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Improved fuzzy clustering algorithm using adaptive particle swarm optimization for nonlinear system modeling and identification

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Abstract

In this paper, an improved Type2-PCM clustering algorithm based on improved adaptive particle swarm optimization called Type2-PCM-IAPSO is proposed. Firstly, a new clustering algorithm called Type2-PCM is proposed. The Type2-PCM algorithm can solve the problems encountered by fuzzy c-means algorithm (FCM), Gustafson-Kessel algorithm (G-K), possibilistic c-means algorithm (PCM) and NPCM (sensitivity to noise or aberrant points and local minimal sensitivity). . . etc. Secondly, we combined our Type2-PCM algorithm with the improved adaptive particle swarm optimization algorithm (IAPSO) to ensure proper convergence to a local minimum of the objective function. The effectiveness of the two proposed algorithms Type2-PCM and Type2-PCM-IAPSO was tested on a system described by a different equation, Box-Jenkins gas furnace, dryer system and the convection system. The validation tests used showed good performance of these algorithms. However, their average square error test (MSE) shows a better behaviour of the Type2-PCM-IAPSO algorithm compared to the FCM, G-K, PCM, FCM-PSO, Type2-PCM-PSO, RKPFCM and RKPFCM-PSO algorithms.

Keywords: Improved adaptive particle swarm optimization (IAPSO), Type2-PCM algorithm, Type2-PCM-IAPSO algorithm, fuzzy identification, fuzzy clustering.

1 Introduction

In the field of engineering, to find the appropriate law to control non-linear systems, one must go through the steps of modeling and identification. The modeling step is used to find a model with the same dynamic characteristics of the real system. When the system is nonlinear and complex, it is very difficult to find the mathematical laws that describe its behaviour [15, 34]. This way, the modeling of nonlinear systems by conventional methods is difficult and not efficient [4, 35]. Other unconventional methods based on fuzzy logic are used to model this type of system in order to approximate its behaviour [18, 36, 37].

In the literature, several fuzzy modeling approaches are developed; the Takagi Sugeno fuzzy model was used in this approach. Indeed, this model is defined by the if-then type rules. Each rule includes a fuzzy antecedent and consequence parameters represent the behaviour of the system in each region [1, 4, 34]. The identification step makes it possible to obtain the parameters of the Takagi Sugeno fuzzy model. In this context, many techniques have been developed, such as, heuristic approaches, nearest neighbor clustering and vector learning mechanisms. The fuzzy clustering algorithms are most widely used in modeling, such as C-Means Fuzzy (FCM), Gustafson-Kessel (GK) and Possibilistic c-means (PCM).Therefore, to find the consequent parameters, there are several identification methods in the literature, such as Weighted Least Squares (WLS), Recursive Weighted Least Squares (RWLS) and Orthogonal Least Squares (OLS).

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Among the clustering algorithms in the literature, the FCM algorithm developed by Bezdek, has been frequently used, but this algorithm has some disadvantages such as the sensitivity to noise or aberrant points and local minimal sensitivity [6, 10, 32]. However, the noise in the sets of measurements can negatively affect the quality of the identification and compel the algorithm to converge towards many inauthentic minima. To solve these problems, several researchers have proposed other algorithms, such as the GK algorithm proposed by Gustafson-Kessel [12] and the PCM algorithm proposed by Krishnapuram and Keller [13]. However, in the framework of identification of the nonlinear system these algorithms are not efficient and lead to atrocious results when there are outliers and noise. The objective of this research is to solve this problem by means of the Type2 adhesion functions are used. A new extension of the PCM algorithm using the Type2 method permits to fuzzy the membership values of the input measures again. The lower the value of belonging to a measure is, the less certain compared to a weak belonging. For measurements that are low, they are considered to have greater uncertainty [14].
The higher the memberships value for a pattern is, the greater the contribution of the measure to determine the location of a cluster center. This is consistent with conventional PCM to determine the location of cluster centers. In this paper, the use of the Type2 membership functions improves the classic PCM algorithm (with the type1 membership functions) in two ways [4, 21, 22, 23, 30]:

- With this algorithm all the data points should not have the same contribution in the calculation of the cluster centers. Instead, data points with a higher membership value should dominate the cluster center calculation.
- In the presence of noise, the cluster centers estimated according to the type2 membership functions have the potential to converge towards the right position than the cluster centers obtained by a PCM method with a Type1 membership functions.

Our first contribution in this work is based on the use of a Type2 membership function instead of the Type1 function in the PCM fuzzy clustering algorithm; with the Type2 splicing function the PCM algorithm becomes more efficient in the case of noisy measurements and in the presence of aberrant points. The simulation results show the advantages of our proposed Type2-PCM algorithm. The Type2-PCM algorithm overcomes the initialization problem but can converge to a local minimum because of its iterative nature. To solve these problems, several solutions exist in the literature. For example, the combination between the clustering algorithm and a heuristic optimization method. This way, several methods of evolutionary calculations based on optimization of particle species (PSO) have been proposed [9, 33, 38].
Our second contribution in this work is based on the use of a new heuristic optimization algorithm called IAPSO [13]. The IAPSO algorithm solves the problems of the convergence towards a local optimum of the optimization algorithms which exist in the literature (PSO, APSO ... etc) [19, 24, 29], because of the use of a new adaptive weight of inertia [13]. To improve the efficiency of our proposed algorithm in the first contribution, we combined this algorithm with the IAPSO algorithm. Our second proposed algorithm called Type2-PCM-IAPSO. The simulations results clearly show the advantages of our new Type2-PCM-IAPSO algorithm compared to other algorithms.
The two proposed algorithms Type2-NPCM and Type2-NPCM-IAPSO has been tested on a system described by a difference equation, the furnace gas box and Jenkins and dryer system and the convection system. This research has been arranged into four parts: first it presents the Takagi-Sugeno fuzzy model. Second, it identifies the parameters of this model where the proposed algorithm Type2-PCM and the PCM algorithm are used. The third part is reserved to present our proposed algorithm based on the IAPSO algorithm. The fourth part highlights the identification of the consequent parameters of Takagi-Sugeno fuzzy model. The results of the simulations and the validity of the model of our algorithms (Type2-PCM and Type2-PCM-IAPSO) are presented in the ultimate. Finally, we resume this document by giving an overview about the different parts discussed through this paper.

2 Takagi-Sugeno Fuzzy model

The Takagi-Sugeno fuzzy model is an appropriate model for approximating nonlinear systems [11, 31, 37]. It is constructed on a rule-based of the type If ... Then ... in which the consequent uses numerical variables rather than linguistic variables (Mamdani) [3, 6, 35]. The result can be expressed as a constant, a polynomial or a differential equation depending on the antecedent variables.
A rule of fuzzy Takagi-Sugeno type is given as follows:

$$R_i : \text{if } x_1 \text{ is } A_{i1}, \ldots, x_n \text{ is } A_{in} \text{ then } y_i = {a_i}^T x_k + b_i \quad (1)$$

Where: $i = 1, ..., C$: $C$is a rules number (or clusters number);
R_i : Represents the ith fuzzy rule; $x = [x_1 x_2 .. x_n]^T \in R^n$: Is the input vector; $y \in R$: Is the output variable.

$a_i \in R^n$,$b_i \in R$: Are the consequent parameters.
Special case: if $b_i = 0$, expression of the rule becomes:

$$R_i : \text{If } x \text{ is } A_i \text{ then } y_i = {a_i}^T x. \tag{2}$$

This type of model is called homogeneous model of Takagi-Sugeno [3]. It has a capacity of approximation more limited than the the general model of equation 1. Its advantage is that it facilitates the synthesis of law control [38]. The general output of the non-linear system is equal to the output average corresponding to the rules multiplied by the degree of completion of the antecedent β_i [36] which is described by the following form:

$$\hat{y} = \frac{\sum_{i=1}^{C} \beta_i (x)\, y_i}{\sum_{i=1}^{C} \beta_i (x)}, \tag{3}$$

with:

$$\beta_{\mathrm{i}} = \mu_{i1} (x_1) \wedge \mu_{i2} (x_2) \wedge ... \mu_{in} (x_n) . \tag{4}$$

Eitherλ_i: Reproduces the standard of achievement described by this expression:

$$\lambda_{\mathrm{i}} = \frac{\beta_i (x)}{\sum_{i=1}^{C} \beta_i (x)}. \tag{5}$$

The estimated production of the Takagi-Sugeno fuzzy model can be described by this expression:

$$\hat{y} = \sum_{i=1}^{C} \lambda_i(x) \left[{a_i}^T + b_i \right] . \tag{6}$$

3 Premise parameters identification by fuzzy clustering algorithm

We used the PCM algorithm and type2-PCM algorithm to identify the Takagi Sugeno fuzzy model parameters described by equation (1).

3.1 Fuzzy possibilistic C-means (PCM)

The PCM algorithm is among the fuzzy clustering algorithms found in the literature; it is based on the minimization of the objective function described by this expression [7, 8]:

$$J_{PCM}(U,V) = \sum_{i=1}^{C} \sum_{k=1}^{N} {\mu^m}_{ik,FCM} {D^2}_{ik} + \sum_{i=1}^{C} \left(\eta_i \sum_{k=1}^{N} (1-\mu_{ik})^m \right). \tag{7}$$

Each point is characterized by a coefficient of belonging μ_{ik} that satisfies the following conditions:

$$0 \leq \mu_{ik} \leq 1 \qquad 1 \leq i \leq C \qquad 1 \leq k \leq N \qquad 0 \leq \sum_{k=1}^{N} \mu_{ik} \leq N, \tag{8}$$

$$0 \leqslant \sum_{k-1}^{N} \mu_{ik} \leqslant N, \tag{9}$$

$${D^2}_{ik} = \left\| x_k - v_i \right\|^2 , \tag{10}$$

$U = [\mu_{ik}]$: Fuzzy partition matrix; $V = [v_1, v_2, \cdots, v_i, \cdots v_c]$: Vector of cluster centers, m: represents the degree of weighting.
η_i**:** Positive numbers described by this expression:

$$\eta_i = \rho \frac{\sum_{k=1}^{N} \mu_{ik,FCM}^m D_{ik}^2}{\sum_{k=1}^{N} \mu_{ik}^m} \qquad , \rho > 0. \tag{11}$$

After the minimization of equation (7), the following equations are obtained:

$$v_i = \frac{\sum_{k=1}^{N} (\mu_{ik,FCM})^m x_k}{\sum_{k=1}^{N} (\mu_{ik,FCM})^m}. \tag{12}$$

$$\mu_{ik,FCM} = \frac{1}{1 + \left(\frac{D^2{}_{ik}}{\eta_i} \right)^{\frac{1}{m-1}}}. \tag{13}$$

The Figure 1 represent the Flowchart of the PCM algorithm.

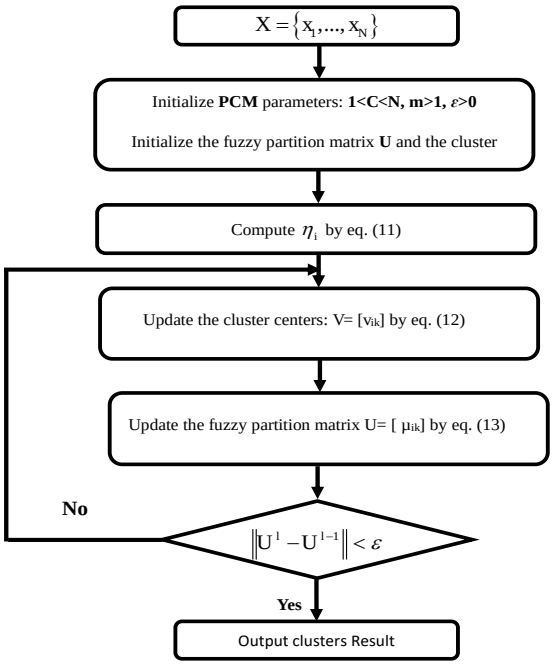


Figure 1: Flowchart of PCM algorithm

3.2 Proposed Type2-possibilistic C-means algorithm (Type2-PCM)

The effectiveness of the PCM algorithm remains limited in the case of strong noisy measurements. To solve the problems found in the PCM algorithm, we used the memberships function Type2. In this context, we proposed a new extension of the Possibilistic C-Means algorithm called Type2-PCM. When membership functions of triangular type are used, the membership functions of Type 2 can be obtained by this expression [30]:

$$\xi_{ik} = \mu_{ik} - \frac{(1 - \mu_{ik})}{2}, \tag{14}$$

where ξ_{ik}: The Type2 memberships; μ_{ik}: The Type1 memberships.
The objective function of Type2-PCM algorithm is described by this expression:

$$J_{Type2-PCM}(U,V) = \sum_{i=1}^{C} \sum_{k=1}^{N} {\xi_{ik}}^m {D^2}_{ik} + \sum_{i=1}^{C} \left(\eta_i \sum_{k=1}^{N} (1 - \xi_{ik})^m \right), \tag{15}$$

where

$$\eta_i = \rho \frac{\sum_{k=1}^{N} (3\mu_{ik,FCM} - 1)^m D_{ik}^2}{\sum_{k=1}^{N} (3\mu_{ik,FCM} - 1)^m} \qquad , \rho > 0. \tag{16}$$

The derivation of the objective function (15) relative to$\mu_{ik,FCM}$ and v_i allows giving the expression of update of cluster centers and the coefficients of belonging:

- ***derivative of*** $J_{Type2-PCM}(U,V)$ ***to get the cluster center update equation*** v_i

$$\frac{\partial J_{Type2-PCM}}{\partial V_i} = \frac{1}{2^m}(\sum_{k=1}^{N}(3\mu_{ik}-1)^m \frac{\partial({x_k}^2 - 2x_k v_i + {v_i}^2)}{\partial V_i}. \tag{17}$$

Equating (17) to zero leads to:

$$\frac{\partial J_{Type2-PCM}}{\partial V_i} = 0 \quad \text{then} \quad \sum_{k=1}^{N}(3\mu_{ik}-1)^m(v_i - x_k) = 0.$$

Then,

$$v_i = \frac{\sum_{k=1}^{N}(3\mu_{ik}-1)^m x_k}{\sum_{k=1}^{N}(3\mu_{ik}-1)^m} \tag{18}$$

- ***derivative of*** $J_{Type2-PCM}(U,V)$***to get the membership functions*** μ_{ik}

We used the Lagrange multiplier to get the expression ofμ_{ik}:

$$\begin{array}{l} J_{Type2-PCM}(U,V) = \frac{1}{2^m}\sum_{i=1}^{N}\sum_{k=1}^{C}(3\mu_{ik}-1)^m {D^2}_{ik} + \\ \frac{1}{2^m}\sum_{i=1}^{C}\left(\eta_i \sum_{k=1}^{N}(1-3\mu_{ik})^m\right)\sum_{k=1}^{N}(\lambda(\sum_{i=1}^{C}(\frac{3\mu_{ik}-1}{2})-1)) \end{array} \tag{19}$$

$$\frac{\partial J_{Type2-PCM}}{\partial \lambda} = -\frac{1}{2}\left[\sum_{k=1}^{N}\sum_{i=1}^{C}(3\mu_{ik}-1)-1\right] = 0, \tag{20}$$

$$\sum_{i=1}^{C}(3\mu_{ik}-1) = 1, \tag{21}$$

$$\frac{\partial J_{Type2-PCM}}{\partial \mu_{ik}} = \frac{m}{2}(3\mu_{ik}-1)^{m-1}{D_{ik}}^2 + \eta_{ik}\frac{m}{2}(3\mu_{ik}-1)^{m-1} - \lambda = 0 \tag{22}$$

From the expression (22), we can write μ_{ik}this form:

$$\mu_{ik} = \frac{1}{3}\left[\left[\frac{\lambda}{\frac{m}{2}\left({D_{ik}}^2 + \eta_i\right)}\right]^{\frac{1}{m-1}} + 1\right] \tag{23}$$

Substituting expression (23) by expression (21):

$$\sum_{i=1}^{C} 3\left[(\frac{1}{3}\left[\frac{\lambda}{\frac{m}{2}\left({D_{ik}}^2 + \eta_i\right)}\right]^{\frac{1}{m-1}} + 1) - 1\right] = 1 \tag{24}$$

Is also

$$\lambda^{\frac{1}{m-1}} = (\frac{m}{2})^{\frac{1}{m-1}}\sum_{i=1}^{C}\left({D_{ik}}^2 + \eta_i\right) \tag{25}$$

Expressions (23) and (25) give the following expression:

$$\mu_{ik} = \frac{1}{3}\frac{\sum_{i=1}^{C}\beta_{ik}}{\beta_{ik}} + \frac{1}{3} \tag{26}$$

Where

$$\beta_{ik} = {D_{ik}}^2 + \eta_i \tag{27}$$

The Figure 2 represent the Flowchart of the Type2-PCM algorithm.

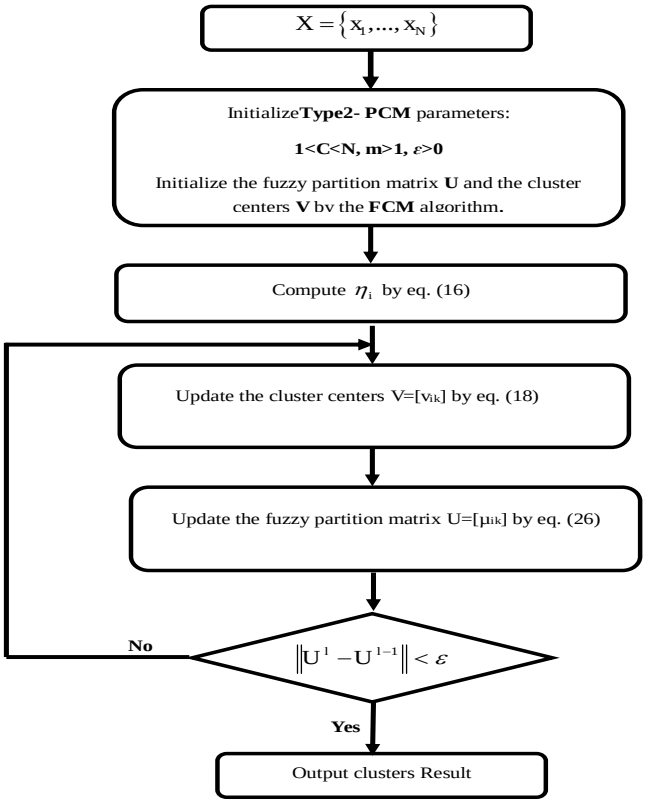


Figure 2: Flowchart of Type2- PCM algorithm

3.3 Proposed Type2-PCM based on improved adaptive particle swarm optimization IAPSO (Type2-PCM-IAPSO)

3.3.1 Improved adaptive particle swarm optimization (IAPSO)

In order to solve the optimization problem, the set of approach parameters of particle swarm optimization has an important impact on the performance of the efficiency of the approach. In the initial stages of the approach, the inertia weight is very important for the PSO performance, that it balances the trade-off between the global and local exploration capability of the swarm [13]. In fact, a small value of inertia weight makes the particle fast converge, but it favors sometimes local crawling that is fine-tuning the current search area [2, 28, 25, 31]. However, a big inertia weight value facilitates exploration, but it makes the particle long time to converge. Then, the acceleration coefficient can be increased to make the particles refer to their own search experience and avoid the algorithm prematurely to fall into a local optimum. In this section, based on the PSO method, the new IAPSO technique is proposed. PSO algorithm is able to find a global optimal solution but requires much computational effort during the optimization process [13].Whereas; IAPSO is proposed to improve the efficiency of the standard PSO algorithm and to avoid the premature convergence. For the proposed approach, we defined a new adaptive inertia weight which can be formulated as follows [13]:

$$w(iter) = (w_{\max} - w_{\min}) * \exp\left(-\frac{iter^{1.1}}{iter_{\max} - iter}\right) + w_{\min} \tag{28}$$

In which iter: is the iterations current number, itermax: is the iterations maximum number and w max and w min: The maximal and minimal weights which are usually set to 0.9 and 0.4 respectively. The dynamic adjustment of velocity and position of each particle can be updated according to the following equations:

$$v_k{}^{(iter+1)} = w(iter)\ v_k{}^{(iter)} + r_1 \times C_1(iter) \times \left(p_k{}^{(iter)} - x_k{}^{(iter)}\right) + r_2 \times C_2(iter) \times \left(p_g{}^{(iter)} - x_k{}^{(iter)}\right) \tag{29}$$

$$x_k{}^{(iter+1)} = x_k{}^{(iter)} + v_k{}^{(iter+1)} \tag{30}$$

g: Sign indicates the best particle in the swarm.
r_1 and r_2: There are two random variables, where$r_1, r_2 \in [0 \quad 1]$.
$C_1(iter)$ and $C_2(iter)$ are the acceleration parametrs described by this two expressions:

$$C_1(iter) = \frac{(C_1 f - C_1 i)}{iter_{\max}} \times iter + C_1 i \qquad C_2(iter) = \frac{(C_2 f - C_2 i)}{iter_{\max}} \times iter + C_2 i \tag{31}$$

3.3.2 Proposed Type2-PCM-IAPSO algorithm

In the case of the identification of nonlinear systems, the implementation of the classical clustering algorithms, like the FCM, GK, PCM and NPCM has several problems, such as convergence towards local optimum, sensitivity to initialization (choice of the number of clusters) and the sensitivity to noise [10, 6]. Although our proposed clustering algorithm based on the Type2 memberships function (Type2-PCM) can solve some of these problems and especially the sensitivity to noise and aberrant points [30, 22, 23, 4, 21], the possibility of convergence to local optimums of this algorithm remains. However; this problem negatively affects classification quality. To improve the classification quality we combined the Type2-PCM fuzzy clustering algorithm with the IAPSO optimization algorithm [13]. The IAPSO algorithm allows a good balance between exploration and exploitation for the increase of the capacity of exploration of the research space and forced the fuzzy clustering algorithm to converge to a global optimum [13]. The new algorithm called Type2-PCM-IAPSO.
The fitness function used in our optimization method is described by this expression:

$$fitness = J_{Type2-PCM}(U, V) \tag{32}$$

$J_{Type2-PCM}(U, V)$: The objective functions of the Type2- PCM fuzzy clustering algorithm.

The Figure 3 represent the Flowchart of the Type2-PCM-IAPSO algorithm.

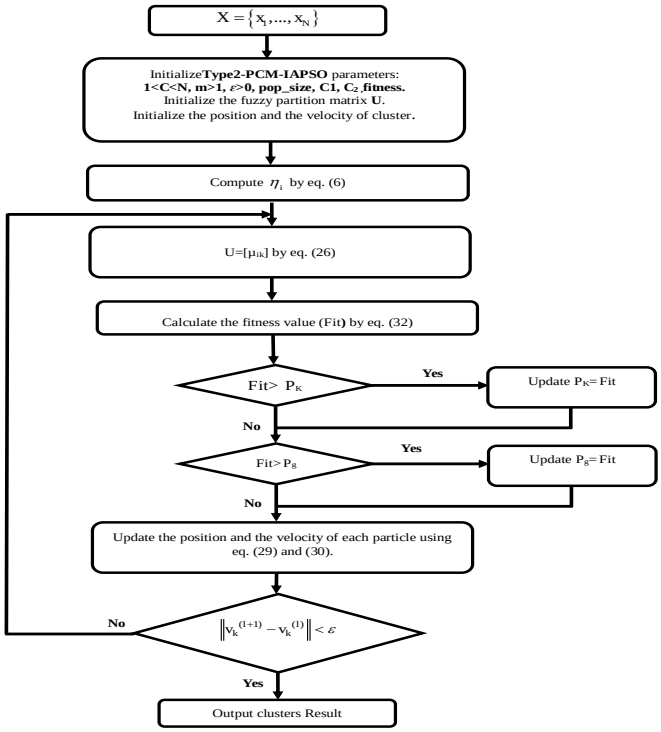


Figure 3: Flowchart of Type2-PCM-IAPSO algorithm

4 Identification for consequent parameters

We used the WLS algorithm to identify the consequent parameters $\theta_i = \left[{a_i}^T; b_i\right]$of the Takagi-Sugeno fuzzy model.

$$\theta_i = \left[{X_e}^T \psi_i X_e\right]^{-1} {X_e}^T \psi_i Y \tag{33}$$

Where $\psi_i = [\mu_{ik}]^{(N\times N)}$: Fuzzy membership matrix; $X_e = [X; 1]$: Regression matrix:$X = [X_1, X_2, ..., X_N]^T$.

$$Y = [y_1, y_2, ..., y_N]^T \text{ : The output vector.}$$

5 Simulation results and discussion

5.1 Square data with outlier

To observe the effectiveness of Type2-PCM and Type2-PCM-IAPSO algorithms versus FCM and PCM we used a data set which contains two classes. The first class has 9 models and the second class has 11 models with a different class density and by adding two noises *x21* and *x22*. The coordinates of data is illustrated in Figure 1.
Figure 4(a): The FCM algorithm finds two almost identical classes. 4(b) shows the classification result by the PCM algorithm. Figure 4(c, d): The Type2-PCM and Type2-PCM-IAPSO algorithms have found two distinct centers and have given best results than the FCM algorithm and the PCM algorithm. However, our proposed algorithm gives desirable results with cluster centers located almost at their prototype location.
The parameters for all the algorithms are: *itermax*=650, *pop_size=60*, e=10^{-7}, *C1i*=2.5, *C1f*=0.5, *C2i*=0.5, *C2f*=2.5, *wmax*=0.9 *and wmin*=0.4.

Pt	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
x1	4	3	5	4	3	5	4	3	5	4	4	10	13	16	10	13	16	10	13	16	8.5	8.5
x2	0	0	0	1	1	1	-1	-1	-1	2	-2	0	0	0	4	4	4	-4	-4	-4	15	0

Table 1: The coordinates of data.

The ideal centers are:$C_{ideal} = \begin{bmatrix} 4 & 0 \\ 13 & 0 \end{bmatrix}$
Table 2 shows the error between the prototype results and the ideal centers clusters of four algorithms using the following expression: $\varepsilon_* = \|C_{ideal} - C_*\|^2$
Where $*$:Are the FCM, PCM, Type2-PCM and Type2-PCM -IAPSO.

Methods	FCM	PCM	Type2-PCM	Type2-PCM-IAPSO
Error (ε_*)	0.532	1.642	**1.18 10^{-6}**	**2.82 10^{-7}**

Table 2: Error between the prototype results and the ideal center.

According to Table 2, the Type2-PCM-IAPSO algorithm offers the best results compared to other algorithms.

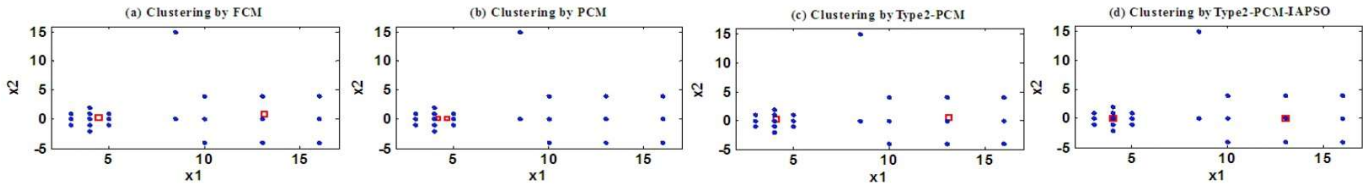


Figure 4: (a) Clustering by FCM, (b) Clustering by PCM, (c) Clustering by Type2-PCM, and (d) Clustering by Type2-PCM-IAPSO.

5.2 Validation model

After applying the identification algorithms, we must test the validity of the Takagi-Sugeno fuzzy model used. Among these tests, we cited the Mean Square Error (MSE) test and the Variance Accounting For (VAF):

$$MSE = \frac{1}{N}\sum_{k=1}^{N}\left(y_{or}\left(k\right) - y_{mo}\left(k\right)\right)^2 \tag{34}$$

$$VAF = 100\%\left[1 - \frac{var\left(y_{or} - y_{mo}\right)}{var(y_{or})}\right] \tag{35}$$

Where 'y_{or}' is the original output and 'y_{mo}' is the model output.

5.2.1 Example1: Box-Jenkins gas furnace

The Box-Jenkins gas furnace is an example of identification of the linear system. The Box-Jenkins data set consists of 296 input-outputs, where the input u (k) is the gas flow velocity in an oven and the output y (k) is the concentration of CO_2 in the gas output [19].In our case we used for this model six inputs {*y(k-1), y(k- 2), y (k- 3), u(k), u(k-1), u(k-2)*} which are selected as input variables to PCM, Type2-PCM and Type2-PCM-IAPSO.To analyze the influence of noise we used the different levels SNR (SNR = 10dB and SNR= 5 dB). In our case we used the first148 input-output training data and the other 148 as test data. The number of the rule of fuzzy model C= 2.
In this example the used parameters for all the algorithms are: $itermax$=650, pop_size=60, e=10^{-7}, C_1i=2.5, C_1f=0.5, C_2i=0.5, C_2f=2.5, w_{max}=0.9 and w_{min}=0.4. The simulation result given by the Type2-PCM-APSO algorithm is illustrated in Figure 5.

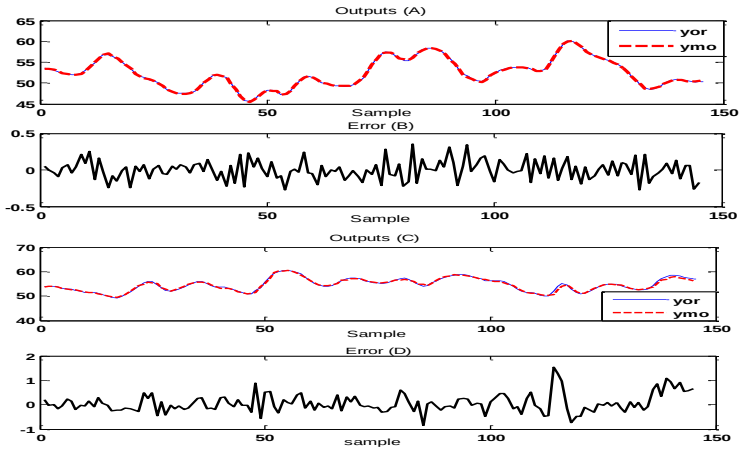


Figure 5: Identification results for the Type2-PCM-IAPSO: (A) and (B) the outputs and error for training data, (C) and (D) the outputs and error for testing data (SNR=10dB)

Methods	**Nb.of rules**	**MSE**$_{-trn}$	**MSE**$_{-test}$	**VAF** (%)
Lin et al. (1995)	5	0.071	0.261	-
Kim et al. (1998)	6	0.034	0.244	-
Tsekouras (2005)	6	0.022	0.236	-
Chaoshun Li, Jianzhong Zhou	3	0.0159	0.1255	-
RKPFCM [23] without noise	2	-	0.1458	-
RKPFCM [23] (SNR=10dB)	2	-	0.2342	-
RKPFCM [23] (SNR=5dB)	2	-	0.2922	-
RKPFCM-PSO [23] without noise	2	-	0.1455	-
RKPFCM-PSO [23](SNR=10dB)	2	-	0.2338	-
RKPFCM -PSO [23] (SNR=5dB)	2	-	0.2902	-

PCM without noise	2	0.0151	0.2933	98.7510
PCM (SNR=10dB)	2	0.0157	0.2942	98.7485
PCM (SNR=5dB)	2	0.0162	0.2951	98.7502
Type2-PCM without noise	**2**	**0.0144**	**0.1425**	**99.1589**
Type2-PCM (SNR=10dB)	**2**	**0.0146**	**0.2145**	**99.1452**
Type2-PCM (SNR=5dB)	**2**	**0.0150**	**0.2189**	**99.1388**
Type2-PCM-IAPSO without noise	**2**	**0.0134**	**0.1089**	**99.6543**
Type2-PCM–IAPSO (SNR=10dB)	**2**	**0.0143**	**0.1147**	**99.6491**
Type2-PCM-IAPSO(SNR=5dB)	**2**	**0.0148**	**0.1198**	**99.6462**

Table 3: Comparison results for Box-Jenkins system.

Table 3 shows that the best *MSE* and the best *VAF* are obtained by the Type2-PCM-IAPSO algorithm. The local linear models identified by the Type2-PCM-IAPSO algorithm are the following:

$$\begin{aligned} y_1(k) &= 0.0212\, y(k-1) - 0.4345 y(k-2) + 1.2678\, y(k-3) \\ &\quad - 0.8689\, u(k) + 0.4152\, u(k-1) - 0.0645 u(k-2) + 9.3758 \end{aligned} \tag{36}$$

$$\begin{aligned} y_2(k) &= 0.0902\, y(k-1) - 0.6245 y(k-2) + 1.3897\, y(k-3) \\ &\quad - 0.7312\, u(k) + 0.1923\ u(k-1) + 0.0389 u(k-2) + 8.2647 \end{aligned}$$

5.2.2 Example 2: Dryer system

The dryer system is a non-linear system widely used in industries. Assuming the supply voltage of the heater is the input and the air temperature is the radiator output. The data set of the drying system comprises 1000 input-output observations, where the input *u (k)* is the supply voltage of the heater and the output *y (k)* is the air temperature [26, 27, 16]. In our case we used the first 500 input-output training data and the other 500 as test data, we used for this model six inputs {*y (k-1), y (k- 2), y(k- 3), u(k), u(k-1),u(k-2)*} which are selected as input variables to PCM, Type2-PCM and Type2-PCM-IAPSO. To analyze the influence of noise we used the different levels *SNR (SNR = 20dB and SNR = 10dB).*

In this example the used parameters for all the algorithms are*:* *itermax*=500, *pop_size*=50, e=10^{-7}, C_1i=2.5, C_1f=0.5, C_2i=0.5, C_2f=2.5, w_{max}=0.9 and w_{min}=0.4.
The Input-Output sequences are illustrated in Figure 6.
The simulation result given by the Type2-PCM-IAPSO algorithm is illustrated in Figure 7.

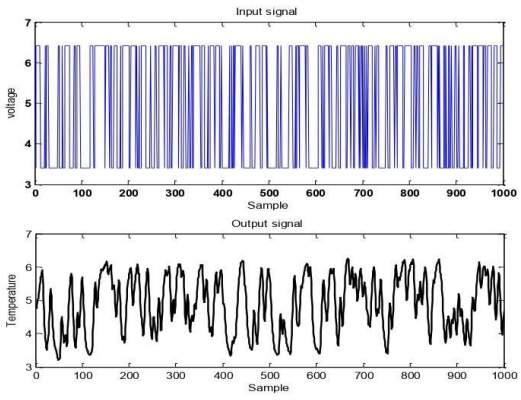


Figure 6: Input-output sequences

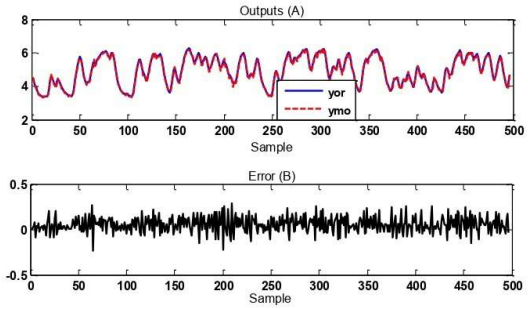


Figure 7: Type2-PCM-IAPSO performances for test data with $SNR = 20$ dB (A: outputs, B: error)

The Figure 7 represents the efficiency of our proposed algorithm in the case of noise ($SNR = 20$ dB).

Methods	**Nb.of rules**	**MSE**$_{-trn}$	**MSE**$_{-test}$	**VAF** (%)
FCM	2	0.0832	0.3126	97.6543
GK	2	0.0542	0.2968	98.0865
PCM	2	0.0219	0.1876	98.8543
Type2-PCM	**2**	**0.0144**	**0.1123**	**99.3387**
Type2-PCM-IAPSO	**2**	**0.0111**	**0.0978**	**99.8248**

Table 4: Comparison results without noise.

Methods	**Nb.of rules**	**MSE**$_{-trn}$	**MSE**$_{-test}$	**VAF** (%)
FCM	2	0.0956	0.3764	96.7585
GK	2	0.0911	0.3621	96.9865
PCM	2	0.0875	0.2876	97.2543
Type2-PCM	**2**	**0.0912**	**0.1389**	**98.7234**
Type2-PCM-IAPSO	**2**	**0.0823**	**0.1205**	**99.1089**

Table 5: Comparison results with SNR=20 dB.

Methods	**Nb.of rules**	**MSE**$_{-trn}$	**MSE**$_{-test}$	**VAF** (%)
FCM	2	0.1103	0.4045	96.4563
GK	2	0.1067	0.3998	96.8875
PCM	2	0.0973	0.3122	97.2137
Type2-PCM	**2**	**0.1024**	**0.2812**	**97.7245**
Type2-PCM-IAPSO	**2**	**0.0925**	**0.2014**	**98.4412**

Table 6: Comparison results with SNR=10 dB.

Table 4 shows that the best *MSE* and the best *VAF* are obtained by the proposed algorithm (Type2-PCM-IAPSO). Tables 5 and 6 present the *MSE* and *VAF* of four identification algorithms with noise influence (SNR= 10dB and 20 dB). However, our proposed algorithm Type2-PCM-IAPSO gives the best performance.
The local linear models identified by the Type2-PCM-IAPSO algorithm are the following:

$$\begin{aligned} y_1(k) = {} & 0.2912y(k-1) - 1.2845y(k-2) + 1.9542\, y(k-3) \\ & - 0.0289\, u(k) + 0.0102\, u(k-1) - 0.0145u(k-2) - 0.0758 \end{aligned} \tag{37}$$

$$\begin{aligned} y_2(k) = {} & 0.3547y(k-1) - 1.4147y(k-2) + 2.0124\, y(k-3) \\ & 0.0547u(k) + 0.0145u(k-1) - 0.0214u(k-2) + 0.0356 \end{aligned}$$

5.2.3 Example 3

We consider the nonlinear system given as [5].

$$y(k) = \frac{y(k-1)(y(k-2)+2)\,(y(k-1)+2.5)}{8.5 + y^2(k-1) + y^2(k-2)} + u(k) + \varphi \tag{38}$$

This is used as a test for identification techniques introduced in this work, to demonstrate the effectiveness of the proposed algorithms. Here y_k is the output, u_k is the input which is uniformly bounded in the region $[-1, 1]$.
The training data set contains 500 input–output pairs while for the testing 1000 data pairs are generated by the following input signal:

$$u_k = \left\{ \begin{array}{ll} \sin(\frac{2k\pi}{260}) & if \quad k\langle = 500 \\ 0.8\sin(\frac{2k\pi}{260}) + 0.2\sin(\frac{2k\pi}{26}) & otherwise \end{array} \right\} \tag{39}$$

The presence of noise and non-trend points is represented by the error modelφdefined by the following expression:

$$\varphi = (1-\gamma)F + \gamma L \tag{40}$$

Where $F = N(0, {\sigma_1}^2)$,$L = N(0, {\sigma_2}^2)$
F and L are distributions of Gaussian white noise, having standard deviations $s_2 > s_1$. The term F represents the presence of noise and the term L represents the presence of aberrant points [40]. The parameter e makes it possible to adjust the proportion of aberrant points in the whole distribution.
In this example, we simulated two cases: Case 1 and Case 2.
Case 1: $s_1 = 0.07$, $s_2 = 5$ *s1* and $\gamma = 0.15$.
Case 2: $s_1 = 0.07$, $s_2 = 5$ *s1* and $\gamma = 0.65$.
The training data set contains 500 input–output pairs while for the testing 500 data pairs.
In our cases we used for this model four inputs {*u (k), u (k-1), y (k-1), y (k-2)*} and a single output *y (k)*.
In this example the used parameters for all the algorithms are: $itermax$=600, pop_size=60, e=10^{-7}, C_1i=2.5, C_1f=0.5, C_2i=0.5, C_2f=2.5, w_{max}=0.9 and w_{min}=0.4.
The shape of the excitation signal used for identification is illustrated in Figure 8.
The simulation result given by the Type2-PCM-IAPSO algorithm is illustrated in Figure 9.

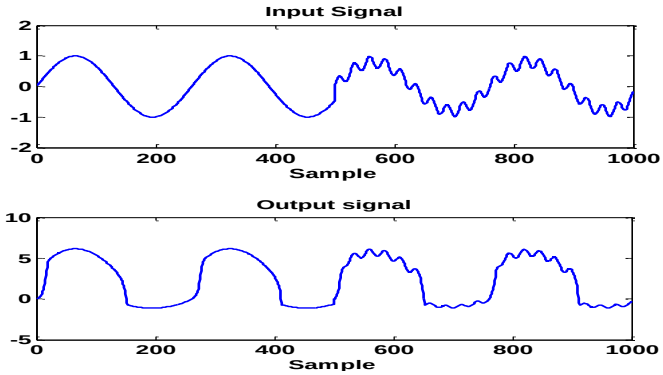


Figure 8: Input-Output sequences

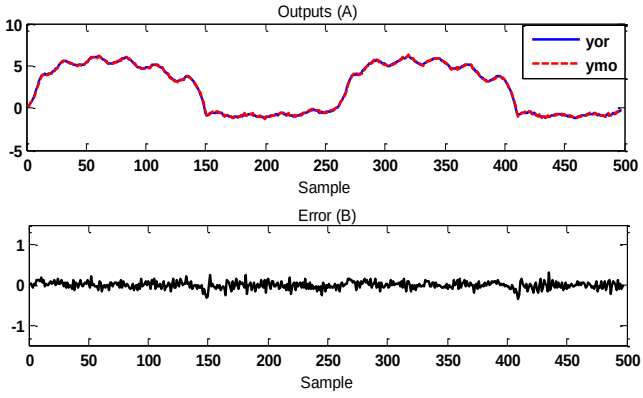


Figure 9: Identification results for the Type2-PCM-IAPSO: (A) and (B) the outputs and error for testing data

Methods	$\mathbf{MSE}_{-trn}$	$\mathbf{MSE}_{-test}$	**VAF** (%)
FCM (Hoppner *et al.* 1999)	0.0090	0.2220	_
GK (Gustafson and Kessel, 1979),	0.0046	0.1347	_
FMI (Chen *et al*, 1998).	0.0013	0.0181	_
NFCRMA(Moz soltani 2012)	5.20e-4	0.0096	_
Type2-PCM	**1.03e-4**	**0.0043**	**99.8658**
Type2-PCM-IAPSO	**2.87e-5**	**0.0027**	**99.9514**

Table 7: **Case1:** Comparison of different models for the nonlinear differential equation.

Methods	$\mathbf{MSE}_{-trn}$	$\mathbf{MSE}_{-test}$	**VAF** (%)
FCM (Hoppner *et al.* 1999)	0.0095	0.2324	_
GK (Gustafson and Kessel, 1979),	0.0061	0.1467	_
FMI (Chen *et al*, 1998).	0.0039	0.0197	_
NFCRMA(Moz soltani 2012)	6.34e-4	0.0107	_
Type2-PCM	**1.21e-4**	**0.0048**	**99.8345**
Type2-PCM-IAPSO	**2.97e-5**	**0.0037**	**99.9574**

Table 8: **Case2:** Comparison of different models for the nonlinear differential equation.

Tables 7, 8 present the *MSE* and *VAF* of identification algorithms with noise and aberrant points influence. However, the Type2-PCM-IAPSO algorithm gives the best performance with a higher noise level.
The local linear models identified by the Type2-PCM-IAPSO algorithm are the following:

$$\begin{aligned}
y_1(k) &= 1.5412y(k-1) - 0.6754y(k-2) + 0.06775u(k) \\
&\quad -0.0478u(k-1) + 0.0415 \\
y_2(k) &= 1.8645y(k-1) - 0.9233\, y(k-2) + 0.0745u(k) \\
&\quad -0.0645u(k-1) + 0.0478 \\
y_3(k) &= 2.1478y(k-1) - 1.1589y(k-2) - 2.7322u(k) \\
&\quad -2.9214u(k-1) + 0.0714 \\
y_4(k) &= 1.8879y(k-1) - 0.9215y(k-2) - 1.1648u(k) \\
&\quad +1.2798u(k-1) + 0.0532
\end{aligned} \tag{41}$$

5.2.4 Example 4: Convection study

The machine used to study the transfer of heat by convection, this model allows students to test and understand this phenomenon.

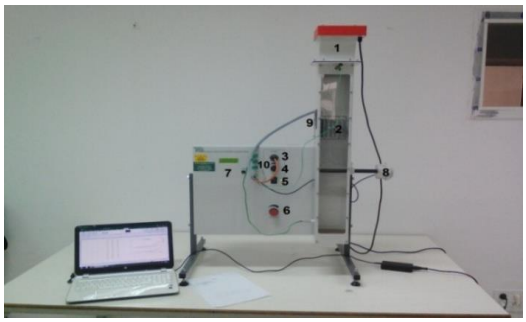


Figure 10: Didactic model for convection study

In this machine, the air passes in the vertical pipe and through the exchanging heat surface. The didactic model is made of 8 components: an electrical resistance (9), an exchange heat surface (2), a ventilator (1), an electrical power converter (4), and three thermocouples to measure the temperature of the exchange surface i.e. the temperatures of the admission and outflow air. The didactic model contains as well an air speed controller (6), an air speed sensor (8), a digital display for 3 temperatures (7) and a power control (5).The data set of the drying system comprises 100 input-output observations, where the input *u (k)* is the electrical power and the output *y (k)* is the air temperature.
In our case we used the first 50 input-output training data and the other 50 as test data, we used for this model four inputs {*y (k-1), y (k- 2), u(k), u(k-1)* } which are selected as input variables to PCM, Type2-PCM and Type2-PCM-IAPSO. To analyze the influence of noise we used the different levels SNR (SNR = 20dB and = 10dB). In this example the parameters for all the algorithms are: C=2, *itermax*=500, *pop_size*=70, e=10^{-7}, C_1i=2.5, C_1f=0.5, C_2i=0.5, C_2f=2.5, w_{max}=0.9, w_{min}=0.4.
The Input-Output sequences are illustrated in Figure 11.
The simulation result given by the Type2-PCM-IAPSO algorithm is illustrated in Figure 12.

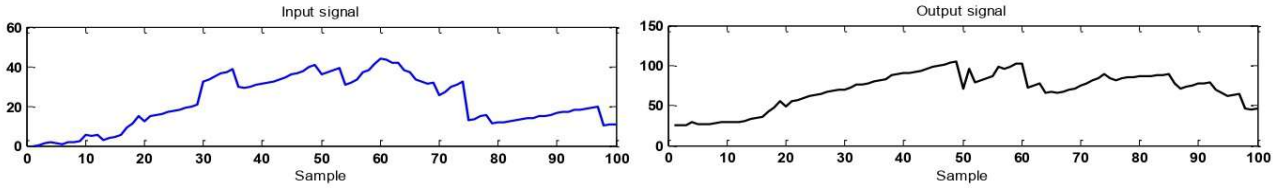


Figure 11: Input-Output Sequences

Figure 11 represents the nonlinear input-output sequences of our real system used to illustrate the efficiency of our contribution.

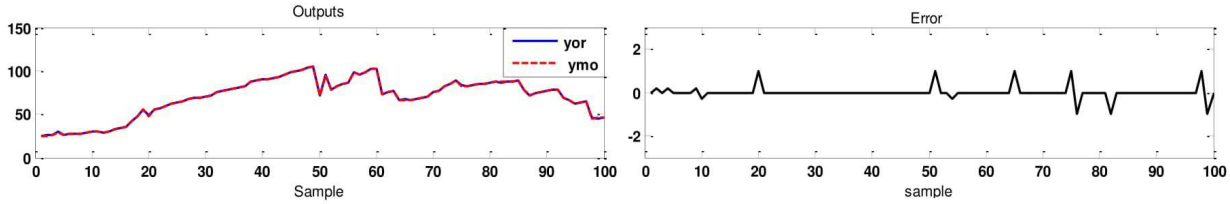


Figure 12: Identification Results for the proposed algorithm Type2-PCM-IAPSO

The convergence of the fitness functions for the two algorithms Type2-PCM and Type2-PCM-IAPSO during the iteration is represented in Figure 13.

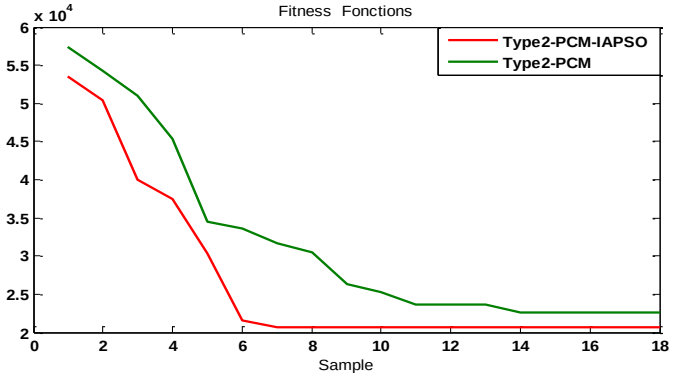


Figure 13: Convergence of the fitness functions values

The measurements and the clusters centers by the Type2-PCM-IAPSO algorithm for test data are represented in Figure 14.

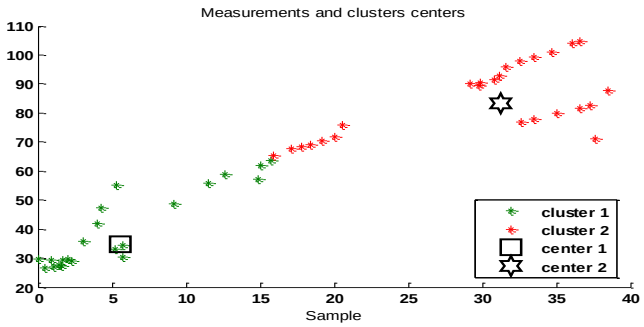


Figure 14: Measurements and clusters centers

Methods	MSE_trn	MSE_test	VAF(%)
FCM	0.1047	0.1174	96.1859
GK	0.1023	0.1162	96.1954
PCM	0.0988	0.1102	96.2140
Type2-PCM	0.0878	0.0844	96.2689
Type2-PCM-IAPSO	0.0823	0.0789	96.2758

Table 9: Comparison Results without noise

Methods	MSE_trn	MSE_test	VAF(%)
FCM	0.0965	0.1114	96.7548
GK	0.0928	0.1077	96.7610
PCM	0.0877	0.1047	96.7647
Type2-PCM	0.0735	0.0944	96.7898
Type2-PCM-IAPSO	0.0714	0.0923	97.2178

Table 10: Comparison Results with SNR=20 dB

Methods	MSE_trn	MSE_test	VAF(%)
FCM	0.1086	0.1196	96.1578
GK	0.1066	0.1173	96.1623
PCM	0.1027	0.1154	96.1688
Type2-PCM	0.0956	0.0968	96.1978
Type2-PCM-IAPSO	0.0912	0.0847	96.2274

Table 11: Comparison Results with SNR=10 dB

Table 9 shows that the best *MSE* and the best *VAF* are obtained by the proposed algorithm (Type2-PCM-IAPSO). The table 10 and 11 present the *MSE* and *VAF* of the identification algorithms with noise influence (SNR = 10dB and 20 dB). However, our proposed algorithm **Type2-PCM-IAPSO** contributes the best performances.

The local linear models identified by the Type2-PCM-IAPSO algorithm are the following:

$$y_1(k) = 1.2285y(k-1) - 0.8521y(k-2) + 0.1347u(k) - 0.0748u(k-1) + 0.0587 \tag{42}$$

$$y_2(k) = 1.5014y(k-1) - 1.1324\,y(k-2) + 0.9144u(k) - 0.0748u(k-1) + 0.0247$$

The clusters centers identified by the Type2-PCM-IAPSO algorithm are the following:

$$clusters_{\text{centers}} = \begin{pmatrix} 36.7848 & 36.5847 & 5.4578 & 5.5875 \\ 82.1235 & 82.2201 & 31.7548 & 31.8247 \end{pmatrix} \tag{43}$$

6 Conclusion

In this work, two fuzzy clustering algorithms Type2-PCM and Type2-PCM-IAPSO are proposed. The first suggested algorithm (Type2-PCM) is based on the recent modification of the PCM where we used a Type2 membership function. The Type2-PCM clustering algorithm can solve some problems related to the classical fuzzy clustering algorithms (FCM, GK...etc) and especially the sensitivity to noise and aberrant points. The second considered algorithm Type2-PCM-IAPSO is based on the combination of the Type2-PCM algorithm with the IAPSO optimization algorithm for the modeling and identification of nonlinear systems in order to obtain a global optimization. Our two contributions proposed in this article allow solving two serious problems of the framework of modeling and identification of nonlinear systems, these two problems are: convergence towards local optimum and sensitivity to noise.
The efficiency of the new clustering algorithms Type2-PCM and Type2-PCM-IAPSO is tested on a system described by a different equation, the electro-hydraulic system, and the convection system. The validation tests used showed good performance of these algorithms. However, their MSE and VAF show a better behaviour of the Type2-PCM-IAPSO algorithm compared to the FCM, GK, PCM, FCM-PSO, RKPFCM, RKPFCM-PSO and Type2-PCM algorithms.
Our approach comprises multiple long-term prospects. To start with, we can improve the quality of the identification and modeling of nonlinear systems using other optimization algorithms, such as the IAPSO algorithm with a new adaptive inertia weight which allows ensuring a good balance between exploitation and exploration in the research field. Added to that, we may as well combine the clustering algorithms with new optimization methods (GPSO, firefly algorithm and BAT algorithm ... etc). Last but not least, to prove the efficiency of our proposed algorithms, we can identify the real non-linear systems (Box and Jenkiss gas furnace, dryer system... etc.). Finally, we will definitely be able to use these algorithms to control of nonlinear systems.

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