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A visual social network group consensus approach with minimum adjustment based on Pythagorean fuzzy set

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Abstract

People's demand for the decision-making space of opinion expression is getting higher, and the methods to determine the threshold value of current consensus still remain elusive. To deal with large and diverse information of users and discuss deeply the threshold in social networks, we establish a new consistency model with a new preference structure. In this paper, the Pythagorean fuzzy numbers (PFNs) are introduced into social network group decision-making for the expression of decision-makers' preference (DMs) and the concepts definition of the distance measurements, consensus index, and threshold indifference curves, respectively. In addition, we establish a Pythagorean fuzzy group consensus model with minimum adjustment through determining the setting rule of threshold value before reaching the consensus. Finally, we use the proposed model to solve the selection of square cabin hospitals.

Keywords: Social network group decision making (SNGDM), Pythagorean fuzzy numbers (PFNs), consensus model, minimum adjustment.

1 Introduction

The emergence of new media such as Facebook and Weibo have enabled people to form a closely connected social network. The increase in the number of information interactions has brought great challenges to issues such as public opinion control and consensus reaching. It is difficult for many decision makers in social networks to give accurate opinions. In many cases, they can only give fuzzy preference information. These studies try to depict the relations among social entities including members of a group, corporations, or nations, and describe the investigated consensus levels [6, 29]. Based on their trust relations, trust score, and knowledge deficit [28, 34], researchers construct a variety of group consensus model to deal with social network group decision-making problems (SNGDM). This kind of problem needs a novel suitable social network group consensus model to solve.

Recently, some researchers have used social network analysis methods to build some group decision making models. Most of the current literatures about SNGDM focus on the expression of trust preferences, aggregation methods, and consensus measurement.

(1) Decision-makers' preferences have different structures, which are expressed in the form of a crisp number, interval number, fuzzy number, linguistic number, and preference matrix, respectively. Sun first defined trust preference in distributed computer networks [25]. Wu defined four-tuple information by unifying trust, distrust, hesitancy, and inconsistency [37]. Cao used the distributed level trust functions (DLTFs) to process online evaluation information, [4]. Many researchers established the incomplete fuzzy preference matrix to complete the incomplete trust network. Wu proposed a comprehensive estimation method for incomplete information. He also defined the type-2 linguistic trust function to model the direct trust relation of group experts in SNGDM by combining the social network trust with the collaborative filtering [31]. Wu investigated a novel dual trust propagation operator based on the t-norm Einstein

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product and the t-conorm Einstein sum [33]. Zheng developed a trust transitivity model for SNGDM, focusing on the uncertainty and attenuation during propagation [46].

(2) To extract and utilize decision-makers' information in SNGDM, some researchers established various aggregation approaches to get the collective decision matrix and reach consensus. To aggregate individual trust relations and obtain their corresponding ranking relations, Wu defined the unorm trust weighted average (UTWA) operator, and unorm trust ordered weighted average (UTWA) operator [37]. Victor proposed four trust score aggregation strategies and two families of operators. This work contributed to the research on trust networks in their infancy [27]. Perez presented three new social network analysis-based IOWA operators. These operators took advantage of the linguistic trustworthiness information gathered from the experts' social network to aggregate the social group preferences [21].

(3) To deal with SNGDM problems, some researchers established distinct group consensus models to reach a much higher satisfactory consensus level in SNGDM. Wu developed a visual interaction consensus model, including a trust-based recommendation mechanism and visible adoption mechanism [33]. Dong built a bridge between opinion dynamics and SNGDM to highlight the importance of the leaders and trust relations improvements on SNGDM problems [9]. Wu analyzed nine consensus approaches based on social networks [35]. Dong provided a review of CRPs in SNGDM and classified them into two paradigms [8]. Considering consensus cost, Ben-Arieh first considered reaching an agreement at a minimum cost, defined the consensus process, and proposed the concept of minimum cost [1]. Wu developed a minimum adjustment cost feedback mechanism [36]. Chen provided a consensus model with minimum adjustments to obtain the optimal adjusted initial opinions and collective consensus opinion [5]. Dong et al. initiated a minimum adjustment consensus model (MACM) [7]. Zhang constructed cost consensus models with a normal distribution. [44]. The complex consensus process is multi-stage, Gou et al. managed consensus by multi-stage optimization models with linguistic preference orderings and double hierarchy linguistic preferences [14]. Decision-makers behavior is also an important factor that should be considered. Dong et al. and Gou et al. construct the consensus model with non-cooperative behaviors [10, 11, 13].

According to the above literatures, we find that existing research still has the following four aspects worthy of study. (1) As social networks include lots of people, the demand for the decision-making space of opinion expression is getting higher. We try to incorporate a novel new preference structure. Pythagorean fuzzy sets [12, 17, 18, 19, 22] both extend and generalize the constraint space of intuitionistic fuzzy sets, which meets the needs of the diversity and complexity of information expression. (2) Although the existing SNGDM approaches and their visual feedback mechanisms are useful to improve the consensus, they [33, 37] neglect the unify distance measurements. As shown in Figure 1, when distance function is general Manhattan distance, the threshold curve's shape is diamond. When distance function is the general Euclidean distance, the threshold curve's shape is a circle. The consensus and threshold curve should rely on the same distance measure. Therefore, we unify the distance measurement methods of consensus and threshold curve. (3) In classic group decision making problems, the threshold is usually set to 0.9 [26], 0.85 [23], 0.8 [38], or 0.75 [16]. However, people seldom think about how to determine the threshold value of current consensus methods. (4) To preserve DMs' information as much as possible, we hope the deviation degree between preferences is minimal before or after the revision. It's represented graphically that inconsistent points move until they are on threshold indifference curve and the movement distance is the shortest simultaneously. Therefore, we adopt the minimum adjustment model to solve the problem.

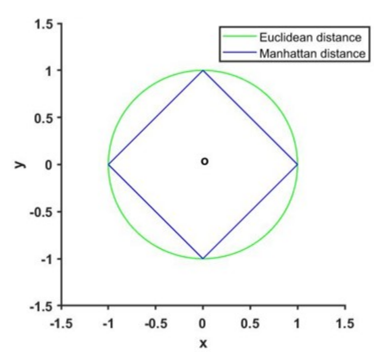


Figure 1: Comparison of Manhattan and Euclidean distance

Hence, we introduce Pythagorean fuzzy numbers into social network group decision-making and establish the consensus model with minimum adjustment. Define threshold indifference curves to analyze the threshold value for a better description of the group decision problems in the social network environment as well as the more convenient and practical management. The rest of this paper is as follows: Section 2 discusses the related theories and methods. Section 3 proposes a novel social network model based on Pythagorean Fuzzy. In Section 4, a case illustrates the proposed methods. Section 5 draws some conclusions.

2 Related theories and methods

Since human preferences and the objects are fuzzy and uncertain, Bellman and Zadeh introduced fuzzy sets into the decision-making problems [3]. To deal with hesitancy information, Atanassov presented the concept of an intuitionistic fuzzy set (IFS) based on membership degree, non-membership degree, and hesitancy degree [2]. In IFS, the sum of the support degree and the against degree is greater than or equal to 0 and less than or equal to 1. However, in reality the sum greater than 1, the quadratic sum is equal to or less than 1, which is not suitable IFS. Thus, Yager generalized the IFS and proposed the Pythagorean fuzzy set (PFS) [24]. As shown in Fig 2, it is evident that the Pythagorean membership space is larger than the intuitive membership space. PFSs have a more vital uncertainty modeling ability than IFSs. To better understand PFS, Peng proposed two operations of division and subtraction [20]. Zhang defined some novel operational laws of PFS and discussed their desirable properties. [45]. Zhang further extended the Pythagorean fuzzy set and proposed the concept of interval Pythagorean fuzzy set [42].

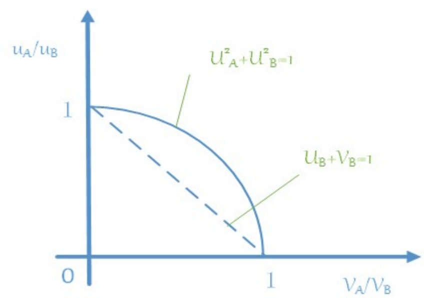


Figure 2: Space comparison of the PFNs and IFNs

In SNGDM, decision-makers' trust and distrust often take on some fuzzy and hesitancy information. To describe decision-makers' preference in SNGDM, we will use Pythagorean fuzzy numbers (PFNs) to express the decision-makers' trust and distrust in this paper. To begin with, we know some preliminaries in this section.

2.1 Pythagorean fuzzy sets

Similar to the definition of IFSs, we give some concepts and definitions of PFSs in this section.

Definition 2.1. [24] *Let X be a universe of discourse. A PFS P is given by*

$$P = \{\langle x, \mu_p(x), v_p(x)\rangle \mid x \in X\}, \tag{1}$$

where, the function $\mu_p : X \to [0,1]$ and $v_p : X \to [0,1]$ represent the membership and the non-membership of the element $x \in X$ to the set P, respectively. And for each $x \in X$, it holds that

$$(\mu_p(x))^2 + (v_p(x))^2 \leq 1. \tag{2}$$

For any PFS P and $x \in X$, $\pi_p(x) = \sqrt{1 - \mu_p^2(x) - v_p^2(x)}$ is the degree of indeterminacy of x to P, which means the information loss of x to P. For simplicity, is called a Pythagorean fuzzy number (PFN), expressed by $\beta = P(\mu_\beta, v_\beta)$, where $\mu_\beta, v_\beta \in [0,1]$,$\pi_\beta = \sqrt{1 - \mu_\beta^2 - v_\beta^2}$, and $\mu_\beta^2 + v_\beta^2 \leq 1$.

Definition 2.2. [10, 11] *Let $\beta = P(\mu_\beta, v_\beta)$ be a PFN, then the score function (SF) is defined*

$$s(\beta) = (\mu_\beta)^2 - (v_\beta)^2, \tag{3}$$

where, $s(\beta) \in [-1, 1]$.

Definition 2.3. [24, 30, 40] *For any PFN $\beta = P(\mu_\beta, v_\beta)$, the accuracy function β can be expressed as follows*

$$a(\beta) = (\mu_\beta)^2 + (v_\beta)^2, \tag{4}$$

where, $a(\beta) \in [0,1]$.

And for any two PFNs, β_1, β_2:

If $s(\beta_1) > s(\beta_2)$, then $\beta_1 \succ \beta_2$;

If $s(\beta_1) = s(\beta_2)$, then there are two situations as follows:

If $a(\beta_1) > a(\beta_2)$, then $\beta_1 \succ \beta_2$;

If $a(\beta_1) = a(\beta_2)$, then $\beta_1 \approx \beta_2$.

Definition 2.4. [22] *If $\beta_1 = P(\mu_{\beta_1}, v_{\beta_1})$ and $\beta_2 = P(\mu_{\beta_2}, v_{\beta_2})$ are two PFNs respectively, the Euclidean distance of β_1 between β_2 are as follows:*

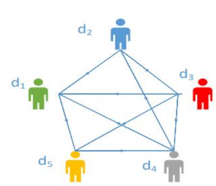
$$d(\beta_1, \beta_2) = \sqrt{\frac{1}{2}\left[\left((\mu_{\beta_1})^2 - (\mu_{\beta_2})^2\right)^2 + \left((v_{\beta_1})^2 - (v_{\beta_2})^2\right)^2 + \left((\pi_{\beta_1})^2 - (\pi_{\beta_2})^2\right)^2\right]}, \tag{5}$$

where, $0 \le d(\beta_1, \beta_2) \le 1$.

2.2 Social network analysis for GDM

Social relationships are regarded as a reliable source to evaluate the importance of decision-makers. The methods to study the relationships among social members adopt social network analysis (SNA), which can establish the relationship model among group members. As an effective method, social network analysis has been widely used in resource optimization and consensus-building. Graphic methods, algebraic representations, and adjacency matrix methods are the three categories commonly used to represent social networks. Details are shown in Table 1.
(1) Social relation matrix representation: the adjacency matrix is also called relation matrix and social matrix. That is, the relation data between decision-makers is presented in the form of the matrix.
(2) Graphic method: the social network is represented by graphs, nodes in the graph represent individual users, and lines are used to describe the relationship between users, that is, a graph composed of nodes connected by lines.
(3) Algebraic representation: A relation expression will be used to express the relationship between nodes, which is used to represent the combination of relations.

Table 1 Elements of social network analysis

Social relation matrix representation	Graphic method	Algebraic representation
$A = \begin{pmatrix} - & a_{12} & a_{13} & 0 & a_{15} \\ 0 & - & a_{23} & a_{24} & 0 \\ 0 & 0 & - & a_{34} & 0 \\ a_{41} & 0 & 0 & - & 0 \\ 0 & 0 & a_{53} & a_{54} & - \end{pmatrix}$		$d_1Rd_2, d_1Rd_3, d_1Rd_3$ d_2Rd_3, d_2Rd_4 d_3Rd_4 d_4Rd_1 d_5Rd_3, d_5Rd_4

Social network analysis (SNA) describe the relationship model among group members to deal with the problem of resource optimization and consensus-reaching, etc. Graphic methods, algebraic representations, and adjacency matrix methods are the three categories commonly used to represent social networks.

In SNGDM, a social network is virtually represented as a graph $G(D,E)$. Nodes refer to decision-makers (DMs), and edges represent the social relations between DMs.

Definition 2.5. [25, 32] *A social network is defined as a directed graph $G(D,E)$, including a set of DMs and a set of ordered pairs of D elements, which means DM di directly trusts DM d_j.*

Definition 2.6. [25, 32] *Let an adjacent matrix* $A = (a_{ij})_{m\times m}$ *to represent* $G(D,E)$*, where*

$$a_{ij} = \begin{cases} 1, (d_i, d_j) \in E \\ 0, (d_i, d_j) \notin E, \end{cases} \tag{6}$$

thus $a_{ij} = 1$ *denotes decision-maker* d_i *directly trust* d_j*.*

In definition 2.6, an adjacent matrix can only describe a trust relationship between each pair of decision-makers. So, we propose an adjacent weighted matrix to describe trust strengths among DMs. For notation simplicity, we still use $A = (a_{ij})_{m\times m}$ to denote an adjacent weighted matrix. And in adjacent matrix A $a_{ij} \in [0,1]$ indicates the trust strength from DM d_i to DM d_j . The larger the value of a_{ij}, the higher the trust d_i has on d_j.

Definition 2.7. *In a social network* $G(D,E)$, *A sequence of edges* $(d_i, d_{i_1})(d_{i_1}, d_{i_2})\,\mathrm{K}\,(d_{i_{n-1}}, d_j)$ *is called a trust path from decision-maker* d_i *to* d_j*.*

Definition 2.8. *The degree centrality index* $C(d_k)$ *of* $\mathrm{DM}d_k$ *defines as:*

$$C(d_k) = \frac{1}{m-1}\sum_{i=1, i\neq k}^{m} a_{ik}. \tag{7}$$

In social network $G(D,E)$, the degree center index of DM reflects its importance. Generally speaking, if the central value of DM is more significant, its priority is higher. In consequence, this value is often used to calculate the weight of each DM in SNGDM.

3 A novel Pythagorean fuzzy consensus model with minimum adjustment

In this section, we introduce a novel Pythagorean fuzzy consensus model in SNGDM. Mainly, Section 3.1 introduces preferences aggregation based on Pythagorean Fuzzy. In Section 3.2, we first define threshold indifference curves to set the optimal consensus threshold. After that, we can calculate three levels of consensus index in Section 3.3. Then we show how to identify the inconsistent experts in Section 3.4. Section 3.5 gives the recommendation mechanism based on the minimum adjustment. Finally, the best alternative is chosen in Section 3.6.

3.1 The preferences aggregation based on Pythagorean Fuzzy

Form Section 2.1, when the elements of the trust relation matrix are represented by PFN,the trust score of each DM is defined as follows:

Definition 3.1. *Assume that there is a social network* $G(D,E,\omega)$*, where* $D = \{d_1, d_2, \mathrm{K}, d_m\}$ *represents the DMs, and a pair of ordered elements of D,* $E \in (d_i, d_j)$ *, which means DM di directly trusts DM* d_j*, and* ω *stands for the weight of DM. If* $A = (a_{ij})_{m\times m}$ *represents the social matrix and* $a_{ij} \in [0,1]$ *expresses the trust strength of* $\mathrm{DM}d_i$ *to DM* d_j *, the degree centrality index* $C(d_k)$ *and trust score of DM* d_k *are defined as follows:*

$$C(d_k) = (\mu_k, v_k) = \frac{1}{m-1}\sum_{i=1, i\neq k}^{m} a_{ik}. \tag{8}$$

$$TS_k = \frac{(\mu_k)^2 - (v_k)^2 + 1}{2}. \tag{9}$$

If the TS value is larger, the importance of DM is higher. Because $0 \le TS_k \le 1$, the Basic Unit- interval Monotone (BUM) membership function $Q(r) = r^\alpha (\alpha \ge 0)$ of the fuzzy linguistic quantifier [39] can be implemented to measure the weight of DM. If two DMs have the same TS, we can use the AF to differentiate.

TS values can be used to assign significant weights to DMs. According to Yager's OWA based procedure [39], the weight of DM d_k can be calculated as

$$\omega_T^{\sigma(k)} = Q\left(\frac{T(\sigma(k))}{T(\sigma(m))}\right) - Q\left(\frac{T(\sigma(k-1))}{T(\sigma(m))}\right), \tag{10}$$

where, $T(\sigma(k))=\sum_{i=1}^{k} TS_{\sigma(i)}$, σ, Q expresses a permutation such that $TS_{\sigma(i)}$ is the l-th most massive value of the set $\{TS_1,\ \mathrm{K}, TS_m\}$ and a Basic Unit interval Monotone (BUM) membership function of the fuzzy linguistic quantifier to implement in the aggregation process: $Q:[0,1]\rightarrow[0,1]$ such that $Q(0)=0, Q(1)=1$ and if $x>y$, then $Q(x)\geq Q(y)$.

In the process of group decision-making, each decision-maker must reach a certain level of consensus. Besides, reaching a group consensus is an interactive dynamic process. In the trusted network based on decision-makers, we should consider trust relations an essential factor for reaching an agreement. Because PFS has a more expansive decision space, decision-makers use PFNs to express a preference. By using TS, we can aggregate each decision matrix into a group as follow:

Definition 3.2. *Assume that* $\left\{R^{(k)}=\left(r_{ij}^{(k)}\right)_{p\times q;}\ k=1,2,\ \mathrm{K}, m\right\}, TS=\{TS_1,\ \mathrm{K}, TS_m\}, \omega_T=\{\omega_1,\ \mathrm{K}, \omega_m\},\ A=\{A_1,\ \mathrm{K}, A_p\}$ *and* $C=\{C_1,\ \mathrm{K}, C_q\}$ *respectively stands for the decision matrix set of the DMs, the importance weights, a group of alternatives, and criteria. Then the individual matrixes can be aggregated as*

$$\bar{r}_{ij}=\sum_{k=1}^{m}\omega_T^k\cdot r_{ij}^k, \tag{11}$$

where $i=1,\ \mathrm{K}, p, j=1,\ \mathrm{K}, q$.

3.2 The optimal consensus threshold

Before the consensus process, it is crucial to realize the purpose of the CRP. The whole CRP is to control the consensus index within a reasonable range. So how to determine the consensus threshold has strategic significance. However, there is no research about that.

Definition 3.3. *When the rally point is* $(\bar{\mu},\bar{v})$ *and the consensus threshold is set to* γ*, the plane's points equal to the consensus degree of the group decision point will be enclosed into a smooth curve for the group on the alternative* A_i *under criterion* C_j*. At each point on the curve, the combination of preferences (horizontal and vertical coordinates) is different, but it represents the same degree of consensus. Therefore, the smooth curve is named based on the concept of indifference curve in economics. According to the expression 5, the threshold indifference curve should be*

$$\left(\mu^2-\bar{\mu}^2\right)^2+\left(v^2-\bar{v}^2\right)^2+\left(\mu^2+v^2-\bar{\mu}^2-\bar{v}^2\right)^2=2(1-\gamma)^2. \tag{12}$$

Suppose $0.8\leq\gamma\leq1$, the graph of the function shows in Fig.3. However, few people think about how the value of γ comes from, why it is 0.8 or 0.9, or other larger numbers. From Fig.3, we can see that with the decrease of γ value, the range of the threshold indifference curve becomes more extensive and broader.

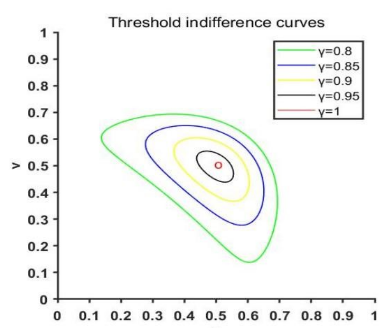


Figure 3: Threshold indifference curves with rally point (0.5,0.5)

Lemma 3.4. $\left(\mu^2-\bar{\mu}^2\right)^2+\left(v^2-\bar{v}^2\right)^2+\left(\mu^2+v^2-\bar{\mu}^2-\bar{v}^2\right)^2=2(1-\gamma)^2$ *be the expression of the threshold indifference curve. With the decrease of* γ *value, two critical values help us distinguish the level of consensus. They are* $\gamma=1-\sqrt{\frac{3}{4}(\bar{\mu})^4}$ *and* $\gamma=1-\sqrt{\bar{\mu}^4+\bar{v}^4+\bar{\mu}^2\bar{v}^2}$.

Proposition 3.5. *When* $\gamma\in\left[1-\sqrt{\frac{3}{4}(\bar{\mu})^4},1\right)$*, the group reaches a high degree of consensus; When* $0\leq\gamma<1-\sqrt{\bar{\mu}^4+\bar{v}^4+\bar{\mu}^2\bar{v}^2}$ *,the group gets a basic agreement; when* $0\leq\gamma<1-\sqrt{\bar{\mu}^4+\bar{v}^4+\bar{\mu}^2\bar{v}^2}$*, the group just goes a low degree of consensus.*

Proof. (1) $\left(\mu^2-\bar{\mu}^2\right)^2+\left(v^2-\bar{v}^2\right)^2+\left(\mu^2+v^2-\bar{\mu}^2-\bar{v}^2\right)^2=2(1-\gamma)^2$ is the expression of the threshold indifference curve, and the function graph is symmetrical with respect to $y=x$, if $\mu=0$, there is only one positive solution for v , that is to say, the equation $\bar{\mu}^4+\left(v^2-\bar{v}^2\right)^2+\left(v^2-\bar{\mu}^2-\bar{v}^2\right)^2-2(1-\gamma)^2=0,\ v,\ \gamma\in(0,1)$) has one and only one resolution. Therefore, there is $\Delta=4\left(\bar{\mu}^2+2\bar{v}^2\right)^2-8\left[\bar{\mu}^4+\bar{v}^4+\left(\bar{\mu}^2+\bar{v}^2\right)-2(1-\gamma)^2\right]=0$, and the answer is $\gamma=1-\sqrt{\frac{3}{4}(\bar{\mu})^4}$.

(2) When $\gamma<1-\sqrt{\frac{3}{4}(\bar{\mu})^4}$, the threshold indifference curve changes, as shown by Fig.4, the consensus range seems to be annular. As the γ value continues to decrease, the inner ring gradually decreases until it happens to become a point $(0,0)$. When $\mu=v=0$, then $\bar{\mu}^4+\bar{v}^4+\left(\bar{\mu}^2+\bar{v}^2\right)^2-2(1-\gamma)^2=0$, and the solution is $\gamma=1-\sqrt{\bar{\mu}^4+\bar{v}^4+\bar{\mu}^2\bar{v}^2}$.

(3) When $\gamma\in\left[1-\sqrt{\frac{3}{4}(\bar{\mu})^4},1\right)$, the change of the threshold indifference curve is shown by Fig.4. In particular, if $\gamma=0$, the consensus range almost fills the whole decision domain $\{(\mu,v)\mid 0\leq\mu\leq 1,0\leq v\leq 1\}$. □

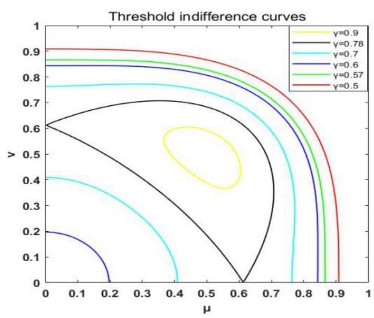


Figure 4: Threshold indifference curves with rally point (0.5,0.5)

All in all, it concludes that when $\gamma\in\left[1-\sqrt{\frac{3}{4}(\bar{\mu})^4},1\right)$, a high degree of consensus reaches; When $0\leq\gamma<1-\sqrt{\bar{\mu}^4+\bar{v}^4+\bar{\mu}^2\bar{v}^2}$, the group gets a basic agreement; And when $0\leq\gamma<1-\sqrt{\bar{\mu}^4+\bar{v}^4+\bar{\mu}^2\bar{v}^2}$, the group just goes a low degree of consensus. Based on the first aggregation results and the problem's actual situation, a reasonable consensus threshold is determined. For example, if the group consensus point is (0.5,0.5) and the consensus index of the first aggregation is 0.3, the threshold indifference curves change, as shown in Fig.4. then $\gamma\in[0.78,1),\gamma\in[0.57,0.78)$ and $\gamma\in[0,0.57)$ is high, basic, and low consensus, respectively. When dealing with urgent decisions, we set the consensus threshold between 0.57 and 0.78.

3.3 Consensus index at three levels

We can calculate three levels (elements level, alternatives level, and decision matrix level) of consensus index for each DM when we obtain the group decision matrix. When the CI at the decision matrix level reaches a preset threshold, we proceed with the selection process of SNGDM. Otherwise, inconsistent DMs will be identified. What's more, to increase group consensus, it recommends modifying those preference values that have less contribution to the consensus [12, 16, 18, 22, 26]. The purpose of this paper is to establish a visual identification rule based on the Euclidean plane to identify those DMs who have less contribution to the consensus. To modify their views from the DM's perspective, we establish a consensus mechanism based on minimum adjustments. The smaller opinion adjustment, the more favorable for DMs. The proposed model in this paper can help decision-makers see the whole consensus process, see their position and know how to modify their opinions with minimum adjustment. The technical route and decision process of the model are shown in Fig. 5.

In SNGDM, the visual consensus process based on minimum adjustment first calculate the TS values and group decision matrix of the DMs and CI at three levels for each DM. Second, we need to obtain visual identification rule based on Euclidean plane. Finally, we make selection process according to recommendation mechanism based on minimum adjustment. The first step has been covered in section 3.1 and 3.2, and the rest will be introduced as follow.

Level 1. Consensus index at elements level. Let be the consensus degree of pair of alternative criteria (A_i,A_j), representing all DMs consensus degree to (A_i,A_j). In other words, CIR_{ij} means the consistency degree of all DMs' evaluation to the element located at (i,j). The consensus index of a DM d_k for the group on the alternatives Ai under

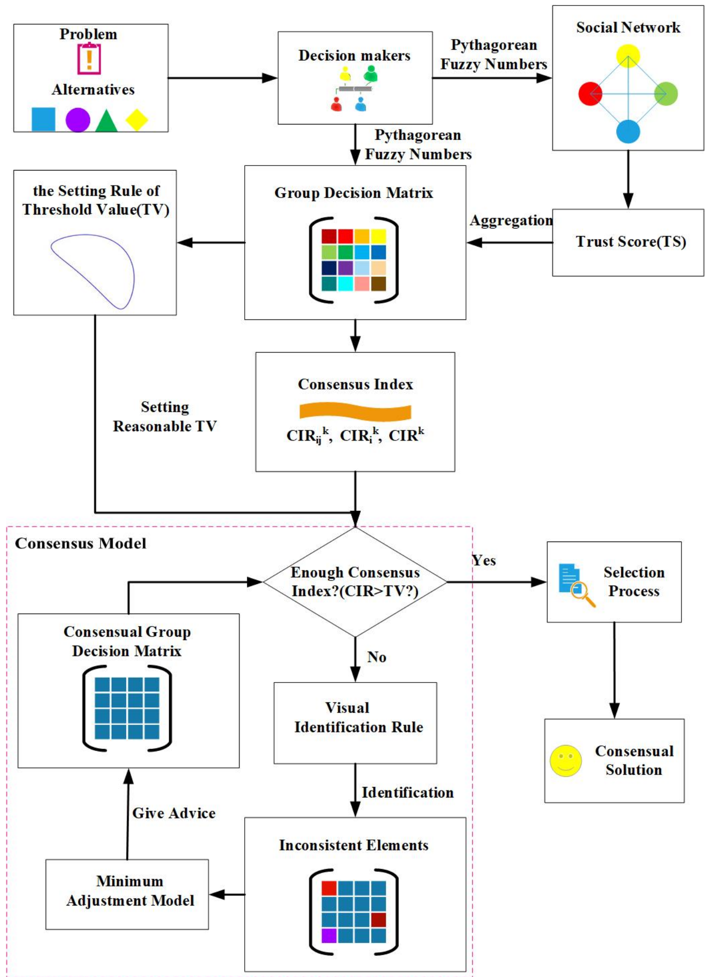


Figure 5: Decision-making process of the proposed approach

criterion C_j is

$$CIR_{ij}^k = 1 - d\left(r_{ij}^k, \bar{r}_{ij}\right) = 1 - \sqrt{\frac{1}{2}\left[\left(\left(\mu_{ij}^k\right)^2 - \left(\bar{\mu}_{ij}\right)^2\right)^2 + \left(\left(v_{ij}^k\right)^2 - \left(\bar{v}_{ij}\right)^2\right)^2 + \left(\left(\pi_{ij}^k\right)^2 - \left(\bar{\pi}_{ij}\right)^2\right)^2\right]}. \tag{13}$$

Level 2. Consensus index at alternatives level. Let A_i be the consensus degree of alternative A_i, then the consensus index of a DM d_k for the group on the alternative A_i is

$$CIR_i^k = \frac{1}{q}\sum_{j=1}^{q} CIR_{ij}^k. \tag{14}$$

Level 3. Consensus index at decision matrix level. Let CIR_k be the consensus degree of decision matrix of d_k, then the consensus index of DM d_k to the group on decision matrix is

$$CIR^k = \frac{1}{p}\sum_{i=1}^{p} CIR_i^k. \tag{15}$$

The larger the value of CIR_k, the higher the consensus index between DM d_k and the group. When $CIR^k = 1$, DM d_k have the same decision matrix. However, this situation is almost impossible in practical decision-making problem. The consensus threshold γ can set according to Lemma 3.4. If the value of CIR_k is lower than the preset threshold value, a visual consensus process based on minimum adjustment is activated. That helps all DMs to see their position. Whats more, to improve the group consensus degree, it recommend those who don't reach consensus to modify their opinions by their the minimum adjustment value. Finally, an appropriate selection process is adopted to obtain the optimal alternative.

3.4 Visual identification rule

The visual identification rule uses visual graphics to help DMs see their position in the consensus process. Fig. 6 shows that when the individual preference point is within the closed curve, the point reaches the consensus threshold. However, when it is outside, this point has less contribution to the group consensus and needs to be modified appropriately with $\gamma = 0.9$.

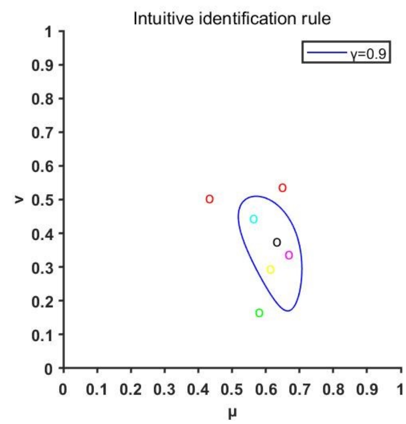


Figure 6: Visual identification rule

The identification steps are as follows mathematically:

Step 1. Identifying the decision-makers who consensus indexes at the decision matrix level are below the preset threshold γ:

$$\text{inconsistent decision maker } s \,\text{set } (IDMS) = \left\{k \mid CIR^k < \gamma\right\}.$$

Step 2. Identifying the alternatives that the consensus indexes at the alternative level are below the preset threshold γ in IDMS identified from Step 1:

$$\text{inconsistent alternatives set } (IAS) = \{(k,i)\} \mid k \in IDMS \cap CIR_i^k < \gamma\}.$$

Step 3. Identifying the preference values that the consensus indexes at elements level are below the preset threshold γ in IAS:

$$\text{inconsistent preference values set } (IPVS) = \left\{(k,i,j) \mid (k,i) \in IAS \cap CIR_{ij}^k < \gamma\right\}.$$

3.5 Recommendation mechanism based on the minimum adjustment

In those models [33, 37], they set the threshold curve as a circle. And the distance between the points on the ring and the center of the ring is equal. The points outside the ring move in the center several times until they are on or inside the circle. However, they all neglect that the shape of the threshold curve has a close relation to the distance function. The following shows a minimum adjustment model based on threshold indifference curves.

Definition 3.6. *Let the preference values before and after the modification of a DM are* (μ_0, v_0) *and* (μ, v), *then the amount of adjustment is defined based on Euclidean distance:*

$$AD = (\mu_0 - \mu)^2 + (v_0 - v)^2. \tag{16}$$

Then the minimum adjustment model is built as follows:

$$\begin{aligned} &\text{Min}\, AD = (\mu_0 - \mu)^2 + (v_0 - v)^2, \\ &s.t. \begin{cases} \left(\mu^2 - \bar{\mu}^2\right)^2 + \left(v^2 - \bar{v}^2\right)^2 + \left(\mu^2 + v^2 - \bar{\mu}^2 - \bar{v}^2\right)^2 = 2(1-\gamma)^2 \\ 0 \le \mu, v \le 1 \\ (\mu_0, v_0) \in IPVS \end{cases} \end{aligned} \tag{17}$$

At last, the results of the minimum adjustment model are introduced into the visual identification rule graph, as shown in Fig.7.

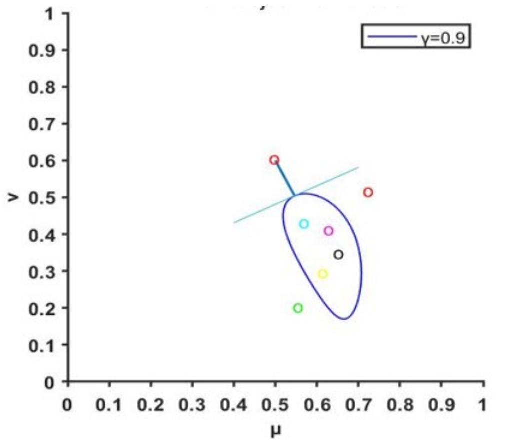


Figure 7: Minimum adjustment model

Furthermore, we can check that the line segment $[(\mu_0, v_0), (\mu, v)]$ and the tangent line of the closed curve $\left(\mu^2 - \bar{\mu}^2\right)^2 + \left(v^2 - \bar{v}^2\right)^2 + \left(\mu^2 + v^2 - \bar{\mu}^2 - \bar{v}^2\right)^2 = 2(1-\gamma)^2$ are perpendicular to each other at the point (μ, v). Therefore, it concludes that point (μ, v) is the preference value point after the minimum adjustment of the point (μ_0, v_0). Since the adjustment value is minimal, decision-makers are more willing to modify their opinions to improve consensus.

When DMs' consensus degrees are below the preset threshold, this mechanism will recommend modifying their opinions in the consensus process. However, when some DMs change preference values, the group decision matrix is not guaranteed to be the same as before in practice. Nonetheless, the actual consensus degrees of those who have revised their preference values will be higher than expected. In other words, the actual outcome will be more favorable than expected.

3.6 Selection process

According to the obtained group decision matrix by modification, the inconsistent preference values and the consensus indexes of all DMs are not lower than the preset threshold. Since each criterion has a corresponding weight, the final TS of each alternative is calculated as follows:

$$\bar{r}_i = \sum_{j=1}^{q} \omega_j \cdot \bar{r}_{ij}. \tag{18}$$

$$TS_{A_i} = \mu_i^2 - v_i^2. \tag{19}$$

Finally, the best alternative is selected according to the TS ordering of all alternatives.

4 Illustrative example

In early 2020, the novel coronavirus (COVID-19) epidemic broke out in Wuhan, China, and many people were infected with the new type of pneumonia and needed emergency treatment. In order to expand the treating capacity, the method of building designated hospitals was adopted in the early stage, but the power was minimal. Therefore, several square cabin hospitals must be established in Wuhan to solve the problem of insufficient beds. Due to the muscular mobility of many mild patients, the square cabin hospital can treat these patients mainly, which is a crucial measure to allow patients to be treated and be isolated at the same time. Moreover, the square cabin hospital can take care of and observe the patients, and once the condition deteriorates, the patients can be transferred to the regular hospital in time. Therefore, forming an orderly hierarchy with the smallest social resources can rapidly expand treatment capacity.

Now the National Health Commission and related units plan to set up five square cabin hospitals $A = \{A_1, A_2, A_3, A_4, A_5\}$ in and around Hongshan District of Wuhan. Their addresses are A_1=Hongshan Gymnasium, A_2=overseas Chinese

Town Primary School Gymnasium, A_3=Wuhan International Convention and Exhibition Center, A_4=Wuhan International Expo Center, and A_5=Wuhan Sports Center. However, due to the limited human resources and material resources, the five square cabin hospitals can only be established one by one. In order to maximize utility, the square cabin hospitals need to be sorted. Five main factors $C = \{C_1, C_2, C_3, C_4, C_5\}$ affecting the square cabin hospital's effectiveness are C_1=the distribution of patients, C_2=the allocation of regional medical and health resources, C_3=traffic conditions, C_4=municipal pipe network communication facilities, and C_5=ecological environment with corresponding importance weight $\omega_C = (0.2, 0, 2, 0, 35, 0, 15, 0, 1)^T$. A group of DMs $D(d_1, d_2, d_3, d_4, d_5)$ from five different departments evaluators offer their own opinions according to the results of each alternative. Since every decision-maker is quarantined because of the epidemic, communication decisions can only be made online (such as Enterprise WeChat). As the environment is uncertain, the DMs group needs to assess the available information by using PFNs. The trust relation matrix between DMs is as follows:

$$A = \begin{pmatrix} - & (0.6, 0.3) & (0.3, 0.6) & (0.5, 0.7) & (0.5, 0.8) \\ (0.7, 0.5) & - & (0.9, 0.2) & (0.8, 0.5) & (0.7, 0.2) \\ (0.6, 0.2) & (0.5, 0.2) & - & (0.6, 0.3) & (0.5, 0.3) \\ (0.4, 0.2) & (0.6, 0.3) & (0.3, 0.4) & - & (0.5, 0.4) \\ (0.6, 0.4) & (0.4, 0.8) & (0.7, 0.6) & (0.5, 0.8) & - \end{pmatrix}.$$

4.1 The calculated results

According to the expression(8), we can compute the degree centrality index $C(d_k)$ of DMs as follows

$$C(d_1) = (0.594, 0.299), C(d_2) = (0.536, 0.346), C(d_3) = (0.684, 0.412), \quad C(d_4) = (0.632, 0.538),$$
$$C(d_5) = (0.565, 0.372).$$

The corresponding TS of DMs are:

$$TS_1 = 0.632, TS_2 = 0.584, TS_3 = 0.649, TS_4 = 0.555, TS_5 = 0.590.$$

By using the BUM function $Q(r) = r^{2/3}$, we can derive the importance weights as follows:

$$\omega_T^1 = 0.21, \omega_T^2 = 0.14, \omega_T^3 = 0.36, \omega_T^4 = 0.13, \omega_T^5 = 0.16.$$

Concerned the decision-making problem, five DMs are invited to evaluate the five possible alternative projects, and then we can obtain the following decision matrixes:

$$R^1 = \begin{pmatrix} (0.6, 0.5) & (0.9, 0.2) & (0.8, 0.1) & (0.5, 0.3) & (0.5, 0.3) \\ (0.9, 0.3) & (0.7, 0.6) & (0.5, 0.8) & (0.6, 0.3) & (0.6, 0.3) \\ (0.6, 0.3) & (0.7, 0.7) & (0.7, 0.3) & (0.7, 0.4) & (0.4, 0.4) \\ (0.7, 0.2) & (0.8, 0.2) & (0.8, 0.4) & (0.6, 0.6) & (0.6, 0.6) \\ (0.8, 0.4) & (0.7, 0.5) & (0.6, 0.2) & (0.7, 0.4) & (0.7, 0.4) \end{pmatrix},$$

$$R^2 = \begin{pmatrix} (0.7, 0.3) & (0.9, 0.2) & (0.8, 0.3) & (0.6, 0.3) & (0.5, 0.4) \\ (0.8, 0.4) & (0.8, 0.6) & (0.6, 0.7) & (0.8, 0.3) & (0.6, 0.5) \\ (0.6, 0.3) & (0.6, 0.6) & (0.7, 0.4) & (0.7, 0.4) & (0.5, 0.4) \\ (0.7, 0.3) & (0.6, 0.4) & (0.9, 0.3) & (0.7, 0.6) & (0.7, 0.1) \\ (0.8, 0.4) & (0.7, 0.5) & (0.7, 0.2) & (0.6, 0.4) & (0.7, 0.3) \end{pmatrix},$$

$$R^3 = \begin{pmatrix} (0.5, 0.6) & (0.9, 0.2) & (0.8, 0.1) & (0.5, 0.3) & (0.4, 0.3) \\ (0.8, 0.6) & (0.7, 0.6) & (0.5, 0.8) & (0.5, 0.5) & (0.6, 0.1) \\ (0.7, 0.4) & (0.7, 0.5) & (0.6, 0.2) & (0.8, 0.3) & (0.5, 0.5) \\ (0.6, 0.1) & (0.8, 0.2) & (0.9, 0.2) & (0.5, 0.6) & (0.6, 0.4) \\ (0.9, 0.2) & (0.5, 0.6) & (0.6, 0.2) & (0.6, 0.1) & (0.7, 0.4) \end{pmatrix},$$

$$R^4 = \begin{pmatrix} (0.6, 0.5) & (0.8, 0.3) & (0.8, 0.2) & (0.6, 0.4) & (0.5, 0.3) \\ (0.8, 0.4) & (0.7, 0.5) & (0.6, 0.7) & (0.7, 0.3) & (0.7, 0.3) \\ (0.3, 0.6) & (0.7, 0.5) & (0.7, 0.5) & (0.8, 0.3) & (0.8, 0.4) \\ (0.7, 0.2) & (0.7, 0.4) & (0.8, 0.4) & (0.6, 0.5) & (0.6, 0.5) \\ (0.5, 0.4) & (0.9, 0.2) & (0.7, 0.3) & (0.6, 0.3) & (0.8, 0.1) \end{pmatrix},$$

$$R^5 = \begin{pmatrix} (0.7,0.4) & (0.8,0.4) & (0.8,0.2) & (0.6,0.5) & (0.6,0.4) \\ (0.8,0.4) & (0.8,0.4) & (0.5,0.7) & (0.7,0.4) & (0.6,0.3) \\ (0.6,0.4) & (0.6,0.5) & (0.6,0.3) & (0.7,0.4) & (0.5,0.6) \\ (0.7,0.3) & (0.6,0.3) & (0.8,0.4) & (0.7,0.5) & (0.5,0.4) \\ (0.8,0.4) & (0.7,0.4) & (0.7,0.3) & (0.7,0.3) & (0.6.0.5) \end{pmatrix}.$$

According to expression (11), we can determine the group decision matrix based on PFNs as follows:

$$\bar{R} = \begin{pmatrix} (0.606,0.480) & (0.878,0.236) & (0.800,0.143) & (0.547,0.338) & (0.488,0.327) \\ (0.828,0.436) & (0.735,0.549) & (0.531,0.755) & (0.644,0.378) & (0.615,0.217) \\ (0.619,0.381) & (0.674,0.550) & (0.653,0.288) & (0.755,0.347) & (0.549,0.463) \\ (0.668,0.176) & (0.743,0.257) & (0.859,0.299) & (0.606,0.569) & (0.603,0.369) \\ (0.828,0.312) & (0.696,0.457) & (0.648,0.225) & (0.642,0.223) & (0.703,0.333) \end{pmatrix}.$$

The consensus indexes at the pair of alternative criterion level are:

$$CIR_{ij}^1 = \begin{pmatrix} 0.982 & 0.966 & 0.990 & 0.935 & 0.985 \\ 0.886 & 0.945 & 0.940 & 0.907 & 0.963 \\ 0.931 & 0.793 & 0.932 & 0.931 & 0.825 \\ 0.951 & 0.922 & 0.912 & 0.967 & 0.778 \\ 0.944 & 0.956 & 0.935 & 0.836 & 0.953 \end{pmatrix}, \quad CIR_{ij}^2 = \begin{pmatrix} 0.868 & 0.966 & 0.930 & 0.947 & 0.940 \\ 0.935 & 0.862 & 0.920 & 0.796 & 0.806 \\ 0.931 & 0.918 & 0.878 & 0.931 & 0.908 \\ 0.911 & 0.833 & 0.928 & 0.856 & 0.874 \\ 0.944 & 0.956 & 0.934 & 0.905 & 0.977 \end{pmatrix},$$

$$CIR_{ij}^3 = \begin{pmatrix} 0.876 & 0.966 & 0.990 & 0.935 & 0.912 \\ 0.847 & 0.945 & 0.940 & 0.855 & 0.951 \\ 0.885 & 0.953 & 0.905 & 0.939 & 0.954 \\ 0.901 & 0.92 & 0.937 & 0.896 & 0.978 \\ 0.892 & 0.794 & 0.935 & 0.921 & 0.953 \end{pmatrix}, \quad CIR_{ij}^4 = \begin{pmatrix} 0.982 & 0.882 & 0.980 & 0.908 & 0.985 \\ 0.935 & 0.912 & 0.920 & 0.933 & 0.862 \\ 0.738 & 0.953 & 0.794 & 0.939 & 0.685 \\ 0.951 & 0.917 & 0.912 & 0.922 & 0.888 \\ 0.592 & 0.718 & 0.904 & 0.953 & 0.871 \end{pmatrix}.$$

$$CIR_{ij}^5 = \begin{pmatrix} 0.893 & 0.880 & 0.980 & 0.826 & 0.845 \\ 0.935 & 0.874 & 0.900 & 0.915 & 0.963 \\ 0.980 & 0.871 & 0.937 & 0.931 & 0.872 \\ 0.911 & 0.818 & 0.912 & 0.893 & 0.896 \\ 0.944 & 0.953 & 0.904 & 0.896 & 0.863 \end{pmatrix}.$$

Then the consensus indexes at the level of the alternative are:

$$CIR_j^1 = (0.972, 0.928, 0.882, 0.906, 0.925);\ CIR_j^2 = (0.930, 0.864, 0.913, 0.880, 0.943),$$
$$CIR_j^3 = (0.936, 0.908, 0.927, 0.927, 0.899);\ CIR_j^4 = (0.947, 0.912, 0.822, 0.918, 0.808),.$$
$$CIR_j^5 = (0.885, 0.917, 0.918, 0.886, 0.912).$$

At last, the consensus indexes at the decision matrix level are:

$$CIR^1 = 0.923, CIR^2 = 0.906, CIR^3 = 0.919, CIR^4 = 0.882, CIR^5 = 0.904.$$

According to Lemma 3.4, the threshold value sets at 0.9, and the DM d_4 needs to modify some of his original preference values by using the visual identification rule. Once the preference values are imported into the graph, the DM d_4 can see the IPVS as follows:

$$IPVS = \{(4,3,1), (4,3,3), (4,3,5), (4,5,1), (4,5,2), (4,5,5)\}.$$

Step 3. The original preference points of DM corresponding to IPVS are respectively:

$$(0.3, 0.6), (0.7, 0.5), (0.8, 0.4), (0.5, 0.4), (0.9, 0.2), (0.8, 0.1).$$

The preference points in the corresponding group decision matrix are respectively:

$$(0.619, 0.381), (0.653, 0.288), (0.549, 0.463), (0.828, 0.312), (0.696, 0.457), (0.703, 0.333).$$

These preference points are respectively put into the constructed minimum adjustment model for calculation. Then the modified points and the corresponding minimum adjustment values are:

$$\begin{array}{llll} (0.525, 0.485) & \text{Min } AD = 0.0638; & (0.649, 0.431) & \text{Min } AD = 0.0074; \\ (0.646, 0.397) & \text{Min } AD = 0.0238; & (0.755, 0.396) & \text{Min } AD = 0.0651; \\ (0.757, 0.317) & \text{Min } AD = 0.0341; & (0.771, 0.105) & \text{Min } AD = 8.7187 \times 10^{-4}. \end{array}$$

A new group decision matrix will be obtained after d4 has modified his preference values:

$$\bar{\bar{R}} = \begin{pmatrix} (0.606, 0.480) & (0.878, 0.236) & (0.800, 0.143) & (0.547, 0.338) & (0.488, 0.327) \\ (0.828, 0.436) & (0.735, 0.549) & (0.531, 0.755) & (0.644, 0.378) & (0.615, 0.217) \\ (0.633, 0.371) & (0.674, 0.550) & (0.645, 0.283) & (0.755, 0.347) & (0.507, 0.462) \\ (0.668, 0.176) & (0.743, 0.257) & (0.859, 0.299) & (0.606, 0.569) & (0.603, 0.369) \\ (0.841, 0.311) & (0.654, 0.486) & (0.648, 0.225) & (0.642, 0.223) & (0.698, 0.335) \end{pmatrix}.$$

At the same time, the consensus results at the new decision matrix level are updated as:

$$CIR^1 = 0.922, \quad CIR^2 = 0.905, \ CIR^3 = 0.923, \quad CIR^4 = 0.916, \ CIR^5 = 0.902.$$

All the consensus degrees are above the threshold value $\gamma = 0.9$, which means the selection process is activated to derive the group solution of consensus.

The collective overall evaluation values of alternatives are calculated according to the expression (18) with the factor weights $\omega_C = (0.2, 0, 2, 0, 35, 0, 15, 0, 1)^T$:

$$\bar{r}_1 = (0.749, 0.248), \quad \bar{r}_2 = (0.684, 0.505), \quad \bar{r}_3 = (0.658, 0.370), \quad \bar{r}_4 = (0.760, 0.294), \quad \bar{r}_5 = (0.707, 0.291).$$

The corresponding TSs of all alternatives are

$$TS_{A_1} = 0.750, TS_{A_2} = 0.606, TS_{A_3} = 0.648, TS_{A_4} = 0.746, TS_{A_5} = 0.708.$$

The DMs reach a high consensus degree and the final ordering: $A_1 > A_4 > A_5 > A_3 > A_2$ the established order of square cabin hospitals is A_1=Hongshan Gymnasium, A_4=Wuhan International Expo Center, A_5=Wuhan Sports Center, A_3=Wuhan International Convention, and Exhibition Center, A_2=overseas Chinese Town Primary School Gymnasium.

4.2 Sensitivity analysis

It is difficult to reach a high degree of consensus in actual emergency decision-making when decision-makers' opinions are quite different. In other words, to achieve a high degree of agreement, each decision-maker may need to adjust their views significantly, which may cause fierce conflicts and make the decision-making process unable to continue. So the threshold can be reduced appropriately after the first aggregation to reach a consensus as soon as possible. For example, assume that the rally point and the consensus indexes are (0.5, 0.5), $CIR^1 = 0.4, CIR^2 = 0.45, CIR^3 = 0.5, CIR^4 = 0.55, CIR^5 = 0.6$ after the first aggregation. We can't use $\gamma = 0.9$ anymore, and the value of γ should be set between 0.57 and 0.78 according to Lemma 3.4 instead of being set subjectively. Setting scientific threshold belongs to strategic application and plays a leading role in actual emergency decision making.

4.3 Comparison and analysis

To further illustrate the proposed approach, we exploit two methods involved in the literature [33] and [43] to calculate the case, respectively.

According to the method involved in the literature [33], we can determine the resultant new collective trust decision-making matrix as follows

$$\bar{\bar{R}} = \begin{pmatrix} (0.606, 0.480) & (0.878, 0.236) & (0.800, 0.143) & (0.547, 0.338) & (0.488, 0.327) \\ (0.828, 0.436) & (0.735, 0.549) & (0.531, 0.755) & (0.644, 0.378) & (0.615, 0.217) \\ (0.628, 0.371) & (0.674, 0.550) & (0.649, 0.280) & (0.755, 0.347) & (0.513, 0.467) \\ (0.668, 0.176) & (0.743, 0.257) & (0.859, 0.299) & (0.606, 0.569) & (0.603, 0.369) \\ (0.835, 0.307) & (0.663, 0.488) & (0.648, 0.225) & (0.642, 0.223) & (0.694, 0.368) \end{pmatrix}.$$

At the same time, we can obtain the consensus results of the new decision matrix as follows: $CIR^1 = 0.923, \quad CIR^2 = 0.905, \quad CIR^3 = 0.923, \quad CIR^4 = 0.908, \quad CIR^5 = 0.903$, and then the corresponding TSs of all alternatives are $TS_{A_1} = 0.750, TS_{A_2} = 0.606, TS_{A_3} = 0.649.TS_{A_4} = 0.746, TS_{A_5} = 0.706$. So, the final ordering is $A_1 > A_4 > A_5 > A_3 > A_2$.

According to the method involved in the literature [43], we get the following individuals' decision matrixes $\bar{R}_1^1 = \bar{R}_1^0 = R_1, \quad \bar{R}_2^1 = \bar{R}_2^0 = R_2, \quad \bar{R}_3^1 = \bar{R}_3^0 = R_3, \quad \bar{R}_5^1 = \bar{R}_5^0 = R_5$, and then $\bar{R}_4^1$ is as follows:

$$\bar{R}_4^1 = \begin{pmatrix} (0.600, 0.500) & (0.806, 0.301) & (0.800, 0.200) & (0.600, 0.400) & (0.500, 0.300) \\ (0.800, 0.400) & (0.700, 0.500) & (0.600, 0.700) & (0.700, 0.300) & (0.687, 0.295) \\ (0.413, 0.543) & (0.700, 0.500) & (0.675, 0.466) & (0.800, 0.300) & (0.723, 0.399) \\ (0.700, 0.200) & (0.700, 0.400) & (0.800, 0.400) & (0.600, 0.500) & (0.598, 0.495) \\ (0.628, 0.398) & (0.829, 0.259) & (0.700, 0.300) & (0.600, 0.300) & (0.786, 0.103) \end{pmatrix}.$$

And then, we can calculate the consensus indexes, which are respectively, $CIR^1 = 0.923, CIR^2 = 0.906,\quad CIR^3 = 0.922,\quad CIR^4 = 0.900,\quad CIR^5 = 0.903$, and then we can determine the corresponding TSs of all alternatives as follows:

$$TS_{A_4} = 0.750, TS_{A_2} = 0.606, TS_{A_3} = 0.648, TS_{A_4} = 0.746, TS_{A_5} = 0.708.$$

And then, the final ordering: $A_1 > A_4 > A_5 > A_3 > A_2$.

Compared with the literature's proposed method [33], although their final result is the same, this model's threshold indifference curve is more practical. It does not need to set feedback parameters and does not require multiple consensus process. Through the model's operation in the opinion after the minimum adjustment directly, the paper [33] needs to judge the value of feedback parameters δ subjectively. What's more, all the preference value points of DM d_4 take the same feedback parameters, while each preference value has different recommendations. The proposed consensus model in this paper can directly reflect the graph's consensus problem and quickly get the consensus result with the least adjustment of opinion, avoiding multiple negotiations, and saving time.

Compared with the literature method [43], the proposed method in this paper can help DMs see their position all the time. What's more, based on the literature method [43], the consensus index of DM d_4 exceeded the threshold at the first adjustment, and it need more times to adjust preference.

In conclusion, compared with other methods, the proposed method in this paper has the following advantages. (1) We define the threshold indifference curves to systematically analyze how to determine an appropriate threshold, which the current study didn't consider. (2) The minimum adjustment model based on threshold indifference curves reduces the number of opinions adjustment and improves the consensus efficiency. And it doesn't need a feedback parameter. (3) The consensus model can directly reflect the position of inconsistent experts in consensus adjustment.

5 Conclusions

To cover the large and heterogeneous user bases and address some shortcomings in SNGDM, we construct a Pythagorean fuzzy group consensus model with a minimum adjustment consensus model by introducing a Pythagorean fuzzy preference structure. According to the analysis of results, the proposed approach can preferably describe and deal with SNGDM problems. The conclusions are as follows:

(1) Introducing Pythagorean fuzzy numbers. Compared with intuitionistic fuzzy sets, Pythagorean fuzzy sets' decision space is more extensive to better express network users' fuzzy preferences. (2) Trust scores reflect the importance of decision-makers in social network relations. The higher the PFNs based trust score, the more critical the decision maker's opinion. (3) The threshold indifference curve can help DMs determine a reasonable threshold strategically, which provides practical significance for the urgent decision in reality. (4) Unifying the distance measures, the proposed approach can clearly express the position of each DMs' preference in the consensus process and help DMs well judge whether to modify their opinions and improve group consensus. Based on the proposed approach, the results meet the preset threshold and minimize the decision-makers adjustment amount, and do not need multiple adjustments.

However, there still some optimization works need to do in the future. For example, DMs do may not fully know each other in real trust network. So, how to determine the propagator of PFNs is worthy of studying. Besides, the model mainly focuses on minimizing the amount of adjustment. it ignores individual differences in unit adjustment costs, and then it is necessary to develop a way to measure unit adjustment costs in SNGDM.

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References

[1] D. B. Arieh, T. Easton, B. Evans, *Minimum cost consensus with quadratic cost functions*, IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans, **39**(1) (2008), 210-217.

[2] K. T. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and System, **20** (1986), 87-96.

[3] R. E. Bellman, L. A. Zadeh, *Decision-making in a fuzzy environment*, Management Science, **17** (1970), 141-164.

[4] Q. W. Cao, L. F. Dai, Q. Sun, *A distributed trust based online evaluation under social network*, Control and Decision, 2019.

[5] X. Chen, Z. G. Ding, Y. C. Dong, H. M. Liang, *Managing consensus with minimum adjustments in group decision making with opinions evolution*, IEEE Transactions on Systems, Man, and Cybernetics: Systems, **51**(4) (2019), 2299-2311.

[6] D. Ciucci, *Ortho pairs: A simple and widely used way to model uncertainty*, Fundamenta Informaticae, **108**(3-4) (2011), 287-304.

[7] Y. C. Dong, Y. F. Xu, H. Y. Li, B. Feng, *The OWA-based consensus operator under linguistic representation models using position indexes*, European Journal of Operational Research, **203**(2) (2010), 455-463.

[8] Y. C. Dong, Q. B. Zha, H. J. Zhang, G. K. H. Fujita, F. Chiclana, E. Herrera-Viedma, *Consensus reaching in social network group decision making: Research paradigms and challenges*, Knowledge-Based Systems, **162** (2018), 3-13.

[9] Y. C. Dong, Q. B. Zha, H. J. Zhang, F. Herrera, *Consensus reaching and strategic manipulation ingroup decision making with trust relations*, IEEE Transactions on Systems, Man and Cybernetics: Systems, **99** (2019), 1-15.

[10] Y. C. Dong, H. J. Zhang, E. Herrera-Viedma, *Integrating experts' weights generated dynamically into the consensus reaching process and its applications in managing non-cooperative behaviors*, Decision Support Systems, **84**(4) (2016), 1-15.

[11] Y. C. Dong, S. H. Zhao, H. J. Zhang, F. Chiclana, E. Herrera-Viedma, *A self-management mechanism for noncooperative behaviors in large-scale group consensus reaching processes*, IEEE Transactions on Fuzzy Systems, **26**(6) (2018), 3276-3288.

[12] X. J. Gou, Z. S. Xu, Q. Lei, *New operational laws and aggregation method of intuitionistic fuzzy information*, Journal of Intelligent and Fuzzy Systems, **30**(1) (2016), 20-21.

[13] X. J. Gou, Z. S. Xu, H. C. Liao, F. Herrera, *Consensus model handling minority opinions and noncooperative behaviors in large-scale group decision-making under double hierarchy linguistic preference relations*, IEEE Transactions on Cybernetics, **51**(1) (2020), 283-296.

[14] X. J. Gou, Z. S. Xu, W. Zhou, *Managing consensus by multi-stage optimization models with linguistic preference orderings and double hierarchy linguistic preferences*, Technological and Economic Development of Economy, **26**(3) (2020), 642-674.

[15] R. A. Hanneman, M. Riddle, *Introduction to social network methods*, University of California, Riverside, 2005.

[16] E. Herrera-Viedma, L. Martnez, F. Mata, F. Chiclana, *A consensus support system model for group decision-making problems with multigranular linguistic preference relations*, IEEE Transactions on Fuzzy Systems, **13**(5) (2005), 644-658.

[17] C. Huang, M. W. Lin, Z. S. Xu, *Pythagorean fuzzy MULTIMOORA method based on distance measure and score function: Its application in multicriteria decision making process*, Knowledge and Information Systems, **62**(11) (2020), 4373-4406.

[18] M. W. Lin, Y. Q. Chen, R. Q. Chen, *Bibliometric analysis on Pythagorean fuzzy sets during 2013-2020*, International Journal of Intelligent Computing and Cybernetics, **14**(2) (2020), 104-121.

[19] M. W. Lin, C. Huang, R. Q. Chen, H. Fujita, X. Wang, *Directional correlation coefficient measures for Pythagorean fuzzy sets: Their applications to medical diagnosis and cluster analysis*, Complex and Intelligent Systems, **7**(2) (2021), 1-19.

[20] X. Peng, Y. Yang, *Some results for Pythagorean fuzzy sets*, International Journal of Intelligent Systems, **30**(11) (2015), 1133-1160.

[21] L. G. Perez, F. Mata, F. Chiclana, *Social network decision making with linguistic trustworthiness-based induced OWA operators*, International Journal of Intelligent Systems, **29**(12) (2014), 1117-1137.

[22] P. J. Ren, Z. S. Xu, X. J. Gou, *Pythagorean fuzzy TODIM approach to multi-criteria decision making*, Applied Soft Computing, **42** (2016), 246-259.

[23] R. M. Rodríguez, L. Labella, G. D. Tré, *A large scale consensus reaching process managing group hesitation*, Knowledge-Based Systems, **159** (2018), 86-97.

[24] R. Ronald, R. Yager, *IFSA world congress and NAFIPS annual meeting*, Pythagorean Fuzzy Subsets, **2013** (2013), 57-61.

[25] Y. L. Sun, W. Yu, Z. Han, K. J. R. Liu, *Information theoretic framework of trust modeling and evaluation for ad hoc networks*, IEEE Journal on Selected Areas in Communications, **24**(2) (2006), 305-317.

[26] J. F. Tian, Z. M. Zhang, M. H. Ha, *An additive-consistency and consensus-based approach for uncertain group decision making with linguistic preference relations*, IEEE Transactions on Fuzzy Systems, **27**(5) (2018), 873-887.

[27] P. Victor, C. Cornelis, M. D. Cock, E. Herrera-Viedma, *Practical aggregation operators for gradual trust and distrust*, Fuzzy Sets and Systems, **84**(1) (2011), 126-147.

[28] P. Victor, C. Cornelis, M. D. Cock, P. P. da Silva, *Pinheiro da Silva, Gradual trust and distrust in recommender systems*, Fuzzy Sets and System, **160**(10) (2009), 367-1382.

[29] S. Wasserman, K. Faust, *Social network analysis: Methods and applications*, Cambridge University Press, 2009.

[30] G. W. Wei, M. Lu, *Pythagorean fuzzy power aggregation operators in multiple attribute decision making*, International Journal of Intelligent Systems, **33**(1) (2018), 169-186.

[31] J. Wu, J. Chang, Q. Cao, C. Y. Liang, *A trust propagation and collaborative filtering-based method for incomplete information in social network group decision making with type-2 linguistic trust*, Computers and Industrial Engineering, **127** (2018), 853-864.

[32] J. Wu, F. Chiclana, *Visual information feedback mechanism and attitudinal prioritization method for group decision making with triangular fuzzy complementary preference relations*, Information Sciences, **279** (2014), 716-734.

[33] J. Wu, F. Chiclana, H. Fujita, E. Herrera-Viedma, *A visual interaction consensus model for social network group decision making with trust propagation*, Knowledge-Based Systems, **122** (2017), 39-50.

[34] J. Wu, F. Chiclana, E. Herrera-Viedma, *Trust based consensus model for social network in an incomplete linguistic information context*, Applied Soft Computing, **35** (2015), 827-839.

[35] J. Wu, M. J. Cobo, K. Samuylov, *Consensus in group decision making and social networks*, Studies in Information and Control, **26**(3) (2017), 259-268.

[36] J. Wu, L. Dai, F. Chiclana, H. Fujita, E. Herrera-Viedmae, *A minimum adjustment cost feedback mechanism-based consensus model for group decision making under social network with distributed linguistic trust*, Information Fusion, **41** (2018), 232-242.

[37] J. Wu, R. Xiong, F. Chiclana, *Uninorm trust propagation and aggregation methods for group decision making in social network with four tuple information*, Knowledge-Based Systems, **96** (2016), 29-39.

[38] Z. B. Wu, J. P. Xu, *Managing consistency and consensus in group decision making with hesitant fuzzy linguistic preference relations*, Omega, **65** (2016), 28-40.

[39] R. R. Yager, *Quantifier guided aggregation using OWA operators*, International Journal of Intelligent Systems, **11**(1) (1996), 49-73.

[40] S. Zeng, *Pythagorean fuzzy multi-attribute group decision making with probabilistic information and OWA approach*, International Journal of Intelligent Systems, **32**(11) (2017), 1136-1150.

[41] S. Zeng, J. Chen, X. Li, *A hybrid method for Pythagorean fuzzy multiple-criteria decision making*, International Journal of Information Technology and Decision Making, **15**(2) (2016), 403-422.

[42] X. L. Zhang, *Multicriteria Pythagorean fuzzy decision analysis: A hierarchical QUALIFLEX approach with the closeness index-based ranking methods*, Information Sciences, **330** (2016), 104-124.

[43] B. Zhang, Y. C. Dong, *Consensus rules with minimum adjustments for multiple attribute group decision making*, Procedia Computer Science, **17**(1) (2013), 473-481.

[44] N. Zhang, Z. W. Gong, F. Chiclana, *Minimum cost consensus models based on random opinions*, Expert Systems with Applications, **89** (2017), 149-159.

[45] X. Zhang, Z. S. Xu, *Extension of TOPSIS to multiple criteria decision making with Pythagorean fuzzy sets*, International Journal of Intelligent Systems, **29**(12) (2014), 1061-1078.

[46] Y. F. Zheng, J. Xu, *A trust transitivity model for group decision making in social network with intuitionistic fuzzy information*, Faculty of Science and Mathematics, **32**(5) (2018), 1937-1945.