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Hierarchical Chinese postman problem with fuzzy travel times

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Abstract

The hierarchical Chinese postman problem (HCPP), one of the kinds of Chinese postman problem (CPP), aims to visit at least once the arcs are classified according to their precedence relations and to find the shortest tour or tours. In HCPP, if the travel time values between two nodes constituting the cost are not crisp, this problem type can be said to be the hierarchical Chinese postman problem with fuzzy travel times (HCPP-FTT). In real-life problems, as the travel time between the nodes is not fixed due to various factors, in this study triangular fuzzy numbers are used at objective function coefficients of a small-size sample problem. Different ranking methods are used to defuzzify the fuzzy numbers and the acquired results are compared. The mathematical model formed is resolved using CPLEX solver GAMS 24.2.3 software. Besides, in this study, a large-size real-life problem was discussed. The objective function coefficients representing the travel time between the two nodes in the case study discussed were considered as trapezoidal fuzzy numbers. Due to the NP-hard nature of the problem, the mathematical model cannot find solutions in large size problems in the appropriate time interval. For this reason, in this study, the heuristic method based on the Greedy Search (GS) algorithm and the meta-heuristic method based on ant colony optimization (ACO) is proposed for the solution of the real-life problem. The proposed algorithms are coded using the Matlab-2019b programming language. Using the developed algorithms, the results of different ranking methods and solution times were compared.

Keywords: Hierarchical Chinese postman problem, arc routing problem, fuzzy number, fuzzy ranking methods.

1 Introduction

The arc routing problems are the problems aiming to form the shortest tour or tours that visit all the arcs on a graph at least once and return to the starting point [12]. CCP that is an arc routing problem, was first revealed when the Chinese mathematician Kwan Mei Ko was interested in the problem with the goal of postal delivery to houses on every street of a quarter using the shortest possible distance [2]. In this kind of problem, there is the obligation of leaving the starting point, visiting all the roads on a graph, and then to return to the starting point. The CPP can be utilized in many fields like routing the newspaper and water delivery, snow plow, road cleaning, school busses, and patrol cars [57].

A special type of CPP, HCPP is the problem to find the shortest tour or tours that shall pass each arc at least once via considering the precedence relations in a hierarchical graph where the arcs are classified according to these precedence relations. In real-life problems like garbage collecting and snow plowing where the roads/arcs are classified according to their precedence relations, the HCPP can be used. In real-life problems, the cost or times may not always be defined. In order to state such uncertainties due to various causes with mathematical models, fuzzy logic is used. The term fuzzy logic was presented to the scientific world for the first time in 1965 by Lotfi Zadeh [67]. We encounter fuzzy logic in situations and events where some state is not known exactly and uncertainty is in question.

The fuzzy logic aims to depict the objects and values closer and more conforming to the reality; this depiction is successful at the rate the mathematics allows. The fuzzy logic provides the chance to perform mathematical calculations

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via using words [68]. In classical logic, there are crisp values like 0 or 1. An element acquires the value 1 if it is an element of a set and 0 if not. However in fuzzy logic, instead of definitions like 0-1 as in classical logic, membership functions are showing the amounts of belonging to these definitions. Such as that the fuzzy logic considers the interim values like 0.1, 0.2, 0.9 between 0 and 1, and using membership functions provide the opportunity to show at what rate an element is included in a set. In fuzzy systems, the most fundamental element of the fuzzy logic theory is the fuzzy set. In a fuzzy set A defined in the universal set, the membership function is stated as $\mu_A : E \rightarrow [0,1]$. For an element x in this fuzzy set A the membership degree is shown as $\tilde{A} = \{(x, \mu_A(x)) \,|x \epsilon E\}$ [72]. In fuzzy sets, to provide operation facilitation, the fuzzy numbers are used in the expression, and assessment of the non-absolute results in real-life problems containing uncertainty. The models calculated with fuzzy data, compared to the models not accepting uncertainties or not including them into calculations, can provide more information on the data [23]. Different fuzzy number types can be used based on the handled subject. Usually, in most studies, it can be said that triangular and trapezoidal fuzzy numbers are used [5]. In some mathematical modeling like optimization and deciding problems, the sorting of the fuzzy numbers is a necessary and important step [47]. In fuzzy logic, for the defuzzification of the numbers, various ordering methods are present. The fuzzy logic aiming to model the decision-making mechanisms for giving consistent and accurate decisions in light of information not complete and absolute has various application areas like medicine, artificial intelligence, robotics, engineering and routing problems [35].

The study is organized as follows: In the second section, firstly, CPP and HCPP studies in the literature are mentioned, and then vehicle routing problems (VRP) and traveling salesman problem (TSP) studies, which are handled with the fuzzy logic approach, are summarized and the solution methods used in these studies are examined. In the third section, first of all, HCPP and its mathematical model are considered, then the sorting techniques used in these studies are explained. In the same section, the developed heuristic algorithm and meta-heuristic algorithm are presented and its details are provided. In the fourth section numerical example of a small-size hierarchical network and a case study is presented. In the fifth section, the results are given and recommendations for future studies are provided.

2 Literature survey

The CPP considered firstly by a Chinese mathematician Mei-Ko Kwan in 1962 [40] was reached through the wish of a postman to deliver the letters he got from the post office by traveling through the shortest path possible via visiting all the street in the town [2]. In time some other constraints are added to this problem type and so different CPP types (Directed CPP, Undirected CPP, Mixed CPP, k-CPP, Min-Max k-CPP, Hierarchical CPP, Capacitated CPP, and Windy CPP) had been achieved. Until now deterministic CPP types had been handled by many researchers like [1, 13, 16, 25, 31, 33, 44, 48, 54, 58, 62, 66]. In some of these studies the problems are solved with classical methods, whereas in others, important algorithms regarding the solution are developed. However, when the related literature is examined, it is seen that the studies related to the uncertain parameters of CPP are rare. Tan et al. [55], had considered CPP in stochastic networks. In these networks the travel times are stochastic. They had proposed an algorithm regarding the solution of the related problem. Zhang and Peng [70] had handled three uncertain programming models of the CPP with Uncertain Weights. They had shown that these models are transformable to deterministic form and that Dijkstras algorithm and Fleurys algorithm can be used as a solution approach to a sample problem. Majumder et al. [43] have proposed, for the undirected connected network, a multi-objective CPP under the framework of uncertainty theory. An expected value model was proposed regarding the problem. They have formulated the expected value model of the proposed UMCPP and determine the deterministic transformation of the model using the 999-method. The model had been solved with two classical methods as the global criterion method and fuzzy programming method. Also, two different multi-objective genetic algorithms (GA) are proposed regarding the solution to the problem.

HCCP was first introduced by Dror et al.[22]. In that study, it is shown that HCPP resides in the NP-hard problem type but the problem can be solved within the polynomial-time if its precedent relations are linear and all its sub-networks are connected. Alfa and Liu [3] had handled a graph that is closer to real-life problems with hierarchical classes with no interconnection. In the study they had conducted, they had proposed a heuristic algorithm that creates a huge tour solving the rural postman problem and subsequently providing the linear precedence relations. Damodaran [17] had developed a new heuristic method to connect the hierarchical classes without interconnection. In the study, it is indicated that the new heuristic method developed shows a better performance than the heuristic method of Alfa and Liu [3] that has the same goal. Ghiani and Improta [30] had developed an algorithm in their study that gives an HCPP definite solution and requires relatively lower calculation effort compared to others. Cabral et al. [7] had developed a method that provides the HCPP to be transformed into an equivalent Rural Postman Problem. They had suggested a splitting intuitive for unidirectional HCPP with linear precedence relations. In order to solve HCPP that has two goals as a hierarchical purpose and production time, the transformed problem was subjected to a branch-and-cut algorithm. Korteweg and Volgenant [37], proposed a new algorithm that finds the minimum cost route in the hierarchical graph according to the order of precedence. In this newly developed algorithm, after the minimum route of the sub-networks with the highest precedence considering the hierarchy, the minimum cost routes of each sub-networks are separately calculated according to this order of precedence. Damodaran et al. [18], had presented an intuitive method that gives a better lower threshold for HCPP and shown that the problem is solved in less time with the help of definite search algorithms. Perrier et al. [50], presented a fundamental model and two intuitive solution approaches for the VRP in snow plowing operations. This problem can be considered as m-HCPP. The proposed method produces routes that are more appropriate than the producing current directing plan. Sayata and Desai [52], had presented an algorithm that removes the $O\left(kn^5\right)$ time chaos suggested by Dror et al. [22]. In their study, to reduce the edge count of a graph with $O\left(k^2n^2\right)$ time chaos, the Kruskal method is used. Colombi et al. [11] had developed a branch-and-cut algorithm producing exact solutions for the hierarchical mixed rural postman problem. Yılmaz [65] had handled the problem of defining the shortest length route on the unidirectional roads for a maintenance vehicle working at the Highways 12^{th} Regional Directorate. To solve this large scale problem, he had developed an intuitive algorithm based on searching the closest neighbor. odur and Yılmaz [15], had handled time-dependent HCPP. For the solution to this new problem type, the mixed-integer mathematical model is developed. Also for the solution of large-scale problems two meta-intuitive algorithms are proposed as a GA and hybrid Simulated Annealing. In the literature, when the studies are conducted on HCPP, it is seen that in general the methods providing certain solutions and heuristic methods are applied.

Fuzzy logic forms the other aspect of the present research. To the best of our knowledge, no study handles HCPP with a fuzzy logic approach. However, in the studies examined, it is seen that fuzzy logic solution approaches are applied to VRP and TSP. Studies on this field were reached by entering the word group fuzzy routing problems into the Web of Science (WoS) database. Keyword network analysis was done using the VOSviewer (Version 1.6.9) package program. According to the bibliometric analysis made using the VOSviewer program, the most used keywords are shown in Figure 1.

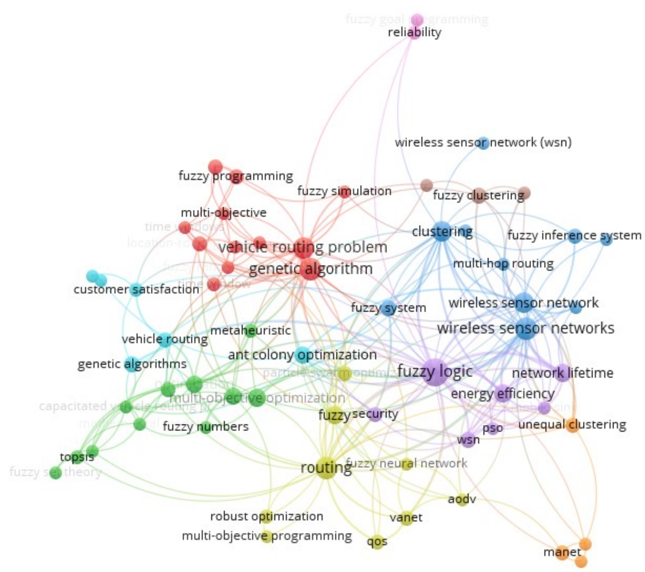

Figure 1: Keywords network analysis (VOSviewer)

As can be seen from Figure 1, among the most used keywords in fuzzy routing problems are vehicle routing, time windows, fuzzy numbers, fuzzy programming, fuzzy simulation. In addition, it is seen that metaheuristic algorithms such as GA and ACO used in the solution of large-scale problems are among these keywords. Some of the studies related to the fuzzy routing problems are given in this section.

Teodorovic and Pavkovic [56], had used the fuzzy set theory approach for the VRP where the requests at nodes are uncertain. The request variable in the study was taken as triangular fuzzy numbers. For the solution of the problem in the study, an intuitive algorithm is suggested. Wang and Wen [60], had handled a time frame restricted CPP. As time limitations are indefinite, they had used a fuzzy set theorem. Lu and Ni [42], in their study, had considered the time instead of distance as it is closer to the real-life problems and as the travel time from one city to another varies due to the air and traffic conditions. In the TSP they handled, they introduced the travel times as fuzzy variables. Fuzzy random simulations and GAs are integrated and a hybrid intelligent algorithm is designed. Zheng and Liu [71] assumed the travel times in a VRP problem as fuzzy variables. Using triangular fuzzy numbers, the fuzzy chance-constrained programming model was presented for the fuzzy VRPs. The GA and fuzzy simulation are integrated and a hybrid intelligent algorithm was designed and the problem was solved. Crisan and Nechita [14], had handled the fuzzy TSP. They had used triangular fuzzy numbers for fuzzy numbers and the Center of Mass Method for the defuzzification technique. For the solution of the problem, they had utilized the ACO. Botzheim et al. [6], in their study, had used triangular fuzzy numbers at objective function coefficients of a fuzzy road-transport TSP. They utilized the Center of Gravity (COG) method to defuzzify the fuzzy numbers. They had proposed the eugenic bacterial memetic algorithm for the solution of the problem. Erbau and Mingyong [24], had used triangular fuzzy numbers in customer requests in a VRP and created a fuzzy chance-constrained program model. For the solution of the problem a hybrid intelligent algorithm was designed and the problem was solved. Cao and Lai [8], in their study, had taken the customer request in an open VRP as triangular fuzzy numbers. Establishing a fuzzy chance-constrained program model and using stochastic simulation, the model is solved. Kumar and Gupta [38], two new methods had been suggested for the solution of the fuzzy assignment problems and fuzzy TSP. In their study, trapezoidal fuzzy numbers were used, and to defuzzify these fuzzy numbers, they had applied Yagers Ranking Technique. Kuo et al. [39], in a capacity VRP, had handled the request variable as triangular fuzzy numbers. In order to solve the problem they had suggested a hybrid particle swarm optimization with a GA. Nirmala and Anju [46], handled a TSP with travel costs as fuzzy variables. The lingual variables in the study were represented with triangular fuzzy numbers. To defuzzify the used fuzzy numbers the Robusts Ranking Method (RRM) is used. Dhanasekar et al. [19] had suggested a new algorithm to solve fuzzy TSP. In their study, to prove the accuracy and efficiency of the suggested algorithm they performed applications on two examples. Using the triangular and trapezoidal fuzzy numbers, they had solved the problem via the Fuzzy Hungarian Method. Zarandi et al. [69], in their study, had handled the travel time between two nodes as triangular fuzzy numbers. To solve the handled location-routing problem a simulated annealing procedure was presented. Changdar et al. [9], had modeled a multi-objective TSP in uncertainty medium using triangular fuzzy numbers. It is indicated that it is more efficient to model the travel time and cost of TSP as fuzzy numbers as it reflects the probability better. The problem was solved using a GA. Dingar and Sundari [20] in order to achieve the optimal fuzzy solution in the fuzzy TSP, had proposed a new method. In this method, the intuitive trapezoid fuzzy numbers were used to find the optimal fuzzy solution. As the defuzzification method, the Linear Ranking Function was preferred. Ghannadpour et al. [28], had studied a multi-objective dynamic VRP where the travel times are handled as triangular fuzzy numbers. In order to transform the fuzzy numbers to crisp values, they used a procedure. The problem is solved with GA. Yahya Mohammed and Divya [64] in their study utilizing the octagonal fuzzy numbers, had solved a TSP problem to maximize the total profit. Using the RRM, they transformed the fuzzy valued numbers into crisp values. When solved using octagonal numbers, it was proven that a better solution is achieved compared to solving via using trapezoid fuzzy numbers. Fazayeli et al. [26], in their study, had taken the request parameter in a routing problem as a triangular fuzzy number. They had used the rating technique presented by Jimenez [34] and suggested a GA for the solution of the problem. The suggested mathematical model was solved with GAMS software. Tohidifard et al. [59] in a Multi-depot VRP, as there was uncertainty in drug requests of the patients, had used triangular fuzzy numbers. In the study GA and Particle swarm optimization, meta-heuristics were suggested. Ghasemkhani et al. [29] present an integrated production inventory routing problem in their paper. As the requests are uncertain in the study, triangular fuzzy numbers are used. Pekel and Kara [49] had studied a fuzzy capacitated location routing problem. In the study, customer requests are taken as fuzzy variables. Also in this study, triangular fuzzy numbers were used, a hybrid heuristic method was presented and applied on a case study. Keskin and Yılmaz [36], as the service costs vary in real life due to weather conditions, traffic conditions, and similar reasons, had handled variable service cost CPP and windy postman problem in their study. It is considered, in a snow plowing operation, that at each pass of a snowplow through a road the snow amount is reduced depending on the passing order, and thus the service cost changes. For the solution of the problems, two different intuitive methods are proposed as pass iteration heuristic and Lagrangian heuristic. Wang et al.[61], in their study, had handled a VRP with fuzzy travel times. For the solution of the problem an optimized hybrid intelligent algorithm, as well as an improved fuzzy simulation, was proposed. Aliahmadi [4] discussed the capacitated node-routing problem for municipal solid waste collection with multiple tours. The amount of solid waste generated can vary from day to day. Due to this uncertainty, this study was handled with a fuzzy optimization approach and triangular fuzzy numbers were used. The study was applied to a real-life problem in Iran and a GA was used for its solution. When these studies are considered, it is seen that most of the studies that are handled with a fuzzy approach are node routing problems. It is determined that there are quite a limited number of studies dealing with arc routing problems with the fuzzy approach and the gap determined in the literature is tried to be filled with this study.

When the studies conducted in this field in the literature and Figure 1. is examined, it is seen that in some studies new mathematical models are proposed as the solution approach for the problems, whereas in most of the studies, for large-scale problems, the heuristic/meta-heuristic methods like GA, particle swarm optimization, lagrangian heuristic, hybrid simulated annealing and ACO are frequently used.

To the best of our knowledge, there are no studies performed previously handling the HCPP, one of the arc routing problems, with the fuzzy logic approach. In this study, a new problem type that is closer to the real-life problems and aims to minimize the total travel time that is called the Hierarchical Chinese Postman Problem With Fuzzy Travel Times (HCPP-FTT) is handled. In real-life problems, factors like traffic intensity, weather conditions, traffic accidents, maintenance, and repair works significantly change the travel time between two centers. Thus, in this study, travel times are considered instead of distance. In the small size network under consideration, the objective function coefficients that represent the travel time between the nodes are handled as triangular fuzzy numbers. To defuzzify the triangular fuzzy numbers used, the RRM, Centroid Ranking Technique (CRT), Kwong-Bai Method, and Best Nonfuzzy Performance (BNP) Value ranking methods which are frequently used in the literature are used. The results achieved with these methods are compared. On the other hand, as the travel times between the centers change depending on the traffic density at different time frames of the day, in the real-life problem that is considered, one day is divided into four different time frames. According to the traffic density in the time frames, the arrival times are given using trapezoidal fuzzy numbers. For the defuzzification of trapezoidal fuzzy numbers used RRM and CRT are used. For solving medium and large-scale problems in a reasonable time, a heuristic algorithm and a meta-heuristic algorithm are proposed.

3 Methodology

3.1 Mathematical model of hierarchical Chinese postman problem with fuzzy travel times

In HCPP that is a CPP within the NP-hard problem class, the roads/arcs are classified and a precedence relation is defined in these classes [22]. I.e. in this problem type in which order the roads on the graph shall be rendered service is predetermined. In HCPP, the aim is to create the shortest tour/tours via passing at least once from each arc/road in a hierarchical graph where these precedence relations are considered. We can give the vehicle route determination in real-life problems like garbage collection, snow plowing and salting in winter, etc. where the streets are classified according to their precedence degree of the streets (urban or rural, main road, side road, etc.)as examples of HCPP. For example, in salting operations held in winter first of all the salting of the main roads like highways is conducted, then the side roads are started. In that case, the main roads are placed in the first hierarchical class and the side roads, etc. are placed in the second hierarchical class. HCPP classical mathematical model can be used in many fields however it is not possible to state the uncertainty due to the real-life conditions with mathematical models. Thus, to include this uncertainty into mathematical models fuzzy logic is used in our study. When the objective function coefficient values showing the travel time between two nodes in HCPP are taken as fuzzy numbers, the problem transforms into the HCPP-FTT. Whereas V is showing the set of nodes and E the set of unidirectional arcs (roads); a $G(V, E)$ graph can be divided into m different sub-networks. Here each of the sub-networks corresponds to a hierarchical level. In this regard, a hierarchical network consists of arcs (roads) belonging to different precedence levels and it is represented as $G(V, E) = G_1(V_1, E_1) \cup G_2(V_2, E_2) \cup ... \cup G_m(V_m, E_m)$ (see [65]). In these notations, $G_1(V_1, E_1)$ is the sub-networks showing the arcs/roads and nodes to be passed in the 1^{st} precedence and similarly, $G_m(V_m, E_m)$ shows the arcs/roads and nodes to be passed in the m^{th} precedence. Compared to the other types of CPP, the mathematical model of HCPP which had drawn less attention, that applies to real-life problems was developed first time by odur and Yılmaz [15]. However, the said model can only produce optimum solutions for small-scale problems and the solution of the large scale problems, even it does not guarantee the optimal solution, it is required to utilize heuristic or meta-heuristic algorithms that can provide close to optimal solutions in a reasonable time.

The indexes, sets, parameters and decision variables used in the HCPP integer linear programming model are shown below under respective titles (see [15]).

Indexes;

i, j: nodes on network,

t : number of the edge traversal in the route (steps),

h : hierarchical levels.

Sets;

$i, j/V$: Set of vertices $i, j = \{1, 2, ..., n\}$,

h/H :Set of hierarchies $\{1, 2, ..., h\}$,

$t \leq L$: number of steps $L = 2\,\|E\|$,

E_h :Set of edges of h^{th} hierarchy class,
E : Set of all edges in the network, $E = \{E_1 \cup E_2 \cup ... \cup E_h\}$,
$\|E_h\|$:The total number of edges of h^{th} hierarchy class,
$\delta_h(i)$: The edges of E_h set that exist from node i, $\delta_h(i) = \{j | (i,j) \epsilon E_h\}$,
$\delta(i)$: Set of all edges that exist from node $i, \delta(i) = \bigcup_{h \epsilon H} \delta_h(i) = \{\delta_1(i) \cup \delta_2(i) \cup ... \cup \delta_h(i)\}$.

Parameters;
C_{ij}: Distance matrix of the edge (i,j),
B_{ij}: Connection matrix of the edge (i,j),
O_{ij}: Precedence matrix of the edge (i,j),
M: Is a sufficiently large number,
n: The total number of nodes in the graph.

Variables;
$x_{ij}^t = \begin{cases} 1 \\ 0 \end{cases}$ if traversed from node i to node j at t^{th} step1, otherwise 0.
$y_{ij}^t = \begin{cases} 1 \\ 0 \end{cases}$ if traversed from node i to node j at t^{th} step for the first time 1, otherwise 0.
$\phi_{ij} = \begin{cases} 1 \\ 0 \end{cases}$ if traversed from all edges of h^{th} hierarchy in t^{th} step completed 1, otherwise 0.

Using this information, HCPPs mixed-integer linear programming model can be written as follows [15]. In this model, when the c_{ij} jobjective function coefficient values are taken as fuzzy numbers, the problem transforms into an HCPP-FTT.

$$Min\tilde{Z}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{t=1}^{T}\tilde{c_{ij}}\overset{t}{x}_{ij}. \tag{1}$$

Subject to;

$$\sum_{t=1}^{L}\left(\overset{t}{x}_{ij} + \overset{t}{x}_{ji}\right) \geq 1 \qquad \forall (i,j)\,\epsilon E, \tag{2}$$

$$\sum_{i}\overset{t-1}{x}_{ij} = \sum_{s}\overset{t}{x}_{js} \qquad \forall_{j,t} \qquad t > 1 \qquad i,j,s\epsilon V, \tag{3}$$

$$\sum_{(i,j)}\overset{1}{x}_{ij} \geq 1 \qquad (i,j)\,\epsilon\delta_1(i_0), \tag{4}$$

$$\sum_{j}\overset{t}{x}_{ji} \geq 1 \qquad t = t_{end}, i = i_0, \tag{5}$$

$$\sum_{(i,j)\epsilon E}\overset{t}{x}_{ij} = 1 \qquad \forall t, \tag{6}$$

$$\overset{t}{x}_{ij} + \overset{t}{x}_{ji} \geq \overset{t}{y}_{ij} \qquad \forall i,j,t \qquad i < j, \tag{7}$$

$$\sum_{t'<t}\left(\overset{t'}{x}_{ij} + \overset{t'}{x}_{ji}\right) \leq M * \left(1 - \overset{t}{y}_{ij}\right) \qquad \forall i,j,t, \qquad i < j, \tag{8}$$

$$\sum_{t=1}^{L}\overset{t}{y}_{ij} = 1 \qquad \forall (i,j)\,\epsilon E \qquad i < j, \tag{9}$$

$$\sum_{t^1 \leq t}\sum_{(i,j)\epsilon E_h}\overset{t'}{y}_{ij} \geqslant \|E_h\| * \phi_{ht} \qquad \forall h,t, \tag{10}$$

$$\sum_{t^1 \leq t}\sum_{(i,j)\epsilon E_{h+1}}\left(\overset{t'}{x}_{ij} + \overset{t'}{x}_{ji}\right) \leq M * \phi_{ht} \qquad \forall t, h = 1,...H-1, \tag{11}$$

$$\overset{t}{x}_{ij}, \overset{t}{y}_{ij}, \phi_{ht}\epsilon\{0,1\} \qquad \forall i,j,t,h. \tag{12}$$

Equation 1 represents the objective function aiming the minimizing the total tour time. Equation 2 is restriction representing the passage from all road on the graph at least once, Equation 3 the provision of continuity among the nodes, Equation 4 the passage from any one of the roads belonging to the 1^{st} hierarchy that can be traveled from the i_0 start node in the first step ($t = 1$), Equation 5 the return to the start node ($i_0 = 1$) at the last step, Equation 6 the passage from only one road in each step, Equation 7 - Equation 9 the assignment of value to the y_{ij}^{t} variable defining the step on which the (i, j) arc is first passed from. Equation 10 checks whether all the roads belonging to the h hierarchy is passed from in the t^{th} step. Equation 11 shows that without completing the roads in the h^{th} hierarchy, the roads of the $(h+1)^{th}$ hierarchy are not passed from, and Equation 12 shows that the road usage variables must receive integer values 0,1 [15].

3.2 Defuzzification of Fuzzy Numbers

Defuzzification is the transformation transaction performed for the fuzzy information to become crisp results. This transaction is very important as it produces an outcome very sensitive to the method used. Because reducing the fuzzy data to a single value with a defuzzification method conforming to it changes the success of the system significantly [45]. There are many methods in literature used widely for defuzzification. In this part of the study, the defuzzification methods used for the solution of the sample problem and case study are given.

3.2.1 Robust ranking method (RRM)

The Robust Ranking Method (RRM) defined firstly by Yager [63] is a ranking technique with linearity and additivity properties. For a convex fuzzy number $\tilde{a}$, the Robusts Ranking Index is defined by Prabha and Vimala [51];

$$R(\tilde{a}) = \int_0^1 (0.5) \left(a_\alpha^L, a_\alpha^U\right) da. \tag{13}$$

Where $\left(a_\alpha^L, a_\alpha^U\right) = \{(b-a)\alpha + a, c - (c-b)\alpha\}$ which is the α level cut of the fuzzy number $\tilde{a}$.

3.2.2 Centroid ranking method (CRT)

Ganesh and Jayakumar [27], in their study, suggested an approach to defuzzify (order) the trapezoidal fuzzy numbers using the centroid point method. This centers of mass for this method are shown in the Figure 2 below. Centroid

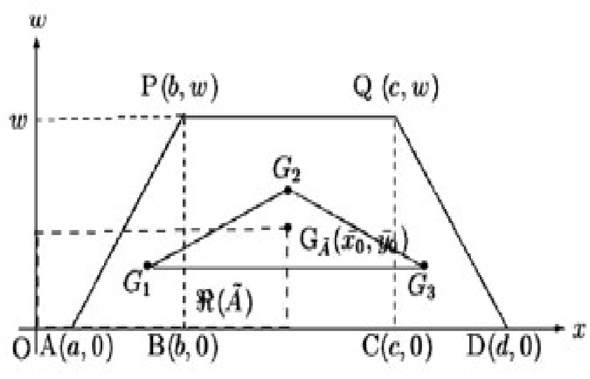

Figure 2: Centroid of a trapezium

of the centroids G_1, G_2 and G_3 is taken as the solution point to define the ranking of generalized trapezoidal fuzzy numbers. When a generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$ is assumed, the centroid of these plane figures are $G_1 = \left(\frac{(a+2b)}{3}, \frac{w}{3}\right), G_2 = \left(\frac{(b+c)}{2}, \frac{w}{2}\right), G_3 = \left(\frac{(2c+d)}{3}, \frac{w}{3}\right)$ respectively. Equation of the line $\overline{G_1G_3}$ can be given as $y = \frac{w}{3}$ and G_2 will not be on the $\overline{G_1G_3}$ line. Thus G_1, G_2 and G_3 are non collinear thus forming a triangle. Ganesh and Jayakumar [27] define the centroid $G_{\tilde{A}}$ $(x_0,\ y_0)$ of the triangle with vertices G_1, G_2 and G_3 for the generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$ as

$$G_{\tilde{A}}(x_0, y_0) = \left(\frac{(2a + 7b + 7c + 2d)}{18}, \frac{7w}{18}\right). \tag{14}$$

Later on, Christi [10] had developed this approach to be used in triangular fuzzy numbers. Accordingly, at a $\tilde{A} = (a, b, d; w)$ triangular fuzzy number (provided that $c = b$), the generalized form of this approach is as follows:

$$G_{\tilde{A}}(x_0, y_0) = \left(\frac{(2a + 7b + d)}{9}, \frac{7w}{18}\right). \tag{15}$$

3.2.3 Kwong-Bai method

Kwong and Bai [41] showed that the triangular fuzzy numbers can be defuzzified using the formula below. As $\tilde{M} = (l, m, u)$ is a triangular fuzzy number, the defuzzification operation is performed as such:

$$M_d = \frac{l + 4m + u}{6}. \tag{16}$$

3.2.4 Best nonfuzzy performance value (BNP)

(l, m, u) parameters, in respective order, are the lower, mid and upper values of the triangular fuzzy number, the defuzzification operation is performed with the following formula using the BNP suggested by Hsieh et al. [32].

$$(BNP)_i = \frac{(u_i - l_i) + (m_i - l_i)}{3} + l_i. \tag{17}$$

3.3 Heuristic algorithm

In the solution of NP-hard problems that cannot be solved with crisp methods, that need intense calculation complexity and solution time, heuristic methods are utilized. In the scope of this study, as the considered real-life problem is on large scale, for reaching an acceptable solution to the problem in a reasonable time, a heuristic algorithm based on a greedy algorithm is developed. The greedy algorithm advances at each step by selecting the best among the alternative options. At each stage, the best solution value found using the local information can be the global optimal solution value. In this study, the logic of the heuristic algorithm developed can be summarized as follows: Following the hierarchies,

Algorithm 1. Pseudo Code of Heuristic Algorithm

```
position: Current position
h: current hierarchy level value
MH : Maximum hierarchy value
route: keeps the entire route that the algorithm shall achieve as a result.
M : Distance matrix
% Assignment of start value
position = 1;
h=1;
        M = callfromfile ();
        for i=1:length(M)
        for j=1:length(M)
                value = M[i,j]
        end
        end
        while true
                unvisitedNodes = unvisitedFind (position,h)
                if ~isempty(unvisitedNodes)
                        [minCost, index]= minCall(unvisitedNodes)
                else
                        optionNodes = CallAllOptions (position,h)
                        [minCost, index]= minCall(optionNodes)
                totalDistance = totalDistance + minCost
                route = routeAdd(index)
                position = index
                if unusedArcControl () == false
                        if h == MH && position == 1
                                break;
                        else if h < MH
                                h = h + 1;
                        end
                end
        end
```

Figure 3: Pseudo code of heuristic algorithm

firstly the list of the unvisited roads is kept for the vehicle getting on the way from the starting node. If there is an element of this list; the vehicle chooses the closest path to itself with minimum time among these roads that have not been visited. If there are no previously non-selected roads, i.e. if all the roads had been used, the road with the shortest time is selected among all the selected roads. In any one of these two stages, a node or an arc shall be selected. Then the selected node is added to the route and the cost value of the selected arc is added to the total cost value and an update is performed. When no unvisited roads are left for any of the hierarchies on the graph, the vehicle shall return to the point where it is present to the starting point. The pseudo-code of the heuristic algorithm developed using the programming language Matlab-2019b is illustrated in Figure 3.

3.4 Meta-Heuristic algorithm

Meta-heuristic algorithms developed driven by natural phenomena are generally preferred for the solution of complex optimization problems with NP-hard structure that cannot be resolved in a reasonable time with precise solution methods as the problem size rises. One of the meta-heuristics commonly applied in literature is for this purpose is ACO. In 1992, prompted by the natural food-finding process of ants, ACO was first developed by Marco Dorigo [21]. The fundamental logic of ACO is briefly as follows. In order to find the shortest route between the nest and food, ants make use of the pheromone substance that provides communication between themselves. The substance that ants release and leave their way to create scent is called Pheromone. On the short routes between food and nest, the amount of scent emitted by this substance is more powerful. Ants seem to prefer the path where this scent is intense, by identifying the pheromone scents generated by other ants to locate food, and this path is deemed as the shortest route. The routes with lesser pheromones are also maintaining a possibility to be chosen by the ants, but the ones with stronger smell density are more likely to be chosen. Selection of paths with less scent avoids the use of the same path by all ants and enables new and shorter paths to be found [53]. In addressing search and optimization problems with artificial systems, these natural feeding activities of real ants have been used. The logic of the meta-heuristic algorithm used in this research and established based on ACO is similar to ACO, yet this algorithm has very distinct features compared to ACO. In this algorithm, the amount of pheromones used in ACO does not matter. Because a path/edge can be passed over more than one time in HCPP. If each edge was passed only once, we could then assign a certain amount of pheromones to each road. Since a route can be used more than once, the pheromone amount does not matter here. For this purpose, "visit data" is used in this algorithm instead of concentrating on the amount of pheromone, indicating how many times a road has passed. In short, since this ACO-based algorithm does not work on the pheromone amount, visit data is used to minimize costs and avoid facing an obstacle with local minimums. Hence, the visit data matrix was created with the logic of attempting to persuade the ant to use the road less since passing from a particular road less would mean that the overall cost would be low as well. Basically, the algorithm consists of 3 main steps such as beginning, selection process of ant and update, and ending. The number of ants in the algorithm was estimated to be 300 in the initial state and the number of iterations was decided to be 1000. At the selection process, the ant can make a selection through three different options which are the routes in the database, the remaining route, or the visit data. The values of visit data, effect, and distance effect parameters used for the ant database path selection process are 4 and 0.3, respectively. The role of the visit data effect as stated earlier theoretically is the opposite of the pheromone effect in ACO. The distance effect is the parameter that defines how the distance value affects the probability of a specific edge is selected. Furthermore, according to the ant's visit results, 4 parameters, alpha, beta, theta, and gamma, are used in the selection process. Of these, alpha is the effect of the final visit data on the selection and its value is 0.7. Beta is the effect of edge length or cost on the selection and its value is 0.3. While the theta is the effect of the global visit data on the selection, and its value is 0.6. Finally, gamma is the effect of the local visit data on the selection and its value is 0.4. The global visit data used in this selection process is the common matrix in which the mean score of visit data of all ants is determined and a certain amount of it has evaporated as well. The local visit data reflects the visit data obtained so far by only the current ant. In other words, the ant takes into account both the number of times it has taken this path previously and how often it has been used on average by other ants when making a selection. The final pheromone amount (FPA) is calculated as follows.

$$FPA = (local \quad pheromone \quad amount^{gamma}) * \left(global \quad pheromone \quad amount^{theta}\right). \tag{18}$$

The local visit data pertaining to the edge the ant visited, the overall distance traveled, and the cost values of the path within the current hierarchy are updated after the selection process of the ant is finished. The node selected is added to the ant's route. At the end of the iteration, after all the ants completed their route, the visit data are collected and updated. The pseudo-code of the meta-heuristic algorithm developed using the programming language Matlab-2019b is illustrated in Figure 4.

4 Application

In this section of the study, in order to show the applicability of the methods and the difference of the route and cost results, firstly a small-size hierarchical network and then a large-scale real-life problem is handled. The results acquired

Algorithm 2. Pseudo Code of Meta-Heuristic Algorithm

```
HN ←Number of hierarchies in the data set. h = 1,2,3,4,5…,HN
NN ← Total number of nodes in the data set
CM ← Cost / distance / time matrix between nodes
HM ← dimensional matrix with the size of DSxDS that shows which hierarchy the roads between nodes have
VD (Visit Data) ← Matrix which has DSxDS dimensions to be allocated to each ant before initiating the route and with initial values as 0.
MVD (Mutual Visit Data) ←Matrix with dimensions of DSxDS which retains the visit data of ants that form a route in an iteration and whose values are reset if
required after the iteration is completed.
GVD (General Visit Data) ← Matrix that has DSxDS dimensions and keeps the mean score of the ants' past visit data.
GHR ← Table with column number of 2* HN, by acknowledging each hierarchy as different routes, the best of these routes their route distances are retained
hierarchically that are received from the ants.
GBR ← Global Best Route (It represents the best route among all results until that moment.)
GBD ← Global Best Distance
k ← An empty ant object-sample
            M = callfromFile();
            for i = 1: DS
             for j = 1: DS
                        value = CM [i,j]
               end
            end
            For iter = 1:totalnumberofiterations
            for antCount = 1:totalNumberofAnts
                        k ← Createant();
                        while true
                                    if makekthchoice()==- 1
                                                break;
                        if kthrouteDistance > globalBestDistance>
                                    GBR = kthRoute
                                    GBD = kthrouteDistance
                        MVD = MVD+ kthVD
                        For j = 1:HS
                                    [hRoute, hCost] = kthhierarchyRoutes (h)
                                    if GHR Findworse (hthRoute) == true && availableControl(hRota)
                                    Change (hRoute, hCost)
            GVD = (GVD*Amountofevaporation + MVD/TotalNumberofAnts)/2
            MVD = 0
Selection function for % ant
NTA(Not Traveled Arc)←It represents the arcs that the ant has not traveled before in the current hierarchy.
alpha ← Coefficient which is determining the effect of GVD and ant's VD on mutual selection for a road.
beta ← Coefficient which is determining the effect of road length on selection
theta ← Coefficient which is determining the effect of the road between 2 nodes in GVD on the selection
gamma ←Coefficient which is determining the effect of the road between 2 nodes and the VD value of the ant on the selection
NTA = FindnotvisitedArc(kthHN);
if NTA == [] && kthHN == maxHierarchy && kthlocation == 1
            return -1;
else if NTA == [] && kthHN ~ = maxHierarchy
            kthHN = kthHN + 1;
else
            NTA = FindnottraveledArc(kthHN);
          if NTA ~ = []
                Options = NTA;
              else
              Options = Findarc(kthHN)
            if (kthImonnewRoute== false) && isempty (kthsingleRoute)
                        for i = 1: length (options)
                                    matchedRoute = min(GHR, [kthRoute + options (i)])
                                    if matchedroute = 0
                                    data (i) = 0;
                             else
                                    data (i) = matchedRoute;
            (data == 0) = 0.0001 * min (data ~ = 0);
                        selectedNode = rouletteSelection(options, data);
                        routes = control (GHR, [kthroute, selectedNode])
                        if length (routes) == 1
                                    kthsingleRoute = routes;
                        else if length (routes) == 0
                              Imonkthnewroute = true;
            elseif isempty (kthsingleRoute) ~ = true
                        s→Deviation Rate
                        selectedNode = kthsingleRoute (1);
                        kthsingleRoute(1) = []
             if generaterandomNumber () <s
                        kthsingleRoute = [];
                        Imonkthnewroute = true;
            Else // Decide Based On Visit Data
                        Options = Findarc(kthHN);
            For i = 1: length (options)
                        M1 = CM[kthLocation, options (i)]
                        X = kthVD[kthlocation, options (i)]
                        Y = GVD [kthlocation, options (i)]
                        if X <= 0 && Y = 0
                        Value (i) = ((1 / M1)^beta);
                        elseif X <= 0 && Y> 0
                                    value (i) = (1 / (Y ^ alpha)) * (1 / (M1 ^ beta));
                        elseif X> 0 && Y> 0
                                    value (i) = ((1/(X ^ gamma * Y^theta))^alpha* (1/(M1 ^beta))
                        else
                                    value(i) = ((1/X) ^alpha) * ((1/D1) ^beta)
            selectedNode = rouletteSelection(value);
kthroute = [kthroute, selectedNode]
kthrouteDistance = kthrouteDistance + CM [kthroute, selectedDugum];
kthVD[kthlocation, selectedNode] + = 1;
return selectedNode;
```

Figure 4: Pseudo code of meta-heuristic algorithm

according to the different ranking techniques are presented in comparison.

4.1 Numerical example on a small-size hierarchical network

In this section, a hierarchical network is used that is developed by Dror et al. [22]. and also used by Ghiani and Importa [30] in their studies. The travel times between the nodes change depending on the variables like weather conditions, traffic intensity, traffic accidents, maintenance, and repair works, so that the objective function coefficients on the sample problem are taken as fuzzy and triangular fuzzy numbers are used. In the problem, the aim is to minimize the total tour time. The network of the problem is given in Figure 5.

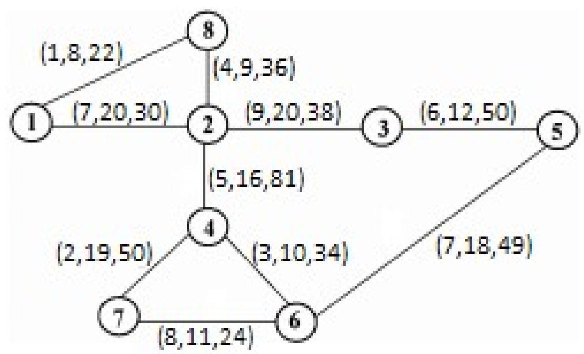

Figure 5: Hierarchical network

In the given unidirectional network, $V = \{1;2;3;4;5;6;7;8\}$ shows the set of nodes and $E = \{E_1 \cup E_2 \cup E_3\}$ shows the set of arcs. $E_1 = \{(1;2);(2;3);(2;4)\}$, $E_2 = \{(3;5);(5;6);(4;6);(4;7);(6;7)\}$, $E_3 = \{(2;8);(1;8)\}$.

Here E_1 indicates the roads to be passed with primary precedence, E_2 indicates the ones with secondary precedence and E_3 indicates the ones with tertiary precedence. In this 8 nodes network, the time passing while going from one node to another is shown in triangular fuzzy numbers in minutes. For example, while under normal traffic conditions the travel time is 20 minutes from point 1 to point 2, in times when the traffic intensity is less (like night time) this time is 7 minutes and when the traffic intensity is high (rush hours, weather conditions etc.) it becomes 30 minutes. The fuzzy cost matrix and the objective function of the problem are given below. The Equation 2 - Equation 12 constraints presented in Section 3.1 are used.

$$\left[\tilde{C}_{ij}\right]_{8x8} = \begin{bmatrix} - & (7,20,30) & - & - & - & - & - & (1,8,22) \\ (7,20,30) & - & (9,20,38) & (5,16,81) & - & - & - & (4,9,36) \\ - & (9,20,38) & - & - & (6,12,50) & - & - & - \\ - & (5,16,81) & - & - & - & (3,10,34) & (2,19,50) & - \\ - & - & (6,12,50) & - & - & (7,18,49) & - & - \\ - & - & - & (3,10,34) & (7,18,49) & - & (8,11,24) & - \\ - & - & - & (2,19,50) & - & (8,11,24) & - & - \\ (1,8,22) & (4,9,36) & - & - & - & - & - & - \end{bmatrix}, \quad (19)$$

$$Min\tilde{Z}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{t=1}^{T}\tilde{c_{ij}}x_{ij}^{t} = R\left(7,20,30\right)x_{121} + R\left(1,8,22\right)x_{181} + ... + R\left(4,9,36\right)x_{82T}. \quad (20)$$

The problem is first solved according to the values of the original times in the cost matrix (when the traffic conditions are assumed normal) and the optimal travel time is found as 185 and the optimal tour route as 1-2-4-2-3-5-6-4-6-7-4-2-8-1. Later on, in order to transform the triangular fuzzy time values on the cost matrix into crisp values, in order, the RRM, the CRT, the Kwong-Bai Method, and the BNP methods are used. Using the net values acquired via defuzzification of the fuzzy time matrixes, a new matrix is formed and on Intel Core i7 2600 CPU 8 GB RAM computer, it is solved using GAMS 24.2.3 version. According to the new cost matrix formed via using these ranking methods, the problem is solved on the GAMS Program and the optimal route and solutions are found as indicated below. When the triangular fuzzy numbers are defuzzified by the RRM, the cost values in Table 1 are acquired. When the problem is solved according to the net cost values acquired via using the RRM, the optimal tour time is found as 218.75 and the optimal tour route as

Table 1: Defuzzifying by Yagers ranking technique

Nodes	1	2	3	4	5	6	7	8
1	-	19.25	-	-	-	-	-	9.75
2	19.25	-	12.75	29.5	-	-	-	14.5
3	-	12.75	-	-	20	-	-	-
4	-	29.5	-	-	-	14.25	22.5	-
5	-	-	20	-	-	23	-	-
6	-	-	-	14.25	23	-	13.5	-
7	-	-	-	22.5	-	13.5	-	-
8	9.75	14.5	-	-	-	-	-	-

Table 2: Defuzzifying by CRT

Nodes	1	2	3	4	5	6	7	8
1	-	7.95	-	-	-	-	-	3.46
2	7.95	-	8.47	8.77	-	-	-	4.62
3	-	8.47	-	-	6.31	-	-	-
4	-	8.77	-	-	-	4.75	8.08	-
5	-	-	6.31	-	-	8.17	-	-
6	-	-	-	4.75	8.17	-	5.05	-
7	-	-	-	8.08	-	5.05	-	-
8	3.46	4.62	-	-	-	-	-	-

1-2-3-2-4-6-4-7-6-5-3-2-8-1. When the triangular fuzzy numbers are defuzzified by the CRT, the cost values in Table 2 are acquired. When the problem is solved according to the net cost values acquired via using the CRT, the optimal tour time is found as 87.32 and the optimal tour route as 1-2-3-2-4-6-7-4-6-5-3-2-8-1. When the triangular fuzzy numbers are defuzzified by the Kwong-Bai Method, the cost values in Table 3 are acquired. When the problem is solved according to the net cost values acquired via using the Kwong-Bai Method, the optimal tour time is found as 228.170, and the optimal tour route as 1-2-3-2-4-6-4-7-6-5-3-2-8-1. When the triangular fuzzy numbers are defuzzified by the BNP, the cost values in Table 4 are acquired. When the problem is solved according to the net cost values acquired via using the BNP Method, the optimal tour time is found as 263.330 and the optimal tour route as 1-2-3-2-4-7-6-4-6-5-3-2-8-1. The values acquired according to the different defuzzification methods are summarized on the comparison in Table 5.

As indicated before, when the problem is solved according to the original values on the cost matrix the optimal travel time was found as 185 and the optimal tour route as 1-2-4-2-3-5-6-4-6-7-4-2-8-1. As seen from the results, when the cost matrix is taken as fuzzy, depending on the defuzzification technique used, the optimal tour time and route change. We see that the cost (travel time) can be minimized by the application of CRT. Also from the results, it can be seen that all the solution times for every method are under 1 minute.

4.2 Case study

In this real-life problem, it is aimed to find the best route or routes to realize the maintenance works of the roads under the task field of the Turkish Highways 12^{th} Regional Directorate with minimum travel time. In such real-life problems, the travel times between the centers, depending on the traffic intensity of the different times within the day, and thus the solution giving the lowest cost is not fixed. Therefore, in this study, considering the traffic intensity, the day is divided into 4 different time frames. Thus, the objective function coefficients representing the travel times between the nodes are taken as trapezoidal fuzzy numbers.

This network considered for determining the shortest distance route on the unidirectional roads by Yılmaz [65] previously, when considering a highway network, can be transformed into a general graph $G = (V, E)$ consisting

Table 3: Defuzzifying by Kwong-Bai method

Nodes	1	2	3	4	5	6	7	8
1	-	19.5	-	-	-	-	-	9.17
2	19.5	-	21.17	25	-	-	-	12.67
3	-	21.17	-	-	17.33	-	-	-
4	-	25	-	-	-	12.83	21.33	-
5	-	-	17.33	-	-	21.33	-	-
6	-	-	-	12.83	21.33	-	12.67	-
7	-	-	-	21.33	-	12.67	-	-
8	9.17	12.67	-	-	-	-	-	-

Table 4: Defuzzifying by BNP

Nodes	1	2	3	4	5	6	7	8
1	-	19	-	-	-	-	-	10.33
2	19	-	22.33	34	-	-	-	16.33
3	-	22.33	-	-	22.67	-	-	-
4	-	34	-	-	-	15.67	23.67	-
5	-	-	22.67	-	-	24.67	-	-
6	-	-	-	15.67	24.67	-	14.33	-
7	-	-	-	23.67	-	14.33	-	-
8	10.33	16.33	-	-	-	-	-	-

Table 5: Comparison of ranking techniques

Methods	Optimal Tour Route	Optimal Tour Time	T^{Cplex} (sn)
RRM	1-2-3-2-4-6-4-7-6-5-3-2-8-1	218.75	28.061
CRT	1-2-3-2-4-6-7-4-6-5-3-2-8-1	87.32	30.563
Kwong-Bai Method	1-2-3-2-4-6-4-7-6-5-3-2-8-1	228.170	10.142
BNP	1-2-3-2-4-7-6-4-6-5-3-2-8-1	263.330	13.575

of main roads, side roads, and intersections. $G = (V, E)$ graph is divided into two sub-networks $G = (V, E) = G^1 \left(V, E^1\right)) \cup G^2 \left(V, E^2\right)$. The state roads, provincial roads, and some district roads where the traffic is intense in general are determined as primary precedence and other roads as secondary precedence [65].

Under the scope of this study, whereas, as it is aimed that the shortest route for a maintenance vehicle traveling along with all the graph considering the precedence relations of the roads at different time frames of the day, the problem is solved as an HCPP-FTT. The graph view of the problem is given in Figure 6. In this network consisting of 75 nodes

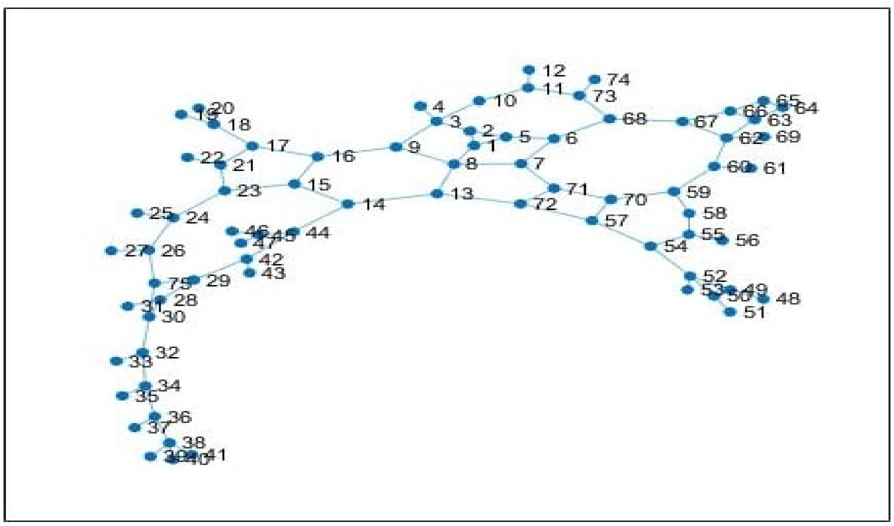

Figure 6: Highways 12^{th} regional directorate task field roads network view

and 90 connections, the first roads to be visited by the maintenance vehicle are defined in the (E_1) first hierarchical class and other roads in (E_2) the second hierarchical class. In the graph, times are used in minutes on the objective function. The fuzzy objective function of the problem is given below and the Equation 2 - Equation 12 constraints given in Section 3.1 are used. (As node count is large, only the start and end values are given in the objective function):

$$Min\tilde{Z} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{t=1}^{T} \tilde{c_{ij}} x_{ij}^{t} = R\left(9, 11, 15, 19\right) x_{121} + R\left(11, 13, 16, 21\right) x_{151} + ... + R\left(19, 25, 32, 35\right) x_{7530T}. \quad (21)$$

The trapezoidal fuzzy numbers here mean the following: For example, while it takes 9 minutes to travel from point 1 to point 2 in the time frame where the traffic intensity is minimum, in the time frame where the traffic intensity is at maximum this time becomes 19 minutes. The interim values (11 and 15 minutes) show the travel times in other time frames based again on the intensity. The fuzzy time matrix in size of 75x75 is defuzzified using the RRM and CRT.

Table 6: Heuristic and meta-heuristic approaches solution results

Algorithms	Methods	Optimal Tour Time($MinZ$)(min)	Computer solution time $T^{SA}(sec)$
Heuristic	**RRM**	198872.5	0.994061
	CRT	38532.2	0.352818
Meta-Heuristic	**RRM**	3374.1	2663.45
	CRT	1541	14496.66

The net values acquired as the result of these calculations are used to form new cost matrixes and objective functions. When the trapezoidal fuzzy numbers are defuzzified using the RRM, the objective function given below is acquired (As node count is large, only the start and end values are). The Equation 2 - Equation 12 constraints given in the Section 3.1 are used.

$$MinZ \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{t=1}^{T} c_{ij}x_{ij}^{t} = 13.5x_{121} + 15.3x_{151} + ... + 27.8x_{7530T}. \tag{22}$$

Defuzzification operation is conducted for all the roads and as an example, the defuzzification operation for road 1-2 in step 1 is given below.

$$R\left(9,11,15,19\right)x_{ij}^{t} = R\left(\tilde{C}_{121}\right) = \int_{0}^{1}(0.5)\left\{(11-9)\,\alpha + 9, 19 - (19-15)\,\alpha\right\}da = 13.5. \tag{23}$$

When the trapezoidal fuzzy numbers are defuzzified using the CRT, the objective function given below is acquired (As node count is large, only the start and end values are). And again the Equation 2 - Equation 12 constraints given in Section 3.1 are used.

$$MinZ \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{t=1}^{T} c_{ij}x_{ij}^{t} = 5.1x_{121} + 5.8x_{151} + ... + 11x_{7530T}. \tag{24}$$

Defuzzification operation is conducted for all the roads and as an example, the defuzzification operation for road 1-2 in step 1 is given below.

$$\tilde{C} = (9,11,15,19)\,x_{ij}^{t} = G_{\tilde{C}}(x_0, y_0) = \left(\frac{(2a+7b+7c+2d)}{18}, \frac{7w}{18}\right) = 5.1. \tag{25}$$

The handled Np-hard class problem is a large-scale one thus an optimum solution could not be reached in a reasonable time in the mathematical model. Therefore, utilizing the heuristic and metaheuristic methods developed in the study is provided for achieving results faster with acceptable solutions. The proposed algorithms aim to minimize the overall travel time of the maintenance vehicles used for road maintenance. For the solution of the problem Matlab-2019b program, a computer with Intel Core i9 9900K processor, 32 GB ram is used, and the results are achieved. By running the algorithms, the best vehicle routes achieved according to both ranking techniques are found. As in the small sample problem handled, also in this real-life problem, different routes are acquired as the result of two methods. However, due to the properties of HCPP, in both techniques, the first starting point of the vehicle is node number 1 and the vehicle returns to the starting point after completing the tour, and the precedence relations are provided for. The minimum total travel times (minutes) and computer solution times TSA (seconds) that are obtained using heuristic and meta-heuristic algorithms are given in Table 6. Due to the large scale of the problem, not the entire routes are added to the table. As can be seen from the results, the cost matrix, when considered as fuzzy, the optimum tour time and route change significantly according to the defuzzification technique used. As similar to the result of the small-scale problem, here also we see that the cost (travel time) can be minimized by applying CRT. Again, which can be seen in Table 6, in terms of the objective function, the meta-heuristic algorithm provides better results than the heuristic and is more effective. Based on the findings obtained using the heuristic algorithm for the RRM method, the objective function value is 58.9 times the value obtained using the meta-heuristic algorithm. Based on the findings obtained using the heuristic algorithm for the CRT method, the objective function value is 25 times the value obtained using the meta-heuristic algorithm. Since metaheuristics carry out an intense and in-depth search in the solution space, higher quality outcomes are expected to take place. While the meta-heuristic algorithm gives better results in a given network, when we look at solution times, heuristic algorithms are better than metaheuristic. It can be seen that the computer solution times for the heuristic are less than 1 minute. For the RRM method, the computer solution time is 2679.36 times the value obtained using the meta-heuristic algorithm, according to the results obtained using the heuristic algorithm. For the CRT method, the computer solution time is 41088.2 times the value obtained using the meta-heuristic algorithm, according to the results obtained using the heuristic algorithm.Utilizing the fuzzy travel times in the objective function, it is aimed to prevent wastage and inconsistencies that may be caused by incorrect planning. With the routing studies performed using two different defuzzification techniques, the unnecessary repetitions of the vehicle on the roads are avoided. Also, by considering the hierarchy of the roads, it is provided that the maintenance operations at the areas where the traffic is intense, that are used more in real life are performed before.

5 Conclusion

In this study, the Hierarchical Chinese Postman Problem With Fuzzy Travel Times (HCPP-FTT), a new type of HCPP that is one of the arc routing problems and aims to minimize the total travel time is handled. In real life, when it is considered that there are passage roads based on precedence for many activities like snow plowing, salting, and garbage collection and when the money and time wastage due to the wrong planning in routing problems is taken into account, it can be said that the problem types on this field carry great importance. When the related literature is examined, it is seen that in many of the studies on the field the distances between the nodes are considered and the travel times from one node to another are ignored. However, in real life, in the inner-city transportation network, the travel times between two centers vary based on various factors like weather conditions, traffic accidents, travel start times, construction and repair works. Due to this uncertainty, in this study, the travel times are represented with fuzzy numbers. In the study, different defuzzification methods present in the literature are used and problems are solved according to the acquired net values. The optimal time and routes are detected to be varying according to the defuzzification methods. When the small-size sample problem results acquired with the help of the GAMS package program are examined it is seen that the optimal time is within the range of 87.32-263.330. Also, in this study determination of the shortest travel time route of a maintenance vehicle used on the roads under the task area of Turkish Highways 12^{th} Regional Directorate is handled. It is considerably hard to represent the travel times between centers with net values based on uncertainties like traffic intensity etc. within different time frames of the day. Therefore in this case study, the day is divided into four-time frames and the travel times are represented by fuzzy numbers based on this. Utilizing Matlab-2019b programming language for the solution of the large-sized NP-hard class problem in question, a heuristic and ACO-based algorithm based on the greedy algorithm is proposed. The algorithms are run for the entire highway network in hand and an efficient result is acquired. When the achieved results are examined, it can be seen that the optimum tour time and the route change considerably. In both problem times, it is seen that the shortest travel time is achieved by the CRT. When comparing the results obtained from algorithms, meta-heuristic as an objective function is proven to provide better results than heuristic. However, it is found that the heuristic reaches fair outcomes in a shorter time than the meta-heuristic when computer solution times are compared. These findings are due to the fact that the heuristic only searches locally, whereas the entire solution space is searched by the meta-heuristic. It is aimed that this is the first study in the literature on the newly developed problem type (HCPP-FTT), the heuristic and meta-heuristic proposed for its solution, and to spread its applicability in real life.

In future studies, the objective function coefficients can be taken as different fuzzy number types and different techniques can be used to defuzzify these numbers and the problem can be resolved as such. Also, the different fuzzy number types can be defuzzified by the method used in this study and the results can be compared with the results of this study. Besides, different types of CPP can be fuzzified and be applied to large scale real-life problems. Also, different heuristic/meta-heuristic algorithm regarding large scale case studies can be developed and compared.

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