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2}{\left(\widetilde{\mu}^u_{\tilde{P}_2}\right)^2 - \left(\widetilde{\mu}^u_{\tilde{P}_2}\right)^2\left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2} \le 1.$$

Based on the above-mentioned conditions, we know that $\widetilde{\nu}^u_{\tilde{P}_2} < 1$ and $\widetilde{\mu}^u_{\tilde{P}_2} \ne 0$; hence $\left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2 < 1$ and $\left(\widetilde{\mu}^u_{\tilde{P}_2}\right)^2 \ne 0$ then $\left(\widetilde{\mu}^u_{\tilde{P}_2}\right)^2 - \left(\widetilde{\mu}^u_{\tilde{P}_2}\right)^2\left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2 = \left(\widetilde{\mu}^u_{\tilde{P}_2}\right)^2\left(1-\left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2\right) > 0$. Then, $\left(\widetilde{\mu}^u_{\tilde{P}_1}\right)^2\left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2 - \left(\widetilde{\mu}^u_{\tilde{P}_2}\right)^2\left(\widetilde{\nu}^u_{\tilde{P}_1}\right)^2 \ge \left(\widetilde{\mu}^u_{\tilde{P}_1}\right)^2 - \left(\widetilde{\mu}^u_{\tilde{P}_2}\right)^2$, which completes the proof of Theorem 2.6. □

2.3 Correlation coefficient of IVPFSs

Let $\tilde{P} = \left\{ \left\langle x_i, \left[\widetilde{\mu}^l_{\tilde{P}}(x_i), \widetilde{\mu}^u_{\tilde{P}}(x_i)\right], \left[\widetilde{\nu}^l_{\tilde{P}}(x_i), \widetilde{\nu}^u_{\tilde{P}}(x_i)\right]\right\rangle \mid x_i \in X\right\}$ be an IVPFS in the universe of the discourse X, where $\widetilde{\mu}^l_{\tilde{P}}(x_i), \widetilde{\mu}^u_{\tilde{P}}(x_i), \widetilde{\nu}^l_{\tilde{P}}(x_i), \widetilde{\nu}^u_{\tilde{P}}(x_i) \in [0,1]$ and $\left(\widetilde{\mu}^u_{\tilde{P}}(x_i)\right)^2 + \left(\widetilde{\nu}^u_{\tilde{P}}(x_i)\right)^2 \le 1$ for every $x_i \in X$, we define the informational energy of the IVPFS $\tilde{P}$ as follows:

$$E_{IVPFS}\left(\tilde{P}\right) = \sum_{i=1}^{n} \frac{\left(\widetilde{\mu}^l_{\tilde{P}}(x_i)\right)^4 + \left(\widetilde{\mu}^u_{\tilde{P}}(x_i)\right)^4 + \left(\widetilde{\nu}^l_{\tilde{P}}(x_i)\right)^4 + \left(\widetilde{\nu}^u_{\tilde{P}}(x_i)\right)^4 + \left(\widetilde{\pi}^l_{\tilde{P}}(x_i)\right)^4 + \left(\widetilde{\pi}^u_{\tilde{P}}(x_i)\right)^4}{2}. \tag{12}$$

Let two IVPFSs P and Q in the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$, where $\widetilde{\mu}^l_{\tilde{P}}(x_i), \widetilde{\mu}^u_{\tilde{P}}(x_i), \widetilde{\nu}^l_{\tilde{P}}(x_i), \widetilde{\nu}^u_{\tilde{P}}(x_i) \in [0,1]$, $\widetilde{\mu}^l_{\tilde{Q}}(x_i), \widetilde{\mu}^u_{\tilde{Q}}(x_i), \widetilde{\nu}^l_{\tilde{Q}}(x_i), \widetilde{\nu}^u_{\tilde{Q}}(x_i) \in [0,1]$, $\left(\widetilde{\mu}^u_{\tilde{P}}(x_i)\right)^2 + \left(\widetilde{\nu}^u_{\tilde{P}}(x_i)\right)^2 \le 1$ and $\left(\widetilde{\mu}^u_{\tilde{Q}}(x_i)\right)^2 + \left(\widetilde{\nu}^u_{\tilde{Q}}(x_i)\right)^2 \le 1$ for every $x_i \in X$. Then, we define the following so-called correlation of IVPFSs $\tilde{P}$ and $\tilde{Q}$:

$$C_{IVPFS}\left(\tilde{P},\tilde{Q}\right) = \frac{1}{2}\Big(\sum_{i=1}^{n} \left(\widetilde{\mu}^l_{P}(x_i)\right)^2.\left(\widetilde{\mu}^l_{\tilde{Q}}(x_i)\right)^2 + \left(\widetilde{\mu}^u_{\tilde{P}}(x_i)\right)^2.\left(\widetilde{\mu}^u_{\tilde{Q}}(x_i)\right)^2 + \left(\widetilde{\nu}^l_{\tilde{P}}(x_i)\right)^2.\left(\widetilde{\nu}^l_{\tilde{Q}}(x_i)\right)^2 + \left(\widetilde{\nu}^u_{\tilde{P}}(x_i)\right)^2.\left(\widetilde{\nu}^u_{\tilde{Q}}(x_i)\right)^2$$
$$+ \left(\widetilde{\pi}^l_{\tilde{P}}(x_i)\right)^2.\left(\widetilde{\pi}^l_{\tilde{Q}}(x_i)\right)^2 + \left(\widetilde{\pi}^u_{\tilde{P}}(x_i)\right)^2.\left(\widetilde{\pi}^u_{\tilde{Q}}(x_i)\right)^2\Big). \tag{13}$$

Definition 2.7. *Let X be the finite universal set, and let $IVPFS(X)$ denote the set of all IVPFSs in X. Between any two IVPFSs P and Q in the universe of discourse X, the correlation coefficient is defined by:*

$$K_{IVPFS}\left(\tilde{P},\tilde{Q}\right) = \frac{C_{IVPFS}\left(\tilde{P},\tilde{Q}\right)}{\sqrt{E_{IVPFS}\left(\tilde{P}\right).E_{IVPFS}\left(\tilde{Q}\right)}}, \tag{14}$$

where $C_{IVPFS}\left(\tilde{P},\tilde{Q}\right)$, $E_{IVPFS}\left(\tilde{P}\right)$ and $E_{IVPFS}\left(\tilde{Q}\right)$ are the correlation of two IVPFSs $\tilde{P}$ and $\tilde{Q}$, the informational energy of $\tilde{P}$ and the informational energy of $\tilde{Q}$, respectively.

Theorem 2.8. *For all $\tilde{P}$, $\tilde{Q} \in IVPFS(X)$, the correlation satisfies:*

1. $C_{IVPFS}\left(\tilde{P},\tilde{P}\right) = E_{IVPFS}\left(\tilde{P}\right)$,
2. $C_{IVPFS}\left(\tilde{P},\tilde{Q}\right) = C_{IVPFS}\left(\tilde{Q},\tilde{P}\right)$.

Proof. The proofs are straightforward. □

Theorem 2.9. *For all $\tilde{P}$, $\tilde{Q} \in IVPFS(X)$, the correlation coefficient satisfies:*

1. $K_{IVPFS}\left(\tilde{P},\tilde{Q}\right) = K_{IVPFS}\left(\tilde{Q},\tilde{P}\right)$,
2. $0 \leq K_{IVPFS}\left(\tilde{P},\tilde{Q}\right) \leq 1$,
3. $\tilde{P} = \tilde{Q} \Longrightarrow K_{IVPFS}\left(\tilde{P},\tilde{Q}\right) = 1$.

Proof. 1. Straightforward.

2. The left- inequality $0 \leq K_{IVPFS}\left(\tilde{P},\tilde{Q}\right)$ is evident. We will prove $K_{IVPFS}\left(\tilde{P},\tilde{Q}\right) \leq 1$.

   By Cauchy–Schwarz inequality, $(x_1y_1+x_2y_2+\cdots+x_ny_n)^2 \leq \left({x_1}^2+{x_2}^2+\cdots+{x_n}^2\right).\left({y_1}^2+{y_2}^2+\cdots+{y_n}^2\right)$, where $(x_1+x_2+\cdots+x_n) \in \mathbb{R}^n$ and $(y_1+y_2+\cdots+y_n) \in \mathbb{R}^n$, we get

$$\left(2\times C_{IVPFS}\left(\tilde{P},\tilde{Q}\right)\right)^2 \leq \sum_{i=1}^{n}\left(\widetilde{\mu}^l_{\tilde{P}}(x_i)\right)^4 + \left(\widetilde{\mu}^u_{\tilde{P}}(x_i)\right)^4 + \left(\widetilde{\nu}^l_{\tilde{P}}(x_i)\right)^4 + \left(\widetilde{\nu}^u_{\tilde{P}}(x_i)\right)^4 + \left(\widetilde{\pi}^l_{\tilde{P}}(x_i)\right)^4 + \left(\widetilde{\pi}^u_{\tilde{P}}(x_i)\right)^4 \times$$

$$\sum_{i=1}^{n}\left(\widetilde{\mu}^l_{\tilde{Q}}(x_i)\right)^4 + \left(\widetilde{\mu}^u_{\tilde{Q}}(x_i)\right)^4 + \left(\widetilde{\nu}^l_{\tilde{Q}}(x_i)\right)^4 + \left(\widetilde{\nu}^u_{\tilde{Q}}(x_i)\right)^4 + \left(\widetilde{\pi}^l_{\tilde{Q}}(x_i)\right)^4 + \left(\widetilde{\pi}^u_{\tilde{Q}}(x_i)\right)^4.$$

   Therefore, $\left(C_{IVPFS}\left(\tilde{P},\tilde{Q}\right)\right)^2 \leq E_{IVPFS}\left(\tilde{P}\right)\times E_{IVPFS}\left(\tilde{Q}\right)$. Since $E_{IVPFS}\left(\tilde{P}\right)$ and $E_{IVPFS}\left(\tilde{Q}\right) \geq 0$, thus $\frac{\left(C_{IVPFS}\left(\tilde{P},\tilde{Q}\right)\right)^2}{E_{IVPFS}\left(\tilde{P}\right)\times E_{IVPFS}\left(\tilde{Q}\right)} = \left(K_{IVPFS}\left(\tilde{P},\tilde{Q}\right)\right)^2 \leq 1$. Hence, $-1 \leq \left(K_{IVPFS}\left(\tilde{P},\tilde{Q}\right)\right) \leq 1$. According to the fact that $0 \leq K_{IVPFS}\left(\tilde{P},\tilde{Q}\right)$, we obtain the following property:

$$0 \leq K_{IVPFS}\left(\tilde{P},\tilde{Q}\right) \leq 1.$$

3. As $\tilde{P} = \tilde{Q}$ this implies that $\left[\widetilde{\mu}^l_{\tilde{P}}(x_i), \widetilde{\mu}^u_{\tilde{P}}(x_i)\right] = \left[\widetilde{\mu}^l_{\tilde{Q}}(x_i), \widetilde{\mu}^u_{\tilde{Q}}(x_i)\right]$ and $\left[\widetilde{\nu}^l_{\tilde{P}}(x_i), \widetilde{\nu}^u_{\tilde{P}}(x_i)\right] = \left[\widetilde{\nu}^l_{\tilde{Q}}(x_i), \widetilde{\nu}^u_{\tilde{Q}}(x_i)\right]$ for $x_i \in X$, thus, $K_{IVPFS}\left(\tilde{P},\widetilde{Q}\right) = 1$, which completes the proof of Theorem 2.9.

□

However, in many practical situations, the different sets may have taken different weights, and thus, weight ω_i of the element $x_i \in X(i = 1, 2, ..., n)$ should be taken into account. In the following, we develop a weighted correlation coefficient between IVPFSs.

2.4 Definition and operations of IVPTrFNs

Definition 2.10. *An interval-valued Pythagorean trapezoidal fuzzy number (IVPTrFN) $\tilde{P} = \left\langle\left(\left[p^1, p^2, p^3, p^4\right]; M_{\tilde{P}}, N_{\tilde{P}}\right)\right\rangle$ is an extension of the IVPFN, whose interval-valued membership and non-membership functions are defined by equation (15-18), respectively.*

$$\mu^l_{\tilde{P}}(x) = \begin{cases} \frac{x-p^1}{p^2-p^1} M^l_{\tilde{P}} & if\ p^1 \leq x < p^2 \\ M^l_{\tilde{P}} & if\ p^2 \leq x \leq p^3 \\ \frac{p^4-x}{p^4-p^3} M^l_{\tilde{P}} & if\ p^3 < x \leq p^4 \\ 0 & \text{others} \end{cases} \tag{15}$$

$$\mu^u_{\tilde{P}}(x) = \begin{cases} \frac{x-p^1}{p^2-p^1} M^u_{\tilde{P}} & if\ p^1 \leq x < p^2 \\ M^u_{\tilde{P}} & if\ p^2 \leq x \leq p^3 \\ \frac{p^4-x}{p^4-p^3} M^u_{\tilde{P}} & if\ p^3 < x \leq p^4 \\ 0 & \text{others} \end{cases} \tag{16}$$

$$\nu^l_{\tilde{P}}(x) = \begin{cases} \frac{p^2-x+N^l_{\tilde{P}}(x-p^1)}{p^2-p^1} & if\ \ p^1 \le x < p^2 \\ N^l_{\tilde{P}} & if\ \ p^2 \le x \le p^3 \\ \frac{x-p^3+N^l_{\tilde{P}}(p^4-x)}{p^4-p^3} & if\ p^3 < x \le p^4 \\ 1 & others \end{cases} \tag{17}$$

$$\nu^u_{\tilde{P}}(x) = \begin{cases} \frac{p^2-x+N^u_{\tilde{P}}(x-p^1)}{p^2-p^1} & if\ \ p^1 \le x < p^2 \\ N^u_{\tilde{P}} & if\ \ p^2 \le x \le p^3 \\ \frac{x-p^3+N^u_{\tilde{P}}(p^4-x)}{p^4-p^3} & if\ p^3 < x \le p^4 \\ 1 & others \end{cases} \tag{18}$$

As depicted in Figure 1, The values $M_{\tilde{P}} = \left[M^l_{\tilde{P}},\ M^u_{\tilde{P}}\right]$ and $N_{\tilde{P}} = \left[N^l_{\tilde{P}},\ N^u_{\tilde{P}}\right]$ represent the bounds of the maximum membership degree and the minimum non-membership degree, respectively, such that they satisfy the conditions $M_{\tilde{P}} \subseteq [0,1]$; $N_{\tilde{P}} \subseteq [0,1]$; $0 \le \left(M^u_{\tilde{P}}\right)^2 + \left(N^u_{\tilde{P}}\right)^2 \le 1$ and $p^l, p^r \in R$.

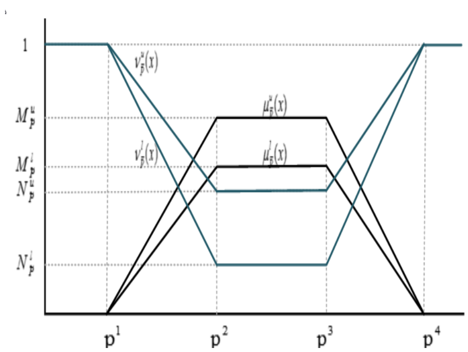

Figure 1: Interval-valued Pythagorean trapezoidal fuzzy number $\tilde{P} = \langle ([p^1, p^2, p^3, p^4]; M_{\tilde{P}}, N_{\tilde{P}}) \rangle$

Definition 2.11. *Let $\tilde{P}_1$ and $\tilde{P}_2$ be two IVPTrFNs and $\lambda \ge 0$, then their arithmetical operations are denoted by equation (19-24).*

$$\tilde{P}_1 \oplus \tilde{P}_2 = \langle [p^1_1+p^1_2, p^2_1+p^2_2, p^3_1+p^3_2, p^4_1+p^4_2]; \left[\sqrt{\left(M^l_{\tilde{P}_1}\right)^2+\left(M^l_{\tilde{P}_2}\right)^2-\left(M^l_{\tilde{P}_1}\right)^2\left(M^l_{\tilde{P}_2}\right)^2}, \sqrt{\left(M^u_{\tilde{P}_1}\right)^2+\left(M^u_{\tilde{P}_2}\right)^2-\left(M^u_{\tilde{P}_1}\right)^2\left(M^u_{\tilde{P}_2}\right)^2}\right], \left[N^l_{\tilde{P}_1}N^l_{\tilde{P}_2}, N^u_{\tilde{P}_1}N^u_{\tilde{P}_2}\right] \rangle, \tag{19}$$

$$\tilde{P}_1 \otimes \tilde{P}_2 = \langle [p^1_1p^1_2, p^2_1p^2_2, p^3_1p^3_2, p^4_1p^4_2]; \left[M^l_{\tilde{P}_1}M^l_{\tilde{P}_2}, M^u_{\tilde{P}_1}M^u_{\tilde{P}_2}\right], \left[\sqrt{\left(N^l_{\tilde{P}_1}\right)^2+\left(N^l_{\tilde{P}_2}\right)^2-\left(N^l_{\tilde{P}_1}\right)^2\left(N^l_{\tilde{P}_2}\right)^2}, \sqrt{\left(N^u_{\tilde{P}_1}\right)^2+\left(N^u_{\tilde{P}_2}\right)^2-\left(N^u_{\tilde{P}_1}\right)^2\left(N^u_{\tilde{P}_2}\right)^2}\right] \rangle, \tag{20}$$

$$\tilde{P}_1 \ominus \tilde{P}_2 = \left\langle [p^1_1-p^4_2, p^2_1-p^3_2, p^3_1-p^2_2, p^4_1-p^1_2]; \left[\sqrt{\frac{\left(\tilde{\mu}^l_{\tilde{P}_1}\right)^2-\left(\tilde{\mu}^l_{\tilde{P}_2}\right)^2}{1-\left(\tilde{\mu}^l_{\tilde{P}_2}\right)^2}}, \sqrt{\frac{\left(\tilde{\mu}^u_{\tilde{P}_1}\right)^2-\left(\tilde{\mu}^u_{\tilde{P}_2}\right)^2}{1-\left(\tilde{\mu}^u_{\tilde{P}_2}\right)^2}}\right], \left[\frac{\tilde{\nu}^l_{\tilde{P}_1}}{\tilde{\nu}^l_{\tilde{P}_2}}, \frac{\tilde{\nu}^u_{\tilde{P}_1}}{\tilde{\nu}^u_{\tilde{P}_2}}\right] \right\rangle, \tag{21}$$

if $\tilde{\mu}^l_{\tilde{P}_1} \ge \tilde{\mu}^l_{\tilde{P}_2}$, $\tilde{\mu}^u_{\tilde{P}_1} \ge \tilde{\mu}^u_{\tilde{P}_2}$, $\tilde{\nu}^l_{\tilde{P}_1} \le \tilde{\nu}^l_{\tilde{P}_2}$ and $\tilde{\nu}^u_{\tilde{P}_1} \le \tilde{\nu}^u_{\tilde{P}_2}$, $\tilde{\mu}^u_{\tilde{P}_2} \ne 1$, $\tilde{\nu}^l_{\tilde{P}_2} \ne 0$, $\left(\tilde{\mu}^u_{\tilde{P}_1}\right)^2\left(\tilde{\nu}^u_{\tilde{P}_2}\right)^2-\left(\tilde{\mu}^u_{\tilde{P}_2}\right)^2\left(\tilde{\nu}^u_{\tilde{P}_1}\right)^2 \le \left(\tilde{\nu}^u_{\tilde{P}_2}\right)^2-\left(\tilde{\nu}^u_{\tilde{P}_1}\right)^2$,

$$\tilde{P}_1\oslash \tilde{P}_2= \left\langle \left[\frac{p_1^1}{p_2^4},\frac{p_1^2}{p_2^3},\frac{p_1^3}{p_2^2},\frac{p_1^4}{p_2^1}\right];\ \left[\frac{\left(\widetilde{\mu}^l_{\tilde{P}_1}\right)^2}{\left(\widetilde{\mu}^l_{\tilde{P}_2}\right)^2},\frac{\left(\widetilde{\mu}^u_{\tilde{P}_1}\right)^2}{\left(\widetilde{\mu}^u_{\tilde{P}_2}\right)^2}\right],\left[\sqrt{\frac{\left(\widetilde{\nu}^l_{\tilde{P}_1}\right)^2-\left(\widetilde{\nu}^l_{\tilde{P}_2}\right)^2}{1-\left(\widetilde{\nu}^l_{\tilde{P}_2}\right)^2}},\sqrt{\frac{\left(\widetilde{\nu}^u_{\tilde{P}_1}\right)^2-\left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2}{1-\left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2}}\right]\right\rangle, \tag{22}$$

if $\widetilde{\mu}^l_{\tilde{P}_1}\leq\widetilde{\mu}^l_{\tilde{P}_2},\ \widetilde{\mu}^u_{\tilde{P}_1}\leq\widetilde{\mu}^u_{\tilde{P}_2},\widetilde{\nu}^l_{\tilde{P}_1}\geq\widetilde{\nu}^l_{\tilde{P}_2}$ and $\widetilde{\nu}^u_{\tilde{P}_1}\geq\widetilde{\nu}^u_{\tilde{P}_2},\ \widetilde{\mu}^l_{\tilde{P}_2}\neq 0\ ,\widetilde{\nu}^u_{\tilde{P}_2}\neq 1,\ \left(\widetilde{\mu}^u_{\tilde{P}_1}\right)^2\left(\widetilde{\nu}^u_{\tilde{P}_2}\right)^2-\left(\widetilde{\mu}^u_{\tilde{P}_2}\right)^2\left(\widetilde{\nu}^u_{\tilde{P}_1}\right)^2\geq\left(\widetilde{\mu}^u_{\tilde{P}_1}\right)^2-\left(\widetilde{\mu}^u_{\tilde{P}_2}\right)^2$,

$$\lambda\tilde{P}_1=\left\langle\left[\lambda p_1^1,\lambda p_1^2,\lambda p_1^3,\lambda p_1^4\right];\left[\sqrt{1-\left(1-\left(M^l_{\tilde{P}_1}\right)^2\right)^\lambda},\sqrt{1-\left(1-\left(M^u_{\tilde{P}_1}\right)^2\right)^\lambda}\right],\left[\left(N^l_{\tilde{P}_1}\right)^\lambda,\left(N^u_{\tilde{P}_1}\right)^\lambda\right]\right\rangle,\ \lambda\geq 0, \tag{23}$$

$$\left(\tilde{P}_1\right)^\lambda=\left\langle\left[{p_1^1}^\lambda,{p_1^2}^\lambda,{p_1^3}^\lambda,{p_1^4}^\lambda\right];\left[\left(M^l_{\tilde{P}_1}\right)^\lambda,\left(M^u_{\tilde{P}_1}\right)^\lambda\right],\left[\sqrt{1-\left(1-\left(N^l_{\tilde{P}_1}\right)^2\right)^\lambda},\sqrt{1-\left(1-\left(N^u_{\tilde{P}_1}\right)^2\right)^\lambda}\right]\right\rangle. \tag{24}$$

Definition 2.12. *For any IVPTrFN $\tilde{P}$, the score function of $\tilde{P}$ is defined by equation (25).*

$$S\left(\tilde{P}\right)=\frac{p^1+p^2+p^3+p^4}{4}.\ \frac{\left(\widetilde{\mu}^l_{\tilde{P}}\right)^2+\left(\widetilde{\mu}^u_{\tilde{P}}\right)^2-\left(\widetilde{\nu}^l_{\tilde{P}}\right)^2-\left(\widetilde{\nu}^u_{\tilde{P}}\right)^2}{2}. \tag{25}$$

Proposition 2.13. *2.1p For any IVPTrFN $\tilde{P}$, the proposed score function $S\left(\tilde{P}\right)\in[-1,1]$.*

Proof. Since $0\leq\left(\widetilde{\mu}^u_{\tilde{P}}(x)\right)^2+\left(\widetilde{\nu}^u_{\tilde{P}}(x)\right)^2\leq 1$, then $\left(\widetilde{\mu}^u_{\tilde{P}}(x)\right)^2-\left(\widetilde{\nu}^u_{\tilde{P}}(x)\right)^2\leq\left(\widetilde{\mu}^u_{\tilde{P}}(x)\right)^2\leq 1$ and $\left(\widetilde{\mu}^u_{\tilde{P}}(x)\right)^2-\left(\widetilde{\nu}^u_{\tilde{P}}(x)\right)^2\geq -1$. Thus, $-1\leq\left(\widetilde{\mu}^u_{\tilde{P}}(x)\right)^2-\left(\widetilde{\nu}^u_{\tilde{P}}(x)\right)^2\leq 1$. Similarly, $0\leq\left(\widetilde{\mu}^l_{\tilde{P}}(x)\right)^2+\left(\widetilde{\nu}^l_{\tilde{P}}(x)\right)^2\leq 1$ results $-1\leq\left(\widetilde{\mu}^l_{\tilde{P}}(x)\right)^2-\left(\widetilde{\nu}^l_{\tilde{P}}(x)\right)^2\leq 1$. Since, p^1,p^2,p^3 *and* $p^4\in[0,1]$, it can be easily seen that $S\left(\tilde{P}\right)\in[-1,1]$. □

Definition 2.14. *The accuracy function of any IVPTrFN $\tilde{P}$ is defined by equation (26).*

$$H\left(\tilde{P}\right)=\frac{p^1+p^2+p^3+p^4}{4}.\ \frac{\left(\widetilde{\mu}^l_{\tilde{P}}\right)^2+\left(\widetilde{\mu}^u_{\tilde{P}}\right)^2+\left(\widetilde{\nu}^l_{\tilde{P}}\right)^2+\left(\widetilde{\nu}^u_{\tilde{P}}\right)^2}{2}. \tag{26}$$

Proposition 2.15. *2.2p For any IVPTrFN $\tilde{P}$, the proposed accuracy function $H\left(\tilde{P}\right)\in[0,1]$.*

Proof. Because $0\leq\left(\widetilde{\mu}^u_{\tilde{P}}(x)\right)^2+\left(\widetilde{\nu}^u_{\tilde{P}}(x)\right)^2\leq 1$, then $0\leq\left(\widetilde{\mu}^l_{\tilde{P}}(x)\right)^2+\left(\widetilde{\nu}^l_{\tilde{P}}(x)\right)^2\leq 1$. We know that p^1,p^2,p^3 *and* $p^4\in[0,1]$. Thus, $H\left(\tilde{P}\right)\in[0,1]$, which completes the proof of this proposition. □

Definition 2.16. *Let $\tilde{P}_1$ and $\tilde{P}_2$ be two IVPTrFNs. According to the defined score and accuracy functions, a prioritized comparison approach is expressed by:*

1. If $S\left(\tilde{P}_1\right)<S\left(\tilde{P}_2\right)$, then $\tilde{P}_1\prec\tilde{P}_2$.
2. If $S\left(\tilde{P}_1\right)>S\left(\tilde{P}_2\right)$, then $\tilde{P}_1\succ\tilde{P}_2$.
3. If $S\left(\tilde{P}_1\right)=S\left(\tilde{P}_2\right)$, then
   a) If $H\left(\tilde{P}_1\right)<H\left(\tilde{P}_2\right)$, then $\tilde{P}_1\prec\tilde{P}_2$.
   b) If $H\left(\tilde{P}_1\right)>H\left(\tilde{P}_2\right)$, then $\tilde{P}_1\succ\tilde{P}_2$.
   c) If $H\left(\tilde{P}_1\right)=H\left(\tilde{P}_2\right)$, then $\tilde{P}_1\sim\tilde{P}_2$.

2.5 Comparison analysis between the different types of fuzzy sets

To carry out a comprehensive comparative analysis, first, a comparison between the different types of fuzzy sets is made by reviewing the definitions and basic properties of them. Figure 2 illustrates the development logic of fuzzy sets [1]. Then, IVPFNs are compared with the interval-valued intuitionistic fuzzy numbers (IVIFNs) to show the advantages of this uncertainty modeling tool in comparison with well-known IVIFNs. Finally, the advantages of IVPTrFNs in comparison with IVPFNs are expressed.

As shown in 2, the main difference between IVPFNs and IVIFNs is their different constraint conditions. According to their definitions, we know that the constraint condition of IVIFN, $\tilde{I}$, is $\widetilde{\mu}_{\tilde{I}}^{u}(x) + \widetilde{\nu}_{\tilde{I}}^{u}(x) \leq 1$, whereas the constraint condition of IVPFN, $\tilde{P}$, is $\left(\widetilde{\mu}_{\tilde{P}}^{u}(x)\right)^2 + \left(\widetilde{\nu}_{\tilde{P}}^{u}(x)\right)^2 \leq 1$. Because for any point $a, b \in [0,1]$, if $a+b \leq 1$, then $a^2+b^2 \leq 1$; Yager [32] showed that the space of the interval-valued Pythagorean membership grade is greater than the space of the interval-valued intuitionistic membership grade. In other words, if one is an IVIFN, then it must also be an IVPFN; however, not all IVPFNs are the IVIFNs. In this regard, IVIFNs have more limitations in expressing the degrees of membership and non-membership in comparison with IVPFNs, and it is not always possible to apply IVIFNs to solve problems that can be solved with IVPFNs. However, IVIFN-based problems can be handled with IVPFN methods because IVPFNs cover wider ranges of uncertainty. Consequently, IVPFNs in comparison with IVIFNs are more flexible and powerful to address uncertainty [20]. However, the domain of IVPFNs is a discrete set, and therefore, it is only used to indicate the extent to which the criterion does or does not belong to some fuzzy concepts. To overcome this flaw, we develop the interval-valued Pythagorean trapezoidal fuzzy numbers (IVPTrFNs), which extend the discrete sets to continuous sets. Applying for such numbers in decision-making problems results in better expressing uncertainty. Nevertheless, this advantage of IVPTrFNs comes at the cost that the operations involving IVPTrFNs are generally more complex than those involving IVIFNs.

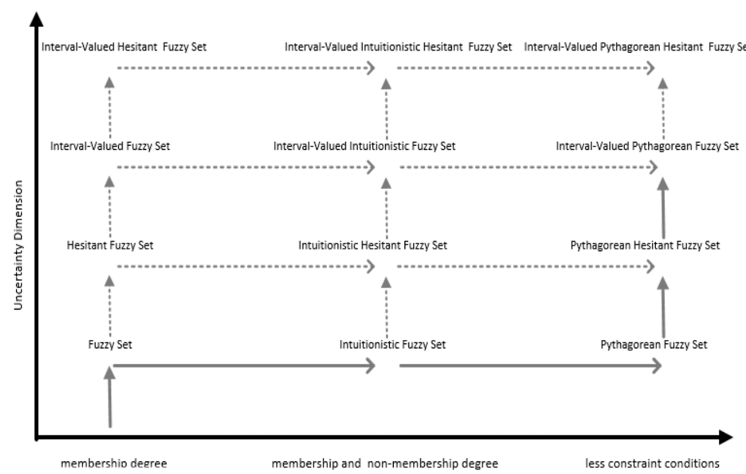

Figure 2: Development logic of fuzzy sets [1]

3 Proposed decision support framework

The proposed framework for the appraisal of transport projects because of sustainability and under uncertainty consists of two parts: CBA and MCDA. These parts make two different types of results from it by using non-monetary criteria of sustainable features in addition to the conventional monetary criteria, respectively. The two parts of the proposed framework are described in the following sub-sections.

3.1 CBA part of the framework

The cost-benefit analysis (CBA) is a method that applies a monetary assessment to measure the positive and negative impacts of transport decisions and is conventionally a part of any transport project appraisal. However, the uncertainties in forecasts and estimations of different types of positive and negative impacts are not usually well-modeled within the process. To map the uncertainties well and improve the robustness of the CBA results, we introduce IVPTrFNs as a new useful tool to deal with ill-known quantities in benefit and cost values. To address the cost-benefit analysis of monetary criteria under uncertainty, since the NPV method presents a comprehensive index, in this paper, it is extended to an IVPTrF environment, and the following is presented by equation (27), where $\tilde{B}_i = \left\langle \left(\left[b_i^1, b_i^2, b_i^3, b_1^4\right], \left[M_{\tilde{B}_i}^l, M_{\tilde{B}_i}^u\right], \left[N_{\tilde{B}_i}^l, N_{\tilde{B}_i}^u\right] \right) \right\rangle$ denotes the benefits cash flow in the duration of i and $\tilde{C}_i = \left\langle \left(\left[c_i^1, c_i^2, c_i^3, c_i^4\right], \left[M_{\tilde{C}_i}^l, M_{\tilde{C}_i}^u\right], \left[N_{\tilde{C}_i}^l, N_{\tilde{C}_i}^u\right] \right) \right\rangle$ shows cost cash flow in the time duration of i. Moreover, N is the planning horizon, and r denotes the discount rate.

$$\text{IVPTrFNPV} = \sum_{i=0}^{N} \frac{\left\langle \left(\left[b_i^1, b_i^2, b_i^3, b_1^4\right], \left[M_{\tilde{B}_i}^l, M_{\tilde{B}_i}^u\right], \left[N_{\tilde{B}_i}^l, N_{\tilde{B}_i}^u\right] \right) \right\rangle}{(1+r)^i} - \sum_{i=0}^{N} \frac{\left\langle \left(\left[c_i^1, c_i^2, c_i^3, c_i^4\right], \left[M_{\tilde{C}_i}^l, M_{\tilde{C}_i}^u\right], \left[N_{\tilde{C}_i}^l, N_{\tilde{C}_i}^u\right] \right) \right\rangle}{(1+r)^i}. \tag{27}$$

3.2 MCDA part of the framework

The MCDA part of the proposed framework is capable of assessing those criteria that are not addressed by the CBA part, but still holds the potential of improving the decision support. It can work based on qualitative inputs from the DMs in the form of linguistic terms. Because often DMs' judgments have some uncertainty and vagueness, the IVPFNs are utilized. It can convey the ambiguity and the uncertainty of the DMs in the MCDM problems effectively. The computational process of the MCDA part takes place in three phases. The detailed description of the computation processes is described subsequently.

3.2.1 Obtaining the DMs' weights

The DMs often possess different specialties. Consequently, each of them employs his/her unique expertise, which is dependent on the level of knowledge and experience. This means that various weights should be assigned to the DMs' judgments to correctly reflect each DM's influence on overall decision results. Therefore, it is essential to search for effective methods to objectively assess proper expert weights by entirely using preference data in decision matrices. Accordingly, this section presents a new correlation coefficient-based technique to calculate the weights of the DMs under an IVPF uncertainty.

It is assumed that each DM's preference over alternatives for attributes is provided in IVPFNs form and the decision information is presented as a matrix, Ψ^k, where $k \in t$ and t shows the group of experts that take part in the decision-making process. The method involves the following steps:

Step 1. Determine the interval-valued Pythagorean fuzzy positive ideal decision (IVPFPID) matrix and the interval-valued Pythagorean fuzzy negative ideal decision (IVPFNID) matrix. We define the average of all individual decisions as to the PID of the group. Hence, $\Psi^+ = \left(\widetilde{\psi}_{ij}^+\right)_{m\times n}$ is defined as the IVPFPID of group opinion, where $\widetilde{\psi}_{ij}^+ = (1/t)\sum_{k=1}^{t} \widetilde{\psi}_{ij}^k$.

$$\Psi^+ = \left(\widetilde{\psi}_{ij}^+\right)_{m\times n} = \left(\tilde{P} \left(\left[\widetilde{\mu}_{\widetilde{\psi}_{ij}^+}^l, \widetilde{\mu}_{\widetilde{\psi}_{ij}^+}^u\right], \left[\widetilde{\nu}_{\widetilde{\psi}_{ij}^+}^l, \widetilde{\nu}_{\widetilde{\psi}_{ij}^+}^u\right] \right) \right)_{m\times n}. \tag{28}$$

The NID of all individual decisions should be with the maximum separation from the PID. As a result, the minimum decision of all individual decisions is computed as an IVPFNID by using the following equation:

$$\Psi_{min}^- = \left(\widetilde{\psi}_{\min_{ij}}^-\right)_{m\times n} = \left(\tilde{P} \left(\left[\widetilde{\mu}_{\widetilde{\psi}_{\min_{ij}}^-}^l, \widetilde{\mu}_{\widetilde{\psi}_{\min_{ij}}^-}^u\right], \left[\widetilde{\nu}_{\widetilde{\psi}_{\min_{ij}}^-}^l, \widetilde{\nu}_{\widetilde{\psi}_{\min_{ij}}^-}^u\right] \right) \right)_{m\times n}. \tag{29}$$

Similarly, we define the maximum decision of all individual decisions as an IVPFNID as follows:

$$\Psi_{max}^- = \left(\widetilde{\psi}_{\max_{ij}}^-\right)_{m\times n} = \left(\tilde{P} \left(\left[\widetilde{\mu}_{\widetilde{\psi}_{\max_{ij}}^-}^l, \widetilde{\mu}_{\widetilde{\psi}_{\max_{ij}}^-}^u\right], \left[\widetilde{\nu}_{\widetilde{\psi}_{\max_{ij}}^-}^l, \widetilde{\nu}_{\widetilde{\psi}_{\max_{ij}}^-}^u\right] \right) \right)_{m\times n}. \tag{30}$$

Also, based on logical reasoning, the complement $(\Psi^+)^{\mathsf{C}}$ of Ψ^+ should have the maximum separation from the IVPFPID Ψ^+, so another IVPFNID should be considered as:

$$\Psi_C^- = \left(\widetilde{\psi}_{C_{ij}}^-\right)_{m\times n} = \left(\tilde{P} \left(\left[\widetilde{\nu}_{\widetilde{\psi}_{ij}^+}^l, \widetilde{\nu}_{\widetilde{\psi}_{ij}^+}^u\right], \left[\widetilde{\mu}_{\widetilde{\psi}_{ij}^+}^l, \widetilde{\mu}_{\widetilde{\psi}_{ij}^+}^u\right] \right) \right)_{m\times n}. \tag{31}$$

Step 2. Compute the informational energy for each element of the individual IVPF decision matrix, Ψ^k (k=1, 2, ... , t), IVPFPID matrices, Ψ^+, and IVPFNID matrices, Ψ^-_{min}, Ψ^-_{max} and $\Psi^-_{\complement}$, respectively by:

$$E_{IVPFS}\left(\widetilde{\psi}^k_{ij}\right)=\frac{\left(\widetilde{\mu}^l_{\widetilde{\psi}^k_{ij}}\right)^4+\left(\widetilde{\mu}^u_{\widetilde{\psi}^k_{ij}}\right)^4+\left(\widetilde{\nu}^l_{\widetilde{\psi}^k_{ij}}\right)^4+\left(\widetilde{\nu}^u_{\widetilde{\psi}^k_{ij}}\right)^4+\left(\widetilde{\pi}^l_{\widetilde{\psi}^k_{ij}}\right)^4+\left(\widetilde{\pi}^u_{\widetilde{\psi}^k_{ij}}\right)^4}{2}. \tag{32}$$

$$E_{IVPFS}\left(\widetilde{\psi}^+_{ij}\right)=\frac{\left(\widetilde{\mu}^l_{\widetilde{\psi}^+_{ij}}\right)^4+\left(\widetilde{\mu}^u_{\widetilde{\psi}^+_{ij}}\right)^4+\left(\widetilde{\nu}^l_{\widetilde{\psi}^+_{ij}}\right)^4+\left(\widetilde{\nu}^u_{\widetilde{\psi}^+_{ij}}\right)^4+\left(\widetilde{\pi}^l_{\widetilde{\psi}^+_{ij}}\right)^4+\left(\widetilde{\pi}^u_{\widetilde{\psi}^+_{ij}}\right)^4}{2}. \tag{33}$$

$$E_{IVPFS}\left(\widetilde{\psi}^-_{\min_{ij}}\right)=\frac{\left(\widetilde{\mu}^l_{\widetilde{\psi}^-_{\min_{ij}}}\right)^4+\left(\widetilde{\mu}^u_{\widetilde{\psi}^-_{\min_{ij}}}\right)^4+\left(\widetilde{\nu}^l_{\widetilde{\psi}^-_{\min_{ij}}}\right)^4+\left(\widetilde{\nu}^u_{\widetilde{\psi}^-_{\min_{ij}}}\right)^4+\left(\widetilde{\pi}^l_{\widetilde{\psi}^-_{\min_{ij}}}\right)^4+\left(\widetilde{\pi}^u_{\widetilde{\psi}^-_{\min_{ij}}}\right)^4}{2}. \tag{34}$$

$$E_{IVPFS}\left(\widetilde{\psi}^-_{\max_{ij}}\right)=\frac{\left(\widetilde{\mu}^l_{\widetilde{\psi}^-_{\max_{ij}}}\right)^4+\left(\widetilde{\mu}^u_{\widetilde{\psi}^-_{\max_{ij}}}\right)^4+\left(\widetilde{\nu}^l_{\widetilde{\psi}^-_{\max_{ij}}}\right)^4+\left(\widetilde{\nu}^u_{\widetilde{\psi}^-_{\max_{ij}}}\right)^4+\left(\widetilde{\pi}^l_{\widetilde{\psi}^-_{\max_{ij}}}\right)^4+\left(\widetilde{\pi}^u_{\widetilde{\psi}^-_{\max_{ij}}}\right)^4}{2}. \tag{35}$$

$$E_{IVPFS}\left(\widetilde{\psi}^-_{\complement_{ij}}\right)=\frac{\left(\widetilde{\mu}^l_{\widetilde{\psi}^-_{\complement_{ij}}}\right)^4+\left(\widetilde{\mu}^u_{\widetilde{\psi}^-_{\complement_{ij}}}\right)^4+\left(\widetilde{\nu}^l_{\widetilde{\psi}^-_{\complement_{ij}}}\right)^4+\left(\widetilde{\nu}^u_{\widetilde{\psi}^-_{\complement_{ij}}}\right)^4+\left(\widetilde{\pi}^l_{\widetilde{\psi}^-_{\complement_{ij}}}\right)^4+\left(\widetilde{\pi}^u_{\widetilde{\psi}^-_{\complement_{ij}}}\right)^4}{2}. \tag{36}$$

Step 3. Compute the correlation between each element of the individual IVPF decision matrix, Ψ^k, $(k = 1,\ 2,\ \ldots,\ t)$, and the corresponding element in the IVPFPID, Ψ^+.

$$\begin{aligned}2.C_{IVPFS}(\widetilde{\psi}^k_{ij},\widetilde{\psi}^+_{ij}) &= (\widetilde{\mu}^l_{\widetilde{\psi}^k_{ij}})^2.(\widetilde{\mu}^l_{\widetilde{\psi}^+_{ij}})^2+(\widetilde{\mu}^u_{\widetilde{\psi}^k_{ij}})^2.(\widetilde{\mu}^u_{\widetilde{\psi}^+_{ij}})^2+(\widetilde{\nu}^l_{\widetilde{\psi}^k_{ij}})^2.(\widetilde{\nu}^l_{\widetilde{\psi}^+_{ij}})^2+(\widetilde{\nu}^u_{\widetilde{\psi}^k_{ij}})^2.(\widetilde{\nu}^u_{\widetilde{\psi}^+_{ij}})^2\\ &+(\widetilde{\pi}^l_{\widetilde{\psi}^k_{ij}})^2.(\widetilde{\pi}^l_{\widetilde{\psi}^+_{ij}})^2+(\widetilde{\pi}^u_{\widetilde{\psi}^k_{ij}})^2.(\widetilde{\pi}^u_{\widetilde{\psi}^+_{ij}})^2, k=1,2,...,t\end{aligned} \tag{37}$$

Step 4. Compute the correlation between each element of the individual IVPF decision matrix, Ψ^k, $(k = 1,\ 2,\ \ldots,\ t)$, and the corresponding elements in the IVPFNIDs, Ψ^-_{min}, Ψ^-_{max} and $\Psi^-_{\complement}$.

$$\begin{aligned}2.C_{IVPFS}(\widetilde{\psi}^k_{ij},\widetilde{\psi}^-_{min_{ij}}) &= (\widetilde{\mu}^l_{\widetilde{\psi}^k_{ij}})^2.(\widetilde{\mu}^l_{\widetilde{\psi}^-_{min_{ij}}})^2+(\widetilde{\mu}^u_{\widetilde{\psi}^k_{ij}})^2.(\widetilde{\mu}^u_{\widetilde{\psi}^-_{min_{ij}}})^2+(\widetilde{\nu}^l_{\widetilde{\psi}^k_{ij}})^2.(\widetilde{\nu}^l_{\widetilde{\psi}^-_{min_{ij}}})^2+(\widetilde{\nu}^u_{\widetilde{\psi}^k_{ij}})^2.(\widetilde{\nu}^u_{\widetilde{\psi}^-_{min_{ij}}})^2\\ &+(\widetilde{\pi}^l_{\widetilde{\psi}^k_{ij}})^2.(\widetilde{\pi}^l_{\widetilde{\psi}^-_{min_{ij}}})^2+(\widetilde{\pi}^u_{\widetilde{\psi}^k_{ij}})^2.(\widetilde{\pi}^u_{\widetilde{\psi}^-_{min_{ij}}})^2, k=1,2,...,t\end{aligned} \tag{38}$$

$$\begin{aligned}2.C_{IVPFS}(\widetilde{\psi}^k_{ij},\widetilde{\psi}^-_{max_{ij}}) &= (\widetilde{\mu}^l_{\widetilde{\psi}^k_{ij}})^2.(\widetilde{\mu}^l_{\widetilde{\psi}^-_{max_{ij}}})^2+(\widetilde{\mu}^u_{\widetilde{\psi}^k_{ij}})^2.(\widetilde{\mu}^u_{\widetilde{\psi}^-_{max_{ij}}})^2+(\widetilde{\nu}^l_{\widetilde{\psi}^k_{ij}})^2.(\widetilde{\nu}^l_{\widetilde{\psi}^-_{max_{ij}}})^2+(\widetilde{\nu}^u_{\widetilde{\psi}^k_{ij}})^2.(\widetilde{\nu}^u_{\widetilde{\psi}^-_{max_{ij}}})^2\\ &+(\widetilde{\pi}^l_{\widetilde{\psi}^k_{ij}})^2.(\widetilde{\pi}^l_{\widetilde{\psi}^-_{max_{ij}}})^2+(\widetilde{\pi}^u_{\widetilde{\psi}^k_{ij}})^2.(\widetilde{\pi}^u_{\widetilde{\psi}^-_{max_{ij}}})^2, k=1,2,...,t\end{aligned} \tag{39}$$

$$\begin{aligned}2.C_{IVPFS}(\widetilde{\psi}^k_{ij},\widetilde{\psi}^-_{\complement_{ij}}) &= (\widetilde{\mu}^l_{\widetilde{\psi}^k_{ij}})^2.(\widetilde{\mu}^l_{\widetilde{\psi}^-_{\complement_{ij}}})^2+(\widetilde{\mu}^u_{\widetilde{\psi}^k_{ij}})^2.(\widetilde{\mu}^u_{\widetilde{\psi}^-_{\complement_{ij}}})^2+(\widetilde{\nu}^l_{\widetilde{\psi}^k_{ij}})^2.(\widetilde{\nu}^l_{\widetilde{\psi}^-_{\complement_{ij}}})^2+(\widetilde{\nu}^u_{\widetilde{\psi}^k_{ij}})^2.(\widetilde{\nu}^u_{\widetilde{\psi}^-_{\complement_{ij}}})^2\\ &+(\widetilde{\pi}^l_{\widetilde{\psi}^k_{ij}})^2.(\widetilde{\pi}^l_{\widetilde{\psi}^-_{\complement_{ij}}})^2+(\widetilde{\pi}^u_{\widetilde{\psi}^k_{ij}})^2.(\widetilde{\pi}^u_{\widetilde{\psi}^-_{\complement_{ij}}})^2, k=1,2,...,t\end{aligned} \tag{40}$$

Step 5. Compute the correlation coefficient between each element of the individual IVPF decision matrix, Ψ^k, and the corresponding element in the IVPFPID matrix, Ψ^+.

$$K_{IVPFS}\left(\widetilde{\psi}^k_{ij},\widetilde{\psi}^+_{ij}\right)=\frac{C_{IVPFS}\left(\widetilde{\psi}^k_{ij},\widetilde{\psi}^+_{ij}\right)}{\sqrt{E_{IVPFS}\left(\widetilde{\psi}^k_{ij}\right).E_{IVPFS}\left(\widetilde{\psi}^+_{ij}\right)}}\ ,\ k=1,\ 2,\ \ldots,\ t \tag{41}$$

Step 6. Compute the correlation coefficient between each element of the individual IVPF decision matrix, Ψ^k, and the corresponding elements in the IVPFNID metrix, Ψ^-_{min}, Ψ^-_{max} and $\Psi^-_{\mathtt{C}}$.

$$K_{IVPFS}\left(\widetilde{\psi}^k_{ij}, \widetilde{\psi}^-_{min_{ij}}\right) = \frac{C_{IVPFS}\left(\widetilde{\psi}^k_{ij}, \widetilde{\psi}^-_{min_{ij}}\right)}{\sqrt{E_{IVPFS}\left(\widetilde{\psi}^k_{ij}\right).E_{IVPFS}\left(\widetilde{\psi}^-_{min_{ij}}\right)}}, \ k = 1,\ 2,\ \ldots,\ t \tag{42}$$

$$K_{IVPFS}\left(\widetilde{\psi}^k_{ij}, \widetilde{\psi}^-_{max_{ij}}\right) = \frac{C_{IVPFS}\left(\widetilde{\psi}^k_{ij}, \widetilde{\psi}^-_{max_{ij}}\right)}{\sqrt{E_{IVPFS}\left(\widetilde{\psi}^k_{ij}\right).E_{IVPFS}\left(\widetilde{\psi}^-_{max_{ij}}\right)}}, \ k = 1,\ 2,\ \ldots,\ t \tag{43}$$

$$K_{IVPFS}\left(\widetilde{\psi}^k_{ij}, \widetilde{\psi}^-_{\mathtt{C}_{ij}}\right) = \frac{C_{IVPFS}\left(\widetilde{\psi}^k_{ij}, \widetilde{\psi}^-_{\mathtt{C}_{ij}}\right)}{\sqrt{E_{IVPFS}\left(\widetilde{\psi}^k_{ij}\right).E_{IVPFS}\left(\widetilde{\psi}^-_{\mathtt{C}_{ij}}\right)}}, \ k = 1,\ 2,\ \ldots,\ t \tag{44}$$

Step 7. Determine the weight of all DMs according to the fact that the more important the DM k will be as his/her opinions be more correlated with the IVPFPID and less correlate with the IVPFNIDs.

$$\lambda_k = \frac{\sum_{i=1}^{m}\left(\sum_{j=1}^{n}\left(\frac{K_{IVPFS}\left(\widetilde{\psi}^k_{ij}, \widetilde{\psi}^+_{ij}\right)}{K_{IVPFS}\left(\widetilde{\psi}^k_{ij}, \widetilde{\psi}^-_{\min_{ij}}\right)+K_{IVPFS}\left(\widetilde{\psi}^k_{ij}, \widetilde{\psi}^-_{\max_{ij}}\right)+K_{IVPFS}\left(\widetilde{\psi}^k_{ij}, \widetilde{\psi}^-_{\mathrm{C}_{ij}}\right)}\right)\right)}{\sum_{t=1}^{k}\left(\sum_{i=1}^{m}\left(\sum_{j=1}^{n}\left(\frac{K_{IVPFS}\left(\widetilde{\psi}^k_{ij}, \widetilde{\psi}^+_{ij}\right)}{K_{IVPFS}\left(\widetilde{\psi}^k_{ij}, \widetilde{\psi}^-_{\min_{ij}}\right)+K_{IVPFS}\left(\widetilde{\psi}^k_{ij}, \widetilde{\psi}^-_{\max_{ij}}\right)+K_{IVPFS}\left(\widetilde{\psi}^k_{ij}, \widetilde{\psi}^-_{\mathrm{C}_{ij}}\right)}\right)\right)\right)} \tag{45}$$

3.2.2 Deriving the importance weights of evaluation criteria

During the decision-making process, an important issue is how to determine the weights of evaluation criteria. In many practical decision-making problems, such as the selection of the best sustainable transport project, the information about evaluation criteria weights is partially known. Hence, the development of effective approaches is needed to compute the criteria weights under limited weight information. In this part, a novel correlation coefficient based technique is established to determine the weights of evaluation criteria under partially known weight information. Let $w = (w_1, w_2, \ldots, w_n)^T$ be the weight vector of the evaluation criteria, where $w_j \geq 0, (j = 1, 2, \ldots, n)$ and $\sum_{j=1}^{n} w_j = 1$, the known weight information on criteria is usually constructed using the following basic ranking forms [9], for $i \neq j$:

$$R_1 = \left\{w_i \geq w_j\right\},\quad R_2 = \left\{w_i - w_j \geq \gamma_j\right\}(\gamma_j > 0),\quad R_3 = \left\{w_i - w_j \geq w_l - w_m\right\}(j \neq l \neq m),$$
$$R_4 = \left\{w_i \geq \gamma_j w_j\right\}\left(0 \leq \gamma_j \leq 1\right) \text{ or } R_5 = \{\gamma_i \leq w_i \geq \gamma_i + \varepsilon_i\}\left(0 \leq \gamma_i \leq \gamma_i + \varepsilon_i\right).$$

With the assumption that R denotes the set of the known weight information of evaluation criteria given by DMs, $R = R_1 \cup R_2 \cup R_3 \cup R_4 \cup R_5$ and the decision information takes the form of IVPFNs; The method involves the following steps:

Step 8. Aggregate all the individual IVPF decision matrices Ψ^k into the overall group IVPF decision matrix Ψ by applying $IPFWA_w$ operator on $\Psi^k = \left(\widetilde{\psi}^k_{ij}\right)_{m\times n}$ and λ_k, $k = 1, 2, \ldots, t$.

$$\Psi = \left(\widetilde{\psi}_{ij}\right)_{m\times n} = \left(\tilde{P}\left(\left[\widetilde{\mu}^l_{\widetilde{\psi}_{ij}}, \widetilde{\mu}^u_{\widetilde{\psi}_{ij}}\right], \left[\widetilde{\nu}^l_{\widetilde{\psi}_{ij}}, \widetilde{\nu}^u_{\widetilde{\psi}_{ij}}\right]\right)\right)_{m\times n}, \tag{46}$$

where $\widetilde{\psi}_{ij} = \sum_{k=1}^{t} \lambda_k\ \widetilde{\psi}^k_{ij}$. So by Defnition $IPFWA_w$ operator, $\widetilde{\psi}_{ij}$ is computed by:

$$\widetilde{\psi}_{ij} = IPFWA_w\left(\widetilde{\psi}^1_{ij}, \widetilde{\psi}^2_{ij}, \ldots, \widetilde{\psi}^t_{ij}\right) = \lambda_1\widetilde{\psi}^1_{ij_1} \oplus \lambda_2\widetilde{\psi}^2_{ij} \oplus \cdots \oplus \lambda_t\widetilde{\psi}^t_{ij}$$
$$= \left\langle\left[\sqrt{1 - \prod_{k=1}^{t}\left(1 - \left(\widetilde{\mu}^l_{\widetilde{\psi}^k_{ij}}\right)^2\right)^{\lambda_k}}, \sqrt{1 - \prod_{k=1}^{t}\left(1 - \left(\widetilde{\mu}^u_{\widetilde{\psi}^k_{ij}}\right)^2\right)^{\lambda_k}}\right], \left[\prod_{k=1}^{t}\left(\widetilde{\nu}^l_{\widetilde{\psi}^k_{ij}}\right)^{\lambda_k}, \prod_{k=1}^{t}\left(\widetilde{\nu}^u_{\widetilde{\psi}^k_{ij}}\right)^{\lambda_k}\right]\right\rangle \tag{47}$$

$(i = \{1, 2, \ldots, m\},\ j = \{1, 2, \ldots, n\},\ k = \{1, 2, \ldots, t\})$.

Step 9. Determine the interval-valued Pythagorean fuzzy positive ideal solution (IVPFPIS) and the interval-valued Pythagorean fuzzy negative ideal solution (IVPFNIS) in the overall group IVPF decision matrix Ψ, while B is a collection of benefit criteria and C is a collection of cost criteria. For $j = 1,\ 2,\ \ldots,\ n$ we have

$$\text{IVPFPIS} = \left\{\left(\left[\max_i \widetilde{\mu}^l_{\widetilde{\psi}_{ij}}\ ,\max_i \widetilde{\mu}^u_{\widetilde{\psi}_{ij}}\ \right]|j\in B\ ,\left[\min_i \widetilde{\mu}^l_{\widetilde{\psi}_{ij}}\ ,\min_i \widetilde{\mu}^u_{\widetilde{\psi}_{ij}}\ \right]|j\in C\right),\left(\left[\min_i \widetilde{\nu}^l_{\widetilde{\psi}_{ij}}\ ,\min_i \widetilde{\nu}^u_{\widetilde{\psi}_{ij}}\ \right]|j\in B\ ,\left[\max_i \widetilde{\nu}^l_{\widetilde{\psi}_{ij}}\ ,\max_i \widetilde{\nu}^u_{\widetilde{\psi}_{ij}}\ \right]|j\in C\right)\right\}. \tag{48}$$

$$\text{IVPFNIS} = \left\{\left(\left[\min_i \widetilde{\mu}^l_{\widetilde{\psi}_{ij}}\ ,\min_i \widetilde{\mu}^u_{\widetilde{\psi}_{ij}}\ \right]|j\in B\ ,\left[\max_i \widetilde{\mu}^l_{\widetilde{\psi}_{ij}}\ ,\max_i \widetilde{\mu}^u_{\widetilde{\psi}_{ij}}\ \right]|j\in C\right),\left(\left[\max_i \widetilde{\nu}^l_{\widetilde{\psi}_{ij}}\ ,\max_i \widetilde{\nu}^u_{\widetilde{\psi}_{ij}}\ \right]|j\in B\ ,\left[\min_i \widetilde{\nu}^l_{\widetilde{\psi}_{ij}}\ ,\min_i \widetilde{\nu}^u_{\widetilde{\psi}_{ij}}\ \right]|j\in C\right)\right\}. \tag{49}$$

Step 10. Calculate the correlating index, $CIij$, of alternative Ai to the ideal solutions on a certain criterion Cj, using the following equation:

$$CIij = 1 - \frac{K_{IVPFS}\left(\widetilde{\psi}_{ij}, \text{IVPFNIS}_j\right)}{K_{IVPFS}\left(\widetilde{\psi}_{ij}, \text{IVPFNIS}_j\right) + K_{IVPFS}\left(\widetilde{\psi}_{ij}, \text{IVPFPIS}_j\right)} \quad i = 1,\ 2,\ \ldots,\ m\ ,\ j = 1,\ 2,\ \ldots,\ n \tag{50}$$

where $K_{IVPFS}\left(\widetilde{\psi}_{ij}, \text{IVPFPIS}_j\right)$ and $K_{IVPFS}\left(\widetilde{\psi}_{ij}, \text{IVPFNIS}_j\right)$ denote the correlation coefficient of A_i with IVPFPIS and IVPFNIS, respectively, on the criterion C_j. $CIij$ is defined to measure the extent to which the alternative A_i is correlated to IVPFPIS and is uncorrelated with IVPFNIS, simultaneously.

Step 11. The weighted correlating index CI_i corresponding to the alternatives A_i on the evaluation criteria can be acquired by:

$$CI_i = \sum_{j=1}^{n} w_j . CI_{ij} \quad ,\ i = 1,\ 2,\ \ldots,\ m. \tag{51}$$

It can be easily seen that the bigger CI_{ij}, the better the alternative A_i on the criterion C_j. Accordingly, for the given weight vector of criteria, the greater the value CI_i, the better the alternative A_i will be.

Step 12. Determine a reasonable weight vector for criteria so that to make all the weighted correlating index CI_i ($i = 1,\ 2,\ \ldots,\ m$) as large as possible, which means to maximize the weighted correlating index vector under the condition $w \in R$. Consequently, it is possible to form the following multiple objective optimization model.

$$(M-1)\begin{cases} \text{Max}F\,(w\ ) \ = (CI_1,\ CI_2,\ \ldots,\ CI_m) \\ \text{s.t.} \quad w\ \in R, \quad \sum_{j=1}^{n} w_j = 1, \quad w_j \geq 0, \quad j = 1, 2, \ldots, n \end{cases}$$

To solve linear programming problems with multiple objectives, many approaches have been proposed in the literature. In this paper, to assist the integration of all the weighted correlating indices CI_i ($i = 1,\ 2,\ \ldots,\ m$) into a single-objective optimization model, the max-min operator is applied [8].

$$(M-2)\begin{cases} \text{Max}\ \ \varrho \\ \quad \text{s.t.} \quad CI_i \leq \varrho, \quad (\ i = 1,\ 2,\ \ldots,\ m) \\ \quad w\ \in R, \quad \sum_{j=1}^{n} w_j = 1, \quad w_j \geq 0, \quad j = 1, 2, \ldots, n \end{cases}$$

By solving the integrated mathematical programming model (M − 2), the optimal solution $w^* = (w_1^*,\ w_2^*,\ \ldots,\ w_n^*)^T$ can be yielded and used as the weight vector of evaluation criteria.

3.2.3 Ranking the preference order of alternatives

To prioritize the sustainable transport project under consideration, in the third phase, we propose a novel extension of the MULTIMOORA method under the IVPF environment, namely interval-valued Pythagorean fuzzy MULTIMOORA (IVPF-MULTIMOORA). The MULTIMOORA method has three parts which are called the Ratio System, the Reference Point, and the Full Multiplicative Form [7]. The IVPF-MULTIMOORA approach involves the following steps:

Step 13. Construct the weighted overall group IVPF decision matrix through multiplying $\Psi = \left(\widetilde{\psi}_{ij}\right)_{m\times n}$ with $(w_1^*,\ w_2^*,\ \ldots,\ w_n^*)^T$.

$$\Phi\ = \left(\widetilde{\varphi}_{ij}\right)_{m\times n} = \left(\tilde{P}\left(\left[\widetilde{\mu}^l_{\widetilde{\varphi}_{ij}}, \widetilde{\mu}^u_{\widetilde{\varphi}_{ij}}\right], \left[\widetilde{\nu}^l_{\widetilde{\varphi}_{ij}}, \widetilde{\nu}^u_{\widetilde{\varphi}_{ij}}\right]\right)\right)_{m\times n}, \tag{52}$$

where $\widetilde{\varphi}_{ij} = w_j^* \widetilde{\psi}_{ij}$. so by Definition 2.3., $\widetilde{\mu}^l_{\widetilde{\varphi}_{ij}} = \sqrt{1-\left(1-\left(\widetilde{\mu}^l_{\psi_{ij}}\right)^2\right)^{w_j^*}}$, $\widetilde{\mu}^u_{\widetilde{\varphi}_{ij}} = \sqrt{1-\left(1-\left(\widetilde{\mu}^u_{\widetilde{\psi}_{ij}}\right)^2\right)^{w_j^*}}$, $\widetilde{\nu}^l_{\widetilde{\varphi}_{ij}} = \left(\widetilde{\nu}^l_{\widetilde{\psi}_{ij}}\right)^{w_j^*}$, $\widetilde{\nu}^u_{\widetilde{\varphi}_{ij}} = \left(\widetilde{\nu}^u_{\widetilde{\psi}_{ij}}\right)^{w_j^*}$ and $(i = \{1, 2, \ldots, m\}\,,\ j = \{1, 2, \ldots, n\}\,,\ k = \{1, 2, \ldots, t\})$

Step 14. Construct the normalized weighted IVPF overall performance decision matrix.

$$\overline{\Phi}=\left(\overline{\widetilde{\varphi}_{ij}}\right)_{m\times n}=\left(\tilde{P}\left(\left[\widetilde{\mu}^l_{\overline{\widetilde{\varphi}_{ij}}}, \widetilde{\mu}^u_{\overline{\widetilde{\varphi}_{ij}}}\right], \left[\widetilde{\nu}^l_{\overline{\widetilde{\varphi}_{ij}}}, \widetilde{\nu}^u_{\overline{\widetilde{\varphi}_{ij}}}\right]\right)\right)_{m\times n}.$$

$$\overline{\widetilde{\varphi}_{ij}} = \begin{cases} \widetilde{\varphi}_{ij} & ; \quad j \in B \\ \left(\widetilde{\varphi}_{ij}\right)^{\complement} & ; \quad j \in C \end{cases}, \tag{53}$$

where, B and C show the set of benefit and cost criteria, respectively, and $\left(\widetilde{\varphi}_{ij}\right)^{\complement}$ is the complement of $\widetilde{\varphi}_{ij}$. These values are bounded to the interval of [0, 1] and therefore do not require an additional normalization.

Step 15. Ratio System of the IVPF-MULTIMOORA: The assessments of a certain alternative are aggregated across the criteria by employing the following equation.

$$y_i = \sum_{j=1}^{n} \tilde{P}\left(\left[\widetilde{\mu}^l_{\overline{\widetilde{\varphi}_{ij}}}, \widetilde{\mu}^u_{\overline{\widetilde{\varphi}_{ij}}}\right], \left[\widetilde{\nu}^l_{\overline{\widetilde{\varphi}_{ij}}}, \widetilde{\nu}^u_{\overline{\widetilde{\varphi}_{ij}}}\right]\right), \tag{54}$$

accordingly, alternatives with higher values of y_i are attributed to higher ranks.

Step 16. Reference Point of the IVPF-MULTIMOORA: Construct the maximal objective reference point matrix, $\overline{\Phi}^{\text{Maximal Objective}}_{\text{reference point}} = \left(\overline{\widetilde{\varphi}}^{\text{Maximal Objective}}_{\text{reference point}\quad j}\right)_m$. The maximum for every criterion is defined as, $\widetilde{\mu}^l_{\overline{\widetilde{\varphi}}^{\text{Maximal Objective}}_{\text{reference point}\ j}} = \max_i\left\{\widetilde{\mu}^l_{\overline{\widetilde{\varphi}_{ij}}}\right\}$, $\widetilde{\mu}^u_{\overline{\widetilde{\varphi}}^{\text{Maximal Objective}}_{\text{reference point}\ j}} = \max_i\left\{\widetilde{\mu}^u_{\overline{\widetilde{\varphi}_{ij}}}\right\}$, $\widetilde{\nu}^l_{\overline{\widetilde{\varphi}}^{\text{Maximal Objective}}_{\text{reference point}\ j}} = \min_i\left\{\widetilde{\nu}^l_{\overline{\widetilde{\varphi}_{ij}}}\right\}$, $\widetilde{\nu}^u_{\overline{\widetilde{\varphi}}^{\text{Maximal Objective}}_{\text{reference point}\ j}} = \min_i\left\{\widetilde{\nu}^u_{\overline{\widetilde{\varphi}_{ij}}}\right\}$, $i \in m,\ j \in n$. More, a correlation coefficient with the reference point, is calculated for each of the alternatives i as follows:

$$\min_j\left\{K_{IVPFS}\left(\tilde{P}\left(\left[\widetilde{\mu}^l_{\overline{\widetilde{\varphi}_{ij}}}, \widetilde{\mu}^u_{\overline{\widetilde{\varphi}_{ij}}}\right], \left[\widetilde{\nu}^l_{\overline{\widetilde{\varphi}_{ij}}}, \widetilde{\nu}^u_{\overline{\widetilde{\varphi}_{ij}}}\right]\right), \overline{\widetilde{\varphi}}^{\text{Maximal Objective}}_{\text{reference point}\quad j}\right)\right\}. \tag{55}$$

Then, the Max- Min- metric of Tchebycheff is used for ranking the alternatives:

$$\max_i\left\{\min_j\left\{K_{IVPFS}\left(\tilde{P}\left(\left[\widetilde{\mu}^l_{\overline{\widetilde{\varphi}_{ij}}}, \widetilde{\mu}^u_{\overline{\widetilde{\varphi}_{ij}}}\right], \left[\widetilde{\nu}^l_{\overline{\widetilde{\varphi}_{ij}}}, \widetilde{\nu}^u_{\overline{\widetilde{\varphi}_{ij}}}\right]\right), \overline{\widetilde{\varphi}}^{\text{Maximal Objective}}_{\text{reference point}\quad j}\right)\right\}\right\}. \tag{56}$$

Step 17. The Full Multiplicative Form of the IVPF-MULTIMOORA, the overall utility of each alternative is determined by the following equation.

$$U_i = \prod_{j=1}^{n} \tilde{P}\left(\left[\widetilde{\mu}^l_{\overline{\widetilde{\varphi}_{ij}}}, \widetilde{\mu}^u_{\overline{\widetilde{\varphi}_{ij}}}\right], \left[\widetilde{\nu}^l_{\overline{\widetilde{\varphi}_{ij}}}, \widetilde{\nu}^u_{\overline{\widetilde{\varphi}_{ij}}}\right]\right). \tag{57}$$

The alternatives are ranked based on the overall utility in a descending order.

Step 18. The ranks obtained in Steps 15-17 are summarized by applying the dominance theory, and the final ranking is obtained.

4 Case study

To show the applicability of the introduced approach in real-world situations, they are applied in a real case study of an Iranian Transport Complex. The company is presented with six alternative marine transport investments, $A = \{A_1, A_2, A_3, A_4, A_5, A_6\}$. Due to the limited investment funds, the company wants to choose the best one of the projects to fund and operate. Their main goal is to improve the selection by focusing on the monetary impacts as well as the non-monetary characters of the alternatives. Hence, the evaluation criteria are classified into monetary and nonmonetary criteria. Monetary criteria include operating benefits and costs. Nonmonetary criteria include benefits to the economy (C_1), productivity (C_2), competency (C_3), safety (C_4), energy consumption (C_5), and greenhouse gas emissions (C_6). These criteria with a brief description are presented in Table1. Due to the confidentiality of the information, a detailed discussion of the projects is not possible.

Table 1: Nonmonetary evaluation criteria

Criteria	Descriptions	Type
Benefits to economy	Benefits to economy, which are the outcome of the transportation system (e.g., employment and resource consumption)	Benefits
Productivity	Ability to reach performance goals	Benefits
Competency	State-of-the-art technology, equipment, and infrastructure used in the transportation system	Benefits
Safety	Safety enhancement of the transportation system	Benefits
Energy consumption	Energy used by the transportation system	Cost
Greenhouse gas emissions	Effects of the transportation system to greenhouse gas emissions	Cost

A group consisting of three experts, $E=\{e_1, e_2, e_3\}$, who had at least eight years of experience in marine transport sectors is formed to assist the process of finding the best sustainable marine transportation project. Questionnaires are applied to get the required data from experts of the aforementioned firm. Due to the available data and the fact that the investments are at the proposal phase, operating benefits and costs are considered as quantifiable monetary criteria, and the rest are directly expressed by experts' judgments. Tables 2 and 3 display the IVPTrF information considering operating costs and benefits while 4 shows linguistic variables are converted into IVPFSs to address nonmonetary criteria values. The linguistic evaluations of alternatives while addressing nonmonetary criteria are depicted in Table 5. The decision-making process for finding a proper sustainable marine transportation project can be explained in detail by steps, which are provided in Sub-sections 3.1 and 3.2. To carry out the financial assessments of the alternatives, equation (27) is used. The DMs have set the value of r equal to 0.05%. Table 6 displays the results. Since the results are in the form of IVPTrF-numbers, the proposed IVPTrF-ranking method, Definition 2.11, should be applied to compare different alternatives on the monetary criteria and produce the final results of the CBA-part. By utilizing the method proposed in Subsection 3.2.1, the weight vector of experts can be obtained through Steps 1 to 7 as follows:

$$\lambda_1 = 0.3399, \lambda_2 = 0.3332, \lambda_3 = 0.3268.$$

Consider that the information about attribute weights given by the DMs is shown by:

$$R = \{w_1 \geq w_3, w_1 \geq w_4, w_2 \geq w_1 - w_3, w_2 \geq 2w_3\}.$$

Based on this information about the weights of the attributes and through Steps 8 to 12, the weight vector of evaluation criteria can be obtained by:

$$w^* = (0.280150, 0.186767, 0.093383, 0.280150, 0.034011, 0.125538)^T.$$

Calculate the weighted overall group IVPF decision matrix Ψ and normalized it as shown in Table 7 through steps 13 and 14. By the ratio system, the Reference Point and the full multiplicative form of the proposed IVPF-MULTIMOORA, the ranking of alternatives is performed through steps 15-18. The results of the three rankings are summarized by using the dominance theory. Therefore, the final ranking is computed and presented in Table 8.

Addressing various evaluations in the process made by various parts of the proposed decision support framework provides a more comprehensive image of the alternative, which perfectly reaches the sustainability framework. As can be seen in Table 9, alternative 3 provides good results from a cost-benefit analysis perspective, while alternatives 4 and 2 perform better in the MCDA. Therefore, this method makes sure that the DMs can have the most informed outcome in their decision-making process by addressing the outcomes of various analysis approaches at the same time.

Table 2: IVPTrF information about operating costs

	i=0	i=1	i=2	i=3	i=4	i=5
A1	([15.0000,17.0000, 18.0000,23.0000], [0.6000,0.7000], [0.3000, 0.4000])	([0.5000,1.0000, 2.0000,2.5000], [0.3000 , 0.4000], [0.6000 , 0.7000])	([2.0000,2.6000, 3.0000,3.2000], [0.3000,0.4000], [0.6000 , 0.7000])	([2.0000,3.0000, 3.5000,3.7000], [0.2000,0.3000], [0.7000 , 0.8000])	([2.0000,3.3000, 3.9000,4.0000], [0.1000,0.2000], [0.8000,0.9000])	([3.0000,3.5000, 3.9000,4.2000], [0.1000,0.2000], [0.8000,0.9000])
A2	([35.0000,37.0000, 38.0000,40.0000], [0.5000,0.6000], [0.4000, 0.5000])	([0.3000,0.4500, 0.5000,0.6000], [0.5000,0.6000], [0.4000 , 0.5000])	([0.3300,0.5000, 0.7000,0.8000], [0.5000,0.6000], [0.4000 , 0.5000])	([0.4000,0.5500, 0.7500,0.8000], [0.5000,0.6000], [0.4000 , 0.5000])	([0.4000,0.6000, 0.7800,0.8000], [0.5000,0.6000], [0.4000,0.5000])	([0.4200,0.6800, 0.8000,0.8000], [0.2000,0.3000], [0.7000, 0.8000])

Table 3: IVPTrF information of operating benefits

	i=0	i=1	i=2	i=3	i=4	i=5
A1	0	([5.0000,7.0000, 12.0000,13.0000], [0.8000 , 0.9000], [0.1000 , 0.2000])	([5.0000,7.0000, 12.0000,13.0000], [0.8000 , 0.9000], [0.1000 , 0.2000])	([5.0000,7.0000, 12.0000,13.0000], [0.7000 , 0.8000], [0.2000 , 0.3000])	([5.0000,7.0000, 12.0000,13.0000], [0.7000 , 0.8000], [0.2000 , 0.3000])	([5.0000,7.0000, 12.0000,13.0000], [0.7000 , 0.8000], [0.2000 , 0.3000])
A2	0	([7.0000,10.0000, 14.0000,16.0000], [0.7000 , 0.8000], [0.2000 , 0.3000])	([7.0000,10.0000, 14.0000,16.0000], [0.7000 , 0.8000], [0.2000 , 0.3000])	([7.0000,10.0000, 14.0000,16.0000], [0.5000 , 0.6000], [0.4000 , 0.5000])	([7.0000,10.0000, 14.0000,16.0000], [0.5000 , 0.6000], [0.4000 , 0.5000])	([7.0000,10.0000, 14.0000,16.0000], [0.5000 , 0.6000] ,[0.4000 , 0.5000])

Table 4: Linguistic variables to address nonmonetary criteria values

Linguistic variables	INPF-numbers
Very poor (VP)	(0.1,0.2), (0.8,0.9)
Poor (P)	(0.2,0.3), (0.7,0.8)
Moderately poor (MP)	(0.3,0.4), (0.6,0.7)
Fair (F)	(0.5,0.6), (0.4,0.5)
Moderately good (MG)	(0.6,0.7), (0.3,0.4)
Good (G)	(0.7,0.8), (0.2,0.3)
Very Good (VG)	(0.8,0.9), (0.1,0.2)

Table 5: Gathered first expert's judgments on nonmonetary criteria - for project 1 and 2

e_1	Benefits to economy	Productivity	Competency	Safety	Energy consumption	Greenhouse gas emissions
A1	F	G	MG	G	VG	MG
A2	G	MG	G	G	VG	G

Table 6: Financial evaluation of monetary criteria

Alternatives	A1	A2	A3	A4	A5	A6
IVPTrFNPV	([-15.5910, 0.7257, 29.6671, 40.5808], [0.9827, 0.9972], [0.0016 , 0.0084])	([-8.7534, 8.5394, 30.3206, 43.2671], [0.7204, 0.8080], [0.3563 , 0.4491])	([22.5546 ,35.821, 50.7899, 66.6078], [0.8404 , 0.9091], [0.1382 , 0.2091])	([56.1246, 62.312, 69.1711, 86.4340], [0.6046 , 0.6769], [0.4644 , 0.4865])	([-2.3767, 34.5171, 40.6463, 68.6854], [0.8875, 0.9589], [0.0591, 0.1381])	([74.5470 , 88.7924, 100.2005, 125.7013], [0.6458 , 0.7281], [0.4747 , 0.5344])
NPV	13.56	7.733	32.295	12.722	29.79	21.229
Ranking	4	6	1	5	2	3

Table 7: Normalized weighted IVPF overall performance decision matrix

	Benefits to economy	Productivity	Competency	Safety	Energy consumption	Greenhouse gas emissions
A1	([0.3318 ,0.4046], [0.7446 ,0.7982])	([0.3250,0.3945], [0.7652 ,0.8164])	([0.1880,0.2323], [0.9349,0.9521])	([0.3770,0.4564], [0.7032, 0.7635])	([0.9421,0.9565], [0.1630 ,0.2031])	([0.8144,0.8551], [0.3139 ,0.3915])
A2	([0.3770 ,0.4564], [0.7032 ,0.7635])	([0.2828,0.3438], [0.7986 , 0.8427])	([0.1935,0.2388], [0.9216, 0.9402])	([0.4454,0.5407], [0.6122 , 0.6939])	([0.9739,0.9806], [0.1404 , 0.1770])	([0.8639 ,0.8941], [0.2398 , 0.2935])

Table 8: Final priority of projects according to the proposed IVPF-MULTIMOORA

Alternatives	Ratio system	Reference point	Full multiplicative form	Final priority
A1	6	3	4	4
A2	1	2	2	2
A3	2	4	3	3
A4	5	1	1	1
A5	4	6	6	6
A6	3	5	5	5

Table 9: Final priority of projects according to the decision support framework

Alternatives	Final ranking of monetary assessment-BCA	Final ranking of nonmonetary assessment-MCDA
A1	4	4
A2	6	2
A3	1	3
A4	5	1
A5	2	6
A6	3	5

5 Conclusions

This paper showed that it was possible to do a composite transport appraisal consisting of both the CBA and MCDA by the presented decision support framework. This framework attempted to contain a wider set of criteria in a transport appraisal than a traditional CBA. To put it differently, it tries to consider and evaluate the multiple criteria, which are also often conflicting and goals that are hard to assess and measure in the monetary sense other than the monetary criteria. Due to the complex socioeconomic conditions, strong decision-making methods for emerging the complicate MCDA and CBA of high vagueness and ambiguity were extended. IVPFSs were used to develop a new MAGDM method with partially known attribute weights and unknown expert weights. Whiles, IVPTrFS as a new uncertainty modeling tool was introduced and applied to address ill-known quantities and inaccuracies existed in the net present value method. The proposed decision support framework was tested in a case study of an Iranian transport complex. The results showed that this framework was promising. Furthermore, it could be used to assess the sustainability of the company's transport projects. For further research, extending the developed decision support framework to support a higher degree of uncertainty in the decision-making process will be an interesting idea.

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