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A new separated fault estimator and fault-tolerant control design strategy for uncertain nonlinear systems using T-S fuzzy modeling

M. Cheraghiyan[1]

[1] *Department of Automation and Instrumentation Engineering, Petroleum University of Technology, Ahwaz, Iran*

mohammad.cheraghiyan@afp.put.ac.ir

Abstract

In this paper, a new separated design of adaptive observer-based fault estimator (FE) and fault-tolerant control (FTC) is introduced for nonlinear systems along with sensor and actuator faults in the presence of external disturbance and parametric modeling uncertainty by T-S fuzzy modeling, which can easily challenge integrated design strategies. Firstly, a new fuzzy adaptive FE estimates actuator/sensor faults, and the system states simultaneously and also can automatically estimate and compensates bi-directional uncertainties between FE function and control system. Secondly, a fuzzy state feedback FTC controller is designed based on these estimations. Then the proposed separated FE/FTC design presented which is solved by linear matrix inequality via H_∞ optimization. Finally, a tutorial example of an inverted pendulum on a cart demonstrates the validity and effectiveness of the proposed design.

Keywords: Separated FE/FTC, T-S fuzzy modeling, fuzzy adaptive fault estimation.

1 Introduction

Fault-Tolerant Control (FTC) is one of the research areas that has attracted a great deal of attention in the two last decades. In the event of part and/or instrument failures, the FTC allows the actual output to be retained close to the desired level [3]. A control system's capacity for accommodation depends on several factors, such as the magnitude of the fault, nominal system robustness, and mechanisms that incorporate redundancy in sensors and actuators. A bibliographic review of the FTC method is published in [23].

The literature defines two main classes of approaches from the viewpoint of FTC strategies: passive and active. Passive FTC consists of a particular robust controller that manages all the potential faults [17]. Contrary to this passive approach, active FTC relies on the real-time fault information provided by the fault estimation module [1, 13]. In this paper, FTC denotes AFTC.

Many fault detection and isolation (FDI) methods are developed in the last decades. FDI approaches can be classified into three different categories, such as model-based, data-based, and hybrid methods [16, 25]. The model-based FDI strategies are presented in [11, 16, 21] and can be categorized into basic approaches, such as unknown input observer (UIO), Kalman filter, algorithms that focused on optimization. Model-based observers are the most popular FDI structure. It not only allows the identification and isolation of different faults but also provides information about the magnitude of the faults and their occurrence time and location.

The Takagi-Sugeno (TS) fuzzy method has become increasingly interested because it is a powerful solution to bridge the gap between linear control systems and nonlinear ones. The T-S fuzzy modeling has a significant advantage due to its universal approximation of any smooth nonlinear functions by mixing of several local linear sub-models, which significantly makes it easy for the complex nonlinear system to analyze and synthesize. There have been several significant analytical and synthesis findings for the TS fuzzy modeling (see [4, 15]).

In real processes, in addition to the faults, there is also modeling uncertainty in the system, and the presence of uncertainty affects both the FE function and the control system, so it may lead to an FE-based FTC with decreased

Corresponding Author: M. Cheraghiyan

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performance when designed separately. The existence of uncertainty in the FE function, along with the uncertainty associated with the control system, leads to the presence of bi-directional robustness interactions between the FE and FTC. Such interactions lead to the need for a joint design of the FE and FTC for robust overall system performance [7]. So, these so-called bi-directional robustness interactions between FE and FTC have naturally lead to the need for integrated FE/FTC strategy [6].

Many separated FE-based FTC designs have been published in the last decades, such as [5, 19], in which descriptor T-S fuzzy systems have been considered with actuator and sensor faults, without any consideration of modeling uncertainty. Li *et al.* [9] proposed a separated design of fast adaptive FE and FTC for uncertain time-delay fuzzy systems which has much computational complexity in the design and also are based upon observer matching conditions that involve the solution of a strict equality constraint, leading to possible infeasibility in some practical problems. Recently, Nagy-Kiss *et al.* [14] has addressed a separated FE/FTC design for linear parameter-varying uncertain multi-model systems in the presence of actuator fault. However, the proposed method brings into a restricted assumption that the uncertainty should be derivative, which might restrict the application of the design for practical scenarios. Zhang *et al.* [24] and Liang *et al.* [10] have proposed separated FE/FTC design approaches for uncertain nonlinear systems, but neither has considered the bi-directional robustness interaction. Hence, the proposed separated FE/FTC designs are sub-optimal solutions of the overall FTC system designs, which can result in the systems with reduced performances. In fact, the separated designs cannot take into account the so-called bi-directional robustness interaction for uncertain systems, which results in the necessity of employing a single-step integrated design approach.

Few integrated FE/FTC designs have been developed in the last few years, such as two-step integrated FE/FTC design for linear parameter varying descriptor system [18], which has not considered modeling uncertainty and, consequently, the bi-directional robustness interactions. Recently, a novel single-step method was proposed by Lan and Patton [8] to integrate FE and FTC in order to reduce the bi-directional robustness interaction for T-S fuzzy systems. However, the proposed method contains considerable design and computational complexity that can be even more challenging in large-scale interconnected systems and online optimization procedures. Besides, some tuning matrices must be set in advance for the proposed LMI conditions. Therefore, the integrated FE/FTC design scheme has much computation and even has its limitation for more complicated systems. Hence, the need for a more functional and much simpler design of FE and FTC for T-S fuzzy systems in the presence of uncertainty, which takes into account the bi-directional robustness interactions been felt. In this paper, the main contributions worth emphasizing are as follow:
I) A new adaptive fuzzy observer-based FE is designed, which is capable of estimation and compensation for these so-called bi-directional robustness interactions concept, described in [7] between the FE and FTC by estimating uncertainty matrix.
II) A separated FTC design strategy is developed based on the proposed adaptive fuzzy FE to robustly stabilize the faulty system via H_∞ optimization with a 2-step linear matrix inequality.
III) The proposed design strategy has less computational complexity compared to both the separated and the proposed integrated FE/FTC approaches in [8].

The rest of this paper is as follows. In Section 2, the problem formulation is presented. Sections 3 and 4, respectively, introduce the fuzzy adaptive fault estimator and fault-tolerant control designs. Then separated design of FE and FTC is provided in Section 5. In Section 6, a comparison experiment is conducted between the proposed separated and the integrated FE/FTC designs proposed in [8] to illustrate the applicability of the proposed design method. In the end, a conclusion is also provided in Section 7.

2 Problem formulation

A nonlinear system influenced by unknown disturbance, actuator and sensor faults, and faced to modeling uncertainty is written as:

$$\begin{aligned} \dot{x}(t) &= h_x\big(x(t), u(t), f_a(t), d(t), \delta(t)\big), \\ y(t) &= h_y\big(x(t), f_s(t)\big), \end{aligned} \tag{1}$$

where $x(t) \in \mathbf{R}^n$, $u(t) \in \mathbf{R}^m$, and $y(t) \in \mathbf{R}^p$ are the state of the system, control input, and output, respectively. $f_a(t) \in \mathbf{R}^q$, $f_s(t) \in \mathbf{R}^{q_1}$ and $d(t) \in \mathbf{R}^l$ denote the actuator fault, sensor fault, and unknown external disturbance, respectively. $\delta(t) \in \mathbf{R}^n$ is the modeling uncertainty, $h_x(.)$ and $h_y(.)$ are continuous nonlinear functions.

Consider a Takagi-Sugeno (T-S) fuzzy system, consisting of a set of If-Then rules derived from the system (1). The *ith* rule of the fuzzy model is as follows:

Plant Rule i $(i = 1, ..., h)$

If θ_1 is μ_{i1} and ... and θ_s is μ_{is}, then:

$$\begin{aligned}\dot{x}(t) &= (A_i + \Delta A_i)x(t) + B_i u(t) + F_{ai} f_a(t) + D_i d(t),\\ y(t) &= Cx(t) + F_s f_s(t),\end{aligned} \tag{2}$$

where A_i, B_i, F_{ai}, D_i, C, F_s are known real constant matrices with appropriate dimensions. $\Delta A_i \in \mathbf{R}^{n\times n}$ denotes for the bounded uncertainty matrix satisfying $\|\Delta A_i\| \le \varepsilon_i$. In general, θ_j $(j = 1, 2, , s)$ denote premise variables, which can be some measurable variables of the system states or the function of states. Moreover, μ_{ij} are the fuzzy sets that are characterized by a membership function, h is the number of sub-models, and s is the number of premise variables. The overall fuzzy model, being achieved from the plant rules (2), can be described as:

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{h} \rho_i\left(\theta(t)\right)\left[(A_i + \Delta A_i)x(t) + B_i u(t) + F_{ai} f_a(t) + D_i d(t)\right],\\ y(t) &= Cx(t) + F_s f_s(t),\end{aligned} \tag{3}$$

in which,

$$\rho_i(\theta) = \frac{\aleph_i(\theta)}{\sum_{w=1}^{h} \aleph_w(\theta)},\ \aleph_i = \prod_{j=1}^{s} \aleph_{ij}(\theta_j),$$

where $\rho_i(\theta(t))$ are membership functions depending on the premise variable vector $\theta(t) = [\theta_1, ..., \theta_s]$, $\aleph_{ij}(\theta_j)$ are grades of the membership of θ_j in the fuzzy sets $\aleph_{ij}$, $0 \le \rho_i(\theta) \le 1$ and $\sum_{i=1}^{h} \rho_i(\theta) = 1$.

Assumption 2.1. *All sub-models (A_i, B_i) and (A_i, C) are controllable and observable, respectively and actuator and sensor faults are observable, which means that the following rank conditions are satisfied:* $\operatorname{rank}(\begin{bmatrix} A_i & F_{ai} \\ C & 0 \end{bmatrix}) = n + q$, $\operatorname{rank}(\begin{bmatrix} A_i & 0 \\ C & F_s \end{bmatrix}) = n + q_1$.

Assumption 2.2. [8] *The actuator fault f_a is matched, i.e.,* $\operatorname{rank}(B_i, F_{ai}) = \operatorname{rank}(B_i)$, $i = 1, 2, , h$.

Assumption 2.3. *The external disturbance $d(t)$, actuator fault $f_a(t)$ and sensor fault $f_s(t)$ fulfill $\|d(t)\| \le d_0$, $\|f_a(t)\| \le \sigma_a$ and $\|f_s(t)\| \le \sigma_s$, respectively, and σ_a, σ_s and d_0 are some unknown positive scalars. Moreover, without loss of generality, it is assumed that the faults have finite first-order time derivative and vector $[d(t)^T\ \dot{f}_a^T(t)\ \dot{f}_s^T(t)]^T$ belongs to $L_2[0, \infty)$.*

Remark 2.4. *Assumption 2.1 ensures that the ith sub-model is controllable and observable, and actuator fault and sensor fault are observable too, and the adaptive observer method proposed in the paper can be used to estimate them. Assumption 2.2 implies that the actuator fault can be compensated through control action. Assumptions 2.3 guarantees that actuator/sensor faults and disturbance are bounded and belong to the L_2-norm bounded set.*

Lemma 2.5. [20] *Let x and y denote two constant vectors, then $x^T y = tr(xy^T)$, where $tr(G)$ is trace of the constant matrix G.*

3 Fuzzy adaptive fault estimator design

Defining $\bar{x} = [x^T\ f_a^T\ f_s^T]^T$ and $\bar{d} = [d^T\ \dot{f}_a^T\ \dot{f}_s^T]^T$; The system 3 can be augmented into:

$$\begin{aligned}\dot{\bar{x}}(t) &= \sum_{i=1}^{h} \rho_i\left(\theta(t)\right)\left[(\bar{A}_i + \Delta\bar{A}_i)\bar{x}(t) + \bar{B}_i u(t) + \bar{D}_i \bar{d}(t)\right],\\ y(t) &= \bar{C}\bar{x}(t),\end{aligned} \tag{4}$$

where:

$$\bar{A}_i = \begin{bmatrix} A_i & F_{ai} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ \Delta\bar{A}_i = \begin{bmatrix} \Delta A_i & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ \bar{B}_i = \begin{bmatrix} B_i \\ 0 \\ 0 \end{bmatrix},\ \bar{D}_i = \begin{bmatrix} D & 0 & 0 \\ 0 & I_q & 0 \\ 0 & 0 & I_{q_1} \end{bmatrix},\ \begin{bmatrix} C & 0 & F_s \end{bmatrix}$$

The estimation of the augmented state vector of (4) can be attained by the fuzzy adaptive observer as follows:

$$\begin{aligned}\dot{\hat{\bar{x}}}(t) &= \sum_{i=1}^{h} \rho_i\left(\theta(t)\right)\left[\bar{A}_i\hat{\bar{x}}(t) + \Delta\hat{\bar{A}}_i\hat{\bar{x}}(t) + \bar{B}_i u(t) + G_i\big(y(t)-\hat{y}(t)\big)\right],\\ \hat{y}(t) &= \bar{C}\hat{\bar{x}}(t),\end{aligned} \tag{5}$$

where $\hat{\bar{x}}(t) \in \mathbf{R}^{n+q+q_1}$ and $\hat{y}(t) \in \mathbf{R}^{p}$ are the estimate of $\bar{x}(t)$ and observer output, respectively. $\Delta\hat{\bar{A}}_i(t)$ is the estimation of $\Delta\bar{A}_i$, and G_i represents the observer gain matrix with appropriate dimensions to be solved, $i = 1,,h$.

State and output estimation errors are defined as follow:

$$\bar{e}(t) = \bar{x}(t) - \hat{\bar{x}}(t), \tag{6}$$

$$e_y(t) = y(t) - \hat{y}(t) = \bar{C}\bar{e}(t), \tag{7}$$

By using (4) to (6), the error dynamics can be written as:

$$\dot{\bar{e}}(t) = \sum_{i=1}^{h} \rho_i\left(\theta(t)\right)\left[(\bar{A}_i - G_i\bar{C})\bar{e}(t) + \Delta\bar{A}_i\bar{x}(t) - \Delta\hat{\bar{A}}_i(t)\hat{\bar{x}}(t) + \bar{D}_i\bar{d}(t)\right]. \tag{8}$$

Now assumed that $\Delta\bar{A}_i$ and $\Delta\hat{\bar{A}}_i(t)$ can be defined as:

$$\Delta\bar{A}_i = N\bar{\varphi}_i, \tag{9a}$$

$$\Delta\hat{\bar{A}}_i(t) = N\hat{\bar{\varphi}}_i(t), \tag{9b}$$

where $N \in \mathbf{R}^{(n+q+q_1)\times p}$ is a constant matrix, and $\bar{\varphi}_i \in \mathbf{R}^{p\times(n+q+q_1)}$ denotes an uncertain constant matrix, satisfying $\|\bar{\varphi}_i\| \le \beta_i$ and $\hat{\bar{\varphi}}_i$ is the estimate of $\bar{\varphi}_i$. Furthermore, Since $\|\Delta A_i\| \le \varepsilon_i$ so it can deduce that $\|\Delta\bar{A}_i\| \le \varepsilon_i$, with known positive scalar ε_i.

By substituting (9a) and (9b) into (8), the error dynamics can be rewritten as:

$$\begin{aligned}\dot{\bar{e}}(t) &= \sum_{i=1}^{h} \rho_i\left(\theta\right)\left[(\bar{A}_i - G_i\bar{C})\bar{e}(t) + N\bar{\varphi}_i\bar{x}(t) - N\hat{\bar{\varphi}}_i(t)\hat{\bar{x}}(t) + \bar{D}_i\bar{d}(t)\right]\\ &= \sum_{i=1}^{h} \rho_i\left(\theta\right)\left[\bar{A}_{1i}\bar{e}(t) + N\bar{\varphi}_i\bar{e}(t) + N\bar{\varphi}_{ei}\hat{\bar{x}}(t) + \bar{D}_i\bar{d}(t)\right].\end{aligned} \tag{10}$$

where $\bar{\varphi}_{ei}(t) = \bar{\varphi}_i - \hat{\bar{\varphi}}_i(t)$ and $\bar{A}_{1i} = (\bar{A}_i - G_i\bar{C})$.

4 Fault-tolerant control design

An FTC is designed for the system (3) as:

$$u = \sum_{i=1}^{h} \rho_i\left(\theta\right) K_i\hat{\bar{x}}, \tag{11}$$

where $K_i = [K_{xi}\ K_{fi}\ 0_{m\times q_1}]$ with K_{xi} and K_{fi} the state-feedback control gains and the actuator fault compensation gains, respectively. By putting (11) into (3) closed-loop system can be written:

$$\dot{x}(t) = \sum_{i=1}^{h}\sum_{j=1}^{h} \rho_i\left(\theta\right)\rho_j\left(\theta\right)\left[(A_i + \Delta A_i)x(t) - B_iK_j\bar{e} + B_iK_{xj}x + B_iK_{fi}f_a + F_{ai}f_a + D_id\right]. \tag{12}$$

Under Assumption 2.2, K_{fi} can be chosen as $K_{fi} = -B_i^{\dagger}F_i$ and $B_i = F_{ai}$, where $B_i^{\dagger}$ defined as left pseudo-inverse of B_i. By putting K_{fi} into (12) the closed-loop system rewritten as:

$$\dot{x}(t) = \sum_{i=1}^{h}\sum_{j=1}^{h} \rho_i\left(\theta\right)\rho_j\left(\theta\right)\left[(A_i + B_iK_{xj})x(t) + E_{ij}e(t) + \Delta A_ix(t) + D_id\right], \tag{13}$$

where $E_{ij} = -B_i K_j = [-B_i K_{xj}\ F_i\ 0]$.

ΔA_i can be expressed as:

$$\Delta A_i = Y_1 \Delta \bar{A}_i Y_2, \tag{14}$$

where $Y_1 = [I_n\ 0_{n\times(q+q_1)}]$ and $Y_2 = Y_1^T$. Substituting (14) and (9a) into (13) gives:

$$\dot{x}(t) = \sum_{i=1}^{h}\sum_{j=1}^{h} \rho_i(\theta)\,\rho_j(\theta)\left[(A_i + B_i K_{xj})x(t) + E_{ij}e(t) + Y_1 N \bar{\varphi}_i Y_2 x(t) + D_i d\right]. \tag{15}$$

5 Separated design of FE/FTC

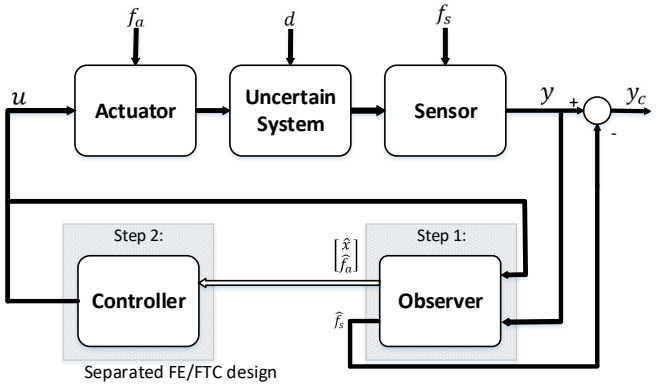


Figure 1: The separated FE/FTC design framework.

The separated design method is by first designing the fault estimator and then the FTC controller (see Fig. 1). This separate FE/FTC design concept is accomplished based on the satisfactory of the Separation Principle as a result of the proposed fuzzy adaptive observer-based FE, which reduces the bi-directional robustness interactions. In this respect, the error dynamics are:

$$\begin{aligned}\dot{\bar{e}}(t) &= \sum_{i=1}^{h} \rho_i(\theta)\left[\bar{A}_{1i}\bar{e}(t) + N\bar{\varphi}_i\bar{e}(t) + N\bar{\varphi}_{ei}(t)\hat{\bar{x}}(t) + \bar{D}_i\bar{d}(t)\right],\\ z_e &= C_e\bar{e},\end{aligned} \tag{16}$$

where $z_e \in \mathbf{R}^{r_2}$ is the measured output and C_e is a known constant matrix of appropriate dimension. It is assumed that the observer is already stable, i.e., $e(t) = 0$, then the closed-loop system (15) becomes into:

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{h}\sum_{j=1}^{h} \rho_i(\theta)\,\rho_j(\theta)\left[(A_i + B_i K_{xj})x(t) + Y_1 N \bar{\varphi}_i Y_2 x(t) + D_i d(t)\right],\\ z_x &= C_x x,\\ y_c &= y - F_s\hat{f}_s,\end{aligned} \tag{17}$$

where $z_x \in \mathbf{R}^{r_3}$, y_c, $\hat{f}_s$ and C_x are the measured output, corrected system output, the sensor fault estimation, and a known constant of appropriate dimension, respectively.

For separate determination of the observer and controller gain matrices, theorems 5.1 and 5.2 are sufficient prerequisites.

Theorem 5.1. *Given positive scalars γ_1 and ε_i, suppose $\Delta\bar{A}_i$ satisfies (9) with $\|\Delta\bar{A}_i\| \le \varepsilon_i$ and $\|\bar{\varphi}_i\| \le \beta_i$, and the error dynamics (15) are stable with H_∞ optimization $\|G_{z_e\bar{d}}\| < \gamma_1$, if there exists a symmetric positive definite matrix $X \in \mathbf{R}^{n_1\times n_1}$, any symmetric positive definite $\Gamma_i \in \mathbf{R}^{p\times p}$ and $W_{1i} \in \mathbf{R}^{n_1\times p}$, such that:*

$$\begin{bmatrix} \Omega_i & \frac{1}{\gamma_1} X\bar{D}_i \\ * & -I_{(n_2)} \end{bmatrix} < 0, \tag{18}$$

$$\dot{\hat{\bar{\varphi}}}_i(t) = \rho_i(\theta)\Gamma_i\, \bar{C}\, \bar{e}\, \hat{\bar{x}}^T(t), \tag{19}$$

$$\bar{C}^T = XN, \tag{20}$$

where $\Omega_i = sym(X\bar{A}_i) - sym(W_{1i}\bar{C}) + 2\varepsilon_i X + C_e^T C_e$, $n_2 = l+q+q_1$. *Then observer gains are given by:* $G_i = X^{-1} \times W_{1i}$, $N = X^{-1}\bar{C}^T$, $i = 1, 2, , h$.

Proof. Consider a Lyapunov function $V_e = \bar{e}^T(t)X\bar{e}(t) + \sum_{i=1}^{h} tr\left(\bar{\varphi}_{ei}^T(t)\Gamma_i^{-1}\bar{\varphi}_{ei}(t)\right)$. The time derivative of V_e is:

$$\dot{V}_e = \dot{\bar{e}}^T(t)X\bar{e}(t) + \bar{e}^T X\dot{\bar{e}}(t) + \sum_{i=1}^{h} tr\left(\dot{\bar{\varphi}}_{ei}^T(t)\Gamma_i^{-1}\bar{\varphi}_{ei}(t)\right) + \sum_{i=1}^{h} tr\left(\bar{\varphi}_{ei}^T(t)\Gamma_i^{-1}\dot{\bar{\varphi}}_{ei}(t)\right). \tag{21}$$

As $\dot{\bar{\varphi}}_{ei}(t) = \dot{\bar{\varphi}}_i - \dot{\hat{\bar{\varphi}}}_i(t) = -\dot{\hat{\bar{\varphi}}}_i(t)$ then

$$\begin{aligned} \dot{V}_e = &\sum_{i=1}^{h} \rho_i(\theta)\left[\bar{e}^T(t)(\bar{A}_{1i}^T X + X\bar{A}_{1i})\bar{e}(t)\right] + 2\sum_{i=1}^{h} \rho_i(\theta)\left(\bar{e}^T(t)XN\bar{\varphi}_i\bar{e}(t)\right) \\ &+ \sum_{i=1}^{h} \rho_i(\theta)\left(sym(\bar{e}^T(t)X\bar{D}_i\bar{d}(t))\right) + 2\sum_{i=1}^{h} \rho_i(\theta)\left(\bar{e}^T(t)XN\bar{\varphi}_{ei}(t)\hat{\bar{x}}(t)\right) \\ &- 2\sum_{i=1}^{h} tr\left(\bar{\varphi}_{ei}^T(t)\Gamma_i^{-1}\dot{\hat{\bar{\varphi}}}_i(t)\right). \end{aligned} \tag{22}$$

If (19) and (20) hold, by Lemma 2.5, the sum of the last two terms of (22) will be zero. Thus, (22) is reduced to:

$$\dot{V}_e = \sum_{i=1}^{h} \rho_i(\theta)\left[\bar{e}^T(t)(\bar{A}_{1i}^T X + X\bar{A}_{1i})\bar{e}(t)\right] + \sum_{i=1}^{h} \rho_i(\theta)\left(sym(\bar{e}^T(t)X\bar{D}_i\bar{d}(t))\right) + Y_{1i}, \tag{23}$$

where:

$$Y_{1i} = 2\sum_{i=1}^{h} \rho_i(\theta)\left(\bar{e}^T(t)XN\bar{\varphi}_i\bar{e}(t)\right). \tag{24}$$

Since $\|\Delta\bar{A}_i\| \le \varepsilon_i$ and (9a), the following can be written:

$$\begin{aligned} Y_{1i} &\le 2\sum_{i=1}^{h} \rho_i(\theta)\left(\bar{e}^T(t)X\|N\bar{\varphi}_i\|\bar{e}(t)\right) = 2\sum_{i=1}^{h} \rho_i(\theta)\left(\bar{e}^T(t)X\|\Delta\bar{A}_i\|\bar{e}(t)\right) \\ &\le 2\sum_{i=1}^{h} \rho_i(\theta)(\bar{e}^T(t)X\varepsilon_i\bar{e}(t)), \end{aligned} \tag{25}$$

By using (23) and (25):

$$\dot{V}_e \le \sum_{i=1}^{h} \rho_i(\theta)\left[\bar{e}^T\left(sym(X\bar{A}_{1i}) + 2\varepsilon_i X + C_e^T C_e\right)\bar{e} + sym\left(\bar{e}^T X\bar{D}_i\bar{d}\right)\right] - z_e^T(t)z_e(t). \tag{26}$$

The H_∞ performance $\|G_{z_e\bar{d}}\|_\infty \le \gamma_1$ is represented by

$$J = \int_0^\infty (z_e^T z_e - \gamma_1^2 \bar{d}^T\bar{d})dt < 0. \tag{27}$$

Under zero initial conditions,

$$J=\int_0^\infty (z_e^T z_e-\gamma_1^2\bar{d}^T\bar{d}+\dot{V}_e)dt-\big(V_e(\infty)-V_e(0)\big)\leq\int_0^\infty (z_e^T z_e-\gamma_1^2\bar{d}^T\bar{d}+\dot{V}_e)dt<0,$$

Then, a sufficient condition for (27) is:

$$J_1=z_e^T z_e-\gamma_1^2\bar{d}^T\bar{d}+\dot{V}_e<0. \tag{28}$$

By using (28):

$$\begin{aligned}J_1&\leq\sum_{i=1}^h\rho_i(\theta)\left[\bar{e}^T\left(sym(X\bar{A}_{1i})+2\varepsilon_i X+C_e^TC_e\right)\bar{e}+sym(\bar{e}^TX\bar{D}_i\bar{d})\right]-\gamma_1^2\bar{d}^T\bar{d}\\&=\sum_{i=1}^h\rho_i(\theta)\left[\bar{e}^T\left(sym(X\bar{A}_{1i})+2\varepsilon_i X+C_e^TCe\right)\bar{e}\right]\\&+\sum_{i=1}^h\rho_i(\theta)\left[-\left(\gamma_1\bar{d}-\frac{1}{\gamma_1}\bar{D}_i^TX\bar{e}\right)^T\left(\gamma_1\bar{d}-\frac{1}{\gamma_1}\bar{D}_i^TX\bar{e}\right)+\frac{1}{\gamma_1^2}\bar{e}^TX\bar{D}_i\bar{D}_i^TX\bar{e}\right]<0.\end{aligned} \tag{29}$$

Then:

$$\sum_{i=1}^h\rho_i(\theta)\left[\bar{e}^T(sym(X\bar{A}_{1i})+2\varepsilon_i X+C_e^TC_e)\bar{e}\right]+\sum_{i=1}^h\rho_i(\theta)\left[\bar{e}^TX\bar{D}_i\bar{D}_i^TX\bar{e}\right]<0 \tag{30}$$

By applying the Schur complement to (30) and denoting $W_{1i}=XG_i$, (18) will be achieved. □

Theorem 5.2. *Given positive scalars γ_2 and ε_i, suppose $\Delta\bar{A}_i$ satisfies (8) with $\|\Delta\bar{A}_i\|\leq\varepsilon_i$ and $\|\bar{\varphi}_i\|\leq\beta_i$, and the closed-loop system (16) is stable with H_∞ optimization $\|G_{z_xd}\|_\infty<\gamma_2$, if there exists a symmetric positive definite matrix $Q\in\mathbf{R}^{n\times n}$ and $W_{2j}\in\mathbf{R}^{m\times n}$, such that:*

$$\begin{bmatrix}\Omega_{ij} & QC_x^T\\ * & -I_{(n)}\end{bmatrix}<0, \tag{31}$$

where $\Omega_{ij}=sym(A_iQ)+sym(B_iW_{2j})+sym(Y_1\varepsilon_iY_2Q)+\frac{1}{\gamma_2^2}(D_iD_i^T)$. Then controller gains are given by: $K_{xj}=W_{2j}\times Q^{-1}$ and $i,j=1,2,,h$.

Proof. Consider a Lyapunov function $V_x=x^TPx$. The time derivative of V_x is:

$$\dot{V}_x=\dot{x}^TPx+x^TP\dot{x}. \tag{32}$$

By substituting (16) into (32):

$$\dot{V}_x=\sum_{i=1}^h\sum_{j=1}^h\rho_i(\theta)\rho_j(\theta)\left[x^Tsym\big(P(A_i+B_iK_{xj})\big)x+sym(x^TPD_id)+Y_{2i}\right], \tag{33}$$

where:

$$Y_{2i}=x^Tsym(PY_1N\bar{\varphi}_iY_2)x, \tag{34}$$

Similarly, since $\|\Delta\bar{A}_i\|\leq\varepsilon_i$ and (9a), the following can be written:

$$Y_{2i}\leq\sum_{i=1}^h\rho_i(\theta)x^Tsym(PY_1\|N\bar{\varphi}_i\|Y_2)x=\sum_{i=1}^h\rho_i(\theta)x^Tsym(PY_1\|\Delta\bar{A}_i\|Y_2)x\leq\sum_{i=1}^h\rho_i(\theta)x^Tsym(PY_1\varepsilon_iY_2)x, \tag{35}$$

By using (33) and (35):

$$\dot{V}_x\leq\sum_{i=1}^h\sum_{j=1}^h\rho_i(\theta)\rho_j(\theta)\Big[x^T\Big(sym\big(P(A_i+B_iK_{xj})\big)+sym(PY_1\varepsilon_iY_2)+C_x^TC_x\Big)x+sym(x^TPD_id)\Big]-z_x^T(t)z_x(t). \tag{36}$$

The H_∞ performance $\|G_{z_x d}\|_\infty \leq \gamma_2$ is represented by

$$J = \int_0^\infty (z_x^T z_x - \gamma_2^2 d^T d)dt < 0. \tag{37}$$

Under zero initial conditions,

$$J = \int_0^\infty (z_x^T z_x - \gamma_2^2 d^T d + \dot{V}_x)dt - \big(V_x(\infty) - V_x(0)\big) \leq \int_0^\infty (z_x^T z_x - \gamma_2^2 d^T d + \dot{V}_x)dt < 0.$$

Then, a sufficient condition for (37) is

$$J_2 = z_x^T z_x - \gamma_2^2 d^T d + \dot{V}_x < 0. \tag{38}$$

By using (38):

$$\begin{aligned} J_2 &\leq \sum_{i=1}^{h}\sum_{j=1}^{h} \rho_i(\theta)\rho_j(\theta)\Big[x^T\Big(sym\Big(P(A_i + B_iK_{xj})\Big) + sym(PY_1\varepsilon_i Y_2) + C_x^T C_x\Big)x + sym(x^T P D_i d)\Big] - \gamma_2^2 d^T d \\ &= \sum_{i=1}^{h}\sum_{j=1}^{h} \rho_i(\theta)\rho_j(\theta)\Big[x^T\Big(sym\Big(P(A_i + B_iK_{xj})\Big) + sym(PY_1\varepsilon_i Y_2) + C_x^T C_x\Big)x\Big] \\ &+ \sum_{i=1}^{h}\sum_{j=1}^{h} \rho_i(\theta)\rho_j(\theta)\Big[- \Big(\gamma_2 d - \frac{1}{\gamma_2} D_i^T P x\Big)^T \Big(\gamma_2 d - \frac{1}{\gamma_2} D_i^T P x\Big) + \frac{1}{\gamma_2^2} x^T P D_i D_i^T P x\Big] < 0. \end{aligned} \tag{39}$$

Then:

$$\sum_{i=1}^{h}\sum_{j=1}^{h} \rho_i(\theta)\rho_j(\theta) x^T\Big[sym\Big(P(A_i + B_iK_{xj})\Big) + sym(PY_1\varepsilon_i Y_2) + C_x^T C_x + \frac{1}{\gamma_2^2} P D_i D_i^T P\Big]x < 0. \tag{40}$$

A congruence transformation with P^{-1} and denoting $Q = P^{-1}$ and $W_{2j} = K_{xj}Q$, turn (40) into:

$$\sum_{i=1}^{h}\sum_{j=1}^{h} \rho_i(\theta)\rho_j(\theta) x^T\Big[sym(A_iQ + B_iW_{2j}) + sym(Y_1\varepsilon_i Y_2 Q) + QC_x^T C_x Q + \frac{1}{\gamma_2^2} D_i D_i^T\Big]x < 0. \tag{41}$$

By applying the Schur complement to (41), (31) will be achieved. □

Remark 5.3. *According to [2] to define computational complexity ($N(\varepsilon)$), an ε-accurate solution of the LMIs is required as follow:*

$$N(\varepsilon) = RN^3 \log(V/\varepsilon),$$

where R and N defined as the total row size and the total number of scalar decision variables of the LMI systems in theorems 5.1 and 5.2, respectively, and V is a data-dependent scaling factor. The parameters R and N for the proposed separated FE/FTC design method are calculated as $R = 2n + q + q_1 + (3n + 2q + 2q_1 + l)h$ and $N = [n(n+1) + (n+q+q_1)(n+q+q_1+1)]/2 + ((n+q+q_1)p + mn)h$. Compared to both the separated and the proposed integrated FE/FTC approaches in [8], the proposed separated FE/FTC has less computational complexity.

6 Numerical example

In this section, a tutorial example of an inverted pendulum on a cart borrowed from [22], has been given in the nonlinear model as follows:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{g\sin(x_1) - aml x_2^2 \sin(2x_1)/2 - a\cos(x_1)u}{4l/3 - aml\cos^2(x_1)} \\ y &= [x_1\ x_2]^T, \end{aligned} \tag{42}$$

where x_1 and x_2 denote the pendulum angle (rad) from the vertical and the angular velocity (rad/s), respectively. g, m, M and $2l$ stand for gravity constant, pendulum mass, cart mass, and pendulum length, respectively, u denotes the applied force to the cart, and $a = 1/(m+M)$. In this study, parameters of the model are chosen as follows, $m = 2.0kg$, $M = 8.0kg$, and $2l = 1.0m$. The T-S fuzzy model derived from (42), assumed to have sensor/actuator faults, external disturbance, and modeling uncertainty. The nonlinear pendulum system can be modeled by the following two-rule fuzzy system, which according to [22], in the controllable region $x_1 \in (-\pi/2, \pi/2)$ is valid.

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{2} \rho_i(x_1)\left[(A_i + \Delta A_i)x + B_i(u + f_a) + D_i d\right], \\ y &= Cx + F_s f_s, \end{aligned} \tag{43}$$

where $\rho_1(x_1) = 1 - \frac{2}{\pi}|x_1|$, $\rho_2(x_1) = \frac{2}{\pi}|x_1|$,

$$A_1 = \begin{bmatrix} 0 & 1 \\ \frac{g}{4l/3 - aml} & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ \frac{-a}{4l/3-aml} \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ \frac{2g}{\pi(4l/3-aml\beta^2)} & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ \frac{-a\beta}{4l/3-aml\beta^2} \end{bmatrix}, C = I_2,$$

$$D_1 = D_2 = \begin{bmatrix} 0 \\ 0.01 \end{bmatrix}, F_s = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}, \beta = \cos(88^\circ).$$

The faults and disturbance are:

$$f_a = \begin{cases} 1, & 0s \le t \le 5s \\ \sin(t), & 5s < t \le 20s \\ 1, & 20s < t \le 30s \end{cases}, \; f_s = \begin{cases} 0.1, & 0s \le t \le 14s \\ 0.2, & 14s < t \le 23s \\ 0.1, & 23s < t \le 30s \end{cases}, \; d = 0.01\sin(10t).$$

To demonstrate the effectiveness of the proposed separated FE/FTC design strategy, a comparison with the same initial angles is conducted between the integrated FE/FTC design mentioned in [8] and the proposed separated design in this paper.

The proposed separated FE based FTC design for the system (43) is solved with the following parameters: $\varepsilon_i = 0.1$, $\gamma_1 = 1$, $\gamma_2 = 1$, $C_x = I_2$, $C_e = 0.1 \times I_4$, $\Gamma_i = I_2$ and $i = 1, 2$. Solving Theorem 5.1 and 5.2 with the chosen parameters and by using the lmilab solver in Yalmip [12] of MATLAB, gives the following observer and control gains:

$$K_{x1} = [4500.75\ 1533.8],\ K_{x2} = [4493.86\ 1528.9],$$

$$G_1 = \begin{bmatrix} -711.94 & 314.145 \\ -2331.69 & 1020.92 \\ 15532.47 & -5311.51 \\ 6738.14 & -2862.93 \end{bmatrix}, G_2 = \begin{bmatrix} 14.57 & 51.11 \\ 50.237 & 157.94 \\ 539.5 & 10.576 \\ -109.64 & -385.97 \end{bmatrix}, N = \begin{bmatrix} 0.00293 & 0.01168 \\ 0.00862 & 0.0366 \\ 0.0575 & 0.0258 \\ -0.0206 & -0.09252 \end{bmatrix},$$

and the matrix gains for the proposed integrated Augmented State Unknown Input Observer (ASUIO)-based FE/FTC areas the same mentioned in [8].

The initial conditions for the integrated ASUIO-based FE/FTC design are chosen $Z(0) = 0.1 \times ones(7, 1)$ and for the proposed separated FE/FTC design are chosen $\hat{\bar{x}}(0) = 0.1 \times ones(4, 1)$, $\hat{\bar{\varphi}}_i = (0) = 0.1 \times ones(2, 4)$ $(i = 1, 2)$. It should be noted that the uncertainty is defined as parametric uncertainty on the parameters m, M, and l as follows:

$$m = \bar{m} + \delta m,\ M = \bar{M} + \delta M, \text{ and } l = \bar{l} + \delta l,$$

where $\bar{m}$, $\bar{M}$ and $\bar{l}$ are used for nominal values of the pendulum mass, the cart mass and half of the pendulum length, respectively, and δm, δM and δl for their corresponding variations, which are $\delta m = 0.01\cos(t)$, $\delta M = 0.01\sin(t)$ and $\delta l = 0.01\cos(t)$.

A comparison has been made with the same initial state conditions ($x(0) = [18^\circ, 0]$). From Figs. 2-5, it has been observed that the proposed separated FE based FTC design has better performance and covers a wider range of initial angles than the one proposed in [8].

In the face of both faults, unknown external disturbance, and with the same uncertainties, the proposed separated FE based FTC design able to balance the pendulum angle in a broader controllable range $|x_1(0)| \le 39.3\,deg$ $(x_2(0) = 0)$. While the proposed Integrated ASUIO-based FE/FTC in [8] only able to balance the pendulum angle in a range $|x_1(0)| \le 28.8\,deg$ $(x_2(0) = 0)$. Besides, it can be observed from Figs. 2-5 that the proposed separated design has better

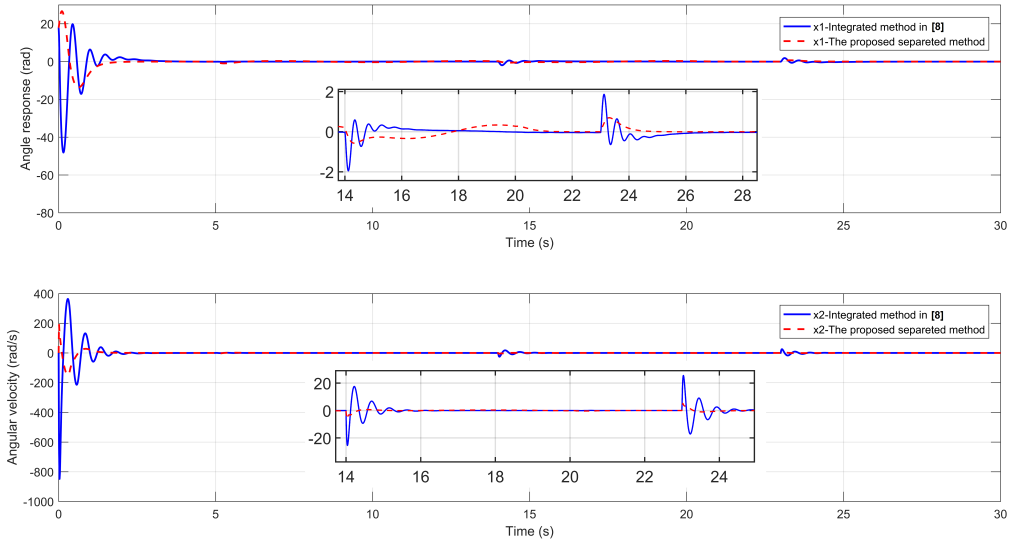


Figure 2: Comparison of first state responses between separated and integrated FE/FTC designs

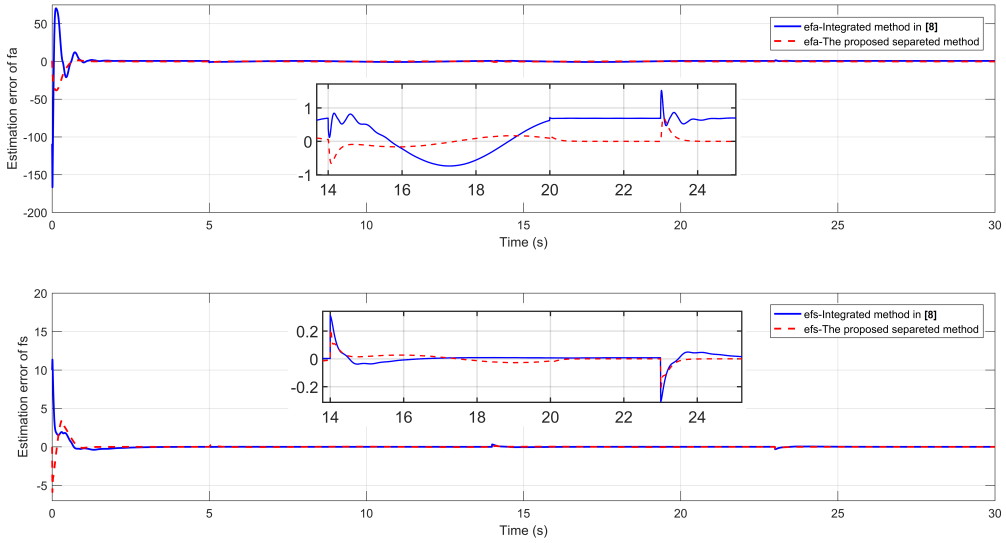


Figure 3: Comparison of actuator and sensor faults estimation errors between separated and integrated FE/FTC designs

performance and more robustness with respect to the proposed integrated FE/FTC design in [8]. Transient responses have better performance for the proposed separated design, and in the moments that fault happens, system states and estimations are less sensitive to faults and showing less oscillation in their responses compared to the integrated design, which is a privilege.

7 Conclusion

Observer and control systems influence each other in the face of uncertainty, and cannot be designed independently. The presence of uncertainty also restricts applications of the Separation Principle. The key reason for using the integrated Fault Estimator (FE) and Fault-Tolerant Control (FTC) framework is these effects, which are called bi-directional robustness interactions. In this paper, a new separated design of adaptive observer-based FE and FTC is introduced for nonlinear systems along with sensor and actuator faults in the presence of external disturbance and parametric modeling uncertainty by T-S fuzzy modeling, which can easily challenge integrated design strategies.

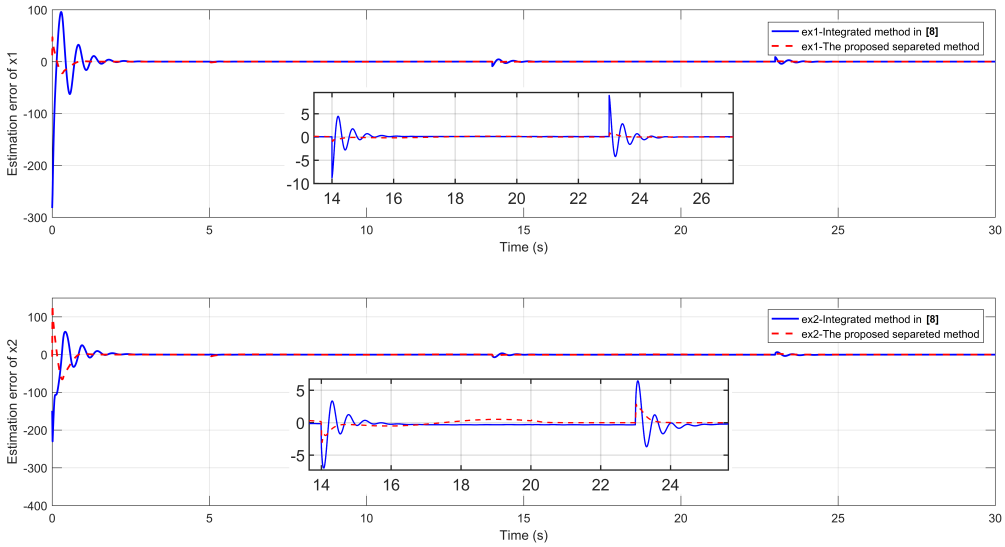


Figure 4: Comparison of states estimation errors between separated and integrated FE/FTC designs

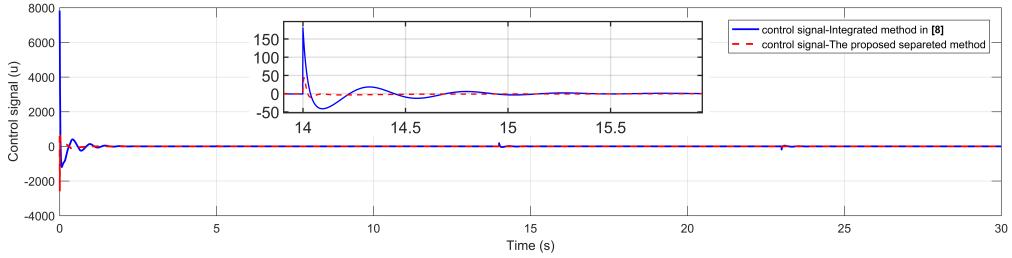


Figure 5: Comparison of control signals between separated and integrated FE/FTC designs

The proposed fuzzy adaptive FE estimates actuator/sensor faults and the system states simultaneously and also can automatically estimate and compensates the bi-directional uncertainties between FE function and control system. Hence, by successfully reducing the bi-directional robustness interactions, it results in a broader control range and much better performance. Then a fuzzy state feedback FTC controller is designed based on these estimations. The proposed separated design has less computational complexity than the integrated ones and shows better performance and more robustness in the face of faults, which makes it more feasible for practical applications.

This method in the future can be extended to large-scale interconnected systems, multi-agent systems, networked systems, descriptor systems, time-delay systems, and even more complicated systems.

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