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Original paper

Robust output feedback adaptive sliding mode control for a class of uncertain nonlinear systems using robust adaptive fuzzy observer

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Abstract

This article presents a new robust adaptive sliding mode controller for a class of uncertain nonlinear systems whereas only the system output is measurable. Firstly, a robust adaptive fuzzy observer is designed for the system in order to estimate its state variables. The robust asymptotic convergence of the proposed observer is proven by Lyapunov direct method. Then based on the observation states, a robust adaptive sliding mode controller is suggested such that the closed loop system to be asymptotically stable. Robust asymptotic stability of the overall system suggested by the controller is also confirmed based on Lyapunov theory. Simulation results illustrate practicality and effectiveness of the proposed technique for controlling uncertain nonlinear systems.

Keywords: State observer, adaptive fuzzy control, adaptive sliding mode control, Lyapunov theory

1 Introduction

Since the most systems are inherently nonlinear and uncertain, choosing a control method with ability to applying to the nonlinear systems which can overcome to the above issues is vital. Many nonlinear control methods can be used to control the nonlinear systems where the sliding mode control (SMC) is one of them. Sliding mode control is an easily understandable nonlinear variable structure method which guarantees stability and robustness of a system. In recent years, SMC has been used as a new control method for a wide range of systems, such as nonlinear, time-varying and fault tolerant systems [12, 15, 23, 31]. In SMC method, when system operates near sliding surface, it causes chattering phenomena and also inherits a discontinuous control action. It is necessary to note that there is a trade-off between chattering and robustness in SMC method. In order to control a system with unknown parameters or dynamics and also to reduce the chattering phenomena, other control methods can be combined with SMC and make more innovative control approaches such as adaptive sliding mode [17, 22], fuzzy sliding mode [18, 29], adaptive fuzzy sliding mode [1, 30], neuro-fuzzy sliding mode [25, 26] and adaptive neuro-fuzzy sliding mode [13].

Recently, many control researches have been done by SMC methods. [20] proposes two model-free sliding mode control structures in the frame work of MIMO system validated by experimental results. In [11], an adaptive sliding mode control has been presented for Markov jump nonlinear systems with actuator faults. Since the bounds of the actuator faults have been considered unknown, the main point of the research has been designing an adaptive sliding mode controller to overcome these issues. Another adaptive sliding mode control has been presented in [16] for a class of uncertain nonlinear systems which is based on dynamic PID sliding mode control. In this research, an adaptive parameter tuning method is used to estimate the bounds of disturbances. Stability of the control structure has been proved by Lyapunov stability theory. Finally, simulation results have been shown for an inverted pendulum system. Sliding mode method has been also used to control nonlinear fractional order systems [3]. In the article, a sliding surface with additional nonlinear part has been extracted by Lyapunov method, which can select the parameters of the switching law so that reaching time to the sliding surface not to be sensitive to initial conditions, although the adaptive

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switching law may not guarantee finite-time convergence. Then, stability of the closed loop system has been discussed through Lyapunov method. By the simulation results, the effectiveness of the proposed method has been illustrated.

In practice, all state variables of a system are not available or measurable. Hence, observer-based methods have been employed to estimate the unavailable and unmeasurable states [8, 9, 10]. Also, several extended observer-based and disturbance observer combined with SMC methods have been already presented [2, 4, 5, 6, 7, 14, 15, 28]. For example, an extended state observer-based sliding mode control has been offered for three-phase power inverters in [15]. The suggested control structure consists of two loops: an inner loop for tracking desired values based on second order sliding mode control (SOSMC) and outer loop for regulating output voltage which consists of an extended state observer. Experimental results on a real power converter prototype have validated the usefulness of the proposed method. Another work in [28], has presented an extended observer-based sliding mode control for DC-DC buck power converter systems. This method has been designed for the system with mismatched disturbances. Results illustrate that the proposed method obtains better disturbance rejection ability in compare with nominal SMC method. This method has been also used in [6] to control the active suspension systems. Due to uncertainties and disturbances produced by nonlinear damper, spring, load variations and road profile, an observer has been employed to estimate them. Efficacy of the proposed method has been verified for three different road profiles applying an experimental setup. An adaptive MIMO extended observer has been designed in [4] for underwater robot. In addition to the estimating unmeasurable states, this observer can also estimate the external disturbances. Then an integral sliding mode controller has been designed by Lyapunov theory for the whole system. Effectiveness of the proposed approach has been compared with conventional PD controller by applying on an underwater robot in a practical experiment.

In this article, a new robust adaptive sliding mode controller is designed for a class of uncertain nonlinear systems. In addition to unknowingness of the nonlinear dynamics of the system, only the output of the system is assumed to be measurable. Therefore, at first, a robust adaptive fuzzy observer is suggested to estimate the system states while the nonlinear dynamics of the system are estimated by fuzzy approximators. Also integrated bound of un-modeled dynamics, disturbance and approximation error of the nonlinear dynamics of the system is also estimated adaptively. Secondly, a robust adaptive sliding mode controller is proposed for the system utilizing the observation states. Robust stability of the suggested observer and controller are demonstrated by Lyapanov theory. Results of the proposed technique on two simulative system will be shown to verify the effectiveness of the method for uncertain nonlinear systems.

This paper is arranged as follows: In Section 2, uncertain nonlinear system is introduced. In Section 3, fuzzy inference system as a universal approximator is explained briefly. In Section 4, the proposed output feedback robust adaptive sliding controller is developed. In Section 5, simulation results of the suggested method are shown on two case studies i.e. cart and pole and a chaos system. Last of all, Section 6 concludes the characteristics of the recommended control method.

2 Problem formulation and assumptions

In this section, the uncertain nonlinear system is described. Consider a SISO system with the following equations

$$x^{(n)} = f(X) + \mathrm{g}(X)\,u + d(t), \tag{1}$$

$$y = x,$$

where $X{=}[x,\ \dot{x},\dots,\ x^{(n-1)}]^T = [x_1,\ x_2,\dots,\ x_n]^T$ is the state vector, y is the output, u is the input, $f(X)$ and $\mathrm{g}(X)$ are the smooth functions and $d(t)$ denotes the bounded uncertainties include un-modeled dynamics and external disturbance. It must be noted that many uncertain nonlinear systems have dynamical model in the form of (1) such as chaotic system [1], Magnetic Levitation System [8], robot manipulator [27], inverted pendulum [21], and so on. Dynamical equation (1) can be rewritten as

$$\dot{X} = AX + B\left(\ f(X) + \mathrm{g}(X)\,u + d(t)\ \right), \tag{2}$$

$$y = C^T X,$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \text{ and } C^T = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

The control objective is to determine a control signal u such that the state x tracks a desired reference x_d in spite of model uncertainties and external disturbances, also only the output of the system i.e. y (or x) is measurable. Thus, the tracking error is defined as $e = x_d - x$.

Assumption 1. The reference state vector $X_d = \left[x_d, \dot{x}_d, \ldots, x_d^{(n-1)}\right]^T$ and its derivative i.e. $\dot{X}_d$ are supposed to be exist and bounded.

Assumption 2. In order to system (1) be controllable, it needs $\mathrm{g}(X) \neq 0$. Therefore, without loss of generality the function $\mathrm{g}(X)$ is assumed to be positive.

Now, a sliding surface is defined as

$$s(E) = K^T E, \tag{3}$$

in which

$$E = \left[e, \dot{e} \ldots,\ e^{(n-1)}\right]^T = X_d - X, \tag{4}$$

is tracking error vector and $K = [k_{n-1},\ k_{n-2}, \ldots, k_1,\ 1]^T$ is to be selected such that the polynomial $\Delta(s) = s^{n-1} + k_1 s^{n-2} + \cdots + k_{n-2}s + k_{n-1}$ is stable i.e. all roots of $\Delta(s)$ are in the open left half of the complex plane. From (3), it can be seen if $s(E) = 0$, then the tracking error asymptotically converges to zero with the following dynamic

$$K^T E = e^{(n-1)} + k_1 e^{(n-2)} + \cdots + k_{n-1}e = 0. \tag{5}$$

Theorem 2.1. *Suppose in the uncertain nonlinear system of the form* (2)*, functions* $f(X)$ *and* $\mathrm{g}(X)$ *and only the bound of* $d(t)$ *are exactly known, and also all the state variables of the system be measurable, then for some* $q > 0$ *and every* $\lambda > 0$*, the following control signal can guarantee the asymptotic stability of the error* e *to zero.*

$$u^* = \frac{1}{\mathrm{g}(X)}\left[-f(X) + K^T\dot{X}_d - K^T A X + \lambda s + q\,\mathrm{sgn}(s)\right], \tag{6}$$

where s *is defined as* (3) *and* $\mathrm{sgn}(.)$ *is standard sign function.*

Proof. Consider the subsequent Lyapunov function for system (2) with control signal (6) $V_1 = \frac{1}{2}s^2$. Then using (3), (4) and (2), the time derivative of V_1 results

$$\dot{V}_1 = s\dot{s} = sK^T\dot{E} = sK^T\left(\dot{X}_d - \dot{X}\right) = sK^T\left(\dot{X}_d - AX - B\left(\ f(X) + \mathrm{g}(X)\,u + d(t)\ \right)\right). \tag{7}$$

By substituting u from (6) in (7), one can get

$$\begin{aligned}\dot{V}_1 &= sK^T\left(\dot{X}_d - AX - B\left(\ K^T\dot{X}_d - K^T AX + \lambda s + q\,\mathrm{sgn}(s) + d(t)\ \right)\right)\\ &= s\left(K^T\dot{X}_d - K^T AX - K^T B\left(\ K^T\dot{X}_d - K^T AX + \lambda s + q\,\mathrm{sgn}(s) + d(t)\ \right)\right),\end{aligned}$$

noticing that $K^T B = 1$, it can be concluded that

$$\dot{V}_1 = -\lambda s^2 - q\,|s| - d(t)\,s \le -\lambda s^2 - q\,|s| + |d(t)|\,|s|\,.$$

Obviously, if q is chosen greater than the bound of $d(t)$, i.e. $q \ge sup\,|d(t)|$, then it can be concluded that

$$\dot{V}_1 \le -\lambda s^2 < 0.$$

This means $\lim_{t\to\infty} s \to 0$, thus from (3) and (5), it can see $\lim_{t\to\infty} e \to 0$. The proof is complete. □

3 Fuzzy inference system as universal approximator

In this paper a fuzzy inference system is used as a universal approximator to approximate the unknown dynamical functions of the system. A zero order TSK fuzzy inference system [24] contains a number of fuzzy IF–THEN rules as

R^l: IF x_1 is F_1^l and ... and x_n is F_n^l THEN $y = \overline{y}^l$, where F_i^l is a fuzzy set, $\overline{y}^l$ is a constant, l indicates the rule number in which l=*1,2,...,M* , and M is the total number of fuzzy rules. $X = [x_1, \ldots, x_n]^T \epsilon R^n$ and $y \epsilon R$ are the input and output of the fuzzy system, respectively. The output of this fuzzy system can be generated by a singleton fuzzification, a product inference and a weighted average i.e.

$$y(X \mid \theta) = \frac{\sum_{l=1}^{M}\ \overline{y}^l\left[\ \prod_{i=1}^{n}\mu_{F_i^l}(x_i)\right]}{\sum_{l=1}^{M}\left[\ \prod_{i=1}^{n}\mu_{F_i^l}(x_i)\right]} = \theta^T\xi(X)\,. \tag{8}$$

Then $\theta = \left[\overline{y}^1, \ldots, \overline{y}^M\right]^T$ is the vector of output singleton membership functions and $\mu_{F_i^l}$ is the membership function of fuzzy set F_i^l. Also $\xi\left(X\right) = \left[\xi^1, \ldots, \xi^M\right]^T$ is the vector of fuzzy basis functions where each element is defined as

$$\xi^l\left(X\right) = \frac{\prod_{i=1}^n \mu_{F_i^l}\left(x_i\right)}{\sum_{l=1}^M \left[\prod_{i=1}^n \mu_{F_i^l}\left(x_i\right)\right]}. \tag{9}$$

In the above fuzzy approximator the optimal parameter vector θ^* is introduced as,

$$\theta^* = \underset{\theta \epsilon \Omega_y}{\operatorname{argmin}} \left(\sup_{X \epsilon R^n} \left| y\left(X\right) - y\left(X \mid \theta\right) \right| \right), \tag{10}$$

where Ω_y is a compact set given as $\Omega_y = \{\theta \ : \|\theta\| \leq m_y \}$. The positive constant of m_y is a design parameter. The minimum approximation error for the fuzzy system may be inscribed as $\omega = y\left(X\right) - y\left(X \mid \theta^*\right)$.

4 Robust adaptive sliding mode control based on robust adaptive fuzzy observer

In this section, an observer-based robust adaptive sliding mode control for system (2) is presented. Firstly, due to the accessibility of only the output of the system, a robust adaptive fuzzy observer is designed in order to estimate the states and unknown dynamical functions of the system. Then based on the designed observer, a robust adaptive sliding mode control is proposed for the system.

Start with the control problem, it can be seen that, the given control signal (6) cannot be implemented on (2), since $f\left(X\right)$, $\mathrm{g}\left(X\right)$ and $d\left(t\right)$ are unknown and also only the output of system i.e. the first element of the vector X, is available. Therefore, fuzzy functions of $\hat{f}\left(\hat{X} \,\middle|\, \theta_f\right)$ and $\widehat{\mathrm{g}}\left(\hat{X} \,\middle|\, \theta_{\mathrm{g}}\right)$ are introduced as the approximations of the unknown functions $f(X)$ and $\mathrm{g}(X)$, where $\hat{X}$ is the estimate of X and θ_f and θ_{g} are tuning parameters vectors. The fuzzy functions are described as follows

$$\hat{f}\left(\hat{X} \,\middle|\, \theta_f\right) = \theta_f^T \xi_f(\hat{X}). \tag{11}$$

$$\widehat{\mathrm{g}}\left(\hat{X} \,\middle|\, \theta_{\mathrm{g}}\right) = \theta_{\mathrm{g}}^T \xi_{\mathrm{g}}(\hat{X}). \tag{12}$$

Also, θ_f^* and θ_{g}^* i.e. the optimal parameters in (11) and (12) are defined as

$$\theta_f^* = \arg\min_{\theta_f \epsilon \Omega_f} \left(\sup_{X,\ \ \hat{X} \epsilon R^n} \left| f\left(X\right) - \hat{f}\left(\hat{X} \,\middle|\, \theta_f\right) \right| \right). \tag{13}$$

$$\theta_{\mathrm{g}}^* = \arg\min_{\theta_{\mathrm{g}} \epsilon \Omega_{\mathrm{g}}} \left(\sup_{X, \hat{X} \epsilon R^n} \left| \mathrm{g}\left(X\right) - \widehat{\mathrm{g}}\left(\hat{X} \,\middle|\, \theta_{\mathrm{g}}\right) \right| \right). \tag{14}$$

Considering the above mentioned definitions, it can proceed to the First designing step.

4.1 Design of robust adaptive fuzzy observer

Now, due to the inaccessibility of the state vector X in system (2), through the input of the system i.e. u and its output $y = x$, the following observer is proposed to estimate X

$$\dot{\hat{X}} = A\hat{X} + B\left(\hat{f}\left(\hat{X} \,\middle|\, \theta_f\right) + \widehat{\mathrm{g}}\left(\hat{X} \,\middle|\, \theta_{\mathrm{g}}\right) u - u_r \right) + L\left(y - \hat{y}\right), \tag{15}$$

$$\hat{y} = C^T \hat{X}.$$

In the above equation, the constant vector L is chosen such that the matrix $A - LC^T$ be Hurwitz and u_r is a robust term for compensating the observer approximation error and plant uncertainties. By defining the state observation error as

$$\tilde{X} = \hat{X} - X = \left[\tilde{e}, \dot{\tilde{e}} \ldots,\ \tilde{e}^{(n-1)}\right]^T, \tag{16}$$

and from (2) and (15), one can write

$$\dot{\tilde{X}} = \left(A - LC^T\right)\tilde{X} + B\left(\left(\hat{f}\left(\hat{X} \,\middle|\, \theta_f\right) - f\left(X\right)\right) + \left(\hat{\mathrm{g}}\left(\hat{X} \,\middle|\, \theta_{\mathrm{g}}\right) - \mathrm{g}\left(X\right)\right)u - d\left(t\right) - u_r\right),$$

$$\tilde{e} = C^T\tilde{X},$$

where $\tilde{e} = \hat{y} - y$. Further, by defining $\omega_o = \left(f\left(X\right) - \hat{f}\left(\hat{X} \,\middle|\, \theta_f^*\right)\right) + \left(\mathrm{g}\left(X\right) - \hat{\mathrm{g}}\left(\hat{X} \,\middle|\, \theta_{\mathrm{g}}^*\right)\right)u$ as the minimum approximation error of observation and using (11) and (12), the observation error dynamic will be

$$\dot{\tilde{X}} = \left(A - LC^T\right)\tilde{X} + B\left(\widetilde{\theta}_f^T\xi_f(\hat{X}) + \widetilde{\theta}_{\mathrm{g}}^T\xi_{\mathrm{g}}(\hat{X})u - \omega_o - d\left(t\right) - u_r\right),$$

$$\tilde{e} = C^T\tilde{X},$$

in which $\widetilde{\theta}_f = \theta_f - \theta_f^*$, $\widetilde{\theta}_{\mathrm{g}} = \theta_{\mathrm{g}} - \theta_{\mathrm{g}}^*$. By introducing $w = \omega_o + d\left(t\right)$ as the uncertainty, the latter can be written as

$$\dot{\tilde{X}} = \left(A - LC^T\right)\tilde{X} + B\left(\widetilde{\theta}_f^T\xi_f\left(\hat{X}\right) + \widetilde{\theta}_{\mathrm{g}}^T\xi_{\mathrm{g}}(\hat{X})u - w - u_r\right), \tag{17}$$

$$\tilde{e} = C^T\tilde{X}.$$

Due to the stability of $A - LC^T$, there exists a stable polynomial $H(p)$ such that the transfer function $G\left(p\right) = C^T\left(pI - A + LC^T\right)^{-1}BH(p) = C^T\left(pI - A + LC^T\right)^{-1}B_h$ be strictly positive real (SPR). Also, based on the property of the SPR transfer functions, this is equal to the existence of positive definite matrices P and Q so that [21]

$$\left(A - LC^T\right)^TP + P\left(A - LC^T\right) = -Q. \tag{18}$$

$$PB_h = C. \tag{19}$$

Therefore (17) can be rewritten as

$$\dot{\tilde{X}} = \left(A - LC^T\right)\tilde{X} + B_h\left(\widetilde{\theta}_f^T\xi_f\left(\hat{X}\right) + \widetilde{\theta}_{\mathrm{g}}^T\xi_{\mathrm{g}}\left(\hat{X}\right)u + w_h - u_r\right), \tag{20}$$

$$\tilde{e} = C^T\tilde{X}, \tag{21}$$

in which $w_h = \left(H^{-1}\left(p\right) - 1\right)\left(\widetilde{\theta}_f^T\xi_f\left(\hat{X}\right) + \widetilde{\theta}_{\mathrm{g}}^T\xi_{\mathrm{g}}(\hat{X})u - u_r\right) - H^{-1}\left(p\right)w$ is a lumped uncertainty. Also suppose that $|w_h| \le \delta_m$, where δ_m is an unknown positive constant. Now, the subsequent theorem is presented.

Theorem 4.1. *Consider the uncertain nonlinear system of the form* (2)*, then using adaptive fuzzy observer* (15)*, the robust term as* (23)*, and adaptation laws* (24)*,* (25) *and* (26)*, the tracking error of the state observation* (16) *will converge to zero asymptotically.*

Proof. The following Lyapunov-like function is candidate

$$V_2 = \frac{1}{2}\tilde{X}^TP\tilde{X} + \frac{1}{2\gamma_f}\widetilde{\theta}_f^T\widetilde{\theta}_f + \frac{1}{2\gamma_{\mathrm{g}}}\widetilde{\theta}_{\mathrm{g}}^T\widetilde{\theta}_{\mathrm{g}} + \frac{1}{2\gamma_\delta}\widetilde{\delta}^2, \tag{22}$$

where γ_f , γ_{g} and γ_δ are positive designing constants. The symmetric positive definite matrix P is given by (18), $\widetilde{\delta} = \delta - \delta_m$, where δ is an estimate of δ_m . Then using (20), the derivative of V_2 with respect to time will result

$$\dot{V}_2 = \frac{1}{2}\tilde{X}^T\left(\left(A - LC^T\right)^TP + P\left(A - LC^T\right)\right)\tilde{X} + \tilde{X}^TPB_h\left(\widetilde{\theta}_f^T\xi_f\left(\hat{X}\right) + \widetilde{\theta}_{\mathrm{g}}^T\xi_{\mathrm{g}}\left(\hat{X}\right)u + w_h - u_r\right) + \frac{1}{\gamma_f}\widetilde{\theta}_f^T\dot{\theta}_f + \frac{1}{\gamma_{\mathrm{g}}}\widetilde{\theta}_{\mathrm{g}}^T\dot{\theta}_{\mathrm{g}} + \frac{1}{\gamma_\delta}\widetilde{\delta}\dot{\delta}.$$

Using (18), (19) and (21), the above equation will become

$$\dot{V}_2 = -\frac{1}{2}\tilde{X}^TQ\tilde{X} + \tilde{e}\left(\widetilde{\theta}_f^T\xi_f\left(\hat{X}\right) + \widetilde{\theta}_{\mathrm{g}}^T\xi_{\mathrm{g}}\left(\hat{X}\right)u + w_h - u_r\right) + \frac{1}{\gamma_f}\widetilde{\theta}_f^T\dot{\theta}_f + \frac{1}{\gamma_{\mathrm{g}}}\widetilde{\theta}_{\mathrm{g}}^T\dot{\theta}_{\mathrm{g}} + \frac{1}{\gamma_\delta}\widetilde{\delta}\dot{\delta}.$$

By proposing

$$u_r = \delta\mathrm{sgn}(\tilde{e}), \tag{23}$$

where sgn(.) is standard sign function and rearranging the latter equation, it will become

$$\dot{V}_2 = -\frac{1}{2}\tilde{X}^T Q\tilde{X} + \left(\tilde{e}\widetilde{\theta}_f^T \xi_f\left(\hat{X}\right) + \frac{1}{\gamma_f}\widetilde{\theta}_f^T \dot{\theta}_f\right) + \left(\tilde{e}\widetilde{\theta}_{\mathrm{g}}^T \xi_{\mathrm{g}}\left(\hat{X}\right)u + \frac{1}{\gamma_{\mathrm{g}}}\widetilde{\theta}_{\mathrm{g}}^T \dot{\theta}_{\mathrm{g}}\right) + \tilde{e}w_h - |\tilde{e}|\,\delta + \frac{1}{\gamma_\delta}\widetilde{\delta}\dot{\delta}$$

Regarding $|w_h| \leq \delta_m$ and noticing $\tilde{e}w_h \leq |\tilde{e}|\,|w_h| \leq |\tilde{e}|\,\delta_m$, it can be concluded that

$$\begin{aligned}\dot{V}_2 &\leq -\frac{1}{2}\tilde{X}^T Q\tilde{X} + \widetilde{\theta}_f^T\left(\tilde{e}\xi_f\left(\hat{X}\right) + \frac{1}{\gamma_f}\dot{\theta}_f\right) + \widetilde{\theta}_{\mathrm{g}}^T\left(\tilde{e}\xi_{\mathrm{g}}\left(\hat{X}\right)u + \frac{1}{\gamma_{\mathrm{g}}}\dot{\theta}_{\mathrm{g}}\right) + |\tilde{e}|\,\delta_m - |\tilde{e}|\,\delta + \frac{1}{\gamma_\delta}\widetilde{\delta}\dot{\delta} \\ &= -\frac{1}{2}\tilde{X}^T Q\tilde{X} + \widetilde{\theta}_f^T\left(\tilde{e}\xi_f\left(\hat{X}\right) + \frac{1}{\gamma_f}\dot{\theta}_f\right) + \widetilde{\theta}_{\mathrm{g}}^T\left(\tilde{e}\xi_{\mathrm{g}}\left(\hat{X}\right)u + \frac{1}{\gamma_{\mathrm{g}}}\dot{\theta}_{\mathrm{g}}\right) - |\tilde{e}|\,\widetilde{\delta} + \frac{1}{\gamma_\delta}\widetilde{\delta}\dot{\delta}.\end{aligned}$$

Now, by suggesting the following adaptive laws

$$\dot{\theta}_f = -\gamma_f\tilde{e}\xi_f\left(\hat{X}\right), \tag{24}$$

$$\dot{\theta}_{\mathrm{g}} = -\ \gamma_{\mathrm{g}}\tilde{e}\xi_{\mathrm{g}}\left(\hat{X}\right)u, \tag{25}$$

$$\dot{\delta}\ = \gamma_\delta\,|\tilde{e}|\,, \tag{26}$$

the above inequality will be

$$\dot{V}_2 \leq -\frac{1}{2}\tilde{X}^T Q\tilde{X} \leq -\frac{1}{2}\lambda_{min}(Q)\left\|\tilde{X}\right\|^2 \leq 0, \tag{27}$$

where $\lambda_{min}(.)$ symbolizes for the minimum eigenvalue of a matrix. From (27) and (22) can deduce $0 \leq V_2(t) \leq V_2(0)$ i.e. $V_2(t)$ is decreasing and bounded. Also, integrating from both sides of (27) results

$$\int_0^\infty \left\|\tilde{X}\right\|^2 \leq \frac{2\left(V_1(0) - V_1(\infty)\right)}{\lambda_{min}(Q),}$$

i.e. $\tilde{X} \in L_2$. Besides, due to the boundedness of $V_2(t)$, (22) concludes $\tilde{X} \in L_\infty$, also θ_f , θ_{g} and δ are bounded. Consequently, from (20), it can be seen that $\dot{\tilde{X}} \in L_\infty$, which means be seen that $\tilde{X}$ is uniformly continuous. Thus, based on the lemma of Barbalat [21], $\lim_{t\to\infty}\tilde{X} \to 0$. This completes the proof. □

Remark 4.2. *One result of the Theorem 2, is boundedness of the vector $\dot{\tilde{X}}$ i.e. $\dot{\tilde{X}} \in L_\infty$. It concludes that the last element of $\dot{\tilde{X}}$ i.e. $\tilde{e}^{(n)}$ is bounded.*

4.2 Design of observer-based robust adaptive sliding mode control

In this section based on the designed observer, a robust adaptive sliding mode control for system (2) is proposed. So, regarding to the ideal control signal (6), the following controller is suggested

$$u = \frac{1}{\widehat{\mathrm{g}}\left(\hat{X}\,\middle|\,\theta_{\mathrm{g}}\right)}\left(-\hat{f}\left(\hat{X}\,\middle|\,\theta_f\right) + K^T\dot{X}_d - K^T A\hat{X} + \lambda\hat{s} + q\,\mathrm{sgn}\left(\hat{s}\right)\right), \tag{28}$$

where $\hat{f}\left(\hat{X}\,\middle|\,\theta_f\right)$ and $\widehat{\mathrm{g}}\left(\hat{X}\,\middle|\,\theta_{\mathrm{g}}\right)$ are the approximations of $f\left(X\right)$ and $\mathrm{g}\left(X\right)$ respectively, which are previously worked out in the designed observer i.e. by Theorem 2. λ is a positive designing constant and q is a positive adaptable parameter. $\hat{s}$ is the estimate of s and it can be defined like as (3), i.e. $\hat{s} = K^T\hat{E}$ in which

$$\hat{E} = X_d - \hat{X}. \tag{29}$$

Now, in order to guarantee the asymptotic stability of the closed loop system through the control signal (28), the following theorem is given.

Theorem 4.3. *Consider the uncertain nonlinear system of the form* (2)*, and proposed robust observer* (15) *in the Theorem 2, then using control signal* (28)*, and adaptation law* (30)*, the tracking error of the closed system tends to zero asymptotically.*

$$\dot{q}{=}\gamma_q\,|\hat{s}|\,, \tag{30}$$

where γ_q is a positive designing constant.

Proof. By time derivation of (3) and using (4) and (2), the dynamic of sliding surface can be inscribed as

$$\dot{s} = K^T \dot{E} = K^T \left(\dot{X}_d - \dot{X}\right) = K^T \left(\dot{X}_d - AX - B\left(\, f(X) + \mathrm{g}(X)\, u + d(t) \, \right)\right). \tag{31}$$

By simple manipulating of (31) and substituting u from (28) in it, (31) will become

$$\dot{s} = K^T \left(\dot{X}_d - AX - B\left(\, f(X) + \left(\mathrm{g}(X) - \widehat{\mathrm{g}}\left(\hat{X} \,\middle|\, \theta_{\mathrm{g}}\right)\right) u \; - \hat{f}\left(\hat{X} \,\middle|\, \theta_f\right) + K^T \dot{X}_d - K^T A\hat{X} + \lambda \hat{s} + q\mathrm{sgn}(\hat{s}) + d(t)\right)\right).$$

Simplifying the latter by noticing $K^T B = 1$, it results

$$\dot{s} = K^T A\left(\hat{X} - X\right) - \left(f(X) - \hat{f}\left(\hat{X} \,\middle|\, \theta_f\right)\right) - \left(\mathrm{g}(X) - \widehat{\mathrm{g}}\left(\hat{X} \,\middle|\, \theta_{\mathrm{g}}\right)\right) u - d(t) - \lambda \hat{s} - q\mathrm{sgn}(\hat{s}). \tag{32}$$

Also, from (16), (4) and (29), it can be seen

$$\tilde{X} = \hat{X} - X = (X_d - X) - \left(X_d - \hat{X}\right) = E - \hat{E} = \tilde{E}. \tag{33}$$

Therefore, by noticing (33), (32) will become

$$\dot{s} = K^T A\tilde{E} - \left(f(X) - \hat{f}\left(\hat{X} \,\middle|\, \theta_f\right)\right) - \left(\mathrm{g}(X) - \widehat{\mathrm{g}}\left(\hat{X} \,\middle|\, \theta_{\mathrm{g}}\right)\right) u - d(t) - \lambda \hat{s} - q\mathrm{sgn}(\hat{s}). \tag{34}$$

Now, consider the following facts

$$\dot{s} = \dot{\hat{s}} + \dot{\tilde{s}}, \tag{35}$$

$$\dot{\tilde{s}} = K^T \dot{\tilde{E}} = K^T \left(A\tilde{E} + B\tilde{e}^{(n)}\right) = K^T A\tilde{E} + \tilde{e}^{(n)}. \tag{36}$$

Thus, by substituting (34) and (36) into (35), it can be written

$$\dot{\hat{s}} = \dot{s} - \dot{\tilde{s}} = -\tilde{e}^{(n)} - \left(f(X) - \hat{f}\left(\hat{X} \,\middle|\, \theta_f\right)\right) - \left(\mathrm{g}(X) - \widehat{\mathrm{g}}\left(\hat{X} \,\middle|\, \theta_{\mathrm{g}}\right)\right) u - d(t) - \lambda \hat{s} - q\mathrm{sgn}(\hat{s}). \tag{37}$$

Let $v = -\tilde{e}^{(n)} - \left(f(X) - \hat{f}\left(\hat{X} \,\middle|\, \theta_f\right)\right) - \left(\mathrm{g}(X) - \widehat{\mathrm{g}}\left(\hat{X} \,\middle|\, \theta_{\mathrm{g}}\right)\right) u - d(t)$ as a lumped uncertainty, accordingly, (37) becomes

$$\dot{\hat{s}} = -\lambda \hat{s} + v - q\mathrm{sgn}(\hat{s}). \tag{38}$$

Note that the boundedness of v is clear, because $d(t)$ based on the assumption of the problem is bounded, the term $\left(f(X) - \hat{f}\left(\hat{X} \,\middle|\, \theta_f\right)\right) + \left(\mathrm{g}(X) - \widehat{\mathrm{g}}\left(\hat{X} \,\middle|\, \theta_{\mathrm{g}}\right)\right) u$ is an modeling approximation error and the term $\tilde{e}^{(n)}$ is the last element of the vector $\dot{\tilde{X}}$, which is bounded according to Remark 1. So, suppose $|v| \le q_m$, where q_m is an unknown positive constant.

Now the subsequent Lyapunov-like function is nominee

$$V_3 = \frac{1}{2}\hat{s}^2 + \frac{1}{2\gamma_q}\tilde{q}^2, \tag{39}$$

where $\tilde{q} = q - q_m$. Using (38), the time derivative of V_3 is found as

$$\dot{V}_3 = -\lambda \hat{s}^2 + v\hat{s} - q\,|\hat{s}| + \frac{1}{\gamma_q}\tilde{q}\dot{q}.$$

Regarding $|v| \le q_m$ and noticing $\hat{s} \le |v|\,|\hat{s}| \le q_m\,|\hat{s}|$, it can be concluded that

$$\dot{V}_3 \le -\lambda \hat{s}^2 + q_m\,|\hat{s}| - q\,|\hat{s}| + \frac{1}{\gamma_q}\tilde{q}\dot{q} = -\lambda \hat{s}^2 - \tilde{q}\,|\hat{s}| + \frac{1}{\gamma_q}\tilde{q}\dot{q}.$$

Using (30), the latter will become $\dot{V}_3 \le -\lambda \hat{s}^2 \le 0$. Such as the stability analysis in Theorem 2, it can be proved that $\lim_{t\to\infty} \hat{s} \to 0$, consequently $\lim_{t\to\infty} \hat{e} \to 0$ and also from Theorem 2, it can be concluded that $\lim_{t\to\infty} \tilde{x} \to 0$. Finally, due to the equality $\tilde{x} = \tilde{e} = e - \hat{e}$, it can be seen that $\lim_{t\to\infty} e \to 0$. The proof is complete. □

Remark 4.4. *The existence of sign function in* (23) *and* (28) *causes to chatter the control and other signals of the system. Therefore, in order to remove this event, usually a continuous function such as saturation or hyperbolic tangent is used instead of the sign function. Of course, this substitution leads to decreasing the final accuracy of the response.*

Block diagram of the observer and the controller of the system is shown in Figure 1.

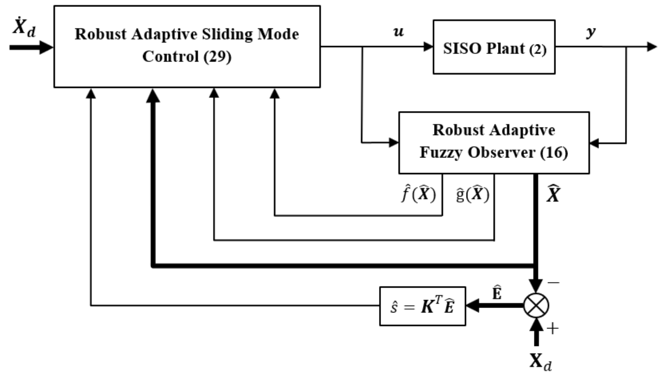

Figure 1: Block diagram of the proposed controller scheme.

5 Case study simulations

In this section the suggested method is applied to two sample uncertain nonlinear systems in class of (1).

Example 5.1. *Cart and pole system is a benchmark nonlinear system which is employed for testing and evaluating of various control approaches. Figure 2 shows a scheme of the system. In this system the aim is to determine the control signal u such that the rod angle θ tracks a desired trajectory. Note that the open loop system is unstable. Mathematical model of the system can be expressed as [21], $\dot{\theta} = \omega$.*

$$\dot{\omega} = f(\theta, \omega) + \mathrm{g}(\theta, \omega)\, u + d(t), \tag{40}$$

where θ is the rod angle, ω is the rod angular velocity, u is the control input, $d(t)$ indicates uncertainty and disturbance and $f(\theta, \omega)$ and $\mathrm{g}(\theta, \omega)$ are as

$$f(\theta, \omega) = \frac{(m_c + m)\, gsin\theta - ml\omega^2 sin\theta cos\theta}{\frac{4}{3}l\,(m_c + m) - mlcos^2\theta}\ , \quad \mathrm{g}(\theta, \omega) = \frac{cos\theta}{\frac{4}{3}l\,(m_c + m) - mlcos^2\theta}.$$

In the above functions $m_c = 1\ kg$ is the cart mass, $m = 0.1\ kg$ is the pole mass, $l = 0.5\ m$ is the rod length and $g = 9.8\ m/s^2$ is the gravity acceleration. Denote $X = [\theta, \omega]^T$, $X_d = [\theta_d, \omega_d]^T$, $K = [k_1, 1]^T$, $E = [e, \dot{e}]^T = [\theta_d - \theta, \omega_d - \omega]^T$, then $s = K^T E = \dot{e} + k_1 e$ and (40) can be written as

$$\dot{X} = AX + B\,(f(X) + \mathrm{g}(X)\, u + d(t)) \quad , \quad A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

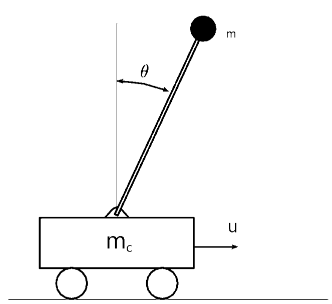

Figure 2: Cart and Pole system

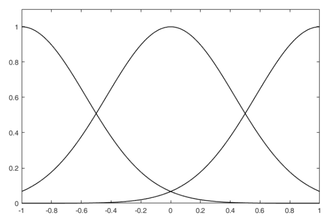

Figure 3: The membership functions for θ and ω

In order to show performance of the proposed technique, a sinusoidal trajectory with frequency of $\frac{2\pi}{3}$ *(rad/sec) and magnitude of 0.2 (rad) is applied to the system, also uncertainty is chosen as* $d(t) = 0.1 sin\pi t + 0.2 sin\frac{3}{2}\pi t$*. Figure 3 shows the membership functions for both* θ *and* ω *which are considered in the interval [-1,1] with completeness of 0.5. Tuning Parameters and initial conditions for the observer are set to*

$$\gamma_f = 10^5, \quad \gamma_g = 10^3, \quad \gamma_\delta = 10^3, \quad L = [400,\ 10^4]^T, \quad \theta_f^T(0) = [0, \ldots, 0]_{1\times 9}\ , \quad \theta_g^T(0) = [0.5, \ldots, 0.5]_{1\times 9}\ , \ \delta(0) = 0.$$

Also, for the controller, they are selected as $\lambda = 10, \quad \gamma_q = 10,\ K = [10,\ 1]^T$ *and* $q(0) = 0$*. Simulation results for the proposed method in compare to an observer-based hybrid adaptive fuzzy control (OHAFC) [9] are illustrated in Figure 4 and Figure 5. Performance of the two observers are displayed in Figure 4.*

As can be seen, in the proposed method, the estimates of the system states converge to their real values rapidly in presence of the uncertainty. Also as shown in Figure 5(a), the output tracking of the proposed approach is better than OHAFC. In Figure 5(b) the error convergence for the two schemes are depicted and their control signals are drawn in Figure 5(c).

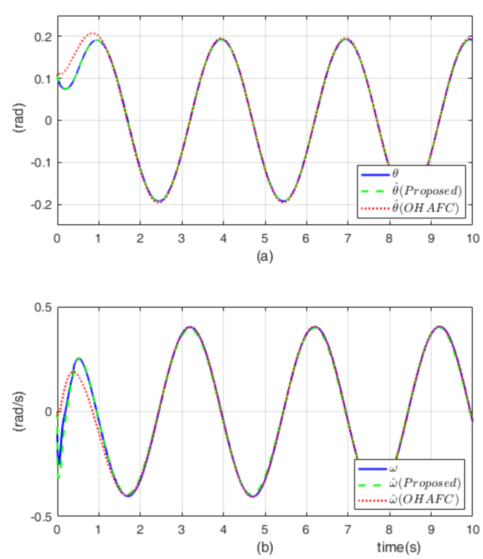

Figure 4: Performance of observer for proposed method and OHAFC a). System output (θ) and its estimate $\left(\widehat{\theta}\right)$. b). System output rate (ω) and its estimate $(\widehat{\omega})$.

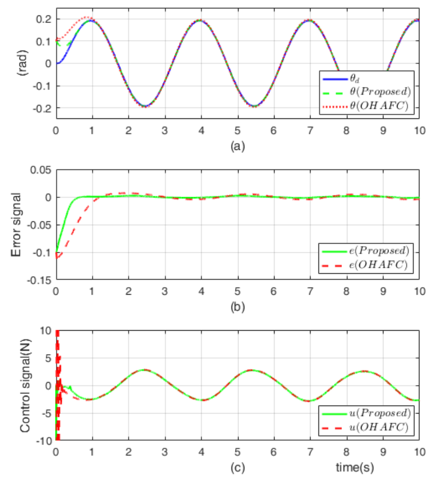

Figure 5: Control performance for proposed method and OHAFC. a). Reference trajectory (θ_d) and system output (θ). b). Error signal (e). c). Control effort (u).

Example 5.2. *In order to show the effectiveness of our proposed control method, as a second case study, trajectory tracking of the following chaos system, known as Chua's system is simulated. Dynamical equations of the system are described as [19]*

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = x_3,$$
$$\dot{x}_3 = \frac{14}{1805}x_1 - \frac{168}{9025}x_2 + \frac{1}{38}x_3 - \frac{2}{45}\left(\frac{28}{361}x_1 + \frac{7}{95}x_2 + x_3\right)^3 + u, \tag{41}$$
$$y = x_1,$$

where u is the control input. Such as previous example, symbolize

$$X = [x_1, x_2, x_3]^T, \quad X_d = [x_{1d}, \dot{x}_{1d}, \ddot{x}_{1d}]^T, \quad K = [k_2, k_1, 1]^T, \quad E = [e, \dot{e}, \ddot{e}]^T = [x_{1d} - x_1, \quad \dot{x}_{1d} - x_2, \quad \ddot{x}_{1d} - x_3]^T,$$

then $s = K^T E = \ddot{e} + k_1\dot{e} + k_2 e$ and (41) can be written as

$$\dot{X} = AX + B\left(f\left(X\right) + \mathrm{g}u\right) \quad , \quad A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

In this case, desired trajectory is $x_{1d} = 0.5sin(\frac{\pi}{3}t)$. The membership functions for all three variables are considered in the interval [-1,1] with completeness of 0.5.

Tuning Parameters and initial conditions for the observer are fixed to

$$\gamma_f = 10, \quad \gamma_{\mathrm{g}} = 10, \quad L = [500, \ 10^4, \ 10^6]^T, \quad \theta_f^T(0) = [0, \ldots, 0]_{1\times 27}, \quad \theta_{\mathrm{g}}(0) = 0.5.$$

The controller's parameters are designated as $\lambda = 10$, $\gamma_q = 5$, $K = [40, \ 14, \ 1]^T$ and $q(0) = 0$.

Simulation results for the proposed method are illustrated in Figure 6 and Figure 7. Performance of the robust adaptive fuzzy observer is displayed in Figure 6. Also as shown in Figure 7(a), the output tracking has been done properly. In Figures 7(b), 7(c) convergence of the error and the control signal are illustrated.

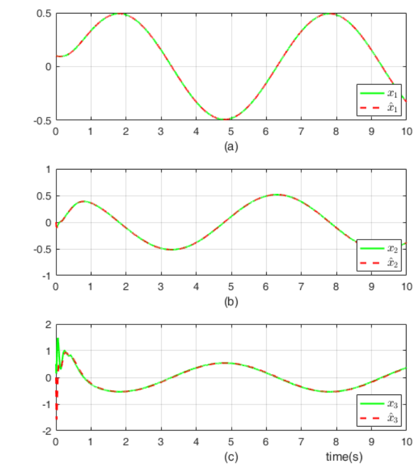

Figure 6: Performance of proposed observer for Chua's system. a). x_1 and its estimate $\hat{x}_1$. b). x_2 and its estimate $\hat{x}_2$. c). x_3 and its estimate $\hat{x}_3$.

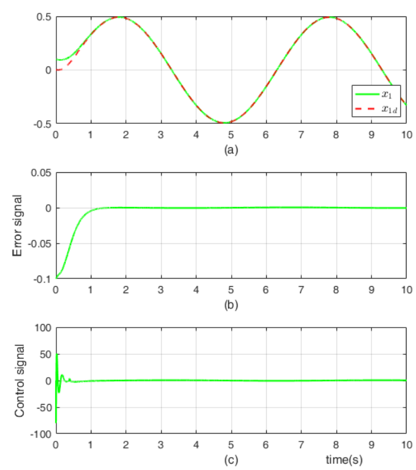

Figure 7: System response for sinusoidal trajectory. a). Reference trajectory x_{1d} and system output x_1. b). Error signal e. c). Control effort u.

6 Conclusion

In this paper, a new output feedback robust adaptive sliding mode control scheme has been proposed for a class of nonlinear systems with uncertainties and disturbances. The proposed method uses a robust adaptive fuzzy observer to estimate the states variables in addition the nonlinear dynamics of the system. Then based on the estimated nonlinear dynamics of the system, a robust controller has been designed. Robust stability of the proposed observer and also controller have been guaranteed by Lyapunov theory. Finally, the suggested method has been applied to two case studies and also compared to other method. The comparing results show that the output feedback robust adaptive sliding mode controller can obtain better performance than the other method and it effectively acquires the good performance as well as attains stability and also attenuates uncertainties and disturbances.

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