Iranian Journal of Fuzzy Systems

Volume *, Number *, (****), pp. **

University of Sistan and Baluchestan

Design and Analysis of Acceptance Sampling Plans Based on Intuitionistic Fuzzy Linguistic Terms

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Abstract

Acceptance sampling plans (ASPs) offer inspection of a small set of items from a lot within a predefined plan to procure a certain output quality level with minimum cost in terms of time, effort, and damage to the inspected items. Although traditional ASPs use crisp plan parameters, quality characteristics of the incoming items or human evaluations about inspection process may contain uncertainties and may not always be defined as crisp values in real life problems. The fuzzy set theory (FST) is one of the most popular techniques to model these uncertainties by defining plan parameters as fuzzy numbers. Despite the advantages, traditional fuzzy sets are not flexible enough to model all kind of uncertainties. For example, it has some disadvantages because of defining the status of any item based on defectiveness or non-defectiveness conditions and presuming the parts as non-defective whose defectiveness is not indeed determined. New extensions of FST can improve the quality of uncertainty modeling of ASPs. Intuitionistic Fuzzy Sets (IFSs) allow slackness for non-determination about the membership and give more sensitive modeling opportunity in human-related evaluations by the help of this ability. Since the inspection procedure of the ASPs depends on human-related judgements, IFSs have been used to define the defectiveness degree of the items in this study. ASPs based on interval-valued IFSs (IVIFSs) have also been designed and some characteristic functions of ASPs, such as acceptance probability (P_a), average sample number (ASN) and average total inspection (ATI) have been reformulated. Intuitionistic binomial and Poisson distributions have been defined to be able to formulate the ASPs. Additionally, the defectiveness of the items has been represented by using linguistic terms to overcome the difficulty of quantifying the verbal evaluation results as numerical measures. The α-cut technique has been combined with the linguistic approach to allow defining with multiple α values for different product segments. Finally, some numerical examples have been presented to analyze the effectiveness of proposed ASPs and discuss the obtained results.

Keywords: Acceptance sampling plans, intuitionistic fuzzy sets, binomial distribution, fuzzy sets, interval intuitionistic fuzzy sets, poisson distribution.

2021 MSC: 03E72.

1 Introduction

Keeping the quality of the manufactured products at a certain level has vital importance in manufacturing process. In that case, the quality of the produced items can be measured statistically by using Acceptance sampling plans (ASPs) that are specific set of rules which offer to inspect a small set of items instead of all of the produced items to reach a pre-determined producer's risk ($\propto$) and consumer's risk (β). Statistically, $\propto$ refers to the probability of rejecting a lot in which the defective ratio (p) of the lot is the same as the producer's acceptable quality level, and β refers to the probability of accepting a lot while p is over the allowed proportion defective [11]. The sample size (n) and acceptable defective item count (c) are main ASP parameters, and they are adjusted to optimize the inspection cost while satisfying

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Received: **; Revised: **; Accepted: **.

the allowed risk margins. Sometimes, tighter risk margins are allowed for the items having high inspection costs or effort. For such cases, one types of ASP named double acceptance sampling plans (DASPs) are used for reaching lower risks with relatively small sample sizes [26]. Especially in real case problems, the inspection process can include many uncertainties as results of many factors, such as human's hesitancy and poorness of measurement methods. Unfortunately, the defectiveness ratio of the lots and the plan parameters have to define as certain values in traditional ASPs. Depending on this, they may not represent realistic, sensitive, and robust risk levels under uncertainties. The fuzzy set theory (FST) is a popular approach to model these uncertainties of the real case problems. Easy to guess, the uncertainty can be caused by multiple factors such as lack of information about the event, human error, inadequacy of measurement method, lack of expertness about the event. FST has been extended recently to model such complicated scenarios better, depending on the nature of the uncertainty. It is built on membership function using a continuous variable between 0 and 1. This approach extends the deterministic statements by replacing the absolute binary variables with uncertain ones [28]. FST has been successfully used in quality problems and acceptance sampling. However, it is clear that traditional, namely type-1 FST is well-performing if the event itself causes the uncertainty.

Atanassov introduced one of the FST extensions called Intuitionistic Fuzzy Sets (IFSs), which offers better modeling in the case of lack of information about fuzziness [9]. IFSs give the flexibility to take into account the non-determinacy about the membership to the FS. In this paper, this advantage has been used for an extend analysis of ASPs. We also know that the formulations of ASPs based on type-1 FSs have drawbacks about sensitive modeling if non-determinacy is possible about the defectiveness of the inspected items. When ASPs are formulated based on IFSs, the inspected items can be classified as defective, non-defective, or non-determinate based on their robustness or defectiveness levels. In this way, more sensitive modeling is reached and it is a better way to represent human's hesitancy. There are some studies in the literature about the design of ASPs based on type-1 FSs, but there is no study for the ASPs based on IFSs. This paper aims to design single and double ASPs based on IFSs and deriving main characteristics of ASPs such as acceptance probability (P_a), average sample number (ASN), average total inspection (ATI), and average outgoing quality (AOQ) for Poisson and binomial distributions. Thus, it is aimed to fill an important gap in the literature. To do this, the intuitionistic Poisson and binomial distributions have been defined first, then the ASP formulations have been derived on these distributions. Even if the ASPs have been analyzed and reformulated based on IFSs, these formulations still reveal a challenge on deciding defectiveness or non-determinacy of items. If operators are responsible for deciding the defectiveness or robustness level of items, it can cause a human-factor-related error in the results of the ASPs. For this reason, a linguistic term set (LTS) is also developed and integrated with these formulations to standardize the definitions of defectiveness classes and minimize the human factor-related classification errors. Turning FSs into interval-valued sets using α-cut approach offers more simple and useful practice in most real-life applications, but working with numeric α-levels has a big drawback. Another contribution of this study to the literature is merging the linguistic approach with α-cut approach, thereby giving the ability to work with multiple linguistic α-levels for different product segments simultaneously.

The rest of this paper has been organized as follows: Section 2 includes brief information about IFSs and α-cut approach. A literature analysis related to FST and ASPs is presented in Section 3. The obtained formulations for ASPs based on IFSs, binomial and poisson distributions are presented, and these are analyzed with some numerical examples in Sections 4 and 5, respectively. The obtained results and future research directions have been discussed into Section 6.

2 Intuitionistic fuzzy sets

Membership ($\mu_{\tilde{A}}(x)$) and non-membership ($\vartheta_{\tilde{A}}(x)$) functions complement each other, and their summation is equal to 1 for type-1 FSs. This means comprehensive information about an event and it is named as complete information case. However, it may not always be possible to decide whether membership and non-membership degrees or equivalent adverse effects may not be observed between membership and non-membership degrees. For example, some factors such as physical obstacles or instructional inabilities can cause hesitation while deciding the defectiveness of an inspected item. As a result of this, it can block the complete information case. IFSs have been offered to model the uncertainties, including such incomplete information cases. An IFS is stated as in Eq. (1) depending on a continuous variable x inside $[0, 1]$ interval [9] [27].

$$\tilde{\tilde{A}} = \{(x, \mu_{\tilde{\tilde{A}}}(x), \vartheta_{\tilde{\tilde{A}}}(x) | x \in X)\} \; and \; \mu_{\tilde{\tilde{A}}}(x) + \vartheta_{\tilde{\tilde{A}}}(x) \leq 1 \tag{1}$$

The accent $\tilde{\tilde{}}$ has been preferred to distinguish the IFSs in equations. On the other hand, type-1 FSs have been represented with the accent $\tilde{}$ throughout the paper.

The uncertainty arising from incomplete information is named as degree of hesitancy or degree of non-determinacy. It is dependent to membership and non-membership functions, and it is represented with the statement as shown in Eq. (2) [9] [27]. Accuracy degree of an IFS is calculated as $1 - \pi_{\tilde{\tilde{A}}}(x)$ so the accuracy degree is big when the accuracy is small [27].

$$\pi_{\tilde{\tilde{A}}}(x) = 1 - (\mu_{\tilde{\tilde{A}}(x)} + \vartheta_{\tilde{\tilde{A}}}(x)) \tag{2}$$

Generally, membership and non-membership are represented with functions for IFSs. However, they can consist of interval-valued numbers in some cases. These sets are named as interval-valued IFSs (IVIFSs) and represented with two independent intervals in $[0, 1]$. IVIFSs are useful in engineering problems because working with interval-valued numbers instead of continuous functions ensures ease of calculation and sense-making. Representation of IVIFSs is shown in Eq. (3) [9] while supremum (sup) is referring to the least upper bound.

$$\begin{gathered}\tilde{\tilde{A}} = (\mu_{\tilde{\tilde{A}}}, \vartheta_{\tilde{\tilde{A}}}) = \langle [\mu_{\tilde{\tilde{A}}_L}, \mu_{\tilde{\tilde{A}}_U}], [\vartheta_{\tilde{\tilde{A}}_L}, \vartheta_{\tilde{\tilde{A}}_U}] \rangle \; and \; \mu_{\tilde{\tilde{A}}}, \vartheta_{\tilde{\tilde{A}}} \in [0, 1] \\ 0 \leq \sup \mu_{\tilde{\tilde{A}}} + \sup \vartheta_{\tilde{\tilde{A}}} \leq 1\end{gathered} \tag{3}$$

2.1 The α-cut approach for fuzzy sets

A threshold value on membership function is decided, and the fuzzy set (FS) is cut from this level horizontally in α-cut approach. More considerable membership degrees are assumed as a full member while the lower membership degrees are ignored [29]. In this way, an interval-valued set is obtained, and the membership value can be stated as a $0 - 1$ crisp value. Triangular FSs (TFSs) are represented with three points as (a_1, a_2, a_3) while a_1 and a_3 are supports and a_2 is core. If α-cut is applied to $\tilde{A}$, the interval $\tilde{A}_\alpha = [a_{1\alpha}, a_{3\alpha}]$ is obtained as show in Figure 1 [15]. Similarly, α-cut of a Trapezoidal FS (TrFS) $\tilde{A} = (a_1, a_2, a_3, a_4)$ is the interval $\tilde{A}_\alpha = [a_{1\alpha}, a_{4\alpha}]$ while a_1, a_4 are supports and $[a_2, a_3]$ is core as show in Figure 1.

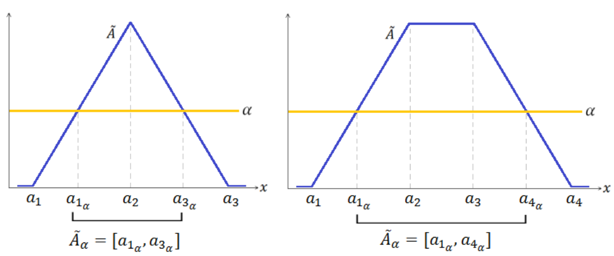

Figure 1: α-cut for an example TFS and TrFS

Any shape of IFS such as triangular IFSs (TIFSs) and trapezoidal IFSs (TrIFSs) can be converted to IVIFSs by using α-cut technique. For an IFS $\tilde{\tilde{A}} = \{(x, \mu_{\tilde{\tilde{A}}}(x), \vartheta_{\tilde{\tilde{A}}}(x) | x \in X)\}$, α-cut is formed as shown in Eq. (4) for an $\alpha \in [0, 1]$ [9]. α-cut for an example TrIFS is also shown in Figure 2.

$$\begin{aligned}\tilde{\tilde{A}}_\alpha =& \{x : x \in X, \mu_{\tilde{\tilde{A}}}(x) \geq \alpha, \vartheta_{\tilde{\tilde{A}}}(x) \leq 1 - \alpha\} \\ =& \langle \tilde{\mu}_\alpha, \tilde{\vartheta}_{1-\alpha} \rangle = \langle [\mu_L, \mu_U], [\vartheta_L, \vartheta_U] \rangle\end{aligned} \tag{4}$$

For given two intervals $D = [d_1, d_3]$ and $E = [e_1, e_3]$ satisfying $\forall d_1, d_3, e_1, e_3 \in \mathbb{R}^+$, addition, subtraction, and multiplication operations are given in Eqs. (5- 7), respectively [15]. These interval operations are applicable to the intervals obtained by α-cut.

$$[d_1, d_3] \oplus [e_1, e_3] = [d_1 + e_1, d_3 + e_3] \tag{5}$$

$$[d_1, d_3] \ominus [e_1, e_3] = [d_1 - e_3, d_3 - e_1] \tag{6}$$

$$[d_1, d_3] \otimes [e_1, e_3] = [d_1 \times e_1, d_3 \times e_3] \tag{7}$$

The operations defined in Eqs. (5- 7) are also arithmetic operations, but they are defined with the help of the ordinary ones. In the rest of this paper, the mathematical symbols of the addition, subtraction, and multiplication

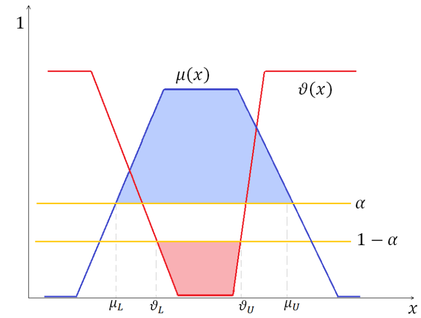

Figure 2: α-cut for an example TrIFS

operations related to FSs and interval-valued numbers have been differentiated from the ordinary ones to not bring any confusion. The symbols $\oplus$,$\ominus$ and $\otimes$ have been used for addition, subtraction, and multiplication operations, respectively related to FSs during this study.

3 Acceptance sampling plans based on fuzzy sets

ASPs consist certain set of rules to reach a specified quality level by inspecting only a small sample instead of all items in a lot. Despite ASPs can be classified into two groups such as variable and attribute, the APSs based on attribute characteristics are more common. While variable ASPs focus on the quality characteristics such as length, weight, etc., the attribute ASPs are interested in the inspected items' defectiveness. They are formulated with Binomial or Poisson distributions by considering the inspection operation as a probabilistic event. For binomial attribute ASPs, plan parameters are sample size (n) and maximum allowed defect count (c), which are decided to reach a specified quality level with minimum cost. The inspector or operator selects only one sample from a lot randomly from a population with a known size (N) and having a known ratio of defective items (p). Then he accepts the lot if the observed defective item count is less than or equal to c. On the other hand, frequency of defective item (λ) is used for Poisson distribution instead of p. Binomial ASP formulation can be converted to Poisson ASP by using the transformation $\lambda = n \times p$ [26]. Sometimes it is needed to reach lower risks with relatively small sample sizes. For these cases, double ASPs (DASPs), two-step ASPs having two sets of plan parameters, are used. For the DASPs, if the first limit for allowable defect count (c_1) is not exceeded in first sample size (n_1) in the first step, the lot is accepted. Similarly, if the second threshold value for defect count (c_2) is exceeded in the first step, the lot is rejected without continuing the second step. In addition to that, the second inspection step is applied with a new sample size (n_2) if the defective item count is observed between c_1 and c_2 [26]. Binomial DASPs can be converted to Poisson DASPs by the conversions $\lambda_1 = (n_1 \times p)$ and $\lambda_2 = (n_2 \times p)$ [26]. ASPs should have lot acceptance probability (P_a) as near as $\propto$ to minimize the inspection cost [26].

Some characteristic functions are also available for ASPs. For example, the average outgoing quality (AOQ) represents the long-term average fraction defective that the consumer will encounter for a given value of p. Another characteristic is average total inspection (ATI) which is the average number of inspections per lot if 100% inspection is performed for rejected lots [26]. Lastly, the average sample number (ASN) to be inspected per lot to reach acceptance or rejection decision is a characteristic for only DASPs [26].

Some studies have been carried out in the literature for attribute ASPs based on binomial and Poisson distributions by using the type-1 FSs [13],[22],[23],[21]. They formulated and solved the sampling plan with the α-cut approach. There are also limited studies based on Neutrosophic Sets (NSs), and there are no studies based on the other FS extensions such as type-2, intuitionistic, hesitant, and Pythagorean FSs on ASPs. Some studies about variable ASPs based on NSs have been performed to determine the neutrosophic plan parameters of n and c by using a non-linear optimization model as a pre-step of sampling. [3] designed the optimization model for the populations whose quality characteristics were introduced as an exponential distribution under the neutrosophic interval statistical method. [2] redesigned this optimization model by using the neutrosophic approach for the process loss function. [7] introduced neutrosophic regression estimator and used it in this optimization model. [6] performed a similar study by considering the measurement error using neutrosophic statistics. [5] rebuilt the optimization model for the population having quality characteristics as normal distribution and it obtained the operating characteristic as neutrosophic statistics. [8] determined the sample size via a similar non-linear model by using Pareto distribution. There are also a few studies about attribute ASPs based on NSs in the literature. [4] offered a single attribute ASP for binomial distribution for the population whose defectiveness is formulated as NS with the help of the standard deviation. Single ASPs have been extended to NSs for Poisson distribution by [19] and double ASPs extended to Neutrosophic Sets (NSs) for binomial distribution by [20].

In this study, ASPs have been extended based on IFSs and IVIFSs for Poisson and binomial distributions. Additionally, the main characteristic functions of ASPs such as P_a, ASN, ATI, and AOQ have been derived based on IFSs. The main objective of this study is to minimize the effects of uncertainty on the inspection procedures and increase the sensitiveness and the flexibility with the help of fuzzy linguistic terms. To reach this objective, a linguistic evaluation procedure has also been suggested for inspection procedure.

3.1 Single acceptance sampling plans based on type-1 fuzzy sets

The single ASPs (SASPs) based on type-1 FSs have been analyzed in the literature. Acceptance probability functions were derived, and ASPs were developed based on binomial and Poisson distributions with fuzzy parameters [23]. In addition, these plans were extended for interval-valued FSs. These formulations are similar to traditional ASPs, but the distributions were switched with the fuzzy ones [23]. When a single ASP was organized with the parameters fuzzy sample size ($\tilde{n}$), and fuzzy maximum allowed defective count ($\tilde{c}$) for a population having fuzzy ratio of defective items ($\tilde{p}$), fuzzy ratio of non-defective items ($\tilde{q} = 1 - \tilde{p}$), the fuzzy acceptance probability of a lot ($\tilde{P}_a$) has been calculated as shown in Eq. (8). The defective item frequency $\tilde{\lambda}$ is also an FS for Poisson distribution, which is equal to $\tilde{n} \otimes \tilde{p}$ [23].

$$\tilde{P}_a = P\{d \leqslant \tilde{c} \mid \tilde{p}, \tilde{q}, \tilde{n}, \tilde{c}\} = \sum_{d=0}^{\tilde{c}} \frac{\tilde{\lambda}^d \otimes e^{-\tilde{\lambda}}}{d!} = \sum_{d=0}^{\tilde{c}} \binom{\tilde{n}}{d} \otimes \tilde{p}^d \otimes \tilde{q}^{\tilde{n} \ominus d} \tag{8}$$

If α-cut is applied to FSs, they turn into interval-valued FSs. Acceptance probability of a lot after α-cut ($\tilde{P}_{a_\alpha}$) is calculated as shown in Eq. (9) [23]. $\tilde{P}_{a_\alpha}$ is obtained as an interval having lower limit P_{al_α} and upper limit P_{ar_α}.

$$\tilde{P}_{a_\alpha} = [P_{al_\alpha}, P_{ar_\alpha}] = \begin{bmatrix} min\{P_a \mid p \in \tilde{p}_\alpha, q \in \tilde{q}_{1-\alpha}, n \in \tilde{n}_\alpha, c \in \tilde{c}_\alpha\}, \\ max\{P_a \mid p \in \tilde{p}_\alpha, q \in \tilde{q}_{1-\alpha}, n \in \tilde{n}_\alpha, c \in \tilde{c}_\alpha\} \end{bmatrix} \tag{9}$$

When $\tilde{n}$, $\tilde{c}$ and $\tilde{p}$ are defined as FSs, the AOQ also becomes an FS ($\widetilde{AOQ}$). $\widetilde{AOQ}$ and the α-cut of it ($\widetilde{AOQ}_\alpha$) can be obtained as shown in Eqs. (10, 11), respectively [23]. ($\widetilde{AOQ}_\alpha$) is also an interval having lower limit AOQ_{l_α} and upper limit AOQ_{r_α}.

$$\widetilde{AOQ} = \tilde{P}_a \otimes \tilde{p} = \tilde{P}_a \otimes (\tilde{\lambda} \oslash \tilde{n}) \tag{10}$$

$$\widetilde{AOQ}_\alpha = [AOQ_{l_\alpha}, AOQ_{r_\alpha}] = \begin{bmatrix} min\{AOQ \mid p \in \tilde{p}_\alpha, q \in \tilde{q}_{1-\alpha}, n \in \tilde{n}_\alpha, c \in \tilde{c}_\alpha\}, \\ max\{AOQ \mid p \in \tilde{p}_\alpha, q \in \tilde{q}_{1-\alpha}, n \in \tilde{n}_\alpha, c \in \tilde{c}_\alpha\} \end{bmatrix} \tag{11}$$

Similarly, $\widetilde{ATI}$ and α-cut ($\widetilde{ATI}_\alpha$) values are obtained as shown in Eqs. (12, 13) for a lot having fuzzy lot size ($\tilde{N}$) while ATI_{l_α} is representing lower limit and ATI_{l_α} is representing the upper limit for $\widetilde{ATI}_\alpha$ [23].

$$\widetilde{ATI} = \tilde{n} \oplus (1 \ominus \tilde{P}_a) \otimes (\tilde{N} \ominus \tilde{n}) \tag{12}$$

$$\widetilde{ATI}_\alpha = [ATI_{l_\alpha}, ATI_{r_\alpha}] = \begin{bmatrix} min\{ATI \mid p \in \tilde{p}_\alpha, q \in \tilde{q}_{1-\alpha}, n \in \tilde{n}_\alpha, c \in \tilde{c}_\alpha\}, \\ max\{ATI \mid p \in \tilde{p}_\alpha, q \in \tilde{q}_{1-\alpha}, n \in \tilde{n}_\alpha, c \in \tilde{c}_\alpha\} \end{bmatrix} \tag{13}$$

3.2 Double acceptance sampling plans based on type-1 fuzzy sets

For a DASP having fuzzy parameters such as $\tilde{n}_1$, $\tilde{c}_1$, $\tilde{n}_2$ and $\tilde{c}_2$, respectively, the $\tilde{P}_a$ and $\tilde{P}_{1_\alpha}$ are obtained as shown in Eqs. (14, 15) [23]. The defective item frequencies for the first and second steps are calculated as follows: $\tilde{\lambda}_1 = \tilde{n}_1 \otimes \tilde{p}$, $\tilde{\lambda}_2 = \tilde{n}_2 \otimes \tilde{p}$

$$\begin{aligned} \tilde{P}_a =& P\{d_1 \leqslant \tilde{c}_1 \mid \tilde{p}, \tilde{q}, \tilde{n}_1, \tilde{n}_2, \tilde{c}_1, \tilde{c}_2\} + P\{\tilde{c}_1 < d_1 \leqslant \tilde{c}_2 \mid d_1 + d_2 \leqslant \tilde{c}_2 \; and \; \tilde{p}, \tilde{q}, \tilde{n}_1, \tilde{n}_2, \tilde{c}_1, \tilde{c}_2\} \\ =& \sum_{d_1=0}^{\tilde{c}_1} \frac{\tilde{\lambda}_1^{d_1} \otimes e^{-\tilde{\lambda}_1}}{d_1!} \oplus \sum_{d_1 > \tilde{c}_1}^{\tilde{c}_2} \left(\frac{\tilde{\lambda}_1^{d_1} \otimes e^{-\tilde{\lambda}_1}}{d_1!} \otimes \sum_{d_2=0}^{\tilde{c}_2 \ominus d_1} \frac{\tilde{\lambda}_2^{d_2} \otimes e^{-\tilde{\lambda}_2}}{d_2!} \right) \\ =& \sum_{d_1=0}^{\tilde{c}_1} \binom{\tilde{n}_1}{d_1} \otimes \tilde{p}^{d_1} \otimes \tilde{q}^{(\tilde{n}_1 \ominus d_1)} \oplus \sum_{d_1 > \tilde{c}_1}^{\tilde{c}_2} \left(\binom{\tilde{n}_1}{d_1} \otimes \tilde{p}^{d_1} \otimes \tilde{q}^{(\tilde{n}_1 \ominus d_1)} \otimes \sum_{d_2=0}^{\tilde{c}_2 \ominus d_1} \binom{\tilde{n}_2}{d_2} \otimes \tilde{p}^{d_2} \otimes \tilde{q}^{(\tilde{n}_2 \ominus d_2)} \right) \end{aligned} \tag{14}$$

$$\tilde{P}_{a_\alpha} = [P_{al_\alpha}, P_{ar_\alpha}] = \begin{bmatrix} min\{P_a \mid p \in \tilde{p}_\alpha, q \in \tilde{q}_{1-\alpha}, n_1 \in \tilde{n}_{1_\alpha}, n_2 \in \tilde{n}_{2_\alpha}, c_1 \in \tilde{c}_{1_\alpha}, c_2 \in \tilde{c}_{2_\alpha}\}, \\ max\{P_a \mid p \in \tilde{p}_\alpha, q \in \tilde{q}_{1-\alpha}, n_1 \in \tilde{n}_{1_\alpha}, n_2 \in \tilde{n}_{2_\alpha}, c_1 \in \tilde{c}_{1_\alpha}, c_2 \in \tilde{c}_{2_\alpha}\} \end{bmatrix} \tag{15}$$

$\widetilde{AOQ}$ is also calculated as shown in Eq. (10) for DASPs then the $\widetilde{AOQ}_\alpha$ is obtained as shown in Eq. (16) [23].

$$\widetilde{AOQ}_\alpha = [AOQ_{l_\alpha}, AOQ_{r_\alpha}] = \begin{bmatrix} min\{AOQ \mid p \in \tilde{p}_\alpha, q \in \tilde{q}_{1-\alpha}, n_1 \in \tilde{n}_{1_\alpha}, n_2 \in \tilde{n}_{2_\alpha}, c_1 \in \tilde{c}_{1_\alpha}, c_2 \in \tilde{c}_{2_\alpha}\}, \\ max\{AOQ \mid p \in \tilde{p}_\alpha, q \in \tilde{q}_{1-\alpha}, n_1 \in \tilde{n}_{1_\alpha}, n_2 \in \tilde{n}_{2_\alpha}, c_1 \in \tilde{c}_{1_\alpha}, c_2 \in \tilde{c}_{2_\alpha}\} \end{bmatrix} \tag{16}$$

Additionally, $\widetilde{ASN}$, $\widetilde{ASN}_\alpha$, $\widetilde{ATI}$ and $\widetilde{ATI}_\alpha$ are also calculated as in shown Eqs. (17, 20) while P^I represents the probability of terminating the sampling in the first step and P^{II} represents the probability of terminating the sampling in the second step as either acceptance or rejection [23].

$$\begin{aligned} \widetilde{ASN} &= \tilde{n}_1 \oplus \tilde{n}_2 \otimes (1 \ominus \tilde{P}^I) \\ &= \tilde{n}_1 \oplus \tilde{n}_2 \otimes (1 \ominus (P\{d_1 \leqslant \tilde{c}_1 \mid \tilde{p}, \tilde{q}, \tilde{n}_1, \tilde{n}_2, \tilde{c}_1, \tilde{c}_2\} \oplus P\{d_1 > \tilde{c}_2 \mid \tilde{p}, \tilde{q}, \tilde{n}_1, \tilde{n}_2, \tilde{c}_1, \tilde{c}_2\})) \end{aligned} \tag{17}$$

$$\widetilde{ASN}_\alpha = [ASN_{l_\alpha}, ASN_{r_\alpha}] = \begin{bmatrix} min\{ASN \mid p \in \tilde{p}_\alpha, q \in \tilde{q}_{1-\alpha}, n_1 \in \tilde{n}_{1_\alpha}, n_2 \in \tilde{n}_{2_\alpha}, c_1 \in \tilde{c}_{1_\alpha}, c_2 \in \tilde{c}_{2_\alpha}\}, \\ max\{ASN \mid p \in \tilde{p}_\alpha, q \in \tilde{q}_{1-\alpha}, n_1 \in \tilde{n}_{1_\alpha}, n_2 \in \tilde{n}_{2_\alpha}, c_1 \in \tilde{c}_{1_\alpha}, c_2 \in \tilde{c}_{2_\alpha}\} \end{bmatrix} \tag{18}$$

$$\begin{aligned} \widetilde{ATI} = \widetilde{ASN} &\oplus (\tilde{N} \ominus \tilde{n}_1) \otimes P\{d_1 > \tilde{c}_2 \mid \tilde{p}, \tilde{q}, \tilde{n}_1, \tilde{n}_2, \tilde{c}_1, \tilde{c}_2\} \\ &\oplus (\tilde{N} \ominus \tilde{n}_1 \ominus \tilde{n}_2) \otimes P\{d_1 + d_2 > \tilde{c}_2 \mid \tilde{p}, \tilde{q}, \tilde{n}_1, \tilde{n}_2, \tilde{c}_1, \tilde{c}_2\} \end{aligned} \tag{19}$$

$$\widetilde{ATI}_\alpha = [ATI_{l_\alpha}, ATI_{r_\alpha}] = \begin{bmatrix} min\{ATI \mid p \in \tilde{p}_\alpha, q \in \tilde{q}_{1-\alpha}, n_1 \in \tilde{n}_{1_\alpha}, n_2 \in \tilde{n}_{2_\alpha}, c_1 \in \tilde{c}_{1_\alpha}, c_2 \in \tilde{c}_{2_\alpha}\}, \\ max\{ATI \mid p \in \tilde{p}_\alpha, q \in \tilde{q}_{1-\alpha}, n_1 \in \tilde{n}_{1_\alpha}, n_2 \in \tilde{n}_{2_\alpha}, c_1 \in \tilde{c}_{1_\alpha}, c_2 \in \tilde{c}_{2_\alpha}\} \end{bmatrix} \tag{20}$$

Eq. (19) can be mathematically rewritten as seen in Eq. (21) where $\tilde{P}_a^I$ represents the probability of terminating the sampling in the first step as acceptance and $\tilde{P}_a^{II}$ represents the probability of terminating the sampling in the second step as acceptance.

$$\widetilde{ATI} = \tilde{n}_1 \otimes \tilde{P}_a^I \oplus (\tilde{n}_1 \oplus \tilde{n}_2) \otimes \tilde{P}_a^{II} \oplus \tilde{N} \otimes (1 - \tilde{P}_a) \tag{21}$$

4 Design of acceptance sampling plans based on intuitionistic fuzzy linguistic terms

In this section, fuzzy single and double ASPs have been extended based on IFSs. For this aim, two well-known statistical distributions of ASPs named binomial and Poisson distributions have been reformulated by using IFSs. The derived formulations are applicable for any shape of IFSs, such as TFSs and TrFSs. Since IFSs permit slackness for non-determination, more precision and flexibility have been provided in formulations. The defectiveness of items has been represented with linguistic terms to overcome with the hardship of quantifying the verbal inspection results. By this way, it has become possible to express the product defection status more clearly by the help of these linguistic terms and a great convenience can be obtained for the inspectors' evaluations.

4.1 Fuzzy linguistic terms

Since the inspection process of ASPs is generally operated by humans, there is difficulty in converting the product quality-related verbal inspection results into numerical expressions. In such circumstances, the linguistic approach is practical. It would be appropriate to use linguistic variables to describe the quality level of the products more accurately. Defectiveness of the items can include fuzziness, so it is more correct to represent it as fuzzy linguistic variable (FLV).

The FLV concept has been offered by [29]. Values of the FLVs are words or sentences in a language. A linguistic variable is represented with a quintuple ($\wp$,$T(\wp)$,U,G,M) in which $\wp$ is the name of the variable, $T(\wp)$ is the term set of $\wp$ which is the collection of linguistic names, U is universe, G is a syntactic rule which generates the terms in $T(\wp)$ and M is a semantic rule to associate the linguistic values and their meanings. $M(X)$ denotes a fuzzy subset of U. The meaning of a linguistic value X is characterized by a compatibility function. The function of the semantic rule is used to relate the compatibilities of the terms with a linguistic value [29]. Deciding a term set means deciding $T(\wp)$, U, G and M, respectively. Fuzzy subsets $M(X)$ can be in any shape, such as triangular and trapezoidal. In addition to that, $M(X)$ can be any FS extension such as hesitant, intuitionistic, and so forth. An example term set with size seven and its triangular type-1 FS semantic rule is given in Eq. (22) [17]. While deciding $M(X)$, superposing the core of the FS of a linguistic term with the support points of the FSs of the previous and the successive linguistic terms are generally preferred as in Eq. (22).

$$\begin{aligned} S = \{&s_0 : Nothing(N), s_1 : Very\,low(VL), s_2 : Low(L), s_3 : Medium(M), \\ &s_4 : High(H), s_5 : Very\,high(VH), s_6 : Perfect(P)\} \\ M(X) = \{&N = (0, 0, 0.167), VL = (0, 0.167, 0.333), L = (0.167, 0.333, 0.5), M = (0.333, 0.5, 0.667), \\ &H = (0.5, 0.667, 0.833), VH = (0.667, 0.833, 1), P = (0.833, 1, 1)\} \end{aligned} \tag{22}$$

Performance of fuzzy modeling is measured with interoperability and accuracy of the characteristics [10]. Linguistic fuzzy modeling (LFM) is good at interoperability, but it has drawback about accuracy as a result of the term set size that is decided at the beginning. If a small-term set is decided, the set may become restrictive and an approximation process is needed for the information does not exactly match any of the initial linguistic terms. This causes loss of information and lack of precision. On the other hand, if the term set is determined as too big, the linguistic statements begin to lose their meanings that cause confusion or become to make no sense.

Several studies have been conducted to increase LFM accuracy in the literature such as tuning the term set size by using data analysis and soft computing techniques [10], have also preferred to change linguistic model structure. For example, double hierarchy LTSs consisting of two independent term sets used as hierarchically have been offered to reach more accurate and comprehensive results [16]. The most popular approach, the 2-tuple linguistic fuzzy approach, has been offered by [17] to minimize the loss of information by making interpolated assessments. In this approach, fuzzy information is represented with a linguistic term and a companion numerical value named "difference of information" in the interval $[-0.5, 0.5)$. For example, the 2-tuple statement $M' = (M, -0.3)$ for the set given in Eq. (24) means that, the information is between L and M but it is closer to M. The graphical representation of M' is seen in Figure 3.

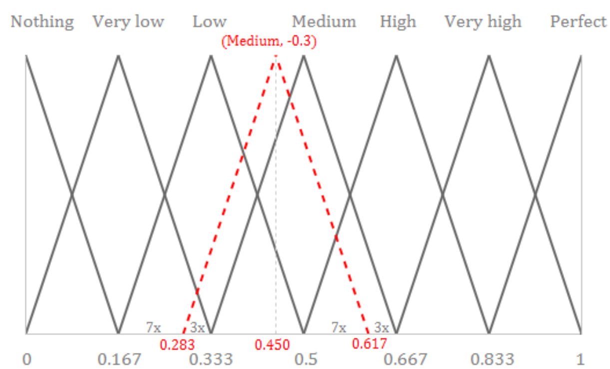

Figure 3: Graphical representation of a 2-tuple linguistic statement $(M, -0.3)$

In this paper, the ordered structure linguistic fuzzy approach with 2-tuple approach is employed to decide the defectiveness information of the population. The formulations of ASPs offered in this study are based on IFSs, so the defectiveness of the population should be represented as IFS to be able to use these formulations. For this reason, an intuitionistic term set has been designed by using the similar term set for both defectiveness and non-defectiveness separately. After deciding the linguistic statements for both, this information is merged to build IFS. For example, the inspector can present an opinion for the defective item ratio as $\vartheta_{\tilde{\tilde{A}}}(x) = (L, -0.5)$ and for non-defective item ratio as $\mu_{\tilde{\tilde{A}}}(x) = (H, 0.3)$ by using the term set given in Eq. (22). In this example, triangular intuitionistic defect information is reached because $\mu_{\tilde{\tilde{A}}}(x) \oplus \vartheta_{\tilde{\tilde{A}}}(x) \leqslant 1$ is satisfied for all x depending on the values $\vartheta_{\tilde{\tilde{A}}}(x) = (L, -0.5) = (0.084, 0.250, 0.417)$ and $\mu_{\tilde{\tilde{A}}}(x) = (H, 0.3) = (0.550, 0.717, 0.883)$. The non-determinacy degree is calculated as $\pi_{\tilde{\tilde{A}}} = 1 \ominus \mu_{\tilde{\tilde{A}}}(x) \ominus \vartheta_{\tilde{\tilde{A}}}(x) = (0.033, 0.033, 0.033) = 0.033$. In the scope of this study, the α-cut technique has also been utilized to obtain IVIFSs. The biggest problem about α-cut technique is working with numerical α values that have no tangible meaning for humans in real case applications. The linguistic approach gives good results when it is combined with α-cut technique to solve this problem. Thus, α level has been decided by using linguistic statements in this study. The produced items may have different types of defects such as color defects, shape defects, and so forth, but some may not be important. In this case, a bigger α value can be acceptable in ASP. Figure. 4 shows an example TrIFS membership function for item defectiveness ratio having non-determinacy level 3%. Assume that, the instruction given to the operator is "At most slightly defective" is acceptable for a standard quality product. In this case, the numerical equivalent of α is determined as $\frac{0.015\times 0.97}{0.025} = 0.582$. Another operator can be instructed as "At most insensibly defective" is acceptable for luxury product" and so the new α is determined as $\frac{0.01\times 0.97}{0.025} = 0.388$. As seen in these examples, this linguistic approach gives the ability to work with multiple α values for different product quality levels in a human-friendly way.

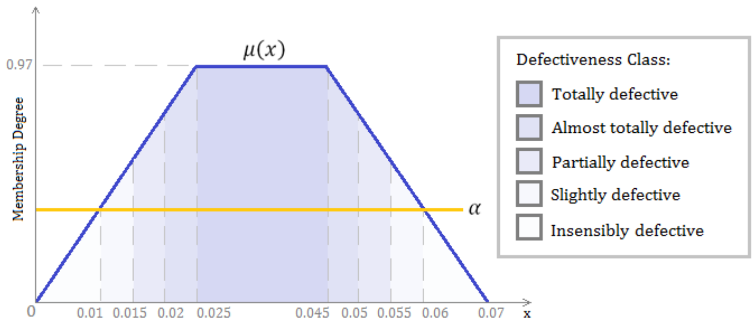

Figure 4: An example membership function for α-cut with linguistic statements

The defectiveness ratio of the items has been considered as IFSs and expressed with linguistic statements. This brings us to the concept of Linguistic IFSs (LIFSs), combination of the concepts of IFSs and LTSs. For a given reference set X, positive integer τ, and LTS $S = \{S_\alpha \mid \alpha \in [0, \tau]\}$, a LIFS is defined as in Eq. (23) while $s_a(x)$ is membership function and $s_b(x)$ is non-membership function [25].

$$I = \{x, s_a(x), s_b(x) \mid x \in X\} \tag{23}$$

Each LIFS satisfies the conditions $0 \leqslant a \leqslant \tau$ and $0 \leqslant b \leqslant \tau$. Correspondingly, the non-determinacy degree is obtained as $s_\pi(x) = s_{\tau-a-b}$ for LIFSs. LIFSs and linguistic Pythagorean FSs (LPFSs) are two special cases of this generalized definition for $q = 1$ and $q = 2$ [25]. Even though, there are studies in the literature such as [18] that develop methods to calculate score function and distance measurement for PFSs to use in decision making (DM) problems, ASP has dissimilar nature to traditional DM problems. For this reason, ASPs based on IFS and PFS must be formulated separately. In this paper, PFSs have not been taken into consideration.

4.2 Intuitionistic binomial and poisson distributions

In the formulation given by [23], $\mu(x) = \tilde{p}$ is defined as the membership and $\vartheta(x) = \tilde{q}$ is defined as non-membership to the defectiveness set. If this formulation is extended for IFSs, $\tilde{p} \oplus \tilde{q} \leqslant 1$ should be satisfied and the hesitancy/non-determinacy degree $(1 \ominus \tilde{p} \ominus \tilde{q})$ should be considered. For an ASP based on IFSs, a random sample selection event should include three states and their concurrencies: defectiveness, non-defectiveness, and non-determination. Depending on this, ASP will have three outcomes: Acceptance, Rejection, and Non-Determinacy. In fuzzy ASPs, the lot is rejected if the observed defective item count exceeds the maximum allowed defective item count threshold ($\tilde{c}$). Even if some defective items are observed but $\tilde{c}$ is not exceeded, the lot is accepted despite the defective items. A similar approach can be applied for non-determinate items by deciding a maximum allowed non-determinacy item count threshold ($\tilde{\tau}$). If $\tilde{\tau}$ is exceeded, then the lot is regarded as non-determinate, and the sampling is repeated. On the other hand, if $\tilde{\tau}$ is not exceeded, the non-determinate items are regarded as belonging to the determinate part. Here, the determinate part statement refers to the defective and non-defective items.

The offered threshold approach is required to be able to consider non-determinacy in the formulation. There are two main reasons for this. Firstly, if there is no $\tilde{\tau}$, it is impossible to achieve the non-determinacy plan outcome. Secondly, if there is no $\tilde{\tau}$ formulations such as $\tilde{P}_a$ will be the same as the one offered by [23]. Let us prove it mathematically. If d items are observed as defective, some of the $(\tilde{n} \ominus d)$ items will be non-defective, and rest of the items will be non-determinate. This means that the ratio of non-defective to non-determinate items is also a binomial event. New variables can be defined for mathematical simplicity as follows: $\tilde{n}_r = \tilde{n} \ominus d$ is the sum of the observed non-defective and non-determinate items, $\tilde{q}_1$ is the non-defectiveness probability, $\tilde{q}_2$ is the non-determinacy probability. Depending on these definitions, the main condition of traditional FSs $(\tilde{p} \oplus \tilde{q} = 1)$ is updated as $\tilde{p} \oplus (\tilde{q}_1 \oplus \tilde{q}_2) = 1$. Hence, $\tilde{P}_a$ without

$\tilde{\tau}$ is determined as follows:

$$\tilde{P}_a = \sum_{d=0}^{\tilde{c}} \binom{\tilde{n}}{d} \otimes \tilde{p}^d \otimes \left[\sum_{i=0}^{\tilde{n}_r} \binom{\tilde{n}_r}{i} \otimes \tilde{q}_1^i \otimes \tilde{q}_2^{(\tilde{n}_r \ominus i)} \right]$$
$$= \sum_{d=0}^{\tilde{c}} \binom{\tilde{n}}{d} \otimes \tilde{p}^d \otimes (\tilde{q}_1 \oplus \tilde{q}_2)^{\tilde{n}_r} \overset{(\tilde{q}_1 \oplus \tilde{q}_2)=(1\ominus\tilde{p})}{\Longrightarrow} \sum_{d=0}^{\tilde{c}} \binom{\tilde{n}}{d} \otimes \tilde{p}^d \otimes (1 \ominus \tilde{p})^{\tilde{n}_r} = \sum_{d=0}^{\tilde{c}} \binom{\tilde{n}}{d} \otimes \tilde{p}^d \otimes (1 \ominus \tilde{p})^{(\tilde{n} \ominus d)}$$

It is obvious from the above proof, $\tilde{\tau}$ is required to differentiate the formulation of ASPs based on IFSs from the formulation of ASPs based on type-1 FSs. This proof can be generalized to obtain intuitionistic binomial distribution (IBD). It can be assumed that the determinate part (defectiveness and non-defectiveness) is primarily interested, and non-determinate part can be ignored until a certain level, namely, the non-determinacy threshold (τ). For a given success probability (p) and failure probability (q) such that $p + q \leqslant 1$, probability facing with exactly x successes in n trials can be calculated as in Eq. (24).

$$P_{IBD}\{x \mid p, n, \tau\} = \binom{n}{x} \times p^x \times \left[\sum_{i=0}^{\tau} \binom{n-x}{i} \times (1-p-q)^i \times q^{n-x-i} \right] \tag{24}$$

If the number of trials (n) is big enough, the binomial distribution converges to Poisson distribution. Similarly, IBD converges to intuitionistic Poisson distribution (IPD) for big trial numbers. For Poisson distribution, frequency values are needed for success and non-determinacy. Success frequency (λ_p) and non-determinacy frequency ($\lambda_{(1-p-q)}$) values are found by using Eq. (25).

$$\lambda_p = n \times p, \;\; \lambda_{(1-p-q)} = n \times (1-p-q) \tag{25}$$

For IPD, the probability of facing with exactly x successes inside n trials is calculated as shown in Eq. (28) with the help of the conversions given in Eqs. (26, 27).

$$\lim_{n\to\infty} \left(\binom{n}{x} \times p^x \right) = \lim_{n\to\infty} \left(\binom{n}{x} \times \left(\frac{\lambda_p}{n} \right)^x \right) = \frac{n}{n} \times \frac{n-1}{n} \times \cdots \times \frac{n-x-1}{n} \times \frac{\lambda_p^x}{x!} = \frac{\lambda_p^x}{x!} \tag{26}$$

$$\lim_{n\to\infty} \left(\binom{n-x}{i} \times (1-p-q)^i \times q^{n-x-i} \right) = \lim_{n\to\infty} \left(\binom{n-x}{i} \times \left(\frac{\lambda_{(1-p-q)}}{n} \right)^i \times \left(1 - \frac{\lambda_p + \lambda_{(1-p-q)}}{n} \right)^{n-x-i} \right)$$
$$= \frac{n}{n} \times \frac{n-1}{n} \times \cdots \times \frac{n-x-i-1}{n} \times \frac{(\lambda_{(1-p-q)})^i}{i!} \times \left(1 - \frac{\lambda_p + \lambda_{(1-p-q)}}{n} \right)^{n-x-i} = \frac{(\lambda_{(1-p-q)})^i}{i!} \times e^{-(\lambda_p + \lambda_{(1-p-q)})} \tag{27}$$

$$P_{IPD}\{x \mid p, n, \tau\} = \frac{\lambda_p^x}{x!} \times \left[\sum_{i=0}^{\tau} \frac{\left(\lambda_{(1-p-q)}\right)^i}{i!} \times e^{-(\lambda_p + \lambda_{(1-p-q)})} \right] \tag{28}$$

4.3 Design of single ASPs based on intuitionistic fuzzy linguistic terms

In this sub-section, the main formulations of SASPs have been analyzed and re-obtained based on IFSs. For an ASP based on IFSs $\widetilde{A\ddot{S}P}$, the main condition $\tilde{p} \oplus \tilde{q} \leqslant 1$ should be satisfied while $\tilde{p} = \mu(x)$ is the membership degree and $\tilde{q} = \vartheta(x)$ is the non-membership degree to the defectiveness set. Accordingly, hesitancy/non-determinacy degree is also determined by the statement $(1 \ominus \tilde{p} \ominus \tilde{q})$. $\tilde{P}_a$ is calculated as shown in Eq. (29) with the help of IBD and IPD while $\tilde{\tau}$ is the non-determinacy threshold and the condition $(\tilde{c} + \tilde{\tau} < \tilde{n})$ is satisfied. Note that the plan parameters $\tilde{n}$, $\tilde{c}$ and $\tilde{\tau}$ can be deterministic values, type-1 FSs or IFSs. In the scope of this study, these have been considered as type-1 FSs.

$$\begin{aligned} \tilde{P}_a &= \tilde{P}\{d \leqslant \tilde{c}, \leqslant \tilde{\tau} \mid \tilde{p}, \tilde{q}, \tilde{n}, \tilde{c}, \tilde{\tau}\} \\ &= \tilde{P}_{a_{Binomial}} = \sum_{d=0}^{\tilde{c}} \left(\binom{\tilde{n}}{d} \otimes \tilde{p}^d \otimes \left[\sum_{i=0}^{\tilde{\tau}} \left(\binom{\tilde{n} \ominus d}{i} \otimes (1 \ominus \tilde{p} \ominus \tilde{q})^i \otimes \tilde{q}^{(\tilde{n} \ominus i \ominus d)} \right) \right] \right) \\ &\approx \tilde{P}_{a_{Poisson}} = \sum_{d=0}^{\tilde{c}} \left(\frac{\tilde{\lambda}_p^d}{d!} \otimes \left[\sum_{i=0}^{\tilde{\tau}} \left(\frac{(\tilde{\lambda}_{(1-p-q)})^i}{i!} \otimes e^{-(\tilde{\lambda}_p \oplus \tilde{\lambda}_{(1-p-q)})} \right) \right] \right) \end{aligned} \tag{29}$$

As seen in Eq. (3), the IFS defectiveness information $\tilde{\tilde{A}} = \{x, \mu_{\tilde{\tilde{A}}}(x) = \tilde{p}, \vartheta_{\tilde{\tilde{A}}}(x) = \tilde{q} \mid x \in X\}$ turns into IVIFS $\tilde{\tilde{A}}_\alpha = \langle \tilde{p}_\alpha, \tilde{q}_{1-\alpha} \rangle = \langle [p_L, p_U], [q_L, q_U] \rangle$ via α-cut while the indices L and U are stating the lower and upper bounds of the intervals, respectively. Similarly, the parameters of $\tilde{n}$, $\tilde{c}$ and $\tilde{\tau}$ turns into interval-valued numbers as $\tilde{n}_\alpha = [n_L, n_U]$, $\tilde{c}_\alpha = [c_L, c_U]$ and $\tilde{\tau}_\alpha = [\tau_L, \tau_U]$ based on α-cuts, respectively. Depending on this, the lot acceptance probability after α-cut ($\tilde{P}_{a_\alpha}$) is obtained as shown in Eq.(30). It should be noted that, $\tilde{q}$ is complement of $\tilde{p}$, so it should be cut by $1-\alpha$.

$$\tilde{P}_{a_\alpha} = [P_{a_L}, P_{a_U}] = \begin{bmatrix} min\{P_a \mid p \in \tilde{p}_\alpha, q \in \tilde{q}_{1-\alpha}, n \in \tilde{n}_\alpha, c \in \tilde{c}_\alpha, \tau \in \tilde{\tau}_\alpha\}, \\ max\{P_a \mid p \in \tilde{p}_\alpha, q \in \tilde{q}_{1-\alpha}, n \in \tilde{n}_\alpha, c \in \tilde{c}_\alpha, \tau \in \tilde{\tau}_\alpha\} \end{bmatrix} \tag{30}$$

Based on IFS, the ASP has three outcomes: acceptance, rejection, and non-determinacy. The non-determinacy outcome is occurred depending on $\tilde{\tau}$ and the rejection outcome is occurred depending on $\tilde{c}$ but both thresholds can be exceeded in some cases. The non-determinacy is a side fact caused by the uncertainty. Hence, it can be assumed that the defectiveness should be dominant to the non-determinacy in calculations such that if $\tilde{c}$ is exceeded, the lot is rejected independently of the non-determinate item count. The rejection probability of a lot ($\tilde{P}_r$) and the rejection probability after α-cut ($\tilde{P}_{r_\alpha}$) are calculated with the help of IBD and IPD, respectively, as seen in Eqs. (31, 32).

$$\begin{aligned} \tilde{P}_r &= \tilde{P}\{d > \tilde{c} \mid \tilde{p}, \tilde{q}, \tilde{n}, \tilde{c}, \tilde{\tau}\} \\ &= \tilde{P}_{r_{Binomial}} = \sum_{d=\tilde{c}\oplus 1}^{\tilde{n}} \left(\binom{\tilde{n}}{d} \otimes \tilde{p}^d \otimes \left[\sum_{i=0}^{\tilde{n}\ominus d} \left(\binom{\tilde{n}\ominus d}{i} \otimes (1 \ominus \tilde{p} \ominus \tilde{q})^i \otimes \tilde{q}^{(\tilde{n}\ominus i \ominus d)} \right) \right] \right) \\ &\approxeq \tilde{P}_{r_{Poisson}} = \sum_{d=\tilde{c}\oplus 1}^{\tilde{n}} \left(\frac{\tilde{\lambda}_p^d}{d!} \otimes \left[\sum_{i=0}^{\tilde{n}\ominus d} \left(\frac{(\tilde{\lambda}_{(1-p-q)})^i}{i!} \otimes e^{-(\tilde{\lambda}_p \oplus \tilde{\lambda}_{(1-p-q)})} \right) \right] \right) \end{aligned} \tag{31}$$

$$\tilde{P}_{r_\alpha} = [P_{r_L}, P_{r_U}] = \begin{bmatrix} min\{P_r \mid p \in \tilde{p}_\alpha, q \in \tilde{q}_{1-\alpha}, n \in \tilde{n}_\alpha, c \in \tilde{c}_\alpha, \tau \in \tilde{\tau}_\alpha\}, \\ max\{P_r \mid p \in \tilde{p}_\alpha, q \in \tilde{q}_{1-\alpha}, n \in \tilde{n}_\alpha, c \in \tilde{c}_\alpha, \tau \in \tilde{\tau}_\alpha\} \end{bmatrix} \tag{32}$$

$\widetilde{AOQ}$ and $\widetilde{AOQ}_\alpha$ are also calculated as in Eqs. (33, 34), respectively:

$$\widetilde{AOQ} = \tilde{P}_a \otimes \tilde{p} = \tilde{P}_a \otimes \tilde{\lambda}_p \oslash \tilde{n} \tag{33}$$

$$\widetilde{AOQ}_\alpha = [AOQ_L, AOQ_U] = \begin{bmatrix} min\{AOQ \mid p \in \tilde{p}_\alpha, q \in \tilde{q}_{1-\alpha}, n \in \tilde{n}_\alpha, c \in \tilde{c}_\alpha, \tau \in \tilde{\tau}_\alpha\}, \\ max\{AOQ \mid p \in \tilde{p}_\alpha, q \in \tilde{q}_{1-\alpha}, n \in \tilde{n}_\alpha, c \in \tilde{c}_\alpha, \tau \in \tilde{\tau}_\alpha\} \end{bmatrix} \tag{34}$$

If $\tilde{\tau}$ is exceeded, the lot is regarded as non-determinate, and sampling is repeated. In other words, sampling is repeated with the probability of $(1 \ominus \tilde{P}_a \ominus \tilde{P}_r)$. However, if it has a minimal probability, the sampling can theoretically continue until all the items in the population are inspected. It is not a desirable approach in terms of cost and time. For this reason, a limitation is required for the sampling at a certain point, such as two repetitions, three repetitions, and so forth. While certain repetitions (such as 1, 2, and 3) are allowed for some organizations, repetition may not be preferred and non-determinate lots may be rejected (non-repetitive) in some other organizations. $\widetilde{ATI}$ for non-repetitive ASPs ($\widetilde{ATI}_{(0)}$) is shown in Eq. (35) and $\widetilde{ATI}$ for repetitive ASPs ($\widetilde{ATI}_{(r)}$) is obtained as in Eq. (36) while r is allowed repetition number. Then $\widetilde{ATI}_\alpha$ is determined by using Eq. (37).

$$\widetilde{ATI}_{(0)} = \tilde{n} \oplus (1 \ominus \tilde{P}_a) \otimes (\tilde{N} \ominus \tilde{n}) \tag{35}$$

$$\widetilde{ATI}_{(r)} = \tilde{n} \oplus \tilde{P}_r \otimes (\tilde{N} \ominus \tilde{n}) \oplus (1 \ominus \tilde{P}_a \ominus \tilde{P}_r) \otimes \widetilde{ATI}_{(r-1)} \tag{36}$$

$$\widetilde{ATI}_\alpha = [ATI_L, ATI_U] = \begin{bmatrix} min\{ATI \mid p \in \tilde{p}_\alpha, q \in \tilde{q}_{1-\alpha}, n \in \tilde{n}_\alpha, c \in \tilde{c}_\alpha, \tau \in \tilde{\tau}_\alpha\}, \\ max\{ATI \mid p \in \tilde{p}_\alpha, q \in \tilde{q}_{1-\alpha}, n \in \tilde{n}_\alpha, c \in \tilde{c}_\alpha, \tau \in \tilde{\tau}_\alpha\} \end{bmatrix} \tag{37}$$

4.4 Design of double ASPs based on intuitionistic fuzzy linguistic terms

For a DASP having fuzzy parameters $(\tilde{n}_1, \tilde{c}_1, \tilde{\tau}_1, \tilde{n}_2, \tilde{c}_2, \tilde{\tau}_2)$ and lot characteristics $(\tilde{N}, \tilde{p})$, $\tilde{P}_a$ and $\tilde{P}_r$ are obtained as shown in Eqs. (38, 41). Where $\tilde{\lambda}_{p_1} = \tilde{n}_1 \otimes \tilde{p}$, $\tilde{\lambda}_{p_2} = \tilde{n}_2 \otimes \tilde{p}$, $\tilde{\lambda}_{(1-p-q)_1} = \tilde{n}_1 \otimes (1 \ominus \tilde{p} \ominus \tilde{q})$ and $\tilde{\lambda}_{(1-p-q)_2} = \tilde{n}_2 \otimes (1 \ominus \tilde{p} \ominus \tilde{q})$ are the defective and non-determinate item frequencies for the first and second step, respectively.

$$\begin{aligned}\tilde{P}_a =&\tilde{P}\{d_1 \leqslant \tilde{c}_1, i_1 \leqslant \tilde{\tau}_1 \mid \tilde{p},\tilde{q},\tilde{n}_1,\tilde{n}_2,\tilde{c}_1,\tilde{c}_2,\tilde{\tau}_1,\tilde{\tau}_2\}\\ &\oplus \tilde{P}\{d_1+d_2 \leqslant \tilde{c}_2, i_1+i_2 \leqslant \tilde{\tau}_2 \mid \tilde{c}_1 < d_1 \leqslant \tilde{c}_2, i_1 \leqslant \tilde{\tau}_2 \; and \; \tilde{p},\tilde{q},\tilde{n}_1,\tilde{n}_2,\tilde{c}_1,\tilde{c}_2,\tilde{\tau}_1,\tilde{\tau}_2\}\\ &\oplus \tilde{P}\{d_1+d_2 \leqslant \tilde{c}_2, i_1+i_2 \leqslant \tilde{\tau}_2 \mid d_1 \leqslant \tilde{c}_1, \tilde{\tau}_1 < i_1 \leqslant \tilde{\tau}_2 \; and \; \tilde{p},\tilde{q},\tilde{n}_1,\tilde{n}_2,\tilde{c}_1,\tilde{c}_2,\tilde{\tau}_1,\tilde{\tau}_2\}\end{aligned}$$

$$\begin{aligned}\tilde{P}_{a_{Binomial}} =& \sum_{d_1=0}^{\tilde{c}_1}\left(\binom{\tilde{n}_1}{d_1}\otimes \tilde{p}^{d_1}\otimes\left[\sum_{i_1=0}^{\tilde{\tau}_1}\binom{\tilde{n}_1\ominus d_1}{i_1}\otimes(1\ominus\tilde{p}\ominus\tilde{q})^{i_1}\otimes\tilde{q}^{(\tilde{n}_1\ominus i_1\ominus d_1)}\right]\right)\\ &\oplus\sum_{d_1=\tilde{c}_1\oplus 1}^{\tilde{c}_2}\left(\binom{\tilde{n}_1}{d_1}\otimes \tilde{p}^{d_1}\otimes\left[\sum_{i_1=0}^{\tilde{\tau}_2}\left(\binom{\tilde{n}_1\ominus d_1}{i_1}\otimes(1\ominus\tilde{p}\ominus\tilde{q})^{i_1}\otimes\tilde{q}^{(\tilde{n}_1\ominus i_1\ominus d_1)}\otimes\right.\right.\right.\\ &\left.\left.\left.\left[\sum_{d_2=0}^{\tilde{c}_2\ominus d_1}\left(\binom{\tilde{n}_2}{d_2}\otimes\tilde{p}^{d_2}\otimes\left[\sum_{i_2=0}^{\tilde{\tau}_2\ominus i_1}\binom{\tilde{n}_2\ominus d_2}{i_2}\otimes(1\ominus\tilde{p}\ominus\tilde{q})^{i_2}\otimes\tilde{q}^{(\tilde{n}_2\ominus i_2\ominus d_2)}\right]\right)\right]\right)\right]\right)\\ &\oplus\sum_{d_1=0}^{\tilde{c}_1}\left(\binom{\tilde{n}_1}{d_1}\otimes \tilde{p}^{d_1}\otimes\left[\sum_{i_1=\tilde{\tau}_1\oplus 1}^{\tilde{\tau}_2}\left(\binom{\tilde{n}_1\ominus d_1}{i_1}\otimes(1\ominus\tilde{p}\ominus\tilde{q})^{i_1}\otimes\tilde{q}^{(\tilde{n}_1\ominus i_1\ominus d_1)}\otimes\right.\right.\right.\\ &\left.\left.\left.\left[\sum_{d_2=0}^{\tilde{c}_2\ominus d_1}\left(\binom{\tilde{n}_2}{d_2}\otimes\tilde{p}^{d_2}\otimes\left[\sum_{i_2=0}^{\tilde{\tau}_2\ominus i_1}\binom{\tilde{n}_2\ominus d_2}{i_2}\otimes(1\ominus\tilde{p}\ominus\tilde{q})^{i_2}\otimes\tilde{q}^{(\tilde{n}_2\ominus i_2\ominus d_2)}\right]\right)\right]\right)\right]\right)\end{aligned} \tag{38}$$

$$\begin{aligned}\tilde{P}_{a_{Binomial}} \approxeq \tilde{P}_{a_{Poisson}} =& \sum_{d_1=0}^{\tilde{c}_1}\left(\frac{\tilde{\lambda}_{p_1}^{d_1}}{d_1!}\otimes\left[\sum_{i_1=0}^{\tilde{\tau}_1}\frac{\tilde{\lambda}_{(1-p-q)_1}^{i_1}}{i_1!}\otimes e^{-(\tilde{\lambda}_{(1-p-q)_1}\oplus\tilde{\lambda}_{p_1})}\right]\right)\\ &\oplus\sum_{d_1=\tilde{c}_1\oplus 1}^{\tilde{c}_2}\left(\frac{\tilde{\lambda}_{p_1}^{d_1}}{d_1!}\otimes\left[\sum_{i_1=0}^{\tilde{\tau}_2}\left(\frac{\tilde{\lambda}_{(1-p-q)_1}^{i_1}}{i_1!}\otimes e^{-(\tilde{\lambda}_{(1-p-q)_1}\oplus\tilde{\lambda}_{p_1})}\otimes\right.\right.\right.\\ &\left.\left.\left.\left[\sum_{d_2=0}^{\tilde{c}_2\ominus d_1}\left(\frac{\tilde{\lambda}_{p_2}^{d_2}}{d_2!}\otimes\left[\sum_{i_2=0}^{\tilde{\tau}_2\ominus i_1}\frac{\tilde{\lambda}_{(1-p-q)_2}^{i_2}}{i_2!}\otimes e^{-(\tilde{\lambda}_{(1-p-q)_2}\oplus\tilde{\lambda}_{p_2})}\right]\right)\right]\right)\right]\right)\\ &\oplus\sum_{d_1=0}^{\tilde{c}_1}\left(\frac{\tilde{\lambda}_{p_1}^{d_1}}{d_1!}\otimes\left[\sum_{i_1=\tilde{\tau}_1\oplus 1}^{\tilde{\tau}_2}\left(\frac{\tilde{\lambda}_{(1-p-q)_1}^{i_1}}{i_1!}\otimes e^{-(\tilde{\lambda}_{(1-p-q)_1}\oplus\tilde{\lambda}_{p_1})}\otimes\right.\right.\right.\\ &\left.\left.\left.\left[\sum_{d_2=0}^{\tilde{c}_2\ominus d_1}\left(\frac{\tilde{\lambda}_{p_2}^{d_2}}{d_2!}\otimes\left[\sum_{i_2=0}^{\tilde{\tau}_2\ominus i_1}\frac{\tilde{\lambda}_{(1-p-q)_2}^{i_2}}{i_2!}\otimes e^{-(\tilde{\lambda}_{(1-p-q)_2}\oplus\tilde{\lambda}_{p_2})}\right]\right)\right]\right)\right]\right)\end{aligned} \tag{39}$$

$$\begin{aligned}\tilde{P}_r =&\tilde{P}\{d_1 > \tilde{c}_2, i_1 \leqslant \tilde{\tau}_1 \mid \tilde{p},\tilde{q},\tilde{n}_1,\tilde{n}_2,\tilde{c}_1,\tilde{c}_2,\tilde{\tau}_1,\tilde{\tau}_2\}\\ &\oplus \tilde{P}\{d_1+d_2 > \tilde{c}_2 \mid \tilde{c}_1 < d_1 \leqslant \tilde{c}_2, i_1 \leqslant \tilde{\tau}_2 \; and \; \tilde{p},\tilde{q},\tilde{n}_1,\tilde{n}_2,\tilde{c}_1,\tilde{c}_2,\tilde{\tau}_1,\tilde{\tau}_2\}\\ &\oplus \tilde{P}\{d_1+d_2 > \tilde{c}_2 \mid d_1 \leqslant \tilde{c}_1, \tilde{\tau}_1 < i_1 \leqslant \tilde{\tau}_2 \; and \; \tilde{p},\tilde{q},\tilde{n}_1,\tilde{n}_2,\tilde{c}_1,\tilde{c}_2,\tilde{\tau}_1,\tilde{\tau}_2\}\end{aligned}$$

$$\begin{aligned}\tilde{P}_{r_{Binomial}} =& \sum_{d_1=\tilde{c}_2\oplus 1}^{\tilde{n}_1}\left(\binom{\tilde{n}_1}{d_1}\otimes \tilde{p}^{d_1}\otimes\left[\sum_{i_1=0}^{\tilde{n}_1\ominus d_1}\binom{\tilde{n}_1\ominus d_1}{i_1}\otimes(1\ominus\tilde{p}\ominus\tilde{q})^{i_1}\otimes\tilde{q}^{(\tilde{n}_1\ominus i_1\ominus d_1)}\right]\right)\\ &\oplus\sum_{d_1=\tilde{c}_1\oplus 1}^{\tilde{c}_2}\left(\binom{\tilde{n}_1}{d_1}\otimes \tilde{p}^{d_1}\otimes\left[\sum_{i_1=0}^{\tilde{\tau}_2}\left(\binom{\tilde{n}_1\ominus d_1}{i_1}\otimes(1\ominus\tilde{p}\ominus\tilde{q})^{i_1}\otimes\tilde{q}^{(\tilde{n}_1\ominus i_1\ominus d_1)}\otimes\right.\right.\right.\\ &\left.\left.\left.\left[\sum_{d_2=\tilde{c}_2\ominus d_1\oplus 1}^{\tilde{n}_2}\left(\binom{\tilde{n}_2}{d_2}\otimes\tilde{p}^{d_2}\otimes\left[\sum_{i_2=0}^{\tilde{n}_2\ominus d_2}\binom{\tilde{n}_2\ominus d_2}{i_2}\otimes(1\ominus\tilde{p}\ominus\tilde{q})^{i_2}\otimes\tilde{q}^{(\tilde{n}_2\ominus i_2\ominus d_2)}\right]\right)\right]\right)\right]\right)\\ &\oplus\sum_{d_1=0}^{\tilde{c}_1}\left(\binom{\tilde{n}_1}{d_1}\otimes \tilde{p}^{d_1}\otimes\left[\sum_{i_1=\tilde{\tau}_1\oplus 1}^{\tilde{\tau}_2}\left(\binom{\tilde{n}_1\ominus d_1}{i_1}\otimes(1\ominus\tilde{p}\ominus\tilde{q})^{i_1}\otimes\tilde{q}^{(\tilde{n}_1\ominus i_1\ominus d_1)}\otimes\right.\right.\right.\\ &\left.\left.\left.\left[\sum_{d_2=\tilde{c}_2\ominus d_1\oplus 1}^{\tilde{n}_2}\left(\binom{\tilde{n}_2}{d_2}\otimes\tilde{p}^{d_2}\otimes\left[\sum_{i_2=0}^{\tilde{n}_2\ominus d_2}\binom{\tilde{n}_2\ominus d_2}{i_2}\otimes(1\ominus\tilde{p}\ominus\tilde{q})^{i_2}\otimes\tilde{q}^{(\tilde{n}_2\ominus i_2\ominus d_2)}\right]\right)\right]\right)\right]\right)\end{aligned} \tag{40}$$

$$\tilde{P}_{r_{Binomial}} \cong \tilde{P}_{r_{Poisson}} = \sum_{d_1=\tilde{c}_2\oplus 1}^{\tilde{n}_1}\left(\frac{\tilde{\lambda}_{p_1}^{d_1}}{d_1!}\otimes\left[\sum_{i_1=0}^{\tilde{n}_1\ominus d_1}\frac{\tilde{\lambda}_{(1-p-q)_1}^{i_1}}{i_1!}\otimes e^{-(\tilde{\lambda}_{(1-p-q)_1}\oplus\tilde{\lambda}_{p_1})}\right]\right)$$
$$\oplus\sum_{d_1=\tilde{c}_1\oplus 1}^{\tilde{c}_2}\left(\frac{\tilde{\lambda}_{p_1}^{d_1}}{d_1!}\otimes\left[\sum_{i_1=0}^{\tilde{\tau}_2}\left(\frac{\tilde{\lambda}_{(1-p-q)_1}^{i_1}}{i_1!}\otimes e^{-(\tilde{\lambda}_{(1-p-q)_1}\oplus\tilde{\lambda}_{p_1})}\otimes\right.\right.\right.$$
$$\left.\left.\left.\left[\sum_{d_2=\tilde{c}_2\ominus d_1\oplus 1}^{\tilde{n}_2}\left(\frac{\tilde{\lambda}_{p_2}^{d_2}}{d_2!}\otimes\left[\sum_{i_2=0}^{\tilde{n}_2\ominus d_2}\frac{\tilde{\lambda}_{(1-p-q)_2}^{i_2}}{i_2!}\otimes e^{-(\tilde{\lambda}_{(1-p-q)_2}\oplus\tilde{\lambda}_{p_2})}\right]\right)\right]\right)\right]\right) \quad (41)$$
$$\oplus\sum_{d_1=0}^{\tilde{c}_1}\left(\frac{\tilde{\lambda}_{p_1}^{d_1}}{d_1!}\otimes\left[\sum_{i_1=\tilde{\tau}_1\oplus 1}^{\tilde{\tau}_2}\left(\frac{\tilde{\lambda}_{(1-p-q)_1}^{i_1}}{i_1!}\otimes e^{-(\tilde{\lambda}_{(1-p-q)_1}\oplus\tilde{\lambda}_{p_1})}\otimes\right.\right.\right.$$
$$\left.\left.\left.\left[\sum_{d_2=\tilde{c}_2\ominus d_1\oplus 1}^{\tilde{n}_2}\left(\frac{\tilde{\lambda}_{p_2}^{d_2}}{d_2!}\otimes\left[\sum_{i_2=0}^{\tilde{n}_2\ominus d_2}\frac{\tilde{\lambda}_{(1-p-q)_2}^{i_2}}{i_2!}\otimes e^{-(\tilde{\lambda}_{(1-p-q)_2}\oplus\tilde{\lambda}_{p_2})}\right]\right)\right]\right)\right]\right)$$

$\tilde{P}_{a_\alpha}$ and $\tilde{P}_{r_\alpha}$ are obtained as shown in Eqs. (42, 43).

$$\tilde{P}_{a_\alpha} = \begin{bmatrix} min\{P_a \mid p\in\tilde{p}_\alpha, q\in\tilde{q}_{1-\alpha}, n_1\in\tilde{n}_{1_\alpha}, n_2\in\tilde{n}_{2_\alpha}, c_1\in\tilde{c}_{1_\alpha}, c_2\in\tilde{c}_{2_\alpha}, \tau_1\in\tilde{\tau}_{1_\alpha}, \tau_2\in\tilde{\tau}_{2_\alpha}\}, \\ max\{P_a \mid p\in\tilde{p}_\alpha, q\in\tilde{q}_{1-\alpha}, n_1\in\tilde{n}_{1_\alpha}, n_2\in\tilde{n}_{2_\alpha}, c_1\in\tilde{c}_{1_\alpha}, c_2\in\tilde{c}_{2_\alpha}, \tau_1\in\tilde{\tau}_{1_\alpha}, \tau_2\in\tilde{\tau}_{2_\alpha}\} \end{bmatrix} \quad (42)$$

$$\tilde{P}_{r_\alpha} = \begin{bmatrix} min\{P_r \mid p\in\tilde{p}_\alpha, q\in\tilde{q}_{1-\alpha}, n_1\in\tilde{n}_{1_\alpha}, n_2\in\tilde{n}_{2_\alpha}, c_1\in\tilde{c}_{1_\alpha}, c_2\in\tilde{c}_{2_\alpha}, \tau_1\in\tilde{\tau}_{1_\alpha}, \tau_2\in\tilde{\tau}_{2_\alpha}\}, \\ max\{P_r \mid p\in\tilde{p}_\alpha, q\in\tilde{q}_{1-\alpha}, n_1\in\tilde{n}_{1_\alpha}, n_2\in\tilde{n}_{2_\alpha}, c_1\in\tilde{c}_{1_\alpha}, c_2\in\tilde{c}_{2_\alpha}, \tau_1\in\tilde{\tau}_{1_\alpha}, \tau_2\in\tilde{\tau}_{2_\alpha}\} \end{bmatrix} \quad (43)$$

$\widetilde{AOQ}$ and $\widetilde{AOQ}_\alpha$ are calculated as in Eqs. (44, 45).

$$\widetilde{AOQ} = \tilde{P}_a\otimes\tilde{p} = \tilde{P}_a\otimes\tilde{\lambda}_{p_1}\oslash\tilde{n}_1 = \tilde{P}_a\otimes\tilde{\lambda}_{p_2}\oslash\tilde{n}_2 \quad (44)$$

$$\widetilde{AOQ}_\alpha = \begin{bmatrix} min\{AOQ \mid p\in\tilde{p}_\alpha, q\in\tilde{q}_{1-\alpha}, n_1\in\tilde{n}_{1_\alpha}, n_2\in\tilde{n}_{2_\alpha}, c_1\in\tilde{c}_{1_\alpha}, c_2\in\tilde{c}_{2_\alpha}, \tau_1\in\tilde{\tau}_{1_\alpha}, \tau_2\in\tilde{\tau}_{2_\alpha}\}, \\ max\{AOQ \mid p\in\tilde{p}_\alpha, q\in\tilde{q}_{1-\alpha}, n_1\in\tilde{n}_{1_\alpha}, n_2\in\tilde{n}_{2_\alpha}, c_1\in\tilde{c}_{1_\alpha}, c_2\in\tilde{c}_{2_\alpha}, \tau_1\in\tilde{\tau}_{1_\alpha}, \tau_2\in\tilde{\tau}_{2_\alpha}\} \end{bmatrix} \quad (45)$$

$\widetilde{ATI}$ for non-repetitive and repetitive ASPs are obtained as shown in Eqs. (46, 47), and $\widetilde{ATI}_\alpha$ is determined by using Eq. (48). Where d_1, d_2, i_1 and i_2 are the observed defective and non-determinate item count in the first and steps, respectively.

$$\widetilde{ATI}_{(0)} = (\tilde{n}_1\otimes\tilde{P}_a^I)\oplus\Big((\tilde{n}_1\oplus\tilde{n}_2)\otimes\tilde{P}_a^{II}\Big)\oplus\Big(\tilde{N}\otimes(1\ominus\tilde{P}_a)\Big) \quad (46)$$

$$\widetilde{ATI}_{(r)} = (\tilde{n}_1\otimes\tilde{P}_a^I)\oplus\Big((\tilde{n}_1\oplus\tilde{n}_2)\otimes\tilde{P}_a^{II}\Big)\oplus\Big(\tilde{N}\otimes\tilde{P}_r\Big)\oplus\Big((1\ominus\tilde{P}_a\ominus\tilde{P}_r)\otimes\widetilde{ATI}_{(r-1)}\Big) \quad (47)$$

$$\widetilde{ATI}_\alpha = \begin{bmatrix} min\{ATI \mid p\in\tilde{p}_\alpha, q\in\tilde{q}_{1-\alpha}, n_1\in\tilde{n}_{1_\alpha}, n_2\in\tilde{n}_{2_\alpha}, c_1\in\tilde{c}_{1_\alpha}, c_2\in\tilde{c}_{2_\alpha}, \tau_1\in\tilde{\tau}_{1_\alpha}, \tau_2\in\tilde{\tau}_{2_\alpha}\}, \\ max\{ATI \mid p\in\tilde{p}_\alpha, q\in\tilde{q}_{1-\alpha}, n_1\in\tilde{n}_{1_\alpha}, n_2\in\tilde{n}_{2_\alpha}, c_1\in\tilde{c}_{1_\alpha}, c_2\in\tilde{c}_{2_\alpha}, \tau_1\in\tilde{\tau}_{1_\alpha}, \tau_2\in\tilde{\tau}_{2_\alpha}\} \end{bmatrix} \quad (48)$$

$\widetilde{ASN}$ and $\widetilde{ASN}_\alpha$ are calculated as shown in Eqs. (49, 50). Where d_1 and i_1 are the observed defective and non-determinate item count in the first step, respectively.

$$\begin{aligned}\widetilde{ASN} &= \tilde{n}_1\oplus\Big(\tilde{n}_2\otimes(1\ominus\tilde{P}^I)\Big) \\ &= \tilde{n}_1\oplus\tilde{n}_2\otimes\left(1\ominus\begin{pmatrix}\tilde{P}\{d_1\leqslant\tilde{c}_1 \mid \tilde{p},\tilde{q},\tilde{n}_1,\tilde{n}_2,\tilde{c}_1,\tilde{c}_2,\tilde{\tau}_1,\tilde{\tau}_2\} \\ \oplus\tilde{P}\{d_1>\tilde{c}_2 \mid \tilde{p},\tilde{q},\tilde{n}_1,\tilde{n}_2,\tilde{c}_1,\tilde{c}_2,\tilde{\tau}_1,\tilde{\tau}_2\} \\ \oplus\tilde{P}\{i_1>\tilde{\tau}_2, d_1\leqslant\tilde{c}_1 \mid \tilde{p},\tilde{q},\tilde{n}_1,\tilde{n}_2,\tilde{c}_1,\tilde{c}_2,\tilde{\tau}_1,\tilde{\tau}_2\}\end{pmatrix}\right)\end{aligned} \quad (49)$$

$$\widetilde{ASN}_\alpha = \begin{bmatrix} min\{ASN \mid p\in\tilde{p}_\alpha, q\in\tilde{q}_{1-\alpha}, n_1\in\tilde{n}_{1_\alpha}, n_2\in\tilde{n}_{2_\alpha}, c_1\in\tilde{c}_{1_\alpha}, c_2\in\tilde{c}_{2_\alpha}, \tau_1\in\tilde{\tau}_{1_\alpha}, \tau_2\in\tilde{\tau}_{2_\alpha}\}, \\ max\{ASN \mid p\in\tilde{p}_\alpha, q\in\tilde{q}_{1-\alpha}, n_1\in\tilde{n}_{1_\alpha}, n_2\in\tilde{n}_{2_\alpha}, c_1\in\tilde{c}_{1_\alpha}, c_2\in\tilde{c}_{2_\alpha}, \tau_1\in\tilde{\tau}_{1_\alpha}, \tau_2\in\tilde{\tau}_{2_\alpha}\} \end{bmatrix} \quad (50)$$

5 An illustrative example from a manufacturing process

Since the defectiveness of items can have different types or levels in real cases, the traditional ASPs do not give the ability to model this variability of item defectiveness. Three defect types and relevant three AQL levels are defined in practice as a workaround solution to overcome this issue. The companies use the following classification and AQL levels: 4% for minor defects, 2.5% for major defects, and 0% for critical defects [1]. The proposed plans have a high potential in terms of efficiency for this kind of real-world applications. This section aims to verify the validity of the proposed formulations by a demonstration based on a numerical example from a pen manufacturer company. The company produces pens in three segments: standard, luxury, and premium. Body of plastic pens are purchased from a supplier, and the acceptance sampling procedure is performed for these pen bodies. Pen bodies might have different types of defects having different significances. For example, screw area defects are considered totally defective. The same pen bodies are used for all segments of products, but acceptable defect types are not the same for all segments. Major color defects and minor form defects (such as micro-cracks) are acceptable for standard pens. In contrast, only minor color defects are acceptable for luxury pens, and none of the defects are accepted for premium pens. The defectiveness levels also vary inside the defect classes. Depending on this, it is sometimes impossible to decide the defectiveness class of the items because of indetermination. The proposed IFS formulation fits well with this scenario.

Deciding on the right FS shape is very important to reach reasonable results. As an exemplary study, [14] has adopted z-shaped FSs for defectiveness of items while deciding fuzzy sample size for control charts. Since z-shaped FSs are suitable for the nature of the defectiveness concept in inspection procedures, we have modified the term set shown in Eq. (22) and Figure 3 by using z-shaped FSs. Z-shaped fuzzy membership functions have been represented with three points as in the format (x, y, z), where two of them are core and support points, and the remaining is the limiting value of the definition space. The employed LTS is shown in Figure 5. Assume that the quality inspector have decided the defectiveness of the pen body population as $\tilde{p} = (Very\,low, -0.4)$ and non-defectiveness as $\tilde{q} = (Very\,high, 0.1)$. Remaining part has been regarded as indeterminate. Additionally, approximate defectiveness ratios of the subsets of defectiveness have been determined as seen in Table 1.

Table 1: Defectiveness ratio of the defect types

Defectiveness Class	Most Significant Defect	Defectiveness Ratio
Insensibly Defective	Minor Color Defect	35%
Slightly Defective	Major Color Defect	55%
Partially Defective	Minor Form Defects	80%
Almost Totally Defective	Major Form Defects	90%
Totally Defective	Screw Area Defects	100%

The company wants to compare the results of single and double ASPs for the lots having 480 items. Acceptance Quality Limit (AQL) has been determined as 2.5% by considering defective items. However, 4% is allowed when indeterminate and defective items are considered together. The company has decided to use similar plan parameters with the MIL-STD-105E normal inspection plan. For this reason, the company has adopted the $H - 2.5\%$ plan for defectiveness and the difference between $H - 2.5\%$ and $H - 4.0\%$ for indeterminacy. Depending on this approach, ASP parameters are obtained as $n = 50$, $c = 3$ and $\tau = 2$ for single ASP, $n_1 = 32$, $n_2 = 32$, $c_1 = 1$, $c_2 = 4$, $\tau_1 = 1$ and $\tau_2 = 2$ for DASP according to MIL-STD-105E tables II-A and III-A. Then the defectiveness information is calculated as a z-shaped IFS such that $\tilde{\tilde{A}} = \{\tilde{p} = (0, 0.100, 0.267), \tilde{q} = (0.683, 0.850, 1) \mid x \in X\}$ by interpolating the FSs of linguistic terms depending on the second part of the 2-tuple statements made by the inspectors. In this case, the non-determinacy degree is obtained as $\pi_{\tilde{\tilde{A}}}(x) = 1 \ominus \mu_{\tilde{\tilde{A}}}(x) \ominus \vartheta_{\tilde{\tilde{A}}}(x) = 0.05$. Membership and non-membership functions are symbolically demonstrated as shown in Fig. 6. Due to the defectiveness information is formed as an IFS, ASP results should be obtained as an IFS such that $\tilde{\tilde{B}} = \{\tilde{P}_r, \tilde{P}_a \mid x \in X\}$.

Results for defectiveness information based on IFS have been determined for the core and support points of $\tilde{p}$ and $\tilde{q}$ as shown in Table 2. To compare the performance of the suggested formulations, the table also shows the results for the scenario in that the non-determinacy is ignored, non-determinate parts are regarded as non-defective. For this benchmark scenario, the formulations offered by [23] have been used. As seen in Table 2, there is a notable difference between the acceptance probabilities of the FS and IFS formulations. It is an expected result because non-determinate items are regarded as non-defective in case of FS. In this example, the non-determinacy is not quite small, so the IFS formulation gives more precautionary results. Acceptance probabilities for SASP are also demonstrated in Figure 6. Rejection probabilities reached by the formulations are similar to each other for SASPs, but not similar to each other for DASPs. The root cause of this finding is that the sampling procedure can continue to the second step when the

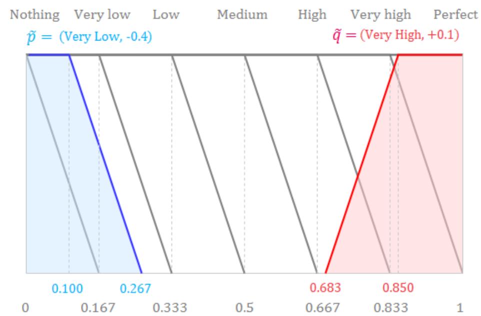

Figure 5: Linguistic defectiveness information for the example

Table 2: Obtained results for FSs and IFSs

	SASP		DASP	
	FS	IFS	FS	IFS
$\{\tilde{P}_a, \tilde{P}_r\}$	$\{(0, 0.250, 1),$ $(0, 0.750, 1)\}$	$\{(0, 0.126, 1),$ $(0, 0.750, 1)\}$	$\{(0.001, 0.277, 1),$ $(0, 0.723, 1)\}$	$\{(0, 0.120, 1),$ $(0, 0.629, 0.986)\}$
$\{\widetilde{AOQ}_{min}, \widetilde{AOQ}_{max}\}$	$\{0, 0.039\}$	$\{0, 0.021\}$	$\{0, 0.042\}$	$\{0, 0.020\}$
$\widetilde{ATI}/\widetilde{ATI}_{(0)}$	$(50, 372.374, 479.913)$	$(50, 425.887, 479.968)$	$(32, 359.959, 479.718)$	$(32, 427.511, 479.898)$
$\widetilde{ATI}_{(1)}$	–	$(50, 372.374, 479.967)$	–	$(32, 307.081, 479.160)$
$\widetilde{ASN}$	–	–	$(32, 33.449, 52.227)$	$(32, 33.012, 49.207)$

indeterminate item count is between τ_1 and τ_2 in the first step. Accordingly, the total rejection probability has a dependency with the non-determinacy degree. It can be deduced that the results of IFS and FS formulations converge to each other more when $\pi_{\tilde{A}}(x)$ is small and $\tilde{\tau}$ is relatively big. This is because, it decreases the indetermination probability. The acceptance and the rejection probabilities for SASP and DASP formulations converge to each other,

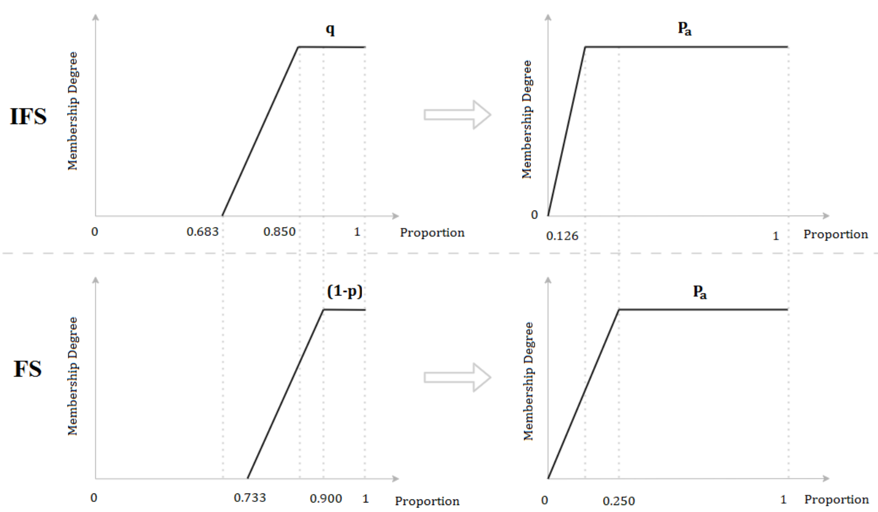

Figure 6: Comparison of SASP results for IFS and type-1 FS formulations

but DASP results are a little bit smaller than SASP results. Since DASPs are used for reaching lower risks with fewer inspections, similar $\tilde{P}_a$ values have been reached with SASPs despite the smaller $\widetilde{ATI}$. $\widetilde{ATI}_{(1)}$ has provided smaller values than $\widetilde{ATI}_{(0)}$. The core of $\widetilde{ATI}_{(0)}$ and $\widetilde{ATI}_{(1)}$ has observed as quite different because the indeterminacy is not too small in the example. It should be noticed that $\tilde{P}_a$ values have been obtained as FSs having too wide range. The reason of this outcome is related to the used LTS. This term set has only seven terms and the adopted linguistic approach divides the 0-1 interval into seven pieces. As a result, $\tilde{p}$ is clarified as a wide fuzzy number. To overcome this problem, linguistic modifiers analyzed by [12], [24] can be applied in the assessment procedure. Calculation of the $\widetilde{AOQ}$ is different from $\tilde{P}_a$ and $\tilde{P}_r$ and it is not sufficient to calculate values for only edge points (core, support, and the limiting value of the definition space) of $\tilde{p}$. According to Eq. (33) and (44), it is calculated by multiplication of $\tilde{P}_a$ and $\tilde{p}$. While the membership function of $\tilde{p}$ is left to right as seen in Figure 5, the membership function of $\tilde{P}_a$ is right to left, as seen in Figure 6. $\widetilde{AOQ}$ gets its biggest value in an intermediate value in the scale. For this reason, only the maximum and minimum values have been presented in Table 2. The maximum value has been reached for $\tilde{p} = 0.058$. Similar to the acceptance probabilities, maximum $\widetilde{AOQ}$ values have been converged to each other for SASP and DASP formulations. IFS formulations have provided smaller $\widetilde{AOQ}$ values.

Interval-valued FSs (IVFSs) are more useful in real-case applications because of their ease of calculation, perception, and interpretation of the results. To use IVIFS formulations, IFSs should be converted to IVIFSs by using a defuzzification technique such as α-cut. However, α-cut has a significant drawback in real-case problems due to the numerical α values. To avoid this problem, linguistic α values have been used. The linguistic α values have been determined and transformed into numerical values to use in formulations with respect to Table 1. These α levels have been demonstrated on the membership function in Figure 7. For example, the biggest defectiveness ratio of "Insensible Defective", "Slightly Defective" and "Partially Defective" items is 0.8 (Partially defective) so, $\alpha_{standard}$ is determined as 0.8. As described in Section 2.1, the non-membership function is cut from $(1-\alpha)$ level while the membership function is cut from α level. Using these values, IVIFS results have been reached as seen in Table 3 for all segment pens having the similar lot sizes.

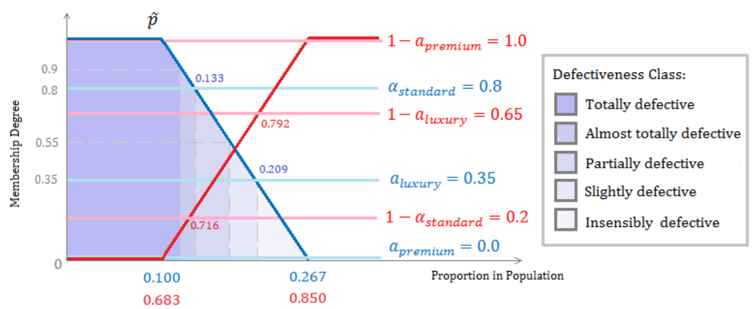

Figure 7: Defectiveness and non-defectiveness α-cuts for the product segments

Due to the high comprehensiveness of $\tilde{p}$ and $\tilde{q}$, too wide results have been reached. Core points are obtained as smaller values in IVIFS formulations. As an interesting finding, while the acceptance probabilities of IVIFS formulations converge to each other for SASP and DASP, the rejection probabilities have been obtained pretty much different. This outcome is also caused by the fact that the sampling can continue to the second step when the indeterminate item count is observed between τ_1 and τ_1 or the defective item count is observed between c_1 and c_2 in the first step. This is not the case for IVFS formulations because the sampling can continue to the second step if and only if the defective item count is observed between c_1 and c_2 in the first step. As a side effect of this situation, rejection probabilities for IVFS and IVIFS formulations have been determined as similar for SASP.

If it is desired to define a stricter control procedure during the inspection phase, it will be necessary to define lower α values. However, the acceptance probability of standard segment product for IVIFS formulation draws a contradictory table with this statement. Because the core point of the acceptance probability for standard segment product has been reached smaller than the luxury segment product, it is not a surprising output. As explained in Section 2.1, both membership and non-membership functions are narrowed by α-cut. Narrowing them increases indeterminacy degree because Eqs. (1) and (2) are satisfied for all IFSs and IVIFSs. If excessively big α values are used in ASPs, the non-determinacy probability can be big enough to decrease the acceptance probability.

Based on the above example, plan parameters n, c, and τ have been determined as real numbers. However, they can also be fuzzy numbers if it is not possible to decide the lot size certainly or AQL is approximately determined. These kinds of issues can be possible for small piece products. If IVFS formulation is used in such circumstances, the fuzzy plan parameters should also be cut from a certain level. When type-1 FSs are used for plan parameters and only a single product segment is available, cutting the defectiveness information and plan parameters from the same α-level gives useful results. However, using the same α values for both defectiveness information and plan parameters in the current scenario brings hardness to compare the results seen in Table 3 for different segment products. A common β-cut can be applied to plan parameters for all segments while the defectiveness's are cut from different α-levels for each product segment. In this way, more comparable results can be reached. Assume that the plan parameters are

Table 3: Results for different quality products based on α-cuts

		SASP		DASP	
		IVFS	IVIFS	IVFS	IVIFS
Standard ($\alpha = 0.8$)	$\{\tilde{P}_a, \tilde{P}_r\}$	$\{[0.086, 1], [0, 0.914]\}$	$\{[0.001, 1], [0, 0.914]\}$	$\{[0.097, 1], [0, 0.903]\}$	$\{[0.001, 1], [0, 0.475]\}$
	$\{\widetilde{AOQ}_{min}, \widetilde{AOQ}_{max}\}$	$\{0, 0.039\}$	$\{0, 0.021\}$	$\{0, 0.042\}$	$\{0, 0.020\}$
	$\widetilde{ATI}/\widetilde{ATI}_{(0)}$	$[50, 442.944]$	$[50, 479.745]$	$[32, 437.635]$	$[32, 479.449]$
	$\widetilde{ATI}_{(1)}$	–	$[50, 479.393]$	–	$[32, 451.949]$
	$\widetilde{ASN}$	–	–	$[32, 48.455]$	$[32, 33.761]$
Luxury ($\alpha = 0.35$)	$\{\tilde{P}_a, \tilde{P}_r\}$	$\{[0.004, 1], [0, 0.996]\}$	$\{[0.004, 1], [0, 0.996]\}$	$\{[0.006, 1], [0, 0.994]\}$	$\{[0.006, 1], [0, 0.994]\}$
	$\{\widetilde{AOQ}_{min}, \widetilde{AOQ}_{max}\}$	$\{0, 0.039\}$	$\{0, 0.021\}$	$\{0, 0.042\}$	$\{0, 0.020\}$
	$\widetilde{ATI}/\widetilde{ATI}_{(0)}$	$[50, 478.392]$	$[50, 478.392]$	$[32, 477.338]$	$[32, 477.340]$
	$\widetilde{ATI}_{(1)}$	–	$[50, 478.392]$	–	$[32, 477.340]$
	$\widetilde{ASN}$	–	–	$[32, 37.320]$	$[32, 37.320]$
Premium ($\alpha = 0.0$)	$\{\tilde{P}_a, \tilde{P}_r\}$	$\{[0, 1], [0, 1]\}$	$\{[0, 1], [0, 1]\}$	$\{[0.001, 1], [0, 0.999]\}$	$\{[0, 1], [0, 0.986]\}$
	$\{\widetilde{AOQ}_{min}, \widetilde{AOQ}_{max}\}$	$\{0, 0.039\}$	$\{0, 0.021\}$	$\{0, 0.042\}$	$\{0, 0.020\}$
	$\widetilde{ATI}/\widetilde{ATI}_{(0)}$	$[50, 479.913]$	$[50, 479.968]$	$[32, 479.718]$	$[32, 479.898]$
	$\widetilde{ATI}_{(1)}$	–	$[50, 479.967]$	–	$[32, 479.160]$
	$\widetilde{ASN}$	–	–	$[32, 33.449]$	$[32, 33.012]$

replaced with triangular type-1 FSs as $\tilde{N} = (460, 480, 500)$,$\tilde{n} = (45, 50, 55)$, $\tilde{c} = (2, 3, 4)$ and $\tilde{\tau} = (1, 2, 3)$ for single ASP, $\tilde{n}_1 = (30, 32, 34)$, $\tilde{n}_2 = (30, 32, 34)$, $\tilde{c}_1 = (0, 1, 2)$, $\tilde{c}_2 = (3, 4, 5)$, $\tilde{\tau}_1 = (0, 1, 2)$ and $\tilde{\tau}_2 = (1, 2, 3)$ for DASP in the above example. The results in Table 4 have been reached for $\beta = 0.35$ by using these plan parameters. Depending on this approach, more narrowed results have been reached than the results seen in Table 3. Nevertheless, no significant differences have been observed between the product segments. This is caused by using a small β-level. Using a small β value means the variability of the plan parameters is high. While this variability is high, variability of $\tilde{P}_a$ increases dramatically, and as a result, the range of $[0, 1]$ is obtained for $\tilde{P}_a$. When the lot size and the plan parameters are getting bigger, the relative effect of the variability (increasing/decreasing c one unit) of the plan parameters over the acceptance probability reduces, the rounding operations in β-cut caused by working small numbers are minimized. In this way, more narrowed intervals can be achieved.

Table 4: Results for premium quality product while plan Parameters $\tilde{N}$, $\tilde{n}$, $\tilde{c}$ and $\tilde{\tau}$ are cut by β=0.35

		SASP		DASP	
		IVFS	IVIFS	IVFS	IVIFS
Standard ($\alpha = 0.8$)	$\{\tilde{P}_a, \tilde{P}_r\}$	$\{[0.022, 1], [0, 0.978]\}$	$\{[0, 1], [0, 0.978]\}$	$\{[0.024, 1], [0, 0.976]\}$	$\{[0, 1], [0, 0.720]\}$
	$\widetilde{ATI}/\widetilde{ATI}_{(0)}$	$[47, 483.513]$	$[47, 492.994]$	$[31, 482.321]$	$[31, 492.992]$
	$\widetilde{ATI}_{(1)}$	–	$[47, 503.545]$	–	$[31, 492.970]$
	$\widetilde{ASN}$	–	–	$[31, 56.78]$	$[31, 38.298]$
Luxury ($\alpha = 0.35$)	$\{\tilde{P}_a, \tilde{P}_r\}$	$\{[0, 1], [0, 1]\}$	$\{[0, 1], [0, 1]\}$	$\{[0.001, 1], [0, 0.999]\}$	$\{[0.001, 1], [0, 0.999]\}$
	$\widetilde{ATI}/\widetilde{ATI}_{(0)}$	$[47, 492.80]$	$[47, 492.804]$	$[31, 492.729]$	$[31, 492.729]$
	$\widetilde{ATI}_{(1)}$	–	$[47, 492.804]$	–	$[31, 492.729]$
	$\widetilde{ASN}$	–	–	$[31, 42.409]$	$[31, 42.409]$
Premium ($\alpha = 0.0$)	$\{\tilde{P}_a, \tilde{P}_r\}$	$\{[0, 1], [0, 1]\}$	$\{[0, 1], [0, 1]\}$	$\{[0, 1], [0, 1]\}$	$\{[0, 1], [0, 0.998]\}$
	$\widetilde{ATI}/\widetilde{ATI}_{(0)}$	$[47, 492.994]$	$[47, 492.999]$	$[31, 492.983]$	$[31, 492.998]$
	$\widetilde{ATI}_{(1)}$	–	$[47, 493]$	–	$[31, 492.998]$
	$\widetilde{ASN}$	–	–	$[31, 36.019]$	$[31, 35.636]$

6 Conclusions

ASPs are prevalent quality control techniques based upon accepting and rejecting the lots by inspecting only a small set to reach a specified consumer's risk and a specified producer's risk. Traditional ASPs use certain parameters, such as the defectiveness ratio of the incoming lot. Nevertheless, they may not always be known certainly in real case problems. FST is used to reformulate the ASPs in the literature to manage the uncertainty of real-case applications. However, sometimes it is more effective to use fuzzy set extensions to model these uncertainties. It is clear that one of the fuzzy extensions named IFS is an efficient approach to model the ASPs having non-determinacy on human evaluation about the defectiveness of the inspected items. Modeling the inspector's hesitancy in deciding the quality class of the product at the inspection stage allows more sensitive results to be revealed. To do this, single and double ASPs having IFSs defection statuses have been designed in this study. The item defectiveness of the lots has been represented as FLVs to improve interoperability of the fuzzy modeling. Additionally, the binomial and poisson distributions have been re-consider for these cases.

As discussed in the literature, the IVIFSs have advantages in engineering problems due to the simplicity in problem formulation and calculations. For this reason, the main parameters of ASPs have been formulated based on IFS and then extended for IVIFSs. The α-cut technique is presented to make the IVIFS formulation usable for the populations having IFS defectiveness. The most crucial criticism about the α-cut technique is that it uses numerical α-levels that do not have tangible counterparts in real life. Combining the linguistic approach with the α-cut technique is another contribution of this paper to the literature. Using linguistic α-values, the inspector can easily decide that the defects of the inspected items are acceptable or not for the given α-level. Moreover, the human error in the transformation of the verbal inspection results into numerical measures is minimized. To compare the performance of the proposed methodologies with the available type-1 FSs for both single and double ASPs, a numerical example from the manufacturing industry has been presented. The obtained results show that the IFSs successfully model the uncertainties and hesitancy on the sampling approach. It is clearly seen that suggested ASPs contain more information than both traditional and type-1 fuzzy ASPs. As a future study, the other types of fuzzy set extensions such as Hesitant and Pythagorean FSs can be used to model the uncertainty of the sampling procedure for ASPs. The formulations can be combined with more sophisticated linguistic fuzzy modeling approaches having high accuracy and interoperability at the same time. Then the obtained results can be compared.

Acknowledgement

The Turkish Academy of Sciences supports this paper under the Outstanding Young Scientists Awards (GEBİP) for the Author, İhsan Kaya.

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