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Original paper

Multi-criteria decision making based on q-rung orthopair fuzzy promethee approach

M. Akram[1] and Shumaiza[2]

[1,2] *Department of Mathematics, University of the Punjab, New Campus, Lahore 4590, Pakistan*

m.akram@pucit.edu.pk, shumaiza00@gmail.com

Abstract

The preference ranking organization method for enrichment of evaluations (PROMETHEE) constitutes a family of outranking multiple-attribute decision-making (MADM) methods that has been adopted by researchers from many areas during the last years. It provides reliable and clear results thanks to the advantages of different types of preference functions. In this paper, we incorporate the benefits of q-rung orthopair fuzzy set (for short, q-ROFS) in this strategy of solution. This model, q-ROFS, is a generalized form of Pythagorean fuzzy set (PFS), as it broadens the space of acceptable orthopairs and has an ability to deal with more elaborate and vague information. The technique of our extension of the PROMETHEE method uses q-rung orthopair fuzzy numbers to render the ratings of alternatives, which allows us to express uncertain and vague information more accurately. The usual criterion preference function has been used to measure the preferences of the alternatives. A partial ordering of alternatives is obtained by considering the outgoing and incoming flows of alternatives, which is known as PROMETHEE I. Furthermore, a complete ordering is accomplished by taking into account the procedure of PROMETHEE II. As a numerical exercise, we consider the selection of a contractor for a construction project. A full analysis is performed to illustrate the application of the technique that stems from our approach. Then we compare the results that we obtain with the results from existing approaches, including q-rung orthopair fuzzy ELECTRE, q-rung orthopair fuzzy TOPSIS, q-rung orthopair fuzzy VIKOR and q-rung orthopair fuzzy aggregation operators. In this way the accuracy and effectiveness of the presented work is conclusively validated .

Keywords: q-rung orthopair fuzzy numbers, PROMETHEE technique, preference functions.

1 Introduction

Decision-making is a process of classifying a set of feasible alternatives or actions, possibly based on different attributes, and depending upon the opinions of one or more decision experts. The source of data , the structure of the alternatives, the form of the 'preferences' or 'ratings', and the goals of the decisions are the most important factors in these problems. Natural decision-making is based on a single criterion, and the set of alternatives is evaluated or compared by this criterion alone [8]. Anyhow, most of the practical problems are characterized and analyzed on the basis of information that stems from multiple sources, have one definite goal, and demand the selection of one final alternative. Motivated by this practical target, in this paper we are concerned with multiple-attribute decision-making (MADM) in which the decision is made on the basis of conflicting criteria. Furthermore, we should get more authentic and accurate results when the decision problem uses data provided by a group of decision experts, a procedure that is therefore called multi-criteria group decision-making (MCGDM).

A variety of MCDM techniques have been proposed by many researchers in order to assess the complex decision problems in various domains including business management [30], operational research [2] and many more. In a broad sense, the fundamental MADM techniques can be divided into two major classes of methodologies, namely, compensatory and outranking decision-making. The former instance includes the analytical hierarchy Process (AHP) [28] and the

Corresponding Author: M. Akram

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technique for the order of preference by similarity to an ideal solution (TOPSIS) [17], among others. Whereas, the preference ranking organization method for enrichment of evaluations (PROMETHEE) [10] and the elimination and choice translating reality (ELECTRE) [7] are the most used classes of outranking MCDM techniques. The outranking approach is based on the pairwise comparison of alternatives such that the least favorable alternatives are eliminated. Sometimes outranking methods are used to obtain the kernel solution of the decision problem, instead of a ranking of the alternatives or an optimal solution.

Improving upon Zadeh's fuzzy set theory [35], Atanassov [5] introduced the concept of intuitionistic fuzzy set (IFS). IFSs first allowed the researcher to deal with the uncertainties of human thinking by means of both membership and non-membership degrees. This simple recourse allowed the decision experts to describe their attitudes towards the actions under consideration in a less restrictive manner. But despite these advantages, Atanassov's IFS still limit the assessments of the experts as the sum of membership and non-membership grades should not exceed one. For instance, when a decision expert independently concludes that the membership grade should be $\sqrt{3}/2$ and the non-membership grade should be $1/2$ then $\sqrt{3}/2 + 1/2 > 1$, which places his opinions outside the scope of Attanasov's theory.

For this reason, Pythagorean fuzzy sets (PFS) were proposed by Yager [33]. The condition that they must meet is that the quadratic sum of each membership and non-membership grades should not be greater than one. For example, the situation explained above satisfies $(\sqrt{3}/2)^2 + (1/2)^2 \leq 1$ thus it belongs to the PFS framework. It is therefore clear that PFSs provide more freedom to decision experts when it comes to allocate the preferences of alternatives with respect to different attributes. Since PFSs were introduced, a number of researchers have conducted in-depth studies [4, 3].

Despite the fact that IFSs and PFSs have been broadly studied, there are still many complex and complicated decision problems where these sets are too limited to give formal support to the evidences about the alternatives. Sometimes the quadratic sum of membership and non-membership grades is exceeds one, but the sum of their cubic or higher powers is lesser than one. This shortcoming of both IFSs and PFSs prompted Yager [34] to introduce the concept of q-rung orthopair fuzzy set (q-ROFS) which impose the weaker condition that the summation of the qth powers of all membership and non-membership grades should not exceed one. For $q = 1$ and $q = 2$, q-ROFSs become IFS and PFS, respectively, so that q-ROFSs generalize both IFS and PFS. It is worth noting that as the parameter q increases, the space of acceptable orthopairs increases, and more orthopairs satisfy the bounding constraint. So we can admit a sufficiently larger range of uncertain information by means of q-ROFSs. In other words, for a given set of evaluations, we can adjust the value of the parameter q in order to fix an adequate range for the expression of the data that encompasses all our evaluations, which definitely makes q-ROFS a completely flexible and adaptable tool for the uncertain environment. Thus the parameter or rung q is an eminent characteristic of the q-ROF model. It comes as no surprise that an extensive study and research has been made by many researchers and practitioners, on the basis of q-rung orthopair fuzzy information. The aggregation operators based on q-ROF information were presented by Liu and Wang [24], and some properties of q-ROF functions were analyzed in [13, 26]. In fact many aggregation operators based on classical operators have been adapted to act on q-ROFS. For example, the power aggregation operators based on q-ROF information have been developed by Ju et al. [18]. Liu et al. [23] proposed the power Maclaurin symmetric mean operators based on generalized orthopair fuzzy information. Further, the Muirhead mean operators under the dual hesitant q-ROF data have been established and applied for making decisions on the basis of multiple criteria [11]. An array of operators including Hamy mean operators [31], Archimedean Muirhead mean operators [15], Heronian mean operators [25], and Choquet integral operators [32], have also been developed for q-ROF information. Furthermore, q-ROF information has been combined with existing techniques to develop multi-criteria decision-making techniques such as TOPSIS and ELECTRE [27]. Krishankumar et al. [21] used q-ROF information for solving the green supplier selection problem with unknown weights.

As said above, the last several decades have witnessed the development of a variety of MADM techniques. Initially, these methodologies were designed to deal with real and exact data. When it became apparent that these crisp methodologies were not able to handle the imprecise and vague situations of real-world problems, some of them were redesigned for the fuzzy environment like the pioneering contribution by Bellman and Zadeh [6]. Since then a number of decision-making methods have been adapted to the requirements of the fuzzy theory and its variants or extensions. Sanayei et al. [29] introduced the fuzzy version of VIKOR technique for the evaluation of suppliers. Chiou et al. [12] presented a comparative study of electronic industry in China by using the fuzzy AHP method. Kannan et al. [20] used a fuzzy TOPSIS technique for the evaluation of suppliers in green environment.

PROMETHEE is considered the most valuable and useful outranking technique of multi-criteria decision analysis. Its different versions provide the kernel set as solution, as well as the ordering of feasible alternatives, on the basis of conflicting and multiple attributes. Its crisp form originated with Brans and Vincle [9] and produces a partial and complete raking of alternatives. Abdullah et al. [1] utilized the methodology of PROMETHEE approach for the selection of suppliers and presented a comparative analysis based on different preference functions. Goumas and Lygerou [16] introduced the fuzzy PROMETHEE technique for the ranking of energy exploitation projects. Krishankumar et al. [22] extended the original PROMETHEE method by applying the intuitionistic fuzzy information. Feng et al. [14] considered new decision-making methods with applications. All the existing versions of PROMETHEE are appropriate to evaluate either crisp or fuzzy data, and even to solve problems from intuitionistic and Pythagorean fuzzy environments, but they are unable to use data in the form of q-ROF information. The main goal of this paper is to overcome this limitation that hinders the applicability of the PROMETHEE technique. To attain this final target we emphasize the following achievements:

1. We develop an effective and comprehensive multi-attribute decision-making approach based on the q-rung orthopair fuzzy weighted averaging (q-ROFWA) operator.

2. The uncertain and ambiguous ratings assigned by the experts are allowed to adopt the form of linguistic terms which are further parameterized by q-ROF information.

3. The version of the PROMETHEE method that we present is not restricted to maximized and benefit type attributes. It can also be applied to decision problems having minimized and cost type attributes, if required.

In our variant of the PROMETHEE approach, all attributes are examined and investigated with the help of the usual criterion preference function. The partial ranking or the solution set of PROMETHEE I is obtained by considering the positive and negative outranking flows of alternatives. In the case of PROMETHEE II, a complete ranking is achieved by taking into account the net outranking flow of alternatives. Then a comparative analysis of the performance of the proposed technique with respect to existing approaches, including q-rung orthopair fuzzy ELECTRE, q-rung orthopair fuzzy TOPSIS, q-rung orthopair fuzzy VIKOR and q-rung orthopair fuzzy aggregation operators, are also presented to validate the accuracy and effectiveness of our proposal.

The main contributions of this research article can be summarized as follows:

1. An extended version of the PROMETHEE technique is presented to solve the decision-making problems having ambiguities in the form of q-rung orthopair fuzzy information. Existing versions of the PROMETHEE technique are currently unable to compute a solution under such kind of imprecision.

2. An aggregation operator called the q-ROFWA operator is used to combine the preferences on the alternatives given by the group of decision experts.

3. As an application, a partial and a complete ranking of contractors for a construction project is obtained. We do this by applying the procedures of PROMETHEE I and PROMETHEE II, respectively.

4. Comparative results show the authenticity and validity of our novel methodology.

The structure of the paper is organized as follows. Section 2 contains some basic definitions related to q-ROF sets. Section 3 contains the methodology of the q-ROF PROMETHEE technique. In Section 4, a numerical example is presented. It consists of the selection of a suitable contractor for a construction project. Section 5 provides the comparative analysis of net results of the PROMETHEE technique with other existing MCDM methods. Section 6 concludes the paper with some discussion.

2 Preliminaries

In this section, a review of basic concepts related to q-rung orthopair fuzzy sets is presented which are related to our developed model.

Definition 2.1. [34] For the universe of discourse $\mathcal{X}$, the q-rung orthopair fuzzy set (q-ROFS) $\mathcal{Q}$ is defined as:

$$\mathcal{Q} = \{\langle x, \mu_{\mathcal{Q}}(x), \nu_{\mathcal{Q}}(x)\rangle \mid x \in \mathcal{X}\}, \tag{2.1}$$

where, $\mu_{\mathcal{Q}}(x), \nu_{\mathcal{Q}}(x) \in [0,1]$ represent the membership and non-membership degrees, respectively, with the condition given below:

$$0 \leq \big(\mu_{\mathcal{Q}}(x)\big)^q + \big(\nu_{\mathcal{Q}}(x)\big)^q \leq 1. \tag{2.2}$$

The hesitation degree of of any member $x \in \mathcal{X}$ can be defined as,

$$\pi_{\mathcal{Q}}(x) = \Big(1 - \big(\mu_{\mathcal{Q}}(x)\big)^q - \big(\nu_{\mathcal{Q}}(x)\big)^q\Big)^{1/q}. \tag{2.3}$$

We see that for $q=1$, the q-ROFS becomes the intuitionistic fuzzy set (IFS) and for $q=2$, it becomes the Pythagorean fuzzy set (PFS).

Definition 2.2. [34] For a q-rung orthopair fuzzy number $\mathcal{Q}=\Big(\mu(x),\nu(x)\Big)$, the score function $s(\mathcal{Q})$ and the accuracy function $h(\mathcal{Q})$ are respectively defined as follows:

$$s(\mathcal{Q}) = \Big(1+\mu(x)^q-\nu(x)^q\Big)\Big/2, \tag{2.4}$$

$$h(\mathcal{Q}) = \mu(x)^q+\nu(x)^q. \tag{2.5}$$

Definition 2.3. [24] For the collection of q-ROFNs, $\alpha_k=\big\langle\mu_k(x),\nu_k(x)\big\rangle$, $(k=1,2,3,\cdots,n)$, the q-rung orthopair fuzzy weighted averaging operator (q-ROFWA) is characterized as follows:

$$q-ROFWA(\alpha_1,\alpha_2,\cdots,\alpha_n)=\left\langle\left(1-\prod_{k=1}^{n}\big(1-\mu_k(x)^q\big)^{\lambda_k}\right)^{1/q},\prod_{k=1}^{n}\nu_k(x)^{\lambda_k}\right\rangle. \tag{2.6}$$

The weight vector $\lambda=(\lambda_1,\lambda_2,\cdots,\lambda_n)^T$ satisfy the condition of normality, i.e., $\lambda_k\in[0,1]$ and $\sum\limits_{k=1}^{n}\lambda_k=1$.

Definition 2.4. [24] For the collection of q-ROFNs, $\alpha_k=\big\langle\mu_k(x),\nu_k(x)\big\rangle$, $(k=1,2,3,\cdots,n)$, the q-rung orthopair fuzzy weighted geometric operator (q-ROFWG) is defined as follows:

$$q-ROFWG(\alpha_1,\alpha_2,\cdots,\alpha_n)=\left\langle\prod_{k=1}^{n}\mu_k(x)^{\lambda_k},\left(1-\prod_{k=1}^{n}\big(1-\nu_k(x)^q\big)^{\lambda_k}\right)^{1/q}\right\rangle. \tag{2.7}$$

The weight vector $\lambda=(\lambda_1,\lambda_2,\cdots,\lambda_n)^T$ satisfy the condition of normality, i.e., $\lambda_k\in[0,1]$ and $\sum\limits_{k=1}^{n}\lambda_k=1$.

The set notations used in this research article are summarized in Table 1.

Table 1: Set of notations and their description

Notations	Description
a_i	ith alternative
c_j	jth criterion
e_k	Decision expert k
Ω_j	Weight vector
$t_{ij}^k=\Big\langle\mu_{ij}^k(x),\nu_{ij}^k(x)\Big\rangle$	q-ROFN assigned to alternative a_i with respect to criterion c_j by decision expert e_k
$\hat{s}(t_{ij})$	Score of q-ROFN
$D_j(a_i,a_l)$	Deviation between alternatives a_i and a_l
$P_j(a_i,a_l)$	Preference function of jth criteria
$\Psi(a_i,a_l)$	Multi-criteria preference index
$\Gamma^+(a_i)$	Positive outranking flow of alternative a_i
$\Gamma^-(a_i)$	Negative outranking flow of alternative a_i
$\Gamma(a_i)$	Net outranking flow of alternative a_i

3 q-Rung orthopair fuzzy PROMETHEE method

Consider a MADM problem in which a collection of ϕ alternatives $a_i;\ i=1,2,3,\cdots,\phi$ is examined with respect to ψ criteria $c_j;\ j=1,2,3,\cdots,\psi$. These alternatives are evaluated by a group of η decision experts $e_k;\ k=1,2,3,\cdots,\eta$ with respect to each criterion. The performance ratings of alternatives a_i with respect to criterion c_j are employed to construct a decision matrix. The procedure of q-rung orthopair fuzzy PROMETHEE method is described in the following steps:

Step 1. Identification of linguistic terms
The linguistic terms are determined for assigning the ratings to the alternatives by each decision expert on the basis of different criteria. The selection of appropriate and relevant linguistic terms is most important factor for the next whole procedure. In this approach, a set of nine linguistic terms are used for the evaluation of considered actions or alternatives. The numeric values of these linguistic terms are given in the form of q-rung orthopair fuzzy numbers. These linguistic terms and their respective values are listed in Table 2.

Table 2: Linguistic terms for alternatives

Linguistic terms	Abbreviations	q-rung orthopair fuzzy numbers
Extremely High	EH	$(0.95, 0.15)$
Very High	VH	$(0.85, 0.25)$
High	H	$(0.75, 0.35)$
Medium High	MH	$(0.65, 0.45)$
Medium	M	$(0.55, 0.55)$
Medium Low	ML	$(0.45, 0.65)$
Low	L	$(0.35, 0.75)$
Very Low	VL	$(0.25, 0.85)$
Extremely Low	EL	$(0.15, 0.95)$

Step 2. Construction of decision matrix
Then, these linguistic terms are converted to the numeric values that are in the form of q-rung orthopair fuzzy numbers. Since, the evaluation of alternatives a_i based on each criterion c_j is made by decision experts e_k; $k = 1, 2, 3, \cdots, \eta$. Therefore, η number of decision matrices are established for each expert in the following way:

$$\mathcal{T} = [t_{ij}^k]_{\phi\times\psi} = \begin{bmatrix} t_{11}^k & t_{12}^k & \cdots & t_{1\psi}^k \\ t_{21}^k & t_{22}^k & \cdots & t_{2\psi}^k \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ t_{\phi 1}^k & t_{\phi 2}^k & ... & t_{\phi\psi}^k \end{bmatrix},$$

where, each entry $t_{ij}^k = \left\langle \mu_{ij}^k(x), \nu_{ij}^k(x) \right\rangle$; $i = 1, 2, 3, \cdots, \phi$, $j = 1, 2, 3, \cdots, \psi$, $k = 1, 2, 3, \cdots, \eta$, represents a q-rung orthopair fuzzy number.

Step 3. Determination of weighted aggregated decision matrix
The choice of criteria weights is one of the most important factors for making decision. In this Step, a weight vector Ω_j, such that $\Omega_j = [\ \Omega_1\ \Omega_2\ \Omega_3\ \cdots\ \Omega_\psi\ ]^T$, is assigned to the criteria by a field expert. Then the ratings of alternatives given by decision experts and the weights of criteria are utilized to construct a weighted aggregated decision matrix. For this purpose, the q-rung orthopair fuzzy weighted averaging (q-ROFWA) operator is applied as follows:

$$q - ROFWA(t_{ij}^k) = \left\langle \left(1 - \prod_{k=1}^{\eta} \left(1 - \mu_{ij}^k(x)^q\right)^{\Omega_j}\right)^{1/q}, \prod_{k=1}^{\eta} \nu_{ij}^k(x)^{\Omega_j} \right\rangle; \quad i = 1, 2, 3, \cdots, \phi, \ j = 1, 2, 3, \cdots, \psi. \qquad (3.1)$$

Then the weighted aggregated decision matrix $\overline{\mathcal{T}}$ is established in the following manner:

$$\overline{\mathcal{T}} = [t_{ij}]_{\phi\times\psi} = \begin{bmatrix} t_{11} & t_{12} & \cdots & t_{1\psi} \\ t_{21} & t_{22} & \cdots & t_{2\psi} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ t_{\phi 1} & t_{\phi 2} & ... & t_{\phi\psi} \end{bmatrix}.$$

This weighted aggregated decision matrix is also consisting of the values in the form of q-rung orthopair fuzzy numbers.

Step 4. Find the score matrix
Furthermore, the aggregated decision values of weighted aggregated decision matrix are converted to the simple crisp values by using the score function of q-rung orthopair fuzzy numbers, which is given as follows:

$$\hat{s}(t_{ij}) = (1 + \mu_{ij}(x)^q - \nu_{ij}(x)^q)/2. \tag{3.2}$$

A score matrix $S = [\hat{s}(t_{ij})]_{\phi\times\psi}$ is then constructed by employing the above crisp values for the further evaluation of alternatives.

Step 5. Computation of the deviation of alternatives
Since the preference structure of PROMETHEE method is established on the basis of pairwise comparison of alternatives. Therefore, in this step, the subtraction arithmetic formula is employed to obtain the deviation between every pair of alternatives on the basis of each criterion as follows:

$$D_j(a_i, a_l) = \hat{s}_j(a_i) - \hat{s}_j(a_l); \quad i, l = 1, 2, 3, \cdots, \phi, \tag{3.3}$$

where the term $D_j(a_i, a_l)$ shows the deviation of alternatives a_i and a_l with respect to criteria j and $\hat{s}_j(a_i)$ and $\hat{s}_j(a_l)$ represent the evaluations of alternatives a_i and a_l, respectively.

Step 6. Selection of an appropriate preference function
Afterward, an appropriate and relevant preference function is chosen to evaluate the preference of each alternative a_i with respect to all other alternatives a_l on the basis of all criteria. The suitable preference function is selected according to the type and nature of criteria. The value of these preferences varies from 0 to 1. The zero or negative preference of an alternative over another alternative shows the indifference of experts towards that pair of alternatives for the respective criteria. On the other hand, a strong preference is achieved for the preference values closest to 1. Regarding above discussion, a preference function of the following form will be selected by the experts.

$$P_j(a_i, a_l) = F_j[d_j(a_i, a_l)], \tag{3.4}$$

such that $0 \leq P_j(a_i, a_l) \leq 1$ and $P_j(a_i, a_l) > 0$ implies that $P_j(a_l, a_i) = 0$. This defined function provides the preferences of alternative a_i over a_l in the case of maximized criteria and has the shape of the following form as shown in (Figure 3.1).

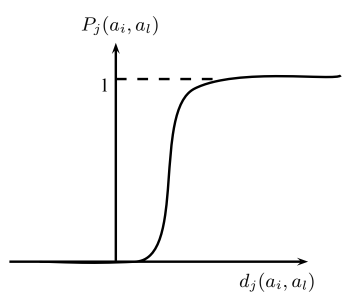

Figure 3.1: Preference function

On the other hand, the preference function in the case of criterion to be minimized can be described as follows:

$$P_j(a_i, a_l) = F_j[-d_j(a_i, a_l)], \tag{3.5}$$

this represents the reverse or the alternate behavior of the preference function instead of original function for such minimized criteria, which is described by the means of negative sign.

Step 7. Calculation of multi-criteria preference index
In this step, the multi-criteria preference index is computed for each pair of alternatives based on above defined preference function. The multi-criteria preference index is basically a weighted average of the corresponding preference function, which is calculated by using the formula as follows:

$$\Psi(a_i, a_l) = \frac{\sum_{j=1}^{\psi} \Omega_j P_j(a_i, a_l)}{\sum_{j=1}^{\psi} \Omega_j}; \quad i \neq l, \; i, l = 1, 2, \cdots, \phi. \tag{3.6}$$

If the selected weight vector is normalized, then the sum of criteria weights will be equal to 1. In this case, the above expression of the multi-criteria preference index can be formulated as follows:

$$\Psi(a_i, a_l) = \sum_{j=1}^{\psi} \Omega_j P_j(a_i, a_l); \quad i \neq l, \ i, l = 1, 2, \cdots, \phi. \tag{3.7}$$

This preference index measures the intensity of the preference of an alternative a_i over another alternative a_l based on all criteria and can be represented with a numeric value lies between 0 and 1, this implies,

- $\Psi(a_i, a_l) \approx 0$ represents that the alternative a_i has a week preference over the alternative a_l with respect to all criteria.
- $\Psi(a_i, a_l) \approx 1$ describes the strong preference of alternative a_i over the alternative a_l with respect to all criteria.

An outranking relation is determined based on this multi-criteria preference index which is further illustrated through an outranking graph. The nodes of this graph demonstrate the alternatives of the considering problem, whereas, the arcs between any two nodes represent the relationship between alternatives.

Step 8. Find out the preference order

Finally, the alternatives are ranked on the basis of preference relationship of alternatives which is determined and described in Step 5. The partial and complete rankings are obtained by following the procedure. The partial order of alternatives is achieved by considering the incoming and outgoing flows of alternatives and named as PROMETHEE I. Further, the complete ranking of alternatives is achieved by following the one more step, which is known as PROMETHEE II.The proceedings of PROMETHEE I and PROMETHEE II are explain as follows.

(i) Partial ranking of alternatives (PROMETHEE I)

The leaving or outgoing flow of each alternative a_i in the outranking graph is characterized as the average value of outgoing arcs from the node a_i to other nodes and calculated by deploying the formula as follows:

$$\Gamma^+(a_i) = \frac{1}{\phi - 1} \sum_{a_l \in Z} \Psi(a_i, a_l); \quad i \neq l, \ i, l = 1, 2, \cdots, \phi. \tag{3.8}$$

The outgoing flow of alternatives is also known as positive flow of outranking graph. Similarly, the incoming or entering flow of each alternative a_i is defined as the average value of the inward arcs of node a_i, which is formulated as follows:

$$\Gamma^-(a_i) = \frac{1}{\phi - 1} \sum_{a_l \in Z} \Psi(a_l, a_i); \quad i \neq l, \ i, l = 1, 2, \cdots, \phi. \tag{3.9}$$

The incoming flow of alternatives is also known as negative flow of outranking graph. If an alternative has a grater amount of leaving flow and a small value of entering flow, then it is chosen as the best possible or most favorable alternative. On the basis of these positive and negative outranking flows, the preferences of alternatives are determined by using the expressions given in Equations 3.10 and 3.11, respectively.

$$\begin{cases} a_i P^+ a_l & \Longleftrightarrow \Gamma^+(a_i) > \Gamma^+(a_l); \quad \forall a_i, a_l \in Z, \\ a_i I^+ a_l & \Longleftrightarrow \Gamma^+(a_i) = \Gamma^+(a_l); \quad \forall a_i, a_l \in Z, \end{cases} \tag{3.10}$$

$$\begin{cases} a_i P^- a_l & \Longleftrightarrow \Gamma^- a_i) < \Gamma^-(a_l); \quad \forall a_i, a_l \in Z, \\ a_i I^- a_l & \Longleftrightarrow \Gamma^-(a_i) = \Gamma^-(a_l); \quad \forall a_i, a_l \in Z. \end{cases} \tag{3.11}$$

The partial ranking $(\breve{P}, \breve{I}, \breve{R})$ of PROMETHEE I is obtained by taking the intersection of these positive and negative preferences of alternatives according to the following criteria:

$$\begin{cases} a_i \breve{P} a_l \ \ (a_i \text{ outranks } a_l) & \text{if } \ a_i P^+ a_l \text{ and } a_i P^- a_l, \\ & \text{or } a_i P^+ a_l \text{ and } a_i I^- a_l, \\ & \text{or } a_i I^+ a_l \text{ and } a_i P^- a_l; \\ a_i \breve{I} a_l \ \ (a_i \text{ is indifferent to } a_l) & \text{iff } a_i I^+ a_l \text{ and } a_i I^- a_l; \\ a_i \breve{R} a_l \ \ (a_i \text{ and } a_l \text{ are incomparable}) & \text{otherwise.} \end{cases} \tag{3.12}$$

The PROMETHEE I partial ranking is not able to compare all alternatives, for this reason, one more step of PROMETHEE II is performed to obtain the net outranking flow of alternatives.

(ii) Complete ranking of alternatives (PROMETHEE II)
The net outranking flow of the alternative a_i is obtained by taking the difference of positive and negative outranking flows of the respective alternative, known as PROMETHEE II, and formulated as follows:

$$\Gamma(a_i) = \Gamma^+(a_i) - \Gamma^-(a_i), \tag{3.13}$$

which provides the complete ranking $(\widetilde{P}, \widetilde{I})$ of PROMETHEE II as given in the expression 3.14.

$$\begin{cases} a_i\widetilde{P}a_l \quad (a_i \text{ outranks } a_l) & \text{iff} \quad \Gamma(a_i) > \Gamma(a_l), \\ a_i\widetilde{I}a_l \quad (a_i \text{ is indifferent to } a_l) & \text{iff} \quad \Gamma(a_i) = \Gamma(a_l). \end{cases} \tag{3.14}$$

Thus, all alternatives are compared on the basis of net outranking flow and a complete ranking of obtained without any incomparability of alternatives. The alternative with greatest net flow is considered as the most favorable alternative or best optimal solution.

4 Contractor selection for construction project

In this section, the developed multi-criteria decision-making approach, such as q-rung orthopair fuzzy PROMETHEE technique, is applied to a numerical problem. The construction project, sometimes simply known as project, is an organized procedure of constructing, renovating or cleaning up a structure, building or infrastructure.

The selection of a suitable contractor is one of the most important decisions for any construction company. Suppose a construction company wants to select a suitable contractor for their up coming projects. For this purpose, the company management designate a set of criteria as a parameter tool for contractors. These criteria are listed as follows:

c_1 = Technical capabilities

c_2 = Educational skills

c_3 = Reputation

c_4 = Financial situation

c_5 = Organizational skills

c_6 = Experience

c_7 = Reliability

c_8 = Security

c_9 = Past track record

c_{10} = Quality assurance

The company choose eight contractors $a_1, a_2, a_3, \cdots a_8$, after initial screening, for further evaluation. A group of three decision experts, e_1, e_2, e_3, is appointed to select the most suitable contractor on the basis of above mentioned criteria.

Step 1. The decision experts have decided to use the linguistic terms to assign the ratings of alternatives. For this reason, a set of nine linguistic terms is defined as given in Table 2. The ratings of alternatives in the form of linguistic terms given by each expert is presented in Table 3.

Table 3: Ratings of alternatives in terms of linguistic terms

Criteria	Contractors	e_1	e_2	e_3	Criteria	Contractors	e_1	e_2	e_3
c_1 (Technical Capabilities)	a_1	H	EH	EH	c_6 (Experience)	a_1	VH	M	M
	a_2	H	MH	VH		a_2	MH	ML	H
	a_3	H	H	M		a_3	M	MH	VH
	a_4	ML	MH	MH		a_4	H	MH	EH
	a_5	MH	VH	M		a_5	M	M	VH
	a_6	VH	H	H		a_6	H	VH	H
	a_7	ML	MH	ML		a_7	H	EH	EH
	a_8	MH	H	MH		a_8	VH	VH	H
c_2 (Educational skills)	a_1	H	VH	H	c_7 (Reliability)	a_1	H	VH	VH
	a_2	L	M	H		a_2	MH	MH	H
	a_3	VH	H	VH		a_3	VH	H	VH
	a_4	EH	MH	M		a_4	H	H	VH
	a_5	ML	MH	ML		a_5	VH	VH	H
	a_6	VH	MH	MH		a_6	L	H	ML
	a_7	H	EH	EH		a_7	H	M	VH
	a_8	H	VH	VH		a_8	VH	VH	EH
c_3 (Reputation)	a_1	H	M	M	c_8 (Security)	a_1	EH	VH	H
	a_2	ML	ML	EH		a_2	H	MH	MH
	a_3	M	H	MH		a_3	VH	H	H
	a_4	H	H	H		a_4	MH	MH	MH
	a_5	H	H	M		a_5	VH	MH	H
	a_6	H	ML	H		a_6	L	VH	ML
	a_7	VH	H	EH		a_7	MH	M	ML
	a_8	MH	VH	M		a_8	VH	VH	H
c_4 (Financial situation)	a_1	H	MH	MH	c_9 (Past track record)	a_1	H	H	EH
	a_2	M	H	VH		a_2	VH	M	MH
	a_3	H	VH	EH		a_3	VH	MH	H
	a_4	H	MH	MH		a_4	L	VH	ML
	a_5	M	VH	MH		a_5	H	MH	EH
	a_6	MH	H	M		a_6	VH	H	MH
	a_7	VH	M	H		a_7	MH	M	M
	a_8	VH	VH	EH		a_8	VH	VH	H
c_5 (Organizational skills)	a_1	M	H	H	c_{10} (Quality assurance)	a_1	H	MH	MH
	a_2	H	H	H		a_2	H	H	EH
	a_3	H	VH	EH		a_3	EH	H	MH
	a_4	VH	H	MH		a_4	MH	VH	M
	a_5	H	MH	H		a_5	MH	M	M
	a_6	VH	H	EH		a_6	H	VH	MH
	a_7	L	MH	H		a_7	ML	L	M
	a_8	H	VH	ML		a_8	VH	VH	H

Step 2. In this step, the linguistic ratings of alternatives are converted to q-rung orthopair fuzzy numbers. Since, three decision experts are involved in this decision-making, so three decision matrices are constructed, such as, $\mathcal{T}_{e_1}, \mathcal{T}_{e_2}$ and $\mathcal{T}_{e_3}$, respectively.

$\mathcal{T}_{e_1} =$

	c_1	c_2	c_3	c_4	c_5
a_1	(0.75, 0.35)	(0.75, 0.35)	(0.75, 0.35)	(0.75, 0.35)	(0.55, 0.55)
a_2	(0.75, 0.35)	(0.35, 0.75)	(0.45, 0.65)	(0.55, 0.55)	(0.75, 0.35)
a_3	(0.75, 0.35)	(0.85, 0.25)	(0.55, 0.55)	(0.75, 0.35)	(0.75, 0.35)
a_4	(0.45, 0.65)	(0.95, 0.15)	(0.75, 0.35)	(0.75, 0.35)	(0.85, 0.25)
a_5	(0.65, 0.45)	(0.45, 0.65)	(0.75, 0.35)	(0.55, 0.55)	(0.75, 0.35)
a_6	(0.85, 0.25)	(0.85, 0.25)	(0.75, 0.35)	(0.65, 0.45)	(0.85, 0.25)
a_7	(0.45, 0.65)	(0.75, 0.35)	(0.85, 0.25)	(0.85, 0.25)	(0.35, 0.75)
a_8	(0.65, 0.45)	(0.75, 0.35)	(0.65, 0.45)	(0.85, 0.25)	(0.75, 0.35)

	c_6	c_7	c_8	c_9	c_{10}
a_1	(0.85, 0.25)	(0.75, 0.35)	(0.95, 0.15)	(0.75, 0.35)	(0.75, 0.35)
a_2	(0.65, 0.45)	(0.65, 0.45)	(0.75, 0.35)	(0.85, 0.25)	(0.75, 0.35)
a_3	(0.55, 0.55)	(0.85, 0.25)	(0.85, 0.25)	(0.85, 0.25)	(0.95, 0.15)
a_4	(0.75, 0.35)	(0.75, 0.35)	(0.65, 0.45)	(0.35, 0.75)	(0.65, 0.45)
a_5	(0.55, 0.55)	(0.85, 0.25)	(0.85, 0.25)	(0.75, 0.35)	(0.65, 0.45)
a_6	(0.75, 0.35)	(0.35, 0.75)	(0.35, 0.75)	(0.85, 0.25)	(0.75, 0.35)
a_7	(0.75, 0.35)	(0.75, 0.35)	(0.65, 0.45)	(0.65, 0.45)	(0.45, 0.65)
a_8	(0.85, 0.25)	(0.85, 0.25)	(0.85, 0.25)	(0.85, 0.25)	(0.85, 0.25)

.

$\mathcal{T}_{e_2} =$

	c_1	c_2	c_3	c_4	c_5
a_1	(0.95, 0.15)	(0.85, 0.25)	(0.55, 0.55)	(0.65, 0.45)	(0.75, 0.35)
a_2	(0.65, 0.45)	(0.55, 0.55)	(0.45, 0.65)	(0.75, 0.35)	(0.75, 0.35)
a_3	(0.75, 0.35)	(0.75, 0.35)	(0.75, 0.35)	(0.85, 0.25)	(0.85, 0.25)
a_4	(0.65, 0.45)	(0.65, 0.45)	(0.75, 0.35)	(0.65, 0.45)	(0.75, 0.35)
a_5	(0.85, 0.25)	(0.65, 0.45)	(0.75, 0.35)	(0.85, 0.25)	(0.65, 0.45)
a_6	(0.75, 0.35)	(0.65, 0.45)	(0.45, 0.65)	(0.75, 0.35)	(0.75, 0.35)
a_7	(0.65, 0.45)	(0.95, 0.15)	(0.75, 0.35)	(0.55, 0.55)	(0.65, 0.45)
a_8	(0.75, 0.35)	(0.85, 0.25)	(0.85, 0.25)	(0.85, 0.25)	(0.85, 0.25)

	c_6	c_7	c_8	c_9	c_{10}
a_1	(0.55, 0.55)	(0.85, 0.25)	(0.85, 0.25)	(0.75, 0.35)	(0.65, 0.45)
a_2	(0.45, 0.65)	(0.65, 0.45)	(0.65, 0.45)	(0.55, 0.55)	(0.75, 0.35)
a_3	(0.65, 0.45)	(0.75, 0.35)	(0.75, 0.35)	(0.65, 0.45)	(0.75, 0.35)
a_4	(0.65, 0.45)	(0.75, 0.35)	(0.65, 0.45)	(0.85, 0.25)	(0.85, 0.25)
a_5	(0.55, 0.55)	(0.85, 0.25)	(0.65, 0.45)	(0.65, 0.45)	(0.55, 0.55)
a_6	(0.85, 0.25)	(0.75, 0.35)	(0.85, 0.25)	(0.75, 0.35)	(0.85, 0.25)
a_7	(0.95, 0.15)	(0.55, 0.55)	(0.55, 0.55)	(0.55, 0.55)	(0.35, 0.75)
a_8	(0.85, 0.25)	(0.85, 0.25)	(0.85, 0.25)	(0.85, 0.25)	(0.85, 0.25)

.

$\mathcal{T}_{e_3} =$

	c_1	c_2	c_3	c_4	c_5
a_1	(0.95, 0.15)	(0.75, 0.35)	(0.55, 0.55)	(0.65, 0.45)	(0.75, 0.35)
a_2	(0.85, 0.25)	(0.75, 0.35)	(0.95, 0.15)	(0.85, 0.25)	(0.75, 0.35)
a_3	(0.55, 0.55)	(0.85, 0.25)	(0.65, 0.45)	(0.95, 0.15)	(0.95, 0.15)
a_4	(0.65, 0.45)	(0.55, 0.55)	(0.75, 0.35)	(0.65, 0.45)	(0.65, 0.45)
a_5	(0.55, 0.55)	(0.45, 0.65)	(0.55, 0.55)	(0.65, 0.45)	(0.75, 0.35)
a_6	(0.75, 0.35)	(0.65, 0.45)	(0.75, 0.35)	(0.55, 0.55)	(0.95, 0.15)
a_7	(0.45, 0.65)	(0.95, 0.15)	(0.95, 0.15)	(0.75, 0.35)	(0.75, 0.35)
a_8	(0.65, 0.45)	(0.85, 0.25)	(0.55, 0.55)	(0.95, 0.15)	(0.45, 0.65)

	c_6	c_7	c_8	c_9	c_{10}
a_1	(0.55, 0.55)	(0.85, 0.25)	(0.75, 0.35)	(0.95, 0.15)	(0.65, 0.45)
a_2	(0.75, 0.35)	(0.75, 0.35)	(0.55, 0.55)	(0.65, 0.45)	(0.95, 0.15)
a_3	(0.85, 0.25)	(0.85, 0.25)	(0.75, 0.35)	(0.75, 0.35)	(0.65, 0.45)
a_4	(0.95, 0.15)	(0.85, 0.25)	(0.65, 0.45)	(0.45, 0.65)	(0.55, 0.55)
a_5	(0.85, 0.25)	(0.75, 0.35)	(0.75, 0.35)	(0.95, 0.15)	(0.55, 0.55)
a_6	(0.75, 0.35)	(0.45, 0.65)	(0.45, 0.65)	(0.65, 0.45)	(0.65, 0.45)
a_7	(0.95, 0.15)	(0.85, 0.25)	(0.45, 0.65)	(0.55, 0.55)	(0.55, 0.55)
a_8	(0.75, 0.35)	(0.95, 0.15)	(0.75, 0.35)	(0.75, 0.35)	(0.75, 0.35)

.

Step 3. Further, a weight vector $\Omega_j = [\ 0.113\ 0.107\ 0.097\ 0.092\ 0.088\ 0.082\ 0.088\ 0.143\ 0.101\ 0.090]^T$ is assigned to the criteria with the condition of normality.

Then, a weighted aggregated decision matrix $\overline{\mathcal{T}}$ is constructed, by utilizing the q-ROFWA operator as given in Equation 3.1, as follows,

$$\overline{\mathcal{T}} = \begin{bmatrix} & c_1 & c_2 & c_3 & c_4 & c_5 \\ a_1 & (0.7335, 0.5785) & (0.5817, 0.6887) & (0.4391, 0.8043) & (0.4698, 0.7839) & (0.4738, 0.7887) \\ a_2 & (0.5708, 0.6938) & (0.4299, 0.8130) & (0.5722, 0.7652) & (0.5234, 0.7564) & (0.5126, 0.7579) \\ a_3 & (0.5123, 0.7373) & (0.6134, 0.6643) & (0.4593, 0.7888) & (0.6478, 0.6712) & (0.6396, 0.6830) \\ a_4 & (0.4308, 0.7952) & (0.6133, 0.7030) & (0.5282, 0.7368) & (0.4698, 0.7839) & (0.5291, 0.7523) \\ a_5 & (0.5333, 0.7302) & (0.3764, 0.8373) & (0.4885, 0.7698) & (0.5866, 0.7386) & (0.5515, 0.7433) \\ a_6 & (0.5912, 0.6744) & (0.5393, 0.7267) & (0.4779, 0.7824) & (0.4257, 0.7085) & (0.6396, 0.6830) \\ a_7 & (0.3831, 0.8290) & (0.7233, 0.5955) & (0.6575, 0.6568) & (0.5234, 0.7564) & (0.4258, 0.8286) \\ a_8 & (0.5011, 0.7415) & (0.6134, 0.6643) & (0.5087, 0.7634) & (0.6682, 0.6508) & (0.5079, 0.7770) \\ \\ & c_6 & c_7 & c_8 & c_9 & c_{10} \\ a_1 & (0.4677, 0.8092) & (0.5790, 0.7144) & (0.7301, 0.5381) & (0.6420, 0.6679) & (0.4666, 0.7880) \\ a_2 & (0.4237, 0.8295) & (0.4633, 0.7922) & (0.5389, 0.6849) & (0.5151, 0.7550) & (0.6211, 0.6979) \\ a_3 & (0.4827, 0.7960) & (0.5790, 0.7144) & (0.6332, 0.6074) & (0.5518, 0.7213) & (0.6072, 0.7138) \\ a_4 & (0.5908, 0.7356) & (0.5484, 0.7358) & (0.5048, 0.7100) & (0.4709, 0.8085) & (0.4970, 0.7785) \\ a_5 & (0.4677, 0.8092) & (0.5790, 0.7144) & (0.6121, 0.6297) & (0.6279, 0.6850) & (0.3910, 0.8357) \\ a_6 & (0.5367, 0.7514) & (0.3886, 0.8559) & (0.5248, 0.7401) & (0.5518, 0.7213) & (0.5327, 0.7474) \\ a_7 & (0.6734, 0.6722) & (0.5162, 0.7657) & (0.4345, 0.7701) & (0.4058, 0.8176) & (0.3054, 0.8882) \\ a_8 & (0.5669, 0.7309) & (0.6600, 0.6630) & (0.6663, 0.5789) & (0.6031, 0.6797) & (0.5829, 0.7089) \end{bmatrix}.$$

Step 4. After that, the aggregated decision values of the aggregated decision matrix is converted to the simple crisp values by applying the score function of q-rung orthopair fuzzy numbers as defined in Equation 3.2. These crisp values are then utilized to formulate a score matrix S for further computations.

Step 5. In this step, the deviation of each alternative towards all other alternatives on the basis of respective criteria is calculated by using the score matrix S.

The deviations of alternatives are computed by applying the subtraction formula as given in Equation 3.3, and the results are presented in Table 4.

$$S = \begin{bmatrix} & c_1 & c_2 & c_3 & c_4 & c_5 \\ a_1 & 0.6005 & 0.4351 & 0.2822 & 0.3110 & 0.3079 \\ a_2 & 0.4260 & 0.2710 & 0.3696 & 0.3553 & 0.3497 \\ a_3 & 0.3668 & 0.4688 & 0.3030 & 0.4847 & 0.4715 \\ a_4 & 0.2886 & 0.4416 & 0.3737 & 0.3110 & 0.3612 \\ a_5 & 0.3812 & 0.2332 & 0.3302 & 0.3995 & 0.3785 \\ a_6 & 0.4500 & 0.3865 & 0.3151 & 0.3607 & 0.4715 \\ a_7 & 0.2433 & 0.5836 & 0.5005 & 0.3553 & 0.2542 \\ a_8 & 0.3591 & 0.4688 & 0.3434 & 0.5114 & 0.3310 \\ \\ & c_6 & c_7 & c_8 & c_9 & c_{10} \\ a_1 & 0.2862 & 0.4147 & 0.6167 & 0.4833 & 0.3061 \\ a_2 & 0.2527 & 0.3011 & 0.4176 & 0.3532 & 0.4498 \\ a_3 & 0.3041 & 0.4147 & 0.5149 & 0.3964 & 0.4301 \\ a_4 & 0.4041 & 0.3833 & 0.3854 & 0.2880 & 0.3255 \\ a_5 & 0.2862 & 0.4147 & 0.4898 & 0.4631 & 0.2381 \\ a_6 & 0.3652 & 0.2158 & 0.3696 & 0.3964 & 0.3668 \\ a_7 & 0.5008 & 0.3443 & 0.3127 & 0.2601 & 0.1639 \\ a_8 & 0.3959 & 0.4980 & 0.5509 & 0.4527 & 0.4209 \end{bmatrix}.$$

Table 4: Deviation of alternatives with respect to criteria

	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
a_1a_2	0.1745	0.1641	−0.1147	−0.0443	−0.0418	0.0335	0.1136	0.1991	0.1301	−0.1437
a_1a_3	0.2337	−0.0337	−0.0208	−0.1737	−0.1636	−0.0179	0.0	0.1018	0.0869	−0.1240
a_1a_4	0.3119	−0.0065	−0.0915	0.0	−0.0533	−0.1179	0.0314	0.2313	0.1953	−0.0194
a_1a_5	0.2193	0.2019	−0.048	−0.0885	−0.0706	0.0	0.0	0.1269	0.0202	0.068
a_1a_6	0.1505	0.0486	−0.0329	−0.0497	−0.1636	−0.079	0.1989	0.2471	0.0869	−0.0607
a_1a_7	0.3572	−0.1485	−0.2183	−0.0443	0.537	−0.2146	0.0704	0.304	0.2232	0.1422
a_1a_8	0.2414	−0.0337	−0.0612	−0.2004	−0.0231	−0.1079	−0.0833	0.0658	0.0306	−0.1148
a_2a_1	−0.1745	−0.1641	0.1147	0.0443	0.0418	−0.0335	−0.1136	−0.1991	−0.1301	0.1437
a_2a_3	0.0592	−0.1978	0.0666	−0.1294	−0.1218	−0.0514	−0.1136	−0.0973	−0.00432	0.0197
a_2a_4	0.1374	−0.1706	−0.0041	0.0443	−0.0115	−0.1514	−0.0822	0.0322	0.0652	0.1243
a_2a_5	0.0448	0.0378	0.0394	−0.0442	−0.0288	−0.0335	−0.1136	−0.0722	−0.1099	0.2117
a_2a_6	−0.024	−0.1155	0.0545	−0.0054	−0.1218	−0.1125	0.0853	0.048	−0.0432	0.083
a_2a_7	0.1827	−0.3126	−0.1309	0.0	0.0955	−0.2481	−0.0432	0.1049	0.0931	0.2859
a_2a_8	0.0669	−0.1978	0.0262	−0.1561	0.0187	−0.1432	−0.1969	−0.1333	−0.0995	0.0289
a_3a_1	−0.2337	0.0337	0.0208	0.1737	0.1636	0.0179	0.0	−0.1018	−0.0869	0.124
a_3a_2	−0.0592	0.1978	−0.0666	0.1294	0.1218	0.0514	0.1136	0.0973	0.0432	−0.0197
a_3a_4	0.0782	0.0272	−0.0707	0.1737	0.1103	−0.1	0.0314	0.1295	0.1084	0.1046
a_3a_5	−0.0144	0.2356	−0.0272	0.0852	0.093	0.0179	0.0	0.0251	−0.0667	0.0192
a_3a_6	−0.0832	0.0823	−0.0121	0.124	0.0	−0.0611	0.1989	0.1453	0.0	0.0633
a_3a_7	0.1235	−0.1148	−0.1975	0.1294	0.2173	−0.1967	0.0704	0.2022	0.1363	0.2662
a_3a_8	0.0077	0.0	−0.0404	−0.0267	0.1405	−0.0918	−0.0833	−0.036	−0.0536	0.0092
a_4a_1	−0.3119	0.0065	0.0915	0.0	0.0533	0.01179	−0.0314	−0.2313	−0.0953	0.0194
a_4a_2	−0.1374	0.1706	0.0041	−0.0443	0.0115	0.1514	0.0822	−0.0322	−0.0652	−0.1243
a_4a_3	−0.0782	−0.0272	0.0707	−0.1737	−0.1103	0.1	−0.0314	−0.1295	−0.1084	−0.1046
a_4a_5	−0.0926	0.2084	0.0435	−0.0885	−0.0173	0.1179	−0.0314	−0.1044	−0.1751	0.0874
a_4a_6	−0.1614	0.0551	0.0586	−0.0497	−0.1103	0.0389	0.1675	0.0158	−0.1084	−0.0413
a_4a_7	0.0453	−0.142	−0.1268	−0.0443	0.107	−0.0967	0.039	0.0727	0.0279	0.1616
a_4a_8	−0.0705	−0.0272	0.0303	−0.2004	0.0302	0.0082	−0.1147	−0.1655	−0.1647	−0.0954
a_5a_1	−0.2193	−0.2019	−0.048	0.0885	0.0706	0.0	0.0	−0.1269	−0.0202	−0.0680
a_5a_2	−0.0448	−0.0378	−0.0394	0.0442	0.0288	0.0335	0.1136	0.0722	0.1099	−0.2117
a_5a_3	0.0144	−0.2356	0.0272	−0.0852	−0.093	−0.0179	0.0	−0.0251	0.0667	−0.192
a_5a_4	0.0926	−0.2084	−0.0435	0.0885	0.0173	−0.1179	0.0314	0.1044	0.1751	−0.0874
a_5a_6	−0.0688	−0.1533	0.0115	0.0388	−0.093	−0.079	0.1989	0.1202	0.0667	−0.1287
a_5a_7	0.1379	−0.3504	−0.1703	0.0442	0.1243	−0.2146	0.0704	0.1771	0.203	0.0742
a_5a_8	0.0221	−0.2356	−0.0132	−0.1119	0.0475	−0.1097	−0.0833	−0.0611	0.0104	−0.1828
a_6a_1	−0.1505	−0.0486	0.0329	0.0479	0.1636	0.079	−0.1989	−0.2471	−0.0869	0.0607
a_6a_2	0.024	0.1155	−0.0545	0.0054	0.1218	0.1125	−0.0853	−0.048	0.0432	−0.083
a_6a_3	0.0832	−0.0823	0.0121	−0.124	0.0	0.0611	−0.1989	−0.1453	0.0	−0.0633
a_6a_4	0.1614	−0.0551	−0.0586	0.0497	0.1103	−0.0389	−0.01675	−0.0158	0.1084	0.0413
a_6a_5	0.0688	0.1533	−0.0151	−0.0388	0.093	0.079	−0.1989	−0.1202	−0.0667	0.1287
a_6a_7	0.2067	−0.1971	−0.1854	−0.0054	0.2173	−0.1356	−0.1285	0.0569	0.1363	0.2029
a_6a_8	0.0909	−0.0823	−0.0283	−0.1507	0.1405	−0.0307	−0.2822	−0.1813	−0.0563	−0.0541
a_7a_1	−0.3572	0.1485	0.2183	0.0443	−0.0537	0.2146	−0.0704	−0.304	−0.2232	−0.1422
a_7a_2	−0.1827	0.3126	0.1309	0.0	0.0955	0.2481	0.0432	−0.1049	−0.0931	−0.2859
a_7a_3	−0.1235	0.1148	0.1975	−0.1294	−0.2173	0.1967	−0.0704	−0.2022	−0.1363	−0.2662
a_7a_4	−0.0453	0.142	0.1288	0.0443	−0.107	0.0967	−0.039	−0.0727	−0.0279	−0.1616
a_7a_5	−0.1379	0.3504	0.1703	−0.0442	−0.1243	0.2146	−0.0704	−0.1771	−0.203	−0.0742
a_7a_6	−0.2067	0.1971	0.1854	−0.0054	−0.2173	0.1356	0.1285	−0.0569	−0.1363	−0.2029
a_7a_8	−0.1158	0.1148	0.1571	−0.1561	−0.0768	0.1049	−0.1537	−0.2382	−0.1926	−0.257
a_8a_1	−0.2414	0.0337	0.0612	0.2004	0.0231	0.1079	0.0833	−0.0658	−0.0306	0.1148
a_8a_2	−0.0669	0.1978	−0.0262	0.1561	−0.0187	0.1432	0.1969	0.1333	0.0995	−0.0289
a_8a_3	−0.0077	0.0	0.0404	0.0267	−0.1405	0.0918	0.0833	0.036	0.0563	−0.0092
a_8a_4	0.0705	0.0272	−0.0303	0.2004	−0.0302	−0.0082	0.1147	0.1655	0.1647	0.0954
a_8a_5	−0.0221	0.2356	0.0132	0.1119	−0.0475	0.1097	0.0833	0.0611	−0.0104	0.1828
a_8a_6	−0.0909	0.0823	0.0283	0.1507	−0.1405	0.0307	0.2822	0.1813	0.0563	0.541
a_8a_7	0.1158	−0.1148	−0.1571	0.1561	0.0768	−0.1049	0.1537	0.2382	0.1926	0.257

Step 6. Since, the technique of PROMETHEE method is based on preference function. Therefore, a suitable and appropriate preference function is selected for each criterion in this step. In this case, the usual criterion preference function is adopted for all criteria, and the results are summarized in Table 5.

Table 5: Usual criterion preference function

	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
a_1a_2	1	1	0	0	0	1	1	1	1	0
a_1a_3	1	0	0	0	0	0	0	1	1	0
a_1a_4	1	0	0	0	0	0	1	1	1	0
a_1a_5	1	1	0	0	0	0	0	1	1	1
a_1a_6	1	1	0	0	0	0	1	1	1	0
a_1a_7	1	0	0	0	1	0	1	1	1	1
a_1a_8	1	0	0	0	0	0	0	1	1	0
a_2a_1	0	0	1	1	1	0	0	0	0	1
a_2a_3	1	0	1	0	0	0	0	0	0	1
a_2a_4	1	0	0	1	0	0	0	1	1	1
a_2a_5	1	1	1	0	0	0	0	0	0	1
a_2a_6	0	0	1	0	0	0	1	1	0	1
a_2a_7	1	0	0	0	1	0	0	1	1	1
a_2a_8	1	0	1	0	1	0	0	0	0	1
a_3a_1	0	1	1	1	1	1	0	0	0	1
a_3a_2	0	1	0	1	1	1	1	1	1	0
a_3a_4	1	1	0	1	1	0	1	1	1	1
a_3a_5	0	1	0	1	1	1	0	1	0	1
a_3a_6	0	1	0	1	0	0	1	1	0	1
a_3a_7	1	0	0	1	1	0	1	1	1	1
a_3a_8	1	0	0	0	1	0	0	0	0	1
a_4a_1	0	1	1	0	1	1	0	0	0	1
a_4a_2	0	1	1	0	1	1	1	0	0	0
a_4a_3	0	0	1	0	0	1	0	0	0	0
a_4a_5	0	1	1	0	0	1	0	0	0	1
a_4a_6	0	1	1	0	0	1	1	1	0	0
a_4a_7	1	0	0	0	1	0	1	1	1	1
a_4a_8	0	0	1	0	1	1	0	0	0	0
a_5a_1	0	0	0	1	1	0	0	0	0	0
a_5a_2	0	0	0	1	1	1	1	1	1	0
a_5a_3	1	0	1	0	0	0	0	0	1	0
a_5a_4	1	0	0	1	1	0	1	1	1	0
a_5a_6	0	0	1	1	0	0	1	1	1	0
a_5a_7	1	0	0	1	1	0	1	1	1	1
a_5a_8	1	0	0	0	1	0	0	0	1	0
a_6a_1	0	0	1	1	1	1	0	0	0	1
a_6a_2	1	1	0	1	1	1	0	0	1	0
a_6a_3	1	0	1	0	0	1	0	0	0	0
a_6a_4	1	0	0	1	1	0	0	0	1	1
a_6a_5	1	1	0	0	1	1	0	0	0	1
a_6a_7	1	0	0	1	1	0	0	1	1	1
a_6a_8	1	0	0	0	1	0	0	0	0	0
a_7a_1	0	1	1	1	0	1	0	0	0	0
a_7a_2	0	1	1	0	0	1	1	0	0	0
a_7a_3	0	1	1	0	0	1	0	0	0	0
a_7a_4	0	1	1	1	0	1	0	0	0	0
a_7a_5	0	1	1	0	0	1	0	0	0	0
a_7a_6	0	1	1	0	0	1	1	0	0	0
a_7a_8	0	1	1	0	0	1	0	0	0	0
a_8a_1	0	1	1	1	1	1	1	0	0	1
a_8a_2	0	1	0	1	0	1	1	1	1	0
a_8a_3	0	0	1	1	0	1	1	1	1	0
a_8a_4	1	1	0	1	0	0	1	1	1	1
a_8a_5	0	1	1	1	0	1	1	1	0	1
a_8a_6	0	1	1	1	0	1	1	1	1	1
a_8a_7	1	0	0	1	1	0	1	1	1	1

Step 7. The multi-criteria preference index of each alternative is computed as the weighted averages of preferences of respective alternative with respect to all other alternatives. The multi-criteria preference index, also known as the total degree of preferences, is computed by deploying the Equation 3.5 and the result are presented in Table 6.

Step 8. In this step, the whole procedure of PROMETHEE method is concluded and the results of partial and complete outranking flows are determined.

a. PROMETHEE I (or Partial ranking of alternatives)

The negative and positive outranking flows of alternatives are determined which are outgoing and incoming flows of alternatives, respectively. The results of positive and negative outranking flows of alternatives are obtained by applying the Equations 3.10 and 3.11, respectively, and the values are summarized in Table 7. Then the intersection of preorders P^+ and P^- provides the partial ranking of contractors, which is given as

Table 6: Multi-criteria preference index

Contractors	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
a_1	–	0.634	0.357	0.445	0.554	0.552	0.535	0.357
a_2	0.367	–	0.3	0.539	0.407	0.418	0.535	0.388
a_3	0.556	0.701	–	0.822	0.602	0.520	0.715	0.291
a_4	0.464	0.462	0.179	–	0.376	0.517	0.623	0.267
a_5	0.180	0.594	0.311	0.625	–	0.521	0.715	0.302
a_6	0.449	0.583	0.292	0.484	0.480	–	0.627	0.201
a_7	0.378	0.374	0.286	0.378	0.286	0.374	–	0.286
a_8	0.644	0.613	0.603	0.734	0.699	0.8	0.715	–

follows,

$$a_1\check{P}a_2, a_1\check{P}a_4, a_1\check{P}a_5, a_1\check{P}a_6, a_1\check{P}a_7, a_2\check{P}a_4, a_2\check{P}a_7, a_3\check{P}a_1, a_3\check{P}a_2, a_3\check{P}a_4, a_3\check{P}a_5, a_3\check{P}a_6,$$
$$a_3\check{P}a_7, a_4\check{P}a_7, a_5\check{P}a_2, a_5\check{P}a_4, a_5\check{P}a_6, a_5\check{P}a_7, a_6\check{P}a_2, a_6\check{P}a_4, a_6\check{P}a_7, a_8\check{P}a_1, a_8\check{P}a_2, a_8\check{P}a_3,$$
$$a_8\check{P}a_4, a_8\check{P}a_5, a_8\check{P}a_6, a_8\check{P}a_7.$$

Table 7: Outgoing and incoming flows

Contractors	$\Gamma^+(a_i)$	$\Gamma^-(a_i)$
a_1	0.49057	0.434
a_2	0.422	0.56586
a_3	0.601	0.33257
a_4	0.41257	0.57529
a_5	0.464	0.48629
a_6	0.44514	0.52886
a_7	0.33743	0.63786
a_8	0.68686	0.29886

b. PROMETHEE II (or complete ranking of alternatives)

The net outranking flow of alternatives is calculated by employing the Equation 3.13, which is the output of the combination of positive and negative outranking flows. The complete ranking of alternatives is obtained without any incomparability, and the results are summarized in Table 8.

Table 8: Net ranking of contractors

Contractors	$\Gamma(a_i)$
a_1	0.05657
a_2	−0.14386
a_3	0.26843
a_4	−0.16272
a_5	−0.02229
a_6	−0.08372
a_7	−0.30043
a_8	0.388

It can be easily observe that the alternative a_8 is chosen as the best contractor for the construction project, and the ordering of contractors is given as follows,

$$a_8 \succ a_3 \succ a_1 \succ a_5 \succ a_6 \succ a_2 \succ a_4 \succ a_7.$$

5 Comparative analysis

5.1 Comparison with q-ROF ELECTRE method

In this subsection, the numerical problem of contractor selection for the construction project is performed under the procedure of existing MCDM technique, namely q-ROF ELECTRE method, which was proposed by Pinar and Bonar [27]. Consider the weighted aggregated decision matrix $\overline{\mathcal{T}}$, and follow the remaining steps of q-ROF ELECTRE method to find out an outranking relation of contractors in account to make a comparison of presented MCDM technique. The computation of q-ROF concordance sets C_{ij}, q-ROF discordance sets D_{ij}, concordance indices $\mathfrak{C}_{ij}$, discordance indices $\mathfrak{D}_{ij}$, concordance dominance $\mathfrak{J}_{ij}$, discordance dominance $\mathfrak{L}_{ij}$, aggregated dominance $\mathfrak{M}_{ij}$ and outranking relations for this problem is briefly summarized in Table 9.

The graph sketch by outranking relations is given in (Figure 5.1). The set of most favorable alternatives is chosen as $\{a_1, a_3, a_8\}$.

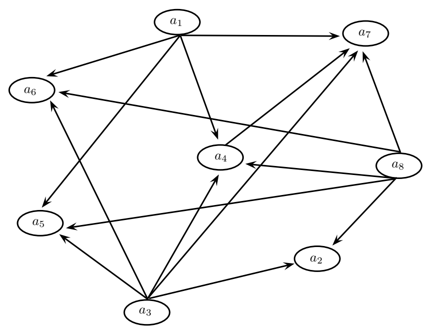

Figure 5.1: Outranking graph of alternatives

5.2 Comparison with q-ROF TOPSIS method

In this subsection, the same numerical problem of contractor selection for the construction project is performed under the procedure of q-ROF TOPSIS method, which was proposed by Pinar and Bonar [27]. Consider the weighted aggregated decision matrix $\overline{\mathcal{T}}$, and follow the remaining steps of q-ROF TOPSIS method to determine the ranking of contractors in account to make a comparison of net results. The positive ideal solution (PIS) and the negative ideal solution (NIS) are computed as follows:

$$PIS = \Big\langle (0.7335, 0.5785), (0.7233, 0.5955), (0.6575, 0.6568), (0.6682, 0.6508), (0.6396, 0.6830), (0.6734, 0.6722), (0.6600, 0.6630), (0.7301, 0.5381), (0.6420, 0.6679), (0.6211, 0.6979) \Big\rangle, \tag{5.1}$$

$$NIS = \Big\langle (0.3831, 0.8290), (0.3764, 0.8373), (0.4391, 0.8043), (0.4257, 0.7085), (0.4258, 0.8286), (0.4237, 0.8295), (0.3886, 0.8559), (0.4345, 0.7701), (0.4058, 0.8176), (0.3054, 0.8882) \Big\rangle. \tag{5.2}$$

The distance of each contractor from positive ideal solution $\mathcal{D}^+$ and the distance of each contractor from negative ideal solution $\mathcal{D}^-$ as well as the values of closeness coefficient are computed, and the results are given in Table 10.

Table 9: q-ROF ELECTRE results for contractor selection

Alternatives compared	C_{ij}	D_{ij}	$\mathfrak{C}_{ij}$	$\mathfrak{D}_{ij}$	$\mathfrak{J}_{ij}$	$\mathfrak{L}_{ij}$	$\mathfrak{M}_{ij}$	Outranking relations
(a_1, a_2)	$\{1, 2, 6, 7, 8, 9\}$	$\{3, 4, 5, 10\}$	0.634	0.8974	1	0	0	Incomparable
(a_1, a_3)	$\{1, 7, 8, 9\}$	$\{2, 3, 4, 5, 6, 10\}$	0.445	0.7740	0	1	0	Incomparable
(a_1, a_4)	$\{1, 4, 7, 8, 9\}$	$\{2, 3, 5, 6, 10\}$	0.537	0.3853	1	1	1	$a_1 \to a_4$
(a_1, a_5)	$\{1, 2, 6, 7, 8, 9, 10\}$	$\{3, 4, 5\}$	0.724	0.4944	1	1	1	$a_1 \to a_5$
(a_1, a_6)	$\{1, 2, 4, 7, 8, 9\}$	$\{3, 5, 6, 10\}$	0.644	0.6824	1	1	1	$a_1 \to a_6$
(a_1, a_7)	$\{1, 5, 7, 8, 9, 10\}$	$\{2, 3, 4, 6\}$	0.623	0.6120	1	1	1	$a_1 \to a_7$
(a_1, a_8)	$\{1, 8, 9\}$	$\{2, 3, 4, 5, 6, 7, 10\}$	0.357	0.8416	0	0	0	Incomparable
(a_2, a_1)	$\{3, 4, 5, 10\}$	$\{1, 2, 6, 7, 8, 9\}$	0.367	1	0	0	0	Incomparable
(a_2, a_3)	$\{1, 3, 10\}$	$\{2, 4, 5, 6, 7, 8, 9\}$	0.300	1	0	0	0	Incomparable
(a_2, a_4)	$\{1, 3, 4, 8, 9, 10\}$	$\{2, 5, 6, 7\}$	0.636	1	1	0	0	Incomparable
(a_2, a_5)	$\{1, 2, 3, 10\}$	$\{4, 5, 6, 7, 8, 9\}$	0.407	0.5198	0	1	0	Incomparable
(a_2, a_6)	$\{3, 4, 7, 8, 10\}$	$\{1, 2, 5, 6, 9\}$	0.510	1	0	0	0	Incomparable
(a_2, a_7)	$\{1, 4, 5, 8, 9, 10\}$	$\{2, 3, 6, 7\}$	0.627	0.9908	1	0	0	Incomparable
(a_2, a_8)	$\{1, 3, 5, 10\}$	$\{2, 4, 6, 7, 8, 9\}$	0.388	1	0	0	0	Incomparable
(a_3, a_1)	$\{2, 3, 4, 5, 6, 7, 10\}$	$\{1, 8, 9\}$	0.644	1	1	0	0	Incomparable
(a_3, a_2)	$\{2, 4, 5, 6, 7, 8, 9\}$	$\{1, 3, 10\}$	0.701	0.4886	1	1	1	$a_3 \to a_2$
(a_3, a_4)	$\{1, 2, 4, 5, 7, 8, 9, 10\}$	$\{3, 6\}$	0.822	0.5879	1	1	1	$a_3 \to a_4$
(a_3, a_5)	$\{2, 4, 5, 6, 7, 8, 10\}$	$\{1, 3, 9\}$	0.690	0.2872	1	1	1	$a_3 \to a_5$
(a_3, a_6)	$\{2, 4, 5, 7, 8, 9, 10\}$	$\{1, 3, 6\}$	0.709	0.4246	1	1	1	$a_3 \to a_6$
(a_3, a_7)	$\{1, 4, 5, 7, 8, 9, 10\}$	$\{2, 3, 6\}$	0.715	0.6832	1	1	1	$a_3 \to a_7$
(a_3, a_8)	$\{1, 2, 5, 10\}$	$\{3, 4, 6, 7, 8, 9\}$	0.398	0.6582	0	1	0	Incomparable
(a_4, a_1)	$\{2, 3, 4, 5, 6, 10\}$	$\{1, 7, 8, 9\}$	0.556	1	1	0	0	Incomparable
(a_4, a_2)	$\{2, 5, 6, 7\}$	$\{1, 3, 4, 8, 9, 10\}$	0.365	0.8082	0	1	0	Incomparable
(a_4, a_3)	$\{3, 6\}$	$\{1, 2, 4, 5, 7, 8, 9, 10\}$	0.179	1	0	0	0	Incomparable
(a_4, a_5)	$\{2, 3, 6, 10\}$	$\{1, 4, 5, 7, 8, 9\}$	0.376	0.7331	0	1	0	Incomparable
(a_4, a_6)	$\{2, 3, 4, 6, 7\}$	$\{1, 5, 8, 9, 10\}$	0.466	1	0	0	0	Incomparable
(a_4, a_7)	$\{1, 5, 7, 8, 9, 10\}$	$\{2, 3, 4, 6\}$	0.623	0.6970	1	1	1	$a_4 \to a_7$
(a_4, a_8)	$\{3, 5, 6\}$	$\{1, 2, 4, 7, 8, 8, 9, 10\}$	0.267	1	0	0	0	Incomparable
(a_5, a_1)	$\{3, 4, 5, 6, 7\}$	$\{1, 2, 8, 9, 10\}$	0.447	1	0	0	0	Incomparable
(a_5, a_2)	$\{4, 5, 6, 7, 8, 9\}$	$\{1, 2, 3, 10\}$	0.594	1	1	0	0	Incomparable
(a_5, a_3)	$\{1, 3, 7, 9\}$	$\{2, 4, 5, 6, 8, 10\}$	0.399	1	0	0	0	Incomparable
(a_5, a_4)	$\{1, 4, 5, 7, 8, 9\}$	$\{2, 3, 6, 10\}$	0.625	1	1	0	0	Incomparable
(a_5, a_6)	$\{3, 4, 7, 8, 9\}$	$\{1, 2, 5, 6, 10\}$	0.521	0.8301	1	0	0	Incomparable
(a_5, a_7)	$\{1, 4, 5, 7, 8, 9, 10\}$	$\{2, 3, 6\}$	0.715	1	1	0	0	Incomparable
(a_5, a_8)	$\{1, 5, 9\}$	$\{2, 3, 4, 6, 7, 8, 10\}$	0.302	1	0	0	0	Incomparable
(a_6, a_1)	$\{3, 5, 6, 10\}$	$\{1, 2, 4, 7, 8, 9\}$	0.357	1	0	0	0	Incomparable
(a_6, a_2)	$\{1, 2, 5, 6, 9\}$	$\{3, 4, 7, 8, 10\}$	0.491	0.7374	0	1	0	Incomparable
(a_6, a_3)	$\{1, 3, 5, 6, 9\}$	$\{2, 4, 7, 8, 10\}$	0.481	1	0	0	0	Incomparable
(a_6, a_4)	$\{1, 5, 8, 9, 10\}$	$\{2, 3, 4, 6, 7\}$	0.535	0.9958	1	0	0	Incomparable
(a_6, a_5)	$\{1, 2, 5, 6, 10\}$	$\{3, 4, 7, 8, 9\}$	0.480	1	0	0	0	Incomparable
(a_6, a_7)	$\{1, 5, 8, 9, 10\}$	$\{2, 3, 4, 6, 7\}$	0.535	0.8451	1	0	0	Incomparable
(a_6, a_8)	$\{1, 5\}$	$\{2, 3, 4, 6, 7, 8, 9, 10\}$	0.201	1	0	0	0	Incomparable
(a_7, a_1)	$\{2, 3, 4, 6\}$	$\{1, 5, 7, 8, 9, 10\}$	0.378	1	0	0	0	Incomparable
(a_7, a_2)	$\{2, 3, 4, 6, 7\}$	$\{1, 5, 8, 9, 10\}$	0.466	1	0	0	0	Incomparable
(a_7, a_3)	$\{2, 3, 6\}$	$\{1, 4, 5, 7, 8, 9, 10\}$	0.286	1	0	0	0	Incomparable
(a_7, a_4)	$\{2, 3, 4, 6\}$	$\{1, 5, 7, 8, 9, 10\}$	0.378	1	0	0	0	Incomparable
(a_7, a_5)	$\{2, 3, 6\}$	$\{1, 4, 5, 7, 8, 9, 10\}$	0.286	0.6117	0	1	0	Incomparable
(a_7, a_6)	$\{2, 3, 4, 6, 7\}$	$\{1, 5, 8, 9, 10\}$	0.466	1	0	0	0	Incomparable
(a_7, a_8)	$\{2, 3, 6\}$	$\{1, 4, 5, 7, 8, 9, 10\}$	0.286	1	0	0	0	Incomparable
(a_8, a_1)	$\{2, 3, 4, 5, 6, 7, 10\}$	$\{1, 8, 9\}$	0.644	1	1	0	0	Incomparable
(a_8, a_2)	$\{2, 4, 6, 7, 8, 9\}$	$\{1, 3, 5, 10\}$	0.613	0.3575	1	1	1	$a_8 \to a_2$
(a_8, a_3)	$\{2, 3, 4, 6, 7, 8, 9\}$	$\{1, 5, 10\}$	0.710	1	1	0	0	Incomparable
(a_8, a_4)	$\{1, 2, 4, 7, 8, 9, 10\}$	$\{3, 5, 6\}$	0.734	0.4535	1	1	1	$a_8 \to a_4$
(a_8, a_5)	$\{2, 3, 4, 6, 7, 8, 10\}$	$\{1, 5, 9\}$	0.699	0.1880	1	1	1	$a_8 \to a_5$
(a_8, a_6)	$\{2, 3, 4, 6, 7, 8, 9, 10\}$	$\{1, 5\}$	0.800	0.4860	1	1	1	$a_8 \to a_6$
(a_8, a_7)	$\{1, 4, 5, 7, 8, 9, 10\}$	$\{2, 3, 6\}$	0.715	0.5539	1	1	1	$a_8 \to a_7$

Table 10: Closeness coefficient to PIS

Contractors	$\mathcal{D}^+$	$\mathcal{D}^-$	Closeness coefficient
a_1	0.3848	0.6896	0.6418
a_2	0.4849	0.3742	0.4356
a_3	0.3428	0.5042	0.5953
a_4	0.4687	0.3553	0.4298
a_5	0.4868	0.3693	0.4314
a_6	0.4641	0.3904	0.4569
a_7	0.5793	0.4311	0.4267
a_8	0.2733	0.5404	0.6641

According to these values of closeness coefficients, the alternatives are ranked in descending order as:

$$a_8 \succ a_1 \succ a_3 \succ a_6 \succ a_2 \succ a_5 \succ a_4 \succ a_7.$$

5.3 Comparison with q-ROF VIKOR method

In this subsection, the numerical problem of contractor selection for the construction project is performed under the procedure of existing q-ROF VIKOR method, which was proposed by Krishankumar et al. [21]. Consider the weighted aggregated decision matrix $\overline{\mathcal{T}}$, and follow the remaining steps of q-ROF VIKOR method to determine the ranking of contractors in order to make a comparison of net results. The best and worst values of criteria are given in Table 11.

Table 11: Best and worst values of criteria

Criteria	Best values	Worst values
c_1	$(0.7335, 0.5785)$	$(0.3831, 0.8290)$
c_2	$(0.7233, 0.5955)$	$(0.3764, 0.8373)$
c_3	$(0.6575, 0.6568)$	$(0.4391, 0.8043)$
c_4	$(0.6682, 0.6508)$	$(0.4257, 0.7085)$
c_5	$(0.6396, 0.6830)$	$(0.4258, 0.8286)$
c_6	$(0.6734, 0.6722)$	$(0.4237, 0.8295)$
c_7	$(0.6600, 0.6630)$	$(0.3886, 0.8559)$
c_8	$(0.7301, 0.5381)$	$(0.4345, 0.7701)$
c_9	$(0.6420, 0.6679)$	$(0.4058, 0.8176)$
c_{10}	$(0.6211, 0.6979)$	$(0.3054, 0.8882)$

Furthermore, the Euclidean distance of q-ROFNs [13] is used to compute the values of $\mathcal{S}_\phi$, $\mathcal{R}_\phi$ and $\mathcal{Q}_\phi$, and the results are presented in Table 12.

Table 12: Values of $\mathcal{S}_\phi$, $\mathcal{R}_\phi$ and $\mathcal{Q}_\phi$

Contractors	$\mathcal{S}_\phi$	$\mathcal{R}_\phi$	$\mathcal{Q}_\phi(\nu = 0.5)$
a_1	0.4328	0.0970	0.3396
a_2	0.6033	0.0924	0.5635
a_3	0.3794	0.0876	0.1903
a_4	0.6131	0.1078	0.6907
a_5	0.5525	0.1070	0.5933
a_6	0.5726	0.1096	0.6426
a_7	0.6478	0.1430	1.0
a_8	0.3167	0.0745	0.0

The results of $\mathcal{Q}_\phi$ satisfy the conditions of compromise solution as the acceptable advantage and the stability of decision. Thus, the ranking of contractors is obtained as follows:

$$a_8 \succ a_3 \succ a_1 \succ a_2 \succ a_5 \succ a_6 \succ a_4 \succ a_7.$$

5.4 Comparison with q-ROFWA and q-ROFWG operators

This subsection provides a comparative analysis of the proposed q-ROF PROMETHEE method with existing aggregation operators, such as q-ROFWA operator and q-ROFWG operator, which was proposed by Liu and Wang [24]. Consider the same problem of contractor selection and solved by the aggregation operators under q-ROF information in order to make the comparison of net results. For this purpose, we consider the weighted aggregated decision matrix $\overline{\mathcal{T}}$ and aggregate all criteria values by applying the q-ROFWA operator and q-ROFWG operator, separately. As a result, a q-ROFN is obtained for each alternative and the results for both operators are summarized in Table 13.

Table 13: Results using q-ROFWA and q-ROFWG operators

Contractors	q-ROFWA	q-ROFWG
a_1	$(0.6016, 0.6949)$	$(0.5617, 0.7251)$
a_2	$(0.5275, 0.7481)$	$(0.5155, 0.7567)$
a_3	$(0.5840, 0.7016)$	$(0.5708, 0.7126)$
a_4	$(0.5232, 0.7514)$	$(0.5129, 0.7563)$
a_5	$(0.5413, 0.7398)$	$(0.5176, 0.7555)$
a_6	$(0.5333, 0.7356)$	$(0.5178, 0.7449)$
a_7	$(0.5443, 0.7527)$	$(0.4847, 0.7781)$
a_8	$(0.5999, 0.6870)$	$(0.5883, 0.6992)$

Further, the score function of each alternative is calculated for both operators and the results are presented in Table 14. The alternatives are ranked on the basis of score function in a descending order. The ranking of contractors through q-ROF TOPSIS, q-ROF VIKOR and q-ROFWA and q-ROFWG operators along with the ranking obtained by proposed q-ROF PROMETHEE method is given in Table 15. It can be easily observed that the ranking of alternatives obtained by all existing MCDM methods are close to the results of our proposed q-ROF PROMETHEE method and the optimal solution a_8 is same for all MCDM methods.

Table 14: Score values of q-ROFWA and q-ROFWG operators

Contractors	q-ROFWA	q-ROFWG
a_1	0.4411	0.3980
a_2	0.3641	0.3519
a_3	0.4269	0.4121
a_4	0.3595	0.3512
a_5	0.3769	0.3537
a_6	0.3768	0.3628
a_7	0.3674	0.3214
a_8	0.4458	0.4309

Table 15: The ranking of contractors

Rankings	Methods				
	q-ROF PROMETHEE	q-ROF TOPSIS	q-ROF VIKOR	q-ROFWA	q-ROFWG
$1st$	a_8	a_8	a_8	a_8	a_8
$2nd$	a_3	a_1	a_3	a_1	a_3
$3rd$	a_1	a_3	a_1	a_3	a_1
$4th$	a_5	a_6	a_2	a_5	a_6
$5th$	a_6	a_2	a_5	a_6	a_5
$6th$	a_2	a_5	a_6	a_7	a_2
$7th$	a_4	a_4	a_4	a_2	a_4
$8th$	a_7	a_7	a_7	a_4	a_7

6 Advantages and disadvantages of q-ROF PROMETHEE method

The main advantages of this research study are as follows:

1. The proposed technique of the PROMETHEE method is based on the q-ROF information that can adjust the range of indication of decision data by changing the value of parameter q for $q \geq 1$. The most dominant factor of q-ROFS is that the sum of q^{th} power of membership grade and the q^{th} power of non-membership grade should not exceed 1 that broadens the space of ambiguous data.

2. Different versions of PROMETHEE technique not only provide the kernel solution of decision problem (PROMETHEE I), but also provide the ranking of feasible actions or alternatives (PROMETHEE II).

3. The usual criterion preference function have been applied to evaluate the preferences of alternatives based on all criteria that simplify the computations of numerical problems and works on the formula "the more the better".

Beside of all above discussion, the presented technique have some limitations which are given as follows:

1. The wights of criteria can be calculated by using an appropriate method to minimize the personal choice or interest of decision experts.

2. In this approach, only the usual criterion preference function have been adopted which can be replaced by the combination of different types of preference functions to check the impact of preference functions on net results.

7 Conclusions

The q-rung orthopair fuzzy sets (q-ROFSs) are attracting the interest of a growing number of experts as they provide a formal setting with a broad space of acceptable orthopairs, which guarantees more flexibility and accuracy of the results. The decision experts can convey their information more effectively by using q-ROFS than PFS. The combination of q-rung orthopair fuzzy numbers with MCDM methods can provide valuable techniques because one can characterize highly complex scenarios in a more authentic way. For this purpose, in this article we combine the capabilities of q-ROF information and the reliability of the PROMETHEE method in order to state and solve decision-making problems that allow for fuzziness and imprecision in the ample range of q-ROF information. The linguistic terms have been used by decision experts to assign the performance ratings of alternatives on the basis of conflicting criteria which are further parameterized by q-ROFNs. Moreover, the q-ROFWA operator has been applied to aggregate the decision values of experts and thus obtain the weighted aggregation of ratings. The usual criterion preference function has been adopted to compute the deviation of each alternative with respect to all criteria. Then, this novel version of the PROMETHEE technique has been applied to select the most suitable contractor for a construction project. Furthermore, we have performed a comparison of the aforementioned technique in contrast to existing MADM methods, namely q-ROF ELECTRE, q-ROF TOPSIS, q-ROF VIKOR and q-ROF aggregation operators. The authenticity and reliability of the new approach is effectively validated by this comparative study. One can observe that the different variants of the PROMETHEE method not only give the kernel solution of the decision problem (PROMETHEE I) but are also able to produce the ranking of the alternatives (PROMETHEE II) as compared to the q-ROF ELECTRE method. A slight difference in the ranking of contractors has been observed, but the optimal solution is the same for all MCDM methods. This research study has a limitation, namely, only the usual criterion preference function is used for all criteria. This limitation will be addressed in future work by taking different preference functions according to the nature and type of criteria.

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