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A new comprehensive project scheduling, monitoring, and management framework based on the critical chain under interval type-2 fuzzy uncertainty

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Abstract

Projects require proper control and management techniques to achieve their defined success criteria. Often projects end in longer periods and higher costs than planned. It is necessary to apply approaches that can keep the project close to its predefined goals and purposes. In this paper, a new framework is presented that is mainly concerned with properly managing project implementation and evaluation. In detail, the method has four main parts. First, a new mathematical programming is offered to address project scheduling. The results of project scheduling are then applied in a critical chain method based approach under interval type-2 fuzzy uncertainty in the second part to both enhance project scheduling and address uncertainty. The third part of the approach is developed to monitor the project implementation process, and new thresholds are defined to discuss corrective measures when necessary. Finally, the fourth part of the approach is designed to evaluate project implementation and reach an understanding of how the project is implemented. To display the applicability of the approach, a practical example is solved, and the process is step by step presented along with the comparative analysis.

Keywords: Project scheduling, critical chain method, mathematical programming, project monitoring, resource constrained project scheduling, interval type-2 fuzzy sets.

1 Introduction

The main elements of project management are planning, scheduling, and control of its activities. Given the criticality of delivering high-quality outcomes on time and within the specified budget, project management is an important and hot topic for both practitioners and researchers [38, 39]. One approach in scheduling is the resource-constrained project scheduling problem (RCPSP) which has been the focus of many scholars and practitioners. In the traditional method of RCPSP, activities are often scheduled with the goal of minimizing the total makespan of the project. This minimization process is subject to various technological and/or organizational precedence constraints and other constraints that address the limitation of required resources [29, 32]. To schedule a project, it is essential to know the respective environment comprehensively. Due to the scarcity of resources, one of the most critical issues in project scheduling is addressing this problem by considering the limitations of resources [14, 23, 26]. In such cases, activities utilize time in addition to certain resource capacities when being implemented [3, 9]. As a result, it is possible to observe delays in the initiation times of activities caused by a lack of required resources of some project activities [10, 24, 25]. One approach to improve project scheduling, given the limitations of resources, is critical chain method (CCM).

Critical chain project management (CCPM) was initially introduced as a novel approach to address project scheduling and management [2]. It is possible to express CCPM as the most innovative technique in project scheduling [11]. The critical chain is defined as the longest path made while balancing resources by addressing the constraints that exist between activities.

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In recent years, the ambiguity which characterizes the project environment has seriously challenged the role of deterministic static project scheduling [4, 36]. Given the dynamic conditions of today's environment, project scheduling is subject to high levels of ambiguities. Another area of uncertainty is the possibility of having to implement new activities or dropping some of the initially existed activities Wang et al. [37]. Most of the project scheduling models assume the existence of full information about scheduling in addition to static deterministic conditions. It is needless to state that real-world situations are much more complicated and are subject to high degrees of ambiguity. Recent studies on project scheduling have resulted in an increasing focus on the concept of ambiguity [8]. Pramanik et al. [31] used fuzzy TOPSIS to reduce investment risks in information system projects. They proposed a framework of business intelligence to obtain the appropriate project with the decision-making procedure.

Kimiagari and Keivanpour [20] focused on the risk of mega projects by using fuzzy group decision making. In their study, projects were dependent, and their tasks were suitable for construction projects. Besides, they regarded a real case study of oil and gas industry. Hazir and Ulusoy [15] proposed a classification of uncertainty in projects. They studied uncertainty in project scheduling and discussed five fundamental approaches of fuzzy scheduling, stochastic scheduling, sensitivity analysis, robust scheduling, and reactive scheduling.

Hu et al. [17] employed the CCM and buffer management to improve the project robustness. They introduced six priority indices to determine the optimal critical chain. In addition, they concentrated on the reactive actions in the proposed framework. Besikci et al. [1] introduced a multi-project problem environment which involved multiple projects with assigned due dates; activities that had alternative resource usage modes; a resource dedication policy that did not allow sharing of resources among projects throughout the planning horizon and a total budget. Ning et al. [28] examined the multi-mode cash flow balanced project scheduling problem with stochastic duration of activities where the objective was to generate a robust baseline schedule to minimize the contractors maximal cumulative gap between cash outflows and cash inflows.

Su et al. [34] addressed an optimal approach to generate the critical chain schedule. A critical chain project scheduling problem was formulated considering both the resource constraints and duration uncertainties. It was hypothesized that the complexity of the networks influenced how efficiently allocated buffers could fulfill their protective role. Therefore, the relationship between complexity indices and the delay-mitigating performance of CCPM were explored. Ghoddousi et al. [12] developed a multi-attribute buffer sizing method aimed at maximizing the robustness of the buffered schedule generated. The project attributes concerning the network complexity, flexibility criteria, criticality index, and robustness measures were considered through the buffer sizing process. The methodology was based on the critical chain buffer management methodology.

A new procedure for generating a proactive baseline schedule for the RCPSP with stochastic activity durations was developed by Lamas and Demeulemeester [21]. RCPSPs with known deterministic renewable resource requirements and uncertain activity durations were investigated by Chakrabortty et al. [7]. Hu et al. [16] proposed project schedule monitoring framework by introducing the activity crucially index and using probability theory. Developing an enhanced CCPM framework for effective implementation of projects related to construction is proposed by Sarkar et al. [33]; they presented a structure that improved buffer sizing by integrating the various uncertainties.

It can be concluded from the above that project scheduling and control is one of the most important aspects of any project. Few studies have simultaneously addressed both of these issues which are mainly caused by the complexities. Another vital point is addressing uncertainty in projects. Projects are unique, and the lack of historical data is a common phenomenon in them; therefore, the utilization of fuzzy sets in projects is a proper task. Another issue that calls for the application of fuzzy sets is that there are many situations, in which using judgments of project experts becomes inevitable. In this paper to address the mentioned issues, a new approach is presented in project scheduling and control.

The introduced approach has four main parts. In the first part, RCPSP with variable duration is carried out and solved with simulated annealing algorithm (SA). In the second part, uncertainty is added to the obtained scheduling through introducing a new buffer sizing approach that utilizes interval type-2 fuzzy sets (IT2FSs). Then, by using novel evaluation indexes and based on the utilization of buffers, the right decisions for the rest of the project are made. Finally, in the last step, new indicators are introduced to investigate how the project has been implemented. The proposed method has framework novelty because the planning, scheduling, and monitoring considered together at the presence of uncertainty. The following presents the main novelties of this paper:

- Presenting a novel framework for critical chain project implementation and control that addresses project scheduling, buffer management, and project implementation evaluation.
- Introducing a new model of RCPSPs in which the durations of activities are considered as intervals, and the exact value is optimized according to minimizing project's makespan.

- Developing a novel method of buffer sizing under IT2FSs. In other words, the deterministic scheduling of the RCPSP model is changed to IT2F-based schedule by adding a buffer.
- Introducing appropriate threshold to monitor and control levels of buffer consumption.
- Introducing new indicator for evaluating the success rate of project implementation to evaluate how successful the project team has been in managing a project.

To conclude this review of recent studies and to highlight the novelties of this paper, Table 1 is presented. The classifications in this table are inspired by the studies of Hazir and Ulusoy [15], Habibi et al.[13], and Ghaffari and Emsley [11]. Scheduling, monitoring, and control at the presence of uncertainty in a comprehensive framework have not been mentioned by other articles.

Table 1 gives a better presentation of the literature gaps and the novelties of this paper. To summarize, the motivations of this paper are as follows:

- Most of the studies try to focus on a specific section of this problem, and only a few of them have presented new frameworks. Having an appropriate framework is an essential issue because in real-world situations such a framework can be beneficial for the project manager.
- In many cases, especially in construction projects, the durations of activities are determined by using time-resource trade-offs. In most of the studies, the durations of activities are considered as fixed while in real-world situations, this rarely happens. Using interval durations is closer to the real condition of projects.
- The CCM is a relatively new method that has been applied in many studies. However, using CCM in a comprehensive framework is an interesting application that has been ignored.
- Many studies have tried to determine the buffer size, while the monitoring procedure is an essential issue to protect the schedule against the uncertainties. So, the combination of buffer sizing and buffer monitoring methods is a practical approach that requires proper presentation.
- Projects usually face uncertain environments [12, 30, 35]. Using fuzzy extensions to address such situations is one of the main motivations of this paper.

Table 1: Literature review on project scheduling, monitoring, and management

Author(s)	year	Uncertain approach			RCPSP		RCPSP				Results
		Fd	Id	Vd	Bm	Bs	Cf	IT2F	P	D	
Long et al.	2008		✓	✓		✓	✓				RCPSP and fuzzy CCM
Beikci et al.	2015		✓							✓	Multi-mode RCPSP
Hu et al.	2016				✓				✓		Buffer monitoring
Su et al.	2016					✓			✓		CCM with schedule network complexities
Oztemel et al.	2017			✓						✓	Multi-mode RCPSP
Bruni et al.	2018	✓							✓		RCPSP with stochastic durations
Pramanik et al.	2019						✓				Fuzzy project selection
Kimiagari et al.	2019						✓				Risk analysis with fuzzy uncertainty
Haziret al.	2019						✓	✓	✓	✓	Project scheduling under uncertainty
Hu et al.	2019				✓	✓			✓		Critical chain sequencing
Tian et al.	2019	✓				✓			✓		RCPSP and CCM
Petroutsatou	2019				✓	✓			✓		CCM and project scheduling
This study	2020	✓	✓	✓	✓	✓	✓	✓			

Fd = Fixed duration, Id = Interval duration, Vd = Variable duration, Bm = Buffer monitoring,
Bs = Buffer sizing, Cf = Conventional fuzzy, P = Probabilistic, D = Deterministic.

The outline of this paper is as follows: In section 2, a methodology is presented, and the introduced framework is given. In section 3, a practical example is shown and solved by using the proposed approach. In this section, comparative analyses and discussion of results are presented. Finally, in section 4 the concluding points of this paper are presented.

2 Introduced project scheduling and control approach

2.1 The entire introduced approach

In this part, a novel approach is presented for project scheduling and control. The method has four main parts. The main steps of the approach are as follows: (1) using an improved form of RCPSP in which duration of activities are not fixed. To solve the mathematical model, a conventional SA is utilized; (2) addressing uncertainty and buffer sizing which is carried out based on the obtained values of the first step; (3) monitoring project execution in which the proposed method focuses on three thresholds to assist the project manager; (4) measuring project performance which is carried out when the project is completed. Figure 1 shows the flowchart of the presented framework.

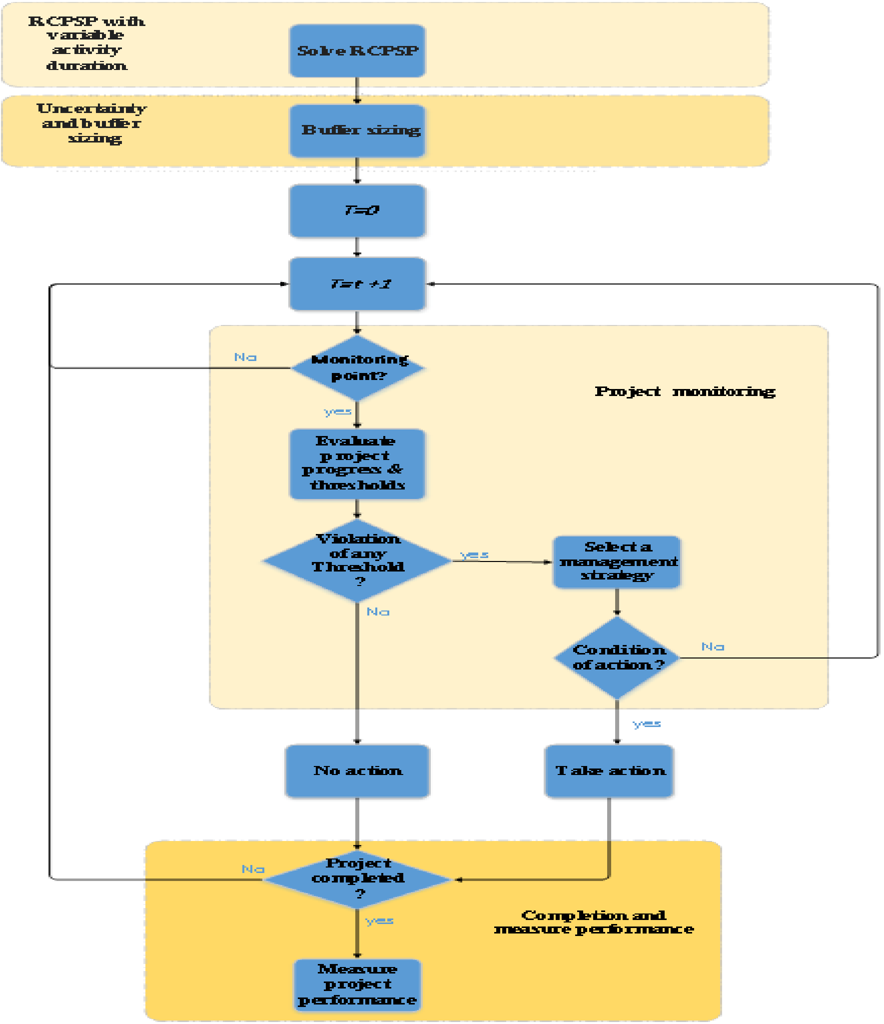

Figure 1: Flowchart of new project monitoring and management approach

2.2 Enhanced RCPSP

First, a mathematical model based on RCPSP is introduced that has variable times for activities. The model addresses the discrete time/resource trade-off problem (DTRTP). In other words, the DTRTP model is first addressed in the model to attend to project scheduling. This method of project scheduling results in project durations which are under considering DTRTP and thus are enhanced while comparing with RCPSP. The model applies the following notations:

$j = 1,2, \ldots, J$	Project activities
dl_j	Minimum duration of activity j
du_j	Maximum duration of activity j
$r_{j,k}$	All the necessary amount of resource k to undertake activity j
$j=J+1$	Dummy activity denotes completion of the project
EF_j	Earliest finish time of activity j
LF_j	Latest finish time of activity j
t	Time period
d_j^*	Assigned duration of activity j
p_j	Precedence of activity j
$R_{k,t}$	Total availability of resource k at day t

Mathematical modeling of the problem is as follows:

$$Min \sum_{t=EF_{J+1}}^{LF_{J+1}} t x_{J+1,t} \tag{1}$$

S.t.:

$$\sum_{t=EF_j}^{LF_j} x_{j,t} = 1, \forall j \tag{2}$$

$$\sum_{t=EF_i}^{LF_i} t x_{i,t} \leq \sum_{t=EF_j}^{LF_j} (t - d_j^*) x_{j,t}, \forall j; i \in p_j \tag{3}$$

$$\sum_{j=1}^{J} \frac{r_{j,k}}{d_j^*} \sum_{b=\max\{t,EF_j\}}^{\min\{t+d_j^*-1,LF_j\}} X_{j,b} \leq R_{k,t}, \forall k, t \tag{4}$$

$$dl_j \leq d_j^* \leq du_j, \forall j \tag{5}$$

$$x_{j,t} = \begin{cases} 1, & \text{if activity } j \text{ is finished at time } t \\ 0, & \text{otherwise} \end{cases}$$

Eq. (1) is the objective function which is presented to minimize project makespan. Eq. (2) guarantees that each activity is executed exactly once. Eq. (3) confirms the precedence relationships of activities. Eq.(4) represents resource limitation, and finally, Eq. (5) represents that activities can require times that are not fixed and change in predefined intervals. Solving the presented model results in obtaining project scheduling while attending to DTRTP. However, the results do not address uncertainty, and the obtained schedule can be enhanced by using IT2FSs and CCM.

2.3 Considering uncertainty and buffers

Many RCPSPs contain levels of uncertainty which are caused by the vagueness of activity durations. Many factors can make uncertainty for activities (e.g., extreme conditions). Such uncertainties can cause disruptions, issues with timings, extra resource consumption, efficiency losses, and disputes among stakeholders. Thus, it is essential to deal with project vagueness. In this paper to address uncertainty of activities, fuzzy sets theory is used. The membership function is set in [0, 1], and is applied to show the degree in which element t belongs to a fuzzy set. The membership function relies on how the expert expresses the availability of resources. The introduced approach employs trapezoidal fuzzy numbers (TrFNs) (a_1, a_2, a_3, a_4) to assess the vagueness in the required time of activities, as depicted in Figure 2.

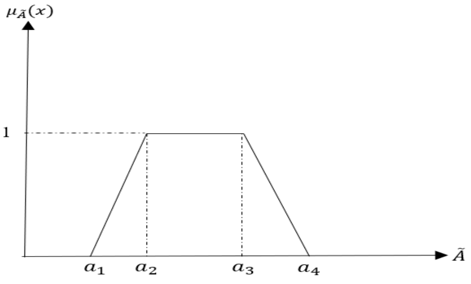

Figure 2: Trapezoidal fuzzy number A

Given the fact that the main issue is with computation and analysis of uncertain project time under limited resources, in the second stage, the crisp activity times obtained in the previous level and experts judgments are applied to form activity duration values in the IT2FSs [23, 26]. Figure 3 illustrates an IT2FN.

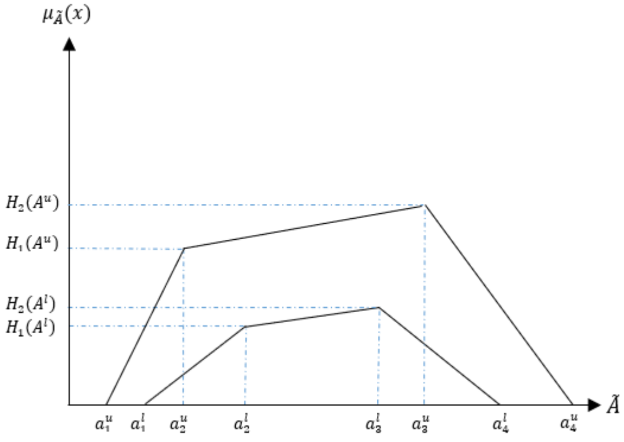

Figure 3: A graphical presentation of the IT2FN A

IT2FSs are used for the following reasons: in cases where uncertainty is high, expressing membership degrees by using crisp values is difficult. Type-2 fuzzy sets are known by two membership degrees which are called the primary and secondary membership degrees. This would give more degrees of freedom and flexibility [5, 6]. However, type-2 fuzzy sets given their computational complexities are difficult to use. Hence, interval type-2 fuzzy sets are applied.

In this part of the proposed method, the deterministic schedule is applied, and project buffer (PB) is added to it to attend to vagueness. To determine the value of PB, first, an appropriate duration is required. This duration should have the highest possible degree. In the proposed method, the time taken at the first stage is regarded as the duration with the maximum possible degree, i.e., d_j^*, which has $\mu = 1$. The reason for this choice is that d_j^* is resulted in addressing all the constraints through solving the mathematical model. So, the values of b and c are obtained by constraint $du_j \leq d_j^* \leq dl_j$. By using d_j^*, dl_j , du_j as reference values and experts' judgments on the criticality and importance of the project, IT2FN is constructed.

The next step is to find the duration that has the highest agreement. Therefore, it is necessary to introduce the agreement index (AI). The AI is applied to compute the possibility between two fuzzy events by calculating the agreement of two fuzzy events [18]. AI is defined as follows:

$$AI(A,B) = {}^{Area(A\cap B)}/_{Area(A)} \tag{6}$$

where $Area(A \cap B) = \int \mu_{A\cap B}(t).dt$

and $Area(A) = \int \mu_A(t).dt$

The Eqs. show that AI denotes what percentage of the fuzzy event (A) is located inside the boundaries of the fuzzy event (B). This index is applied to replace a fuzzy time by a deterministic value. To make sure that an activity with fuzzy times is done within its duration, in this paper, it is considered that the planner will apply the duration (dh^*) which has the highest agreement index (AI = 0.95) to assess activity time. The dh^* is introduced by Long and Ohsato [22]. They used the dh^* to obtain the buffer while employing the conventional fuzzy numbers.

The dh^* is computed by using Eqs. (7-9) where A is depicted by TrFN (*a(j), b(j), c(j),d(j)*) and the expected event B is shown by TrFN (0, 0, dh^u, dh^u) and TrFN (0, 0, dh^l, dh^l). Actually, the value of the dh^* is used to find the duration with the certain AI as follows:

$$AI(A,B)=0.95$$

$$\frac{(c^u(j)-b^u(j)+d^u(j)-a^u(j))/2-(d_j{}^*-dh^u)/(2.(d^u(j)-c^u(j))}{(c^u(j)-b^u(j)+d^u(j)-a^u(j))/2}=0.95 \tag{7}$$

$$\frac{(c^l(j)-b^l(j)+d^l(j)-a^l(j))/2-(d_j{}^*-dh^l)/(2.(d^l(j)-c^l(j))}{(c^l(j)-b^l(j)+d^l(j)-a^l(j))/2}=0.95 \tag{8}$$

Based on the above Eqs., the value of dh^u and dh^l are evaluated; thus, these values need to be unified, and the optimal duration (dh^*) should be calculated which is shown in Figure 4. The value of the dh^* is obtained by Eq. (9).

$$dh^*=\alpha.dh^u+(1-\alpha)dh^l \tag{9}$$

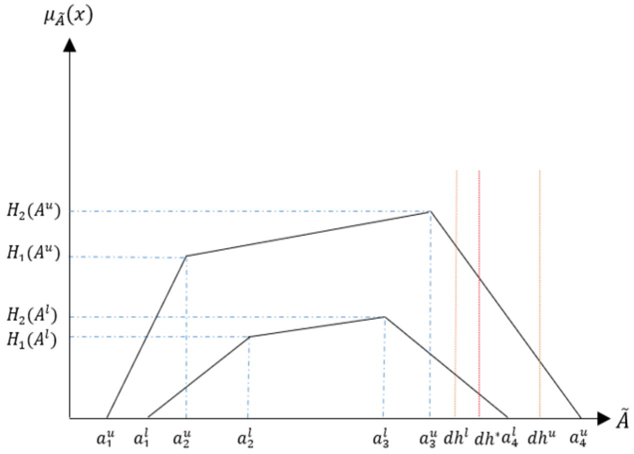

Figure 4: A graphical presentation of dh^*[22]

According to Eq. (9), the value of dh^* is determined, which is somewhere between dh^u and dh^l. The dh^* is dependent on the value of α so when the value of α increases, the dh^* becomes closer to the dh^u and vice versa. To find a buffer size of the project, it is necessary to compute the difference *st(j)* of the proper crisp time $d^*$and the high agreement time dh^*. Eq. (10) yields *st(j)* which is denoted as the safety duration of activity (j). This time is added to the end of the critical chain to absorb possible lateness.

$$st(j)=dh^*{}_j-d_j^* \tag{10}$$

Safety time is obtained for all activities and helps to evaluate the buffer size. The buffer of the critical chain (p) can be obtained by using the squares of the safety time *st (j)* of activities on the chain (p) as follows:

$$PB=\max_{p=1,2,\ldots,P}\sqrt{\sum_{k\in p}St(k)^2} \tag{11}$$

According to Eq. (11), the project buffer is evaluated and added to the end of the critical chain to protect the project plan.

2.4 Monitoring project execution

Project completion time, the schedule risk, and successful utilization of CCM depend on how buffers are monitored [19]. To enhance the existing buffer management (BM) methods, in this part, a novel BM approach is presented. The introduced buffer management method is as follows: three indexes are applied to evaluate and control project in the implementation phase. These indexes provide the project manager with the ability to decide on taking a pessimistic or optimistic stance on the project given the conditions of the project. The indexes are checked so that when the required conditions are not obtained, corrective action would be carried out. There are three main situations for the project manager:

1. Optimistic where the project manager applies corrective action in which all the three indexes are violated.
2. Moderate where the corrective actions are taken when two of the indexes are violated.
3. Pessimistic where even violation of one of the indexes results in taking corrective actions.

Schedule sensitivity index (SSI) combines the activity duration and project duration sample standard deviations with the criticality index. $f_1 = \beta$ is the constant index in which β shows the median of SSI for all project activities. In some studies, other indexes are employed. For example, Hu et al. [16] have used the crucial index (CI). The value of SSI is obtained as follows:

$$SSI = \left[\sqrt{\frac{var(d_j)}{var(C\max)}}\right].CI_j \tag{12}$$

where CI shows the possibility of an activity being on the critical path. The decreasing index decreases when the project is being executed and is obtained as follows:

$$f_1 = \beta - \lambda.PCC \tag{13}$$

where λ is the decreasing slope, and PCC shows the proportion of chain completed. PCC is the ratio of current time to the project completion time that is obtained by solving the mathematical model. As a result, the value of PCC is equal to:

$$PCC = \frac{\text{Current execution time}}{\text{Estimated project completion time}} \tag{14}$$

PBC is the proportion of buffer consumed and is obtained as:

$$PBC = \frac{BC}{BS} \tag{15}$$

where BS is buffer size and BC denotes the estimated activity completion time minus planned activity completion time. β shows the initial value of PBC which is obtained by computing the median of SSI_j for all the activities. For the constant cases, the initial value of SSI is obtained in accordance with the criteria that is used to evaluate how the project has progressed from beginning to the end. The value of this index is obtained as follows:

$$f_2 = \beta \tag{16}$$

The value of β shows the median of SSI_j for all the activities. The increasing index shows that by using any amount of the buffer, the same amount of the project should be consumed. In other words, these criteria equal to $y = x$.

2.5 Project performance measure

As the project is completed, it is necessary to assess how the project has been implemented. This would result in assessing efficiency and leads to better project management in future projects. This evaluation is obtained by comparing the real spent resources and the planned ones while addressing the corrective actions. Therefore, the following performance indicators are applied:

- Number of crashing activities (CAs): This shows the sum of all activities which require correction actions. Larger values of the CA show that more activities needed corrective actions. Therefore, the scheduling and managing of the project are more difficult (Hu et al. [16]).
- Real project duration *(RPD):* This shows the simulated project completion time with or without threshold caused corrective measures (Hu et al. [16]).
- Real completed time to planned completed time (RT/PT): This index shows the quality of the initial scheduling. In other words, values closer to 1 indicate that the scheduling has been closer to reality, thus project implementation and management have been proper since real time was close to planned values.

3 Practical example

An activity network with 11 activities is considered. Figure 5 depicts the network. The total available resources are 25 workers per day. Technological relationships among activities and other input parameters of the problem are given in Table 2 and Figure 5. Figure 5 shows the precedence relationship among activities, and Table 2 provides the resource requirement of activities and maximum and minimum duration of proposed problem example.

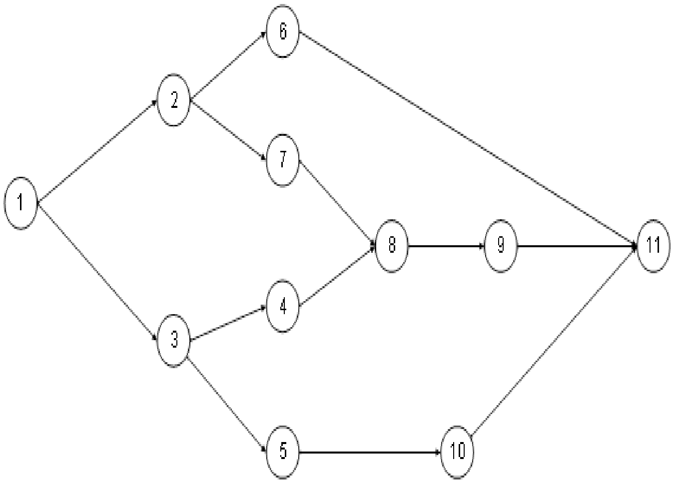

Figure 5: Activity network of the studied project

Table 2: Input data of the project

Activity	$dl(j)$	$du(j)$	$r(j,k)$
1	4	6	82
2	4	6	77
3	4	5	88
4	3	5	33
5	2	4	17
6	6	9	50
7	6	10	50
8	5	8	22
9	3	5	30
10	7	10	66
11	5	8	55

3.1 Result of solving practical example

Given the complexities of the model, SA has been used to solve the model. Therefore, the results depicted in Table 3 are obtained. The obtained values are used to make a Gantt chart of the project which is depicted in Figure 6.

Table 3: The results of project scheduling in the first part of the model

Activity	$dl(j)$	$du(j)$	d^*
1	4	6	82
2	4	6	77
3	4	5	88
4	3	5	33
5	2	4	17
6	6	9	50
7	6	10	50
8	5	8	22
9	3	5	30
10	7	10	66
11	5	8	55

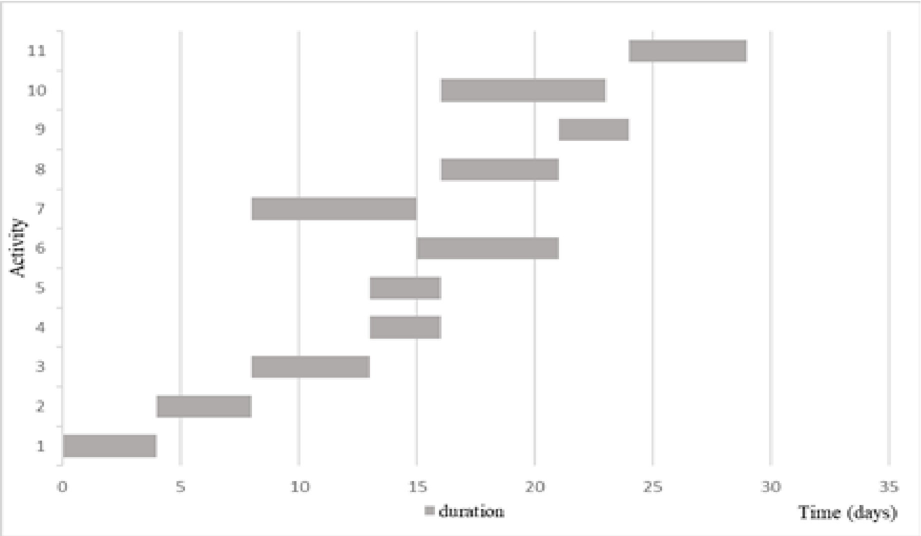

Figure 6: Gantt chart of the project

In the second step of the approach, uncertainty is added to the scheduling. As presented in section 2, IT2FSs are applied. The fuzzy numbers for activities are depicted in Table 4.

Table 4: The IT2FNs for activities

Activity	(a^l, b^l, c^l, d^l)				(a^u, b^u, c^u, d^u)			
Act1	2	3	5	13	4	5	9	25
Act2	2	3	4	12	4	5	9	23
Act3	1	3	4	11	2	5	8	21
Act4	1	2	4	9	2	4	8	18
Act5	1	2	3	7	1	3	6	13
Act6	2	5	7	14	5	8	14	28
Act7	2	5	8	15	5	8	15	30
Act8	2	4	6	8	5	7	12	15
Act9	1	2	4	5	2	4	8	10
Act10	3	6	8	16	6	9	15	31
Act11	2	4	6	10	5	7	12	20

By using *du(j)* and *dl(j)* as b and c and an upper and a lower bound received from project expert, a trapezoidal fuzzy number is formed that is located in the footprint of the uncertainty of the initial IT2FN. Now by using the fuzzy numbers and Eqs. (7-10) and considering $\alpha = 0.4$, the safety time values depicted in Table 5 are obtained. After obtaining the values in Table 5, critical chain and PB should be obtained. The critical chain is obtained as 1-2-7-8-9-11. Also, the PB is computed as 8 days. To implement the third part of the approach, it is needed to obtain the values of CI and SSI in all activities so that by using the median of SSI, the indexes can be obtained. Table 6 presents the values of CI of activities and their SSI. It is worth noting that $f = 0.45$.

λ=0.3 is used to form the decreasing index. Figure 7 depicts the indexes. Since the presented framework has four steps, the fourth step evaluates the project performance by using some indicators. The first indicator is the CA, and in the solved practical example two activities are crashed which are activities 5 and 9. The next indicator is RPD which is obtained at the end of the project. In the solved example, the RPD value is 36. The third indicator which is introduced in this paper is RT/PT. This indicator in the solved practical example is 1.24. This indicator shows the closeness of planned project completion time to the real completion time. The result of the fourth step with the proper description

Table 5: Step-by-step results for $st(j)$

Activity	dh^u	dh^l	dh^*	$st(j)$
Act1	8.88	17.10	12	8
Act2	8.37	16.11	11	7
Act3	7.69	14.80	11	7
Act4	6.55	12.61	9	6
Act5	4.75	9.14	7	5
Act6	10.49	20.14	14	8
Act7	11.10	21.33	15	9
Act8	6.45	12.29	9	4
Act9	4.15	7.91	6	3
Act10	11.79	22.64	16	9
Act11	7.79	14.93	11	6

Table 6: The values of SSI and CI

	Act1	Act2	Act3	Act4	Act5	Act6	Act7	Act8	Act9	Act10	Act11	Median
CI	1	0.71	0.35	0.18	0.19	0.01	0.71	0.85	0.85	0.19	1	
SSI	0.96	0.65	0.26	0.13	0.11	0.01	0.94	0.71	0.45	0.25	0.96	0.45

of the value is provided in Table 7.

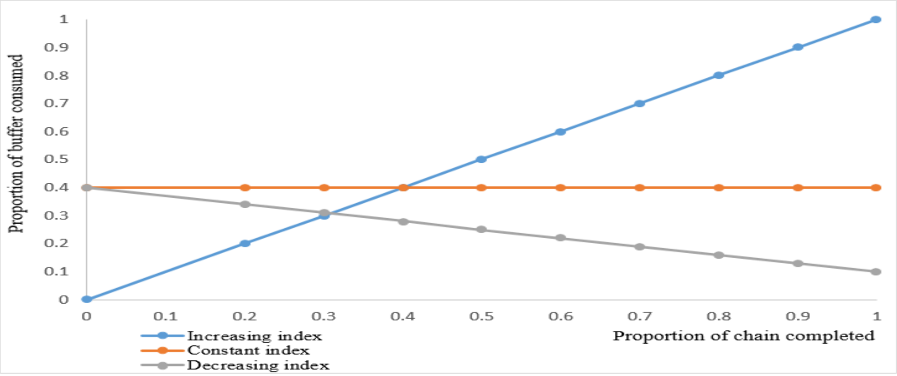

Figure 7: Applied indexes for the project implementation evaluation

3.2 Discussion of results

To better illustrate the application of the proposed approach, different approaches based on optimistic, pessimistic, and moderate conditions are applied and explained in the following: To depict this part of the method, day 9 of the project is selected. On this day, project manager given the uncertainty of the project in the optimistic approach has obtained the Gantt chart depicted in Figure 8.

As it can be viewed on the Gantt chart, on day 9, activity 1 which was supposed to be over in 4 days has taken 6 days to complete. In the optimistic approach, the project manager considers the initial conditions of the project and acknowledges that if the initial issues of the project are resolved; then, the rest of the project would not suffer high levels of delay. Since activities 1 and 2 are on the critical chain, the amount of critical chain from 29 days is extended to 32 days.

Another condition is considered in this example. In this condition, again activities 1 and 2 are delayed as previous

Table 7: Results of the fourth step

	CAs	RPD	RT/PT
Obtained value	2	36	1.24
Description	In the practical example, there are 11 activities, but in the solved model only two of them are crashed which means only 18% of the activities. Lower values of CA are preferable since less crushing costs occur.	The real project completion time is 36 days, while the planned duration is 29 days. Generally, the lower RPD value is better, but rarely the RPD value is equal to the planned duration. Thus, determined RPD in the solved example is 7 days more than planned duration, which is acceptable for most of the projects, especially in the presence of the uncertainty.	In the best situation the RT/PT is equal to one. In the solved example, the value of RT/PT is 1.24; it means 24% deviation from the planned completion time. According to the basic CCM method, the deviation until 33% is due to the casual project uncertainty. In the solved model, this value is 24%, so the project is under control, and the presented framework has provided proper results.

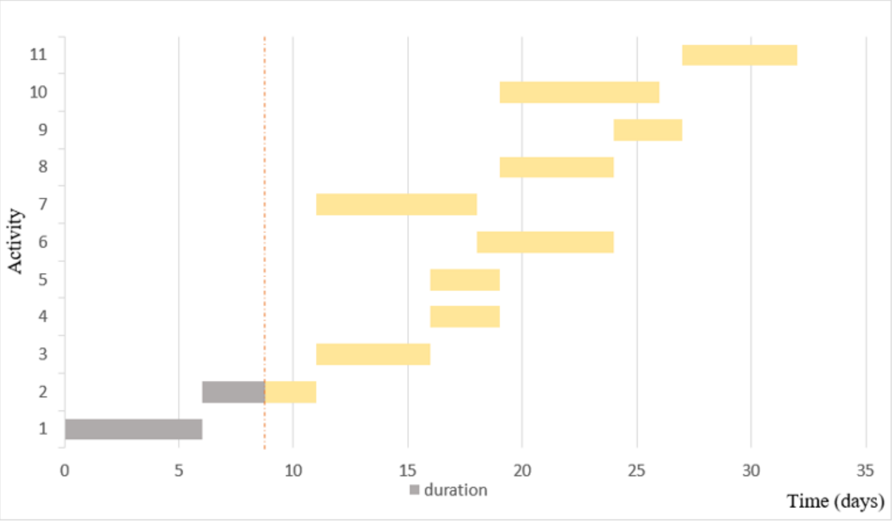

Figure 8: Gantt chart on day 9

conditions. In the moderate condition, the project manager predicts that the next two activities (4 and 5) would have delays, but with proper management of them, the project would not face serious problems. This increase would result in buffer consumption and extension of project durations. Figure 9 presents the Gantt chart of the moderate condition.

Finally, the pessimistic condition is presented. In this condition, project manager reviews the buffer on the 9th day and given the importance of project; the manager can predict the worst case according to the delays. The manager can foresee that after the 9th day, the project would require one extra day. Based on this prediction, the Gantt chart is obtained in Figure 10.

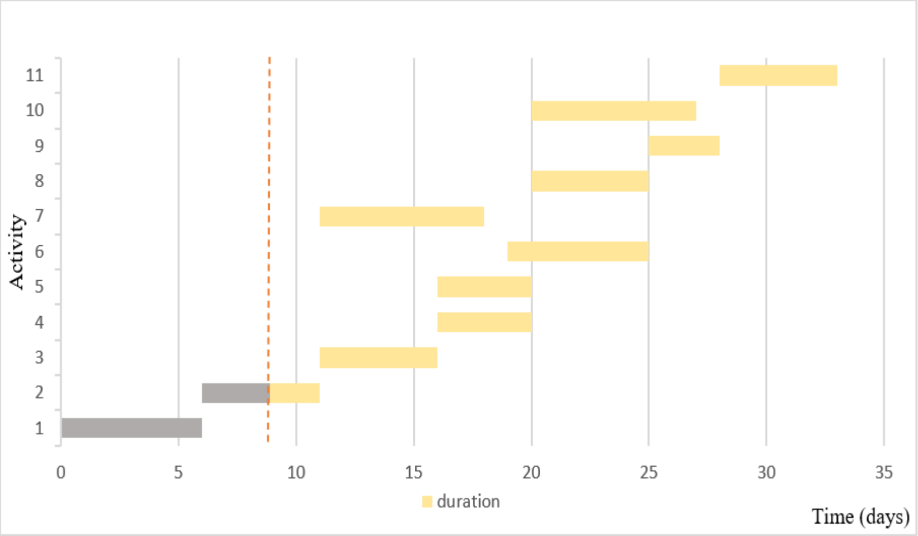

Figure 9: Gantt chart in the moderate situation

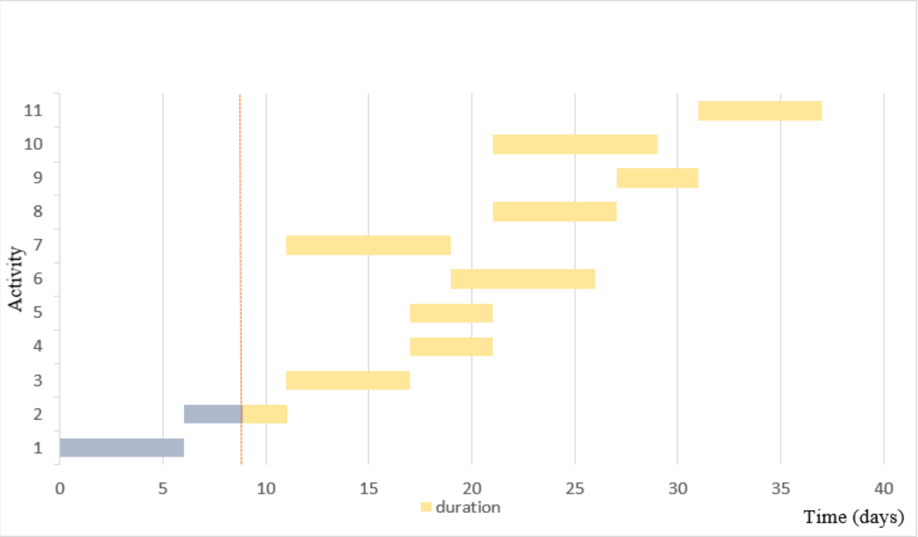

Figure 10: Gantt chart in the pessimistic case

Now, based on these three approaches and the corresponding Gantt charts, project manager can make proper decisions. However, the decision mainly depends on the fact that on the 9^{th} day which index has been violated. Figure 11 shows the condition of the indexes on the 9^{th} day.

The yellow point in Figure 11 shows whether it is necessary to take corrective measures or not. As presented, to take corrective actions in the optimistic case the obtained point should violate all three indexes. However, it is not the case here. In moderate conditions, corrective measures should be taken if two of the indexes were violated. As can be seen, this has happened, and in a moderate stance, it is required to take corrective measures. Obviously, the pessimistic condition has the same outcome. Finally, as the project ends, the stated criteria can be applied to evaluate how the project has been implemented.

Now the following part presents a sensitivity analysis on buffer size and buffer thresholds. The buffer size can be analyzed by α in Eq. (9) and AI in Eqs. (7) and (8). First of all, the value of α effects dh^*, $st(j)$ and PB. So, by changing the value of α, the amount of evaluated project buffer changes as illustrated in Figure 12. The amount of α

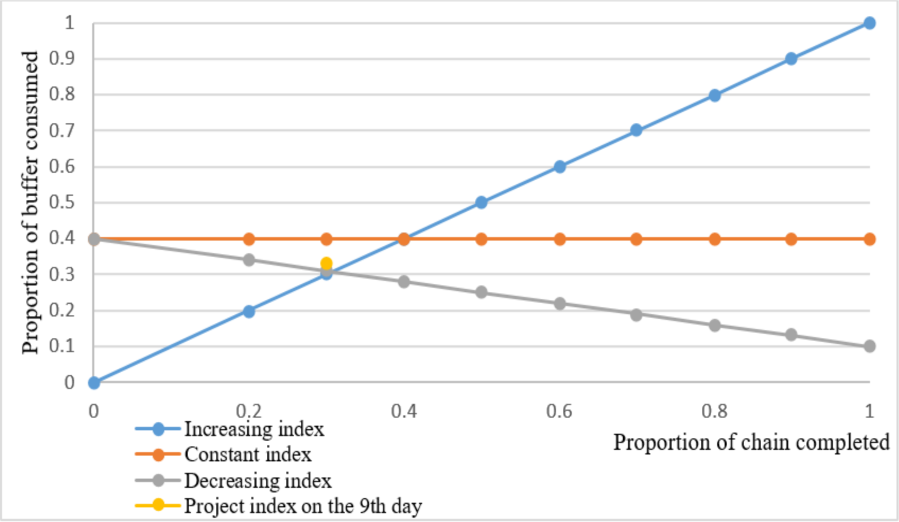

Figure 11: Indexes on the 9^{th} day

changes from 0 to 1 in Figure 12 and as it is evident by increasing the value of dh^* gets closer to dh^u. When $\alpha = 1$ the amount of dh^* equals dh^u. Lowering the value of α moves the value of dh^* closer to dh^l. When $\alpha = 0$ the amount of dh^* becomes equal to dh^l. Buffer size in $\alpha = 0$ is at the highest value because the uncertainty has increased. So, more project buffer should be considered to overcome this uncertainty, and the least buffer size is required when $\alpha = 0$.

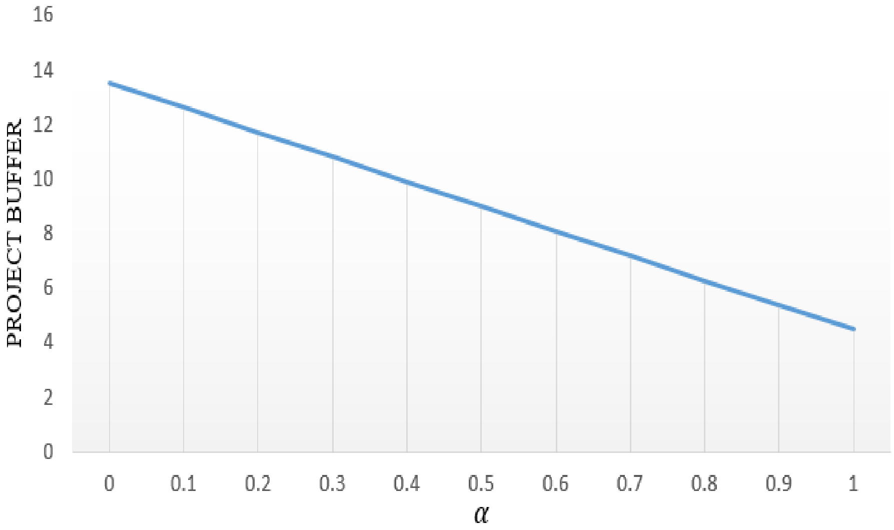

Figure 12: Effect of α on buffer size

Agreement index (AI) shows effects on buffer size based on Eqs. (7) and (8). Figure 14 illustrates the effect of AI against the buffer size. By increasing the AI from 0.75 to 0.95, buffer size also increases. As a result, AI has a direct effect on the value of project buffer. Figure 13 shows the effect of α and AI on project buffer.

In this paper, three different thresholds are proposed. A sensitivity analysis of λ has been done in Figure 14. This figure is provided by the increase of λ from 0.05 to 0.65 which transforms the constant index to the decreasing index. It is clear that by the increase of λ, f_1 becomes closer to f_2 and for $\lambda = 0$, these two thresholds become equal. In Figure 14, the effect of changing the λ on the thresholds is shown by the different colors such that each color indicates the buffer threshold with the particular value of λ. In addition, the increasing index which is shown by the blue line

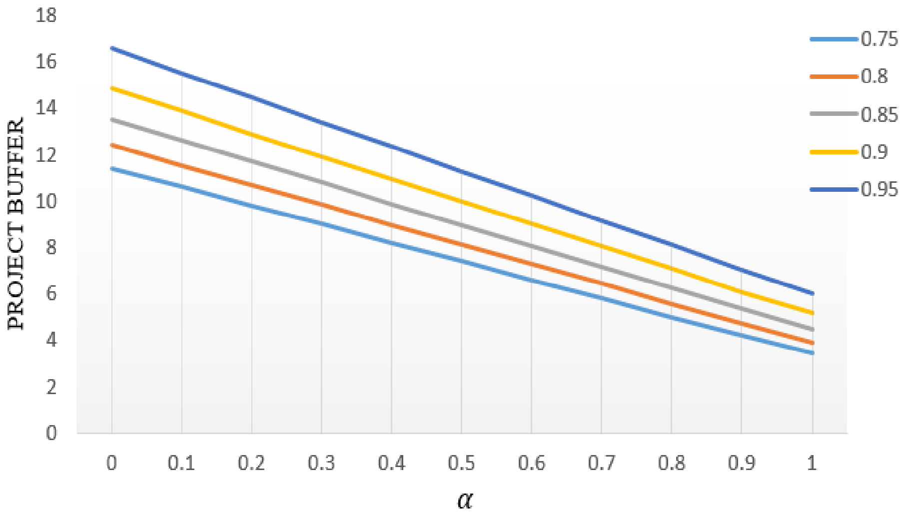

Figure 13: Effect of α and AI on project buffer

is independent of the value of λ. The proper value of λ is important because of its effect on the project managers decisions. For example, a point that needs no planning might require planning or corrective actions by decreasing the value of λ.

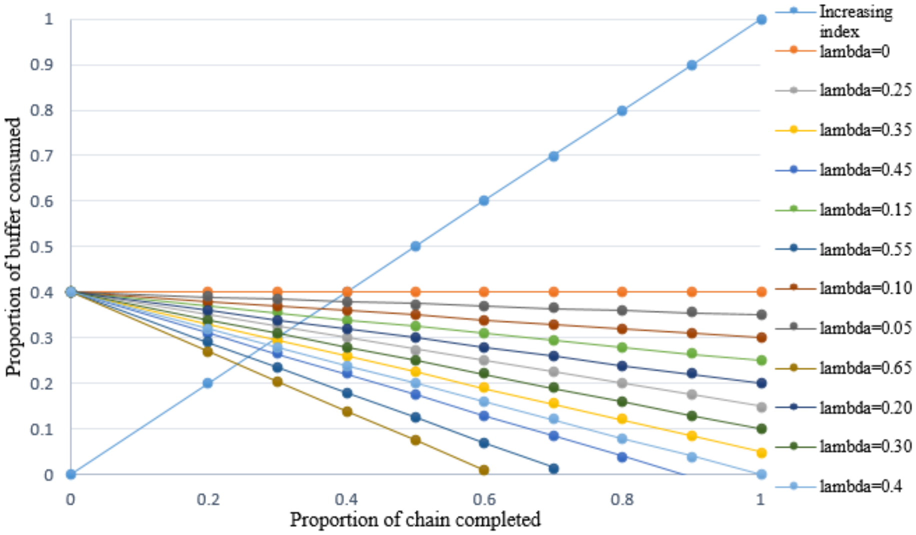

Figure 14: Effect of λ in buffer threshold 1

The results of the proposed mathematical model in this paper and the study of Long and Ohsato [22] are compared. The computational results show that the project completion time in the study of Long and Ohsato [22] is 33 days, while the result of this paper is just 29 days. To better present this comparison, Figure 15 is given. This figure depicts a comparison between the durations of activities in both studies.

Another comparison is carried out based on the presented buffer sizing method. Figure 16 shows the results of the safety times of activities with conventional fuzzy numbers and IT2FNs. It is clear that in the IT2F case since more complexity is depicted, the evaluated safety time for activities should be higher than the conventional fuzzy numbers.

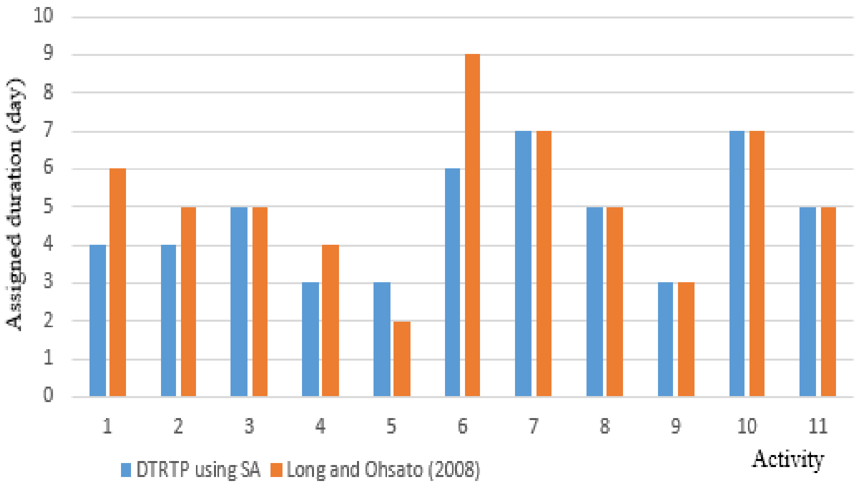

Figure 15: Comparison of assigned duration in this paper and Long and Ohsato [22]

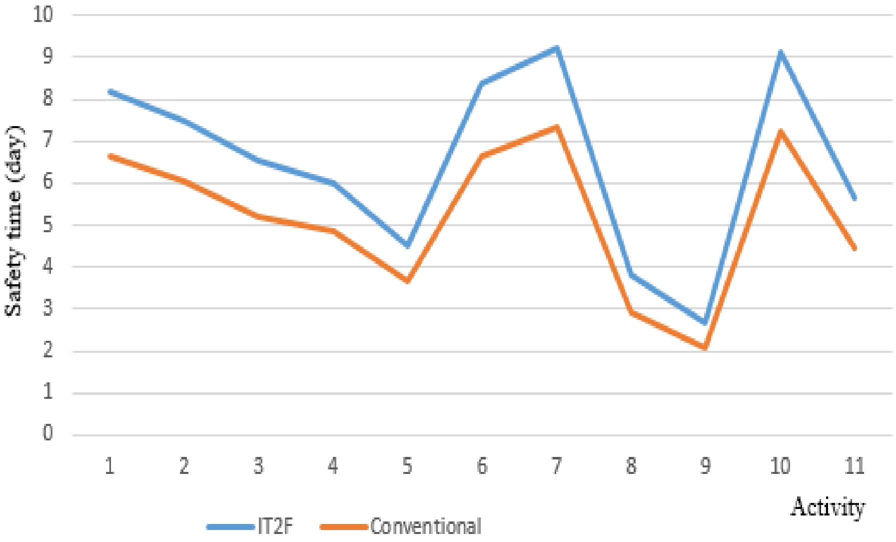

Figure 16: Comparison of safety time by IT2FNs and conventional fuzzy numbers

3.3 Comparative analysis

To compare the presented framework with the existing studies, this section provides several comparisons with two studies that are similar to this paper (i.e., [16, 22]). The study of Long and Ohsato [22] is employed to compare the first and the second steps, while the study of Hu et al. [16] is employed for the third and the fourth steps. Thus, the results of the comparison are reported in Table 8.

Table 8: A summarized comparative analysis

Elements of comparison	Results of comparison
Overall framework	The proposed approach presents a new framework in comparison with the existing papers. Long and Ohsato [22] introduced a framework, but they have no clear monitoring procedure or project performance indexes. Hu et al.[16] proposed a new monitoring method, but the issues, such as obtaining the basic plan and determining the buffer size, were not mentioned. The project planning, scheduling, and controlling is an essential issue that is presented in this paper and was not addressed in the literature.
Mathematical model and solution approaches	The mathematical model in this study is the RCPSP model with consideration of time resource trade-offs. The concept of the model is similar to Long and Ohsato [22], but the formulation is different. Besides, in this paper, the SA algorithm is used to solve the mathematical model, whereas in the study of Long and Ohsato [22] the GA algorithm was applied. The obtained results show that this paper provides a shorter completion time (29 days) in comparison with the study of Long and Ohsato [22] (33 days).
Buffer sizing method	The buffer sizing method was initially introduced by Newbold [27] which determined the project buffer by using safety times. After that Long and Ohsato [22] used the AI to obtain the project buffer. This paper extended the buffer sizing method with IT2FNs. This approach made the model capable of handling complex and highly vague conditions in projects.
Monitoring method	Hu et al. [16] used the CI to build the indexes, but this paper applied the SSI to assess the basic schedule. In addition, the optimistic, moderate, and pessimistic situations for the project manager were not addressed in the study of Hu et al. [16]. Using the SSI index in this paper results in measuring the scheduling sensitivity.
Project performance index	Hu et al. [16] introduced the project performance indicators, but the RT/PT is a new index that is added to the existing indicators. This index is defined by comparing the actual completed time to planned completed time. The newly presented index RT/PT is easy to understand and quickly shows the overall performance of the project manager.

4 Conclusions

In this paper to address the essential task of project scheduling and control, a new approach is presented based on type-2 fuzzy sets (T2FSs). The proposed approach is divided into four main parts: First, project scheduling is addressed through the RCPSP model that discussed discrete-time trade-offs, and solved using the simulated annealing (SA) method. Second, the critical chain method is used where the uncertainty in project durations is added by using T2FSs. Then in the third step, for the implementation phase of projects, three indices are introduced that are employed in different approaches to determine whether it is required to take proper corrective measures or not. Finally, for the completion of the projects, some criteria and measures are presented to evaluate the performance of the project managers. In other words, the presented framework consists of several main steps. In the first step, a new mathematical modeling is offered for the RCPSP with the discrete time-resource trade-off that is solved using the SA. In the second step, the buffer sizing method is developed using the T2FSs. The third step of this paper is extended regarding the existing buffer monitoring method by using schedule sensitivity index (SSI) to evaluate project implementation conditions. After that, in the fourth step to measure the project performance new performance indicator (RT/PT) is introduced.

To show the application of the proposed framework, a practical example is provided and solved. The application demonstrates several advantages. First, it indicates that the approach could address scheduling and vagueness of projects by a novel approach that has the merits of RCPSP, DTRTP, CCM, and IT2FSs, concurrently. Moreover, the approach has a unique perspective on project implementation and provides the project managers with a proper image on the condition of the project and assists them in deciding to take appropriate actions. Besides, the result illustrates that the presented framework is capable of determining if the project has been implemented successfully and efficiently or not.

For further studies, using group decision-making methods for determining the RCPSP parameters can be added to the presented framework. Other considerations in the RCPSP, e.g., multi-mode, multi-project, and overlapping, can be investigated further. In addition, using different meta-heuristic algorithms to solve the problem and comparing the results with this paper can form another interesting future research direction. Finally, to build a robust project schedule, a project buffer can be allocated to the activities.

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