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Spherical fuzzy soft sets: Theory and aggregation operator with its applications

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Abstract

The aim of this paper is to redefine the notion of spherical fuzzy soft sets as a more general concept to make them more functional for solving multi-criteria decision-making problems. We first define the set operations under the new spherical fuzzy soft set environment and obtain some fundamental properties of them. Then, we construct the spherical fuzzy soft aggregation operator which allows establishing a more efficient and useful method to solve the multi-criteria decision-making problems. We establish an algorithm for the decision-making process which is more useful, simple, and easier than the existing methods. After constructing the method for solving the decision-making problem, we give a numerical example based on linguistic terms to show that the validity of the proposed technique. Finally, we analyze the reliability of the results of this method with the help of the comparative studies by applying this to a real-time data set and using the existing methods.

Keywords: Spherical fuzzy sets, spherical fuzzy soft sets, aggregation operator, multi-criteria decision-making problem.

1 Introduction

As we all know, since the fuzzy set theory has been introduced in 1965 by Zadeh detailed in [43], many researchers have concentrated on this theory by applying this theory to different fields of science such that image processing, data mining, engineering, medical sciences, clustering, statistical information theory and etc. But since a fuzzy set has only a membership (positive-membership) degree, it has some restrictions in expressing uncertain data. For this reason, the fuzzy set has been extended to many new types of fuzzy sets in various consideration: Type-2 fuzzy sets by Zadeh [44]; Interval-valued fuzzy sets by Sambuc [31], Zadeh, Jahn and Grattan-Guinness; Intuitionistic fuzzy sets by Atanassov [6]; Fuzzy multi-sets by Yager [40]; Neutrosophic sets by Smarandache [33]; Non-stationary fuzzy sets by Garibaldi and Ozen [15]; Hesitant fuzzy sets by Torra [34]; Pythagorean fuzzy sets by Yager [41]; Picture fuzzy sets by Cuong [10] and Spherical fuzzy sets by Kahraman and Kutlu Gündoğdu [19].

Another extension of the fuzzy set theory is the soft set theory introduced by Molodtsov [27] to deal with the uncertainties and vagueness of many problems that arise in medical science, engineering, social science, economics and etc. This theory has been studied in various areas with significant applications (see [7, 13, 14, 26, 28]). After, with the combination of the extension of fuzzy sets which are mentioned above and soft sets, many different set theories have been obtained: Fuzzy soft sets by Maji et al. [25] and Çagman et al. [11]; Intuitionistic fuzzy soft sets by Çagman and Karataş [12]; Hesitant fuzzy soft sets by Babitha and John [8]; Pythagorean fuzzy soft sets by Peng et al. [29]; Picture fuzzy soft sets by Yang et al. [42]; Spherical fuzzy soft sets by Perveen et al. [30].

If we return to the real-life, the work of scientists, managers, lawyers, engineers that steers the course of society is largely making decisions and solving problems. Decision-making is very important not only mentioned at the level of business organizations but also at the level of our individual lives (buying a house, choosing a school or a career). During the past half-century, major research gains have been made in understanding problem-solving and decision making in

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many fields such as psychology, economics, mathematical statistics, operations research, political science, artificial intelligence, and cognitive science. In a lot of decision-making problems, we may face various types of uncertainties. To handle these situations, appropriate sets of data and methods have been used by following the developments in science. The first application of the fuzzy set theory to decision-making problems was given by Bellman and Zadeh [9] in 1970. After this paper, a lot of studies have been made with two types of methods, one is the traditional decision-making methods such as AHP, VIKOR, WASPAS and another is established on some operators which aggregate the information. In the point of aggregation operators, many theories and applications were improved in recent years by considering the new types of fuzzy sets: For instance, Xu [39] presented the arithmetic weighted aggregation operators for intuitionistic fuzzy numbers. Wei [36, 37] utilized arithmetic and geometric operations to develop some picture fuzzy aggregation operators, also proposed picture fuzzy Hamacher aggregation operators and picture fuzzy Hamacher geometric aggregation operators with applications to enterprise resource planning. Ashraf and Abdullah [2] extended different strict Archimedean triangular norms and triangular conorms to aggregate the spherical fuzzy information and also defined some spherical aggregation operators and applied these operators to multi-criteria group decision-making problems. Different types of aggregation operators for spherical fuzzy sets can be found in [5, 17, 18, 23, 35]. The traditional decision-making methods in spherical fuzzy environment was given in [20–22]. Yang et al. [42] introduced an algorithm based on adjustable soft discernibility matrix by using the level soft set of a picture fuzzy soft set to solve decision-making problems, which can find an order relation of all the objects. Perveen et al. [30] proposed an algorithm based on adjustable soft discernibility matrix to solve the decision-making problem in spherical fuzzy soft environment. Guleria and Bajaj [16] initiated some aggregation operators to merge the spherical fuzzy soft information. Ahmmad et al. [1] presented some average aggregation operators in spherical fuzzy soft environment and their applications in multi-criteria decision process.

In the present work, we redefine the notion of spherical fuzzy soft sets as a more general concept from the definition of [30] to make them more functional and efficient when solving multi-criteria decision-making problems. The new concept of spherical fuzzy soft sets is larger than the concept of spherical fuzzy soft sets given by Perveen et al. [30]. Also, we initiate the set operations under the spherical fuzzy soft set environment and obtain some fundamental properties of them by defining the null sf-set and universal sf-set. Then, we present the spherical fuzzy soft aggregation operator which allows establishing a more efficient and easier method to solve the multi-criteria decision-making problems. After introducing the notion of spherical fuzzy soft aggregation operators, we propose an algorithm based on linguistic terms that is more useful and practical for the decision-making process than the existing methods. Then, we give a numerical example to clarify the proposed technique. Finally, we compare the results of this method with using a real-time data set and the existing methods.

2 Preliminaries

In this section, we yield some fundamental concepts concerned with soft sets, fuzzy soft sets and spherical fuzzy sets which will be necessary for the main sections. Throughout this paper, U will denote an initial universe, E refers to a non-empty parameters set and suppose that $A \subset E$, unless otherwise specified.

Definition 2.1. [27] *A pair (F, E) is called a soft set (s-set) over U if F is a mapping of E into the set of all subsets of the set U (i.e., $F : E \to \mathscr{P}(U)$). Every set $F(e)$ for all $e \in E$, from this family may be considered as the set of $e-$elements of the soft set (F, E), or as the set of $e-$approximate elements of the soft set.*

Definition 2.2. [24] *Let I^U denote the family of all fuzzy sets (f-sets) $f : U \to I$. A pair (F, E) is called a fuzzy soft set (fs-set) over U if $F : E \to I^U$ is a function.*

Obviously, an s-set (F, E) over U can be thought as an fs-set by using the characteristic function of the set $F(e)$:

$$F(e)(a) = \begin{cases} 1, & \text{if } a \in F(e); \\ 0, & \text{otherwise.} \end{cases}$$

Definition 2.3. [4, 10, 19] *Let $\mu : U \to [0, 1]$, $\iota : U \to [0, 1]$ and $\nu : U \to [0, 1]$ be three mappings. A set $S = \{< u, \mu(u), \iota(u), \nu(u) > | u \in U\}$ is called a*

(i) picture fuzzy set (pf-set) over U if $0 \leq \mu(u) + \iota(u) + \nu(u) \leq 1$ for all $u \in U$.

(ii) spherical fuzzy set (sf-set) over U if $0 \leq \mu^2(u) + \iota^2(u) + \nu^2(u) \leq 1$ for all $u \in U$.

The values $\mu(u), \iota(u), \nu(u) \in [0, 1]$ denote the degree of positive-membership, neutral-memberhip and negative-membership of u to S, respectively. We will denote by $PF(U)$ and $SF(U)$ with the set of all pf-sets and sf-sets over U, respectively.

Definition 2.4. [42] *A pair (F, E) is called a picture fuzzy soft set (pfs-set) over U if $F : E \to PF(U)$ is a function.*

Remark 2.5. *It is clear that the family of all sf-sets over U contains the family of all pf-set over U and the family of all pf-set over contains the family of all U f-sets over U. Also, the family of pfs-sets over U contains the family of all fs-sets over U and the family of fs-sets over U contains the family of all s-sets over U.*

We give the definitions of null sf-set and universal sf-set similar to the f-sets as follows:

Definition 2.6. *Let $S \in SF(U)$.*
(1) *S is said to be a null sf-set denoted by $\bar{0}$ if $S = \{< u, 0, 0, 1 >: u \in U\}$.*
(2) *S is said to be a universal sf-set denoted by $\bar{U}$ if $S = \{< u, 1, 0, 0 >: u \in U\}$.*

The set operations under sf-sets are given by Ashraf et al. [4] as follows:

Definition 2.7. *[4] For the sf-sets $S_1 = \{< u, \mu_1(u), \iota_1(u), \nu_1(u) >: u \in U\}$ and $S_2 = \{< u, \mu_2(u), \iota_2(u), \nu_2(u) >: u \in U\}$, the set operations are defined as follows:*
(1) *$S_1 \sqsubseteq S_2$ if $\mu_1(u) \le \mu_2(u)$, $\iota_1(u) \le \iota_2(u)$ and $\nu_1(u) \ge \nu_2(u)$ for all $u \in U$.*
(2) *$S_1 = S_2$ if and only if $S_1 \sqsubseteq S_2$ and $S_2 \sqsubseteq S_1$.*
(3) *The union of S_1 and S_2 is denoted by $S_1 \sqcup S_2$ and defined by*

$$S_1 \sqcup S_2 = \{< u, max\{\mu_1(u), \mu_2(u)\}, min\{\iota_1(u), \iota_2(u)\}, min\{\nu_1(u), \nu_2(u)\} >: u \in U\}.$$

(4) *The intersection of S_1 and S_2 is denoted by $S_1 \sqcap S_2$ and defined by*

$$S_1 \sqcap S_2 = \{< u, min\{\mu_1(u), \mu_2(u)\}, min\{\iota_1(u), \iota_2(u)\}, max\{\nu_1(u), \nu_2(u)\} >: u \in U\}.$$

Lemma 2.8. *(i) $\bar{0} \sqsubseteq S$ for all $S \in SF(U)$.*
(ii) $S \sqsubseteq \bar{U}$ for all $S \in SF(U)$.
(iii) $S \sqcup \bar{U} = \bar{U}$ for all $S \in SF(U)$.
(iv) $S \sqcap \bar{0} = \bar{0}$ for all $S \in SF(U)$.

Definition 2.9. [3] *Let $S \in SF(U)$ where $S = \{< u, \mu(u), \iota(u), \nu(u) > |u \in U\}$. Then the complement of S is denoted by S^c and defined by $S^c = \{< u, \nu(u), \iota(u), \mu(u) > |u \in U\}$.*

Remark 2.10. *Let $S \in SF(U)$. Then $S \sqcup S^c \neq \bar{S}$ and $S \sqcap S^c \neq \bar{0}$.*

The following example is given to illustrate the above remark:

Example 2.11. *Let $U = \{u_1, u_2\}$ and $S = \{< u_1, 0.2, 0.3, 0.1 >, < u_2, 0.5, 0.3, 0.8 >\}$. Then, we obtain the followings*

$$\begin{aligned} S^c &= \{< u_1, 0.1, 0.3, 0.2 >, < u_2, 0.8, 0.3, 0.5 >\} \\ S \sqcup S^c &= \{< u_1, 0.2, 0.3, 0.1 >, < u_2, 0.8, 0.3, 0.5 >\} \\ S \sqcap S^c &= \{< u_1, 0.1, 0.3, 0.2 >, < u_2, 0.5, 0.3, 0.8 >\} \end{aligned}$$

which mean that $S \sqcup S^c \neq \bar{S}$ and $S \sqcap S^c \neq \bar{0}$.

The following definitions are recollected to use for solving multi-criteria decision-making problems as seen in the last sections.

Definition 2.12. *Let $S \in SF(U)$ where $S = \{< u, \mu(u), \iota(u), \nu(u) > |u \in U\}$. The triplet $< \mu(u), \iota(u), \nu(u) >$ is called a $u-element$ of the sf-set S. For simplicity, if $\mu, \iota, \nu \in [0, 1]$ and $\mu^2 + \iota^2 + \nu^2 \le 1$, then the triplet $< \mu, \iota, \nu >$ denotes a spherical fuzzy number (sf-number). We will denote by $SFN(U)$ with the set of all sf-numbers over U.*

Definition 2.13. [4] *Let $S =< \mu, \iota, \nu >, S_1 =< \mu_1, \iota_1, \nu_1 >$ and $S_2 =< \mu_2, \iota_2, \nu_2 >$ be three sf-numbers and $a \ge 0$. Then the operations are defined over the sf-numbers as follows:*
(i) $S_1 \oplus S_2 =< \sqrt{\mu_1^2 + \mu_2^2 - \mu_1^2\mu_2^2}, \iota_1\iota_2, \nu_1\nu_2 >$,
(ii) $S_1 \odot S_2 =< \mu_1\mu_2, \iota_1\iota_2, \sqrt{\nu_1^2 + \nu_2^2 - \nu_1^2\nu_2^2} >$,
(iii) $aS =< \sqrt{1 - (1 - \mu^2)^a}, \iota^a, \nu^a >$,
(iv) $S^a < \mu^a, \iota^a, \sqrt{1 - (1 - \nu^2)^a} >$.

Definition 2.14. [4] *Let $S =< \mu, \iota, \nu >$ be an sf-number.*
(*i*) *A score function $SC : SFN(U) \to [0,1]$ is defined as $SC(S) = \frac{2+\mu-\iota-\nu}{3}$.*
(*ii*) *An accuracy function $AC : SFN(U) \to [-1,1]$ is defined as $AC(S) = \mu - \nu$.*

Definition 2.15. [4] *Let $S_1 =< \mu_1, \iota_1, \nu_1 >$ and $S_2 =< \mu_2, \iota_2, \nu_2 >$ be two sf-numbers. Then the ranking method (comparison technique) as follows:*
(*i*) *If $SC(S_1) < SC(S_2)$, then $S_1 < S_2$,*
(*ii*) *If $SC(S_1) > SC(S_2)$, then $S_1 > S_2$,*
(*iii*) *$SC(S_1) = SC(S_2)$, then*
(*a*) *$AC(S_1) < AC(S_2)$, then $S_1 < S_2$,*
(*b*) *$AC(S_1) > AC(S_2)$, then $S_1 > S_2$,*
(*c*) *$AC(S_1) = AC(S_2)$, then $S_1 = S_2$.*

3 sfs-Sets

In this section, we redefine the notion of spherical fuzzy soft sets in which the parameters set is crisp and the approximation set is sf-set over U by giving their operations.

Definition 3.1. *Let $\delta_A : E \to SF(U)$ be a mapping such that $\delta_A(e) = \bar{0}$ whenever $e \notin A$. Then the set of ordered pairs*

$$\Delta_A = \{(e, \delta_A(e)) : e \in E, \delta_A(e) \in SF(U)\},$$

is called a spherical fuzzy soft set (sfs-set) over U. Here, δ_A is called the spherical fuzzy approximation function, $\delta_A(e)$ is a spherical fuzzy set called e-element of the sfs-set Δ_A and denoted by

$$\delta_A(e) = \{< u, \mu_{\delta_A(e)}(u), \iota_{\delta_A(e)}(u), \nu_{\delta_A(e)}(u) >: u \in U\}.$$

We will denote by $SFS(U)$ with the set of all sfs-sets over U.

If $U = \{u_1, u_2, ..., u_n\}$, $E = \{e_1, e_2, ..., e_m\}$ and $A \subset E$, then $\Delta_A \in SFS(U)$ can be represented by the following table:

Δ_A	e_1	$\dots$	e_m
u_1	$< \mu_{\delta_A(e_1)}(u_1), \iota_{\delta_A(e_1)}(u_1), \nu_{\delta_A(e_1)}(u_1) >$	$\dots$	$< \mu_{\delta_A(e_m)}(u_1), \iota_{\delta_A(e_m)}(u_1), \nu_{\delta_A(e_m)}(u_1) >$
u_2	$< \mu_{\delta_A(e_1)}(u_2), \iota_{\delta_A(e_1)}(u_2), \nu_{\delta_A(e_1)}(u_2) >$	$\dots$	$< \mu_{\delta_A(e_m)}(u_2), \iota_{\delta_A(e_m)}(u_2), \nu_{\delta_A(e_m)}(u_2) >$
$\vdots$	$\vdots$	$\dots$	$\vdots$
u_n	$< \mu_{\delta_A(e_1)}(u_n), \iota_{\delta_A(e_1)}(u_n), \nu_{\delta_A(e_1)}(u_n) >$	$\dots$	$< \mu_{\delta_A(e_m)}(u_n), \iota_{\delta_A(e_m)}(u_n), \nu_{\delta_A(e_m)}(u_n) >$

If $a_{ij} =< \mu_{\delta_A(e_j)}(u_i), \iota_{\delta_A(e_j)}(u_i), \nu_{\delta_A(e_j)}(u_i) >$ for $i = 1, 2..., n$ and $j = 1, 2, ..., m$, then the sfs-set Δ_A is represented by the following matrix

$$\Delta_A = [a_{ij}]_{n\times m} = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{pmatrix}, \quad (1)$$

which is called a matrix form of the sfs-set Δ_A.

Example 3.2. *Let $U = \{u_1, u_2, u_3\}$, $E = \{e_1, e_2, e_3, e_4\}$ and $A = \{e_1, e_3, e_4\}$. Take $\delta_A(e_1) = \{< u_1, 0.2, 0.3, 0.5 >, < u_2, 1, 0, 0 >, < u_3, 0.8, 0.1, 0.5 >\}$, $\delta_A(e_3) = \{< u_1, 0.8, 0.5, 0.4 >, < u_2, 0.9, 0.2, 0.1 >, < u_3, 0.2, 0.7, 0.8 >\}$ and $\delta_A(e_4) = \{< u_1, 0.8, 0.4, 0.2 >, < u_2, 0.3, 0.5, 0.7 >, < u_3, 0.4, 0.5, 0.5 >\}$, then the sfs-set Δ_A is written as follows:*

$$\begin{aligned} \Delta_A = \; & \{(e_1, \{< u_1, 0.2, 0.3, 0.5 >, < u_2, 1, 0, 0 >, < u_3, 0.8, 0.1, 0.5 >\}), \\ & (e_2, \{< u_1, 0, 0, 1 >, < u_2, 0, 0, 1 >, < u_3, 0, 0, 1 >\}), \\ & (e_3, \{< u_1, 0.8, 0.5, 0.4 >, < u_2, 0.9, 0.2, 0.1 >, < u_3, 0.2, 0.7, 0.8 >\}), \\ & (e_4, \{< u_1, 0.8, 0.4, 0.2 >, < u_2, 0.3, 0.5, 0.7 >, < u_3, 0.4, 0.5, 0.5 >\})\}. \end{aligned}$$

The matrix form of the sfs-set Δ_A is given by

$$M_{\Delta_A} = \begin{pmatrix} <0.2,0.3,0.5> & <0,0,1> & <0.8,0.5,0.4> & <0.8,0.4,0.2> \\ <1,0,0> & <0,0,1> & <0.9,0.2,0.1> & <0.3,0.5,0.7> \\ <0.8,0.1,0.5> & <0,0,1> & <0.2,0.7,0.8> & <0.4,0.5,0.5> \end{pmatrix}.$$

Definition 3.3. *Let $\Delta_A \in SFS(U)$.*

(1) *Δ_A is called a null sfs-set denoted by Δ_Φ if $\delta_A(e) = \bar{0}$ for all $e \in E$.*

(2) *Δ_A is called an A-universal sfs-set denoted by $\Delta_{\widetilde{A}}$ if $\delta_A(e) = \bar{U}$ for all $e \in A$. If $A = E$, then the A-universal sfs-set is called universal sfs-set and denoted by $\Delta_{\widetilde{E}}$.*

Definition 3.4. *Let $\Delta_A, \Delta_B \in SFS(U)$. Δ_A is said to be an sfs-subset of Δ_B denoted by $\Delta_A \widetilde{\sqsubseteq} \Delta_B$ if $\delta_A(e) \sqsubseteq \delta_B(e)$ for all $e \in E$. i.e., $\mu_{\delta_A(e)}(u) \leq \mu_{\delta_(e)}(u)$, $\iota_{\delta_A(e)}(u) \leq \iota_{\delta_B(e)}(u)$ and $\nu_{\delta_A(e)}(u) \geq \nu_{\delta_B(e)}(u)$ for all $u \in U$. Δ_A and Δ_B are said to be sfs-equal denoted by $\Delta_A = \Delta_B$ if $\Delta_A \widetilde{\sqsubseteq} \Delta_B$ and $\Delta_B \widetilde{\sqsubseteq} \Delta_A$.*

Proposition 3.5. *Let $\Delta_A, \Delta_B, \Delta_C \in SFS(U)$. Then the followings are satisfied:*

(*i*) $\Delta_\Phi \widetilde{\sqsubseteq} \Delta_A$,
(*ii*) $\Delta_A \widetilde{\sqsubseteq} \Delta_{\widetilde{A}}$,
(*iii*) $\Delta_{\widetilde{A}} \widetilde{\sqsubseteq} \Delta_{\widetilde{E}}$,
(*iv*) *If* $\Delta_A \widetilde{\sqsubseteq} \Delta_B$ *and* $\Delta_B \widetilde{\sqsubseteq} \Delta_B$, *then* $\Delta_A \widetilde{\sqsubseteq} \Delta_C$.

Definition 3.6. *Let $\Delta_A \in SFS(U)$. The complement of Δ_A is written as $\Delta_A^c \in SFS(U)$ and defined by $\Delta_A^c = \{(e, \delta_A^c(e)) : e \in E, \delta_A^c(e) \in SF(U)\}$ where $\delta_A^c(e) = (\delta_A(e))^c$ for all $e \in E$.*

Proposition 3.7. *Let $\Delta_A \in SFS(U)$. Then the followings are satisfied:*

(*i*) $(\Delta_A^c)^c = \Delta_A$,
(*ii*) $\Delta_\Phi^c = \Delta_{\widetilde{E}}$.

Proof. The proofs can be easily obtained by using the spherical fuzzy approximation function of the sfs-sets Δ_Φ, Δ_A and $\Delta_{\widetilde{E}}$. □

Definition 3.8. *Let $\Delta_A, \Delta_B \in SFS(U)$.*

(1) *The union of Δ_A and Δ_B is the sfs-set Δ_C defined by its approximation function such as $\delta_C(e) = \delta_A(e) \sqcup \delta_B(e)$ for all $e \in E$. We will denote it by $\Delta_C = \Delta_A \widetilde{\sqcup} \Delta_B$.*

(2) *The intersection of Δ_A and Δ_B is the sfs-set Δ_C defined by its approximation function such as $\delta_C(e) = \delta_A(e) \sqcap \delta_B(e)$ for all $e \in E$. We will denote it by $\Delta_C = \Delta_A \widetilde{\sqcap} \Delta_B$.*

Proposition 3.9. *Let $\Delta_A, \Delta_B, \Delta_C \in SFS(U)$. Then the followings are satisfied:*

(*i*) $\Delta_A \widetilde{\sqcup} \Delta_A = \Delta_A$,
(*ii*) $\Delta_A \widetilde{\sqcup} \Delta_\Phi = \Delta_A$,
(*iii*) $\Delta_A \widetilde{\sqcup} \Delta_{\widetilde{E}} = \Delta_{\widetilde{E}}$,
(*iv*) $\Delta_A \widetilde{\sqcup} \Delta_B = \Delta_B \widetilde{\sqcup} \Delta_A$,
(*v*) $(\Delta_A \widetilde{\sqcup} \Delta_B) \widetilde{\sqcup} \Delta_C = \Delta_A \widetilde{\sqcup} (\Delta_B \widetilde{\sqcup} \Delta_C)$,
(*vi*) $\Delta_A \widetilde{\sqcap} \Delta_A = \Delta_A$,
(*vii*) $\Delta_A \widetilde{\sqcap} \Delta_\Phi = \Delta_\Phi$,
(*viii*) $\Delta_A \widetilde{\sqcap} \Delta_{\widetilde{E}} = \Delta_A$,
(*ix*) $\Delta_A \widetilde{\sqcap} \Delta_B = \Delta_B \widetilde{\sqcap} \Delta_A$,
(*x*) $(\Delta_A \widetilde{\sqcap} \Delta_B) \widetilde{\sqcap} \Delta_C = \Delta_A \widetilde{\sqcap} (\Delta_B \widetilde{\sqcap} \Delta_C)$,
(*xi*) $\Delta_A \widetilde{\sqcup} (\Delta_B \widetilde{\sqcap} \Delta_C) = (\Delta_A \widetilde{\sqcup} \Delta_B) \widetilde{\sqcap} (\Delta_A \widetilde{\sqcup} \Delta_C)$,
(*xii*) $\Delta_A \widetilde{\sqcap} (\Delta_B \widetilde{\sqcup} \Delta_C) = (\Delta_A \widetilde{\sqcap} \Delta_B) \widetilde{\sqcup} (\Delta_A \widetilde{\sqcap} \Delta_C)$,

Proof. The proofs of these properties can be shown easily by using Definition 3.8. □

Remark 3.10. *Let $\Delta_A \in SFS(U)$. Then $\Delta_A \widetilde{\sqcup} \Delta_A^c \neq \Delta_{\widetilde{E}}$ and $\Delta_A \widetilde{\sqcap} \Delta_A^c \neq \Delta_\Phi$.*

Example 3.11. *Let $U = \{u_1, u_2\}$, $E = \{e_1, e_2, e_3\}$ and $A = \{e_1, e_2\}$. Take the sfs-set $\Delta_A = \{(e_1, \{<u_1, 0.1, 0.2, 0.7>, <u_2, 0.8, 0.3, 0.4>\}), (e_2, \{<u_1, 0.5, 0.3, 0.6>, <u_2, 0.7, 0.5, 0.1>\})\}$. Then, we obtain*

$$\begin{aligned} \Delta_A \widetilde{\sqcup} \Delta_A^c &= \{(e_1, \{<u_1, 0.7, 0.2, 0.1>, <u_2, 0.8, 0.3, 0.4>\}), (e_2, \{<u_1, 0.6, 0.3, 0.5>, <u_2, 0.7, 0.5, 0.1>\})\} \\ \Delta_A \widetilde{\sqcap} \Delta_A^c &= \{(e_1, \{<u_1, 0.1, 0.2, 0.7>, <u_2, 0.4, 0.3, 0.8>\}), (e_2, \{<u_1, 0.5, 0.3, 0.6>, <u_2, 0.1, 0.5, 0.7>\})\}. \end{aligned}$$

Hence, we see that $\Delta_A \widetilde{\sqcup} \Delta_A^c \neq \Delta_{\widetilde{E}}$ and $\Delta_A \widetilde{\sqcap} \Delta_A^c \neq \Delta_\Phi$.

Proposition 3.12. *Let $\Delta_A, \Delta_B, \Delta_C \in SFS(U)$. Then the followings are satisfied:*
(i) $(\Delta_A \tilde{\sqcup} \Delta_B)^c = \Delta_A^c \tilde{\sqcap} \Delta_B^c$,
(ii) $(\Delta_A \tilde{\sqcap} \Delta_B)^c = \Delta_A^c \tilde{\sqcup} \Delta_B^c$.

Proof. (i) For all $e \in E$, we have

$$\begin{aligned}\delta^c_{A\cup B}(e) &= (\delta_A(e) \sqcup \delta_B(e))^c = \{< u, max\{\mu_{\delta_A(e)}, \mu_{\delta_B(e)}\}, min\{\iota_{\delta_A(e)}, \iota_{\delta_B(e)}\}, min\{\nu_{\delta_A(e)}, \nu_{\delta_B(e)}\} >\}^c \\ &= \{< u, min\{\nu_{\delta_A(e)}, \nu_{\delta_B(e)}\}, min\{\iota_{\delta_A(e)}, \iota_{\delta_B(e)}\}, max\{\mu_{\delta_A(e)}, \mu_{\delta_B(e)}\} >\} = \delta^c_A(e) \sqcap \delta^c_B(e),\end{aligned}$$

which means that $(\Delta_A \tilde{\sqcup} \Delta_B)^c = \Delta_A^c \tilde{\sqcap} \Delta_B^c$.
(ii) The proof can be shown similarly to the above. □

4 Application of the sfs-sets to the multi-criteria decision-making problems

4.1 sfs-Aggregation operators

In this subsection, we define the spherical fuzzy soft aggregation operator which allows solving the multi-criteria decision-making problems with an easier method.

Definition 4.1. *Let $\Delta_A \in SFS(U)$. The cardinal set of Δ_A is denoted by $c\Delta_A$ and defined by $c\Delta_A = \{< e, \mu_{c\Delta_A}(e), \iota_{c\Delta_A}(e), \nu_{c\Delta_A}(e) >: e \in E\}$. Here, $c\Delta_A$ is an sf-set over E and the positive-membership function $\mu_{c\Delta_A}$, the neutral-membership function $\iota_{c\Delta_A}$ and the negative-membership function $\nu_{c\Delta_A}$ of $c\Delta_A$ are defined as follows:*

$$\mu_{c\Delta_A} : E \to [0,1], \mu_{c\Delta_A}(e) = \frac{|\mu_{\delta_A(e)}|}{|U|}, \tag{2}$$

where $|\mu_{\delta_A(e)}|$ is the scalar cardinality of the f-set $\mu_{\delta_A(e)}$. i.e., $|\mu_{\delta_A(e)}| = \sum_{u_i \in U} \mu_{\delta_A(e)}(u_i)$.

$$\iota_{c\Delta_A} : E \to [0,1], \iota_{c\Delta_A}(e) = \frac{|\iota_{\delta_A(e)}|}{|U|}, \tag{3}$$

where $|\iota_{\delta_A(e)}|$ is the scalar cardinality of the f-set $\iota_{\delta_A(e)}$. i.e., $|\iota_{\delta_A(e)}| = \sum_{u_i \in U} \iota_{\delta_A(e)}(u_i)$.

$$\nu_{c\Delta_A} : E \to [0,1], \nu_{c\Delta_A}(e) = \frac{|\nu_{\delta_A(e)}|}{|U|}, \tag{4}$$

where $|\nu_{\delta_A(e)}|$ is the scalar cardinality of the f-set $\nu_{\delta_A(e)}$. i.e., $|\nu_{\delta_A(e)}| = \sum_{u_i \in U} \nu_{\delta_A(e)}(u_i)$.
We will denote by $cSFS(U)$ with the set of all cardinal sets of the sfs-sets over U. It is obvious that $cSFS(U) \subseteq SFS(E)$.

Definition 4.2. *Let $\Delta_A \in SFS(U)$ and $c\Delta_A$ be the cardinal set of Δ_A. Then the aggregate sf-set of the sfs-set Δ_A is denoted by Δ_A^* and defined by $\Delta_A^* = \{< u, \mu^*_{\Delta_A}(u), \iota^*_{\Delta_A}(u), \nu^*_{\Delta_A}(u) >: u \in U\}$. The positive-membership function $\mu^*_{\Delta_A}$, the neutral-membership function $\iota^*_{\Delta_A}$ and the negative-membership function $\nu^*_{\Delta_A}$ of ${\Delta_A}^*$ are defined as follows:*

$$\mu^*_{\Delta_A} : U \to [0,1], \mu^*_{\Delta_A}(u) = \sqrt{1 - \prod_{e\in E}\left(1 - \mu^2_{c\Delta_A}(e)\mu^2_{\delta_A(e)}(u)\right)^{\frac{1}{|E|}}} \tag{5}$$

$$\iota^*_{\Delta_A} : U \to [0,1], \iota^*_{\Delta_A}(u) = \left(\prod_{e\in E} \iota_{c\Delta_A}(e)\iota_{\delta_A(e)}(u)\right)^{\frac{1}{|E|}} \tag{6}$$

$$\nu^*_{\Delta_A} : U \to [0,1], \nu^*_{\Delta_A}(u) = \left(\prod_{e\in E} \sqrt{\nu_{c\Delta_A}(e)^2 + \nu_{\delta_A(e)}(u)^2 - \nu_{c\Delta_A}(e)^2\nu_{\delta_A(e)}(u)^2}\right)^{\frac{1}{|E|}} \tag{7}$$

If $U = \{u_1, u_2, ..., u_n\}$, then the aggregate sf-set of $\Delta_A \in SFS(U)$ can be represented by the following table,

Δ_A^*	$<\mu^*_{\Delta_A}, \iota^*_{\Delta_A}, \nu^*_{\Delta_A}>$
u_1	$<\mu^*_{\Delta_A}(u_1), \iota^*_{\Delta_A}(u_1), \nu^*_{\Delta_A}(u_1)>$
u_2	$<\mu^*_{\Delta_A}(u_2), \iota^*_{\Delta_A}(u_2), \nu^*_{\Delta_A}(u_2)>$
$\vdots$	$\vdots$
u_n	$<\mu^*_{\Delta_A}(u_n), \iota^*_{\Delta_A}(u_n), \nu^*_{\Delta_A}(u_n)>$

If $a_{i1} =<\mu_{\Delta_A}(u_i), \iota_{\Delta_A}(u_i), \nu_{\Delta_A}(u_i)>$ *for* $i = 1, 2..., n$, *then the sf-set* Δ_A^* *is represented by the following matrix*

$$\Delta_A^* = [a_{i1}]_{n\times 1} = \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{n1} \end{pmatrix} \tag{8}$$

which is called a matrix form of the aggregate sf-set Δ_A^*.

Definition 4.3. *Let* $\Delta_A \in SFS(U)$ *and* Δ_A^* *be the aggregate sf-set of* Δ_A. *Define an operator* $SFS_{agg} : cSFS(U) \times SFS(U) \to SF(U)$ *as* $SFS_{agg}(c\Delta_A, \Delta_A) = \Delta_A^*$. *Then* SFS_{agg} *is said to be a sfs-aggregation operator.*

Theorem 4.4. *Let* $\Delta_A \in SFS(U)$. *Suppose that* $M_{\Delta_A}, M_{c\Delta_A}$ *and* $M_{\Delta_A^*}$ *represent the matrices of* $\Delta_A, c\Delta_A$ *and* Δ_A^*. *Then*

$$|E|M_{\Delta_A^*} = M_{\Delta_A} \odot M_{c\Delta_A}^T \tag{9}$$

where $|E|$ *is the cardinality of* E *and* $M_{c\Delta_A}^T$ *is the transposition of* $M_{c\Delta_A}$.

Proof. It is clear from $[a_{i1}]_{n\times 1} = [a_{ij}]_{n\times m} \odot [a_{1j}]_{1\times m}^T$. □

4.2 An application

In this subsection, we aim to give an application of the sfs-aggregation operators which satisfies a solution for the multi-criteria decision-making problems. We present an algorithm for multi-criteria decision-making problems in the spherical fuzzy soft environment. The proposed method is shown with the frame diagram (Figure 1) to clarify the organization of this method. Then, we give an illustrative example by using the proposed algorithm.

Step I: Let the decision-maker establish the sfs-set Δ_A over U by filling the decision and criteria evaluation matrices using the linguistic terms given in Table 1. Then from this table, construct the matrix form of the sfs-set $\Delta_A = [a_{ij}]_{n\times m}$ as given in Eq.(1).

Table 1: Corresponding spherical fuzzy numbers to linguistic terms

Linguistic terms	$<\mu, \iota, \nu>$
Absolutely Low Importance (ALI)	$<0.1, 0.9, 0.1>$
Very Low Importance (VLI)	$<0.2, 0.8, 0.2>$
Low Importance (LI)	$<0.3, 0.7, 0.3>$
Slightly Low Importance (SLI)	$<0.4, 0.6, 0.4>$
Equally Importance (EI)	$<0.5, 0.5, 0.5>$
Slightly More Importance (SMI)	$<0.6, 0.4, 0.4>$
High Importance (HI)	$<0.7, 0.3, 0.3>$
Very High Importance (VHI)	$<0.8, 0.2, 0.2>$
Absolutely More Importance (AMI)	$<0.9, 0.1, 0.1>$

Step II: Since the multi-criteria decision-making problem may have some benefit and cost types criteria, the information given by decision-maker is normalized in the following way:

$$s_{ij} = \begin{cases} a_{ij}, & \text{for benefit criteria} \\ (a_{ij})^c, & \text{for cost criteria} \end{cases}$$

for all $i = 1, 2, .., n$ and $j = 1, 2, ..., m$, where $(a_{ij}^{(r)})^c$ is the complement of $a_{ij}^{(r)}$. Then the normalized matrix of the sfs-set Δ_A is constructed as $\Delta'_A = [s_{ij}]_{n\times m}$

Step III: Find the cardinal set $c\Delta_A$ of Δ'_A by using Eq.(2), Eq.(3) and Eq.(4).

Step IV: Find the aggregate sf-set Δ^*_A of Δ'_A by using Eq.(9).

Step V: Calculate the score values of the aggregate sf-set Δ^*_A.

Step VI: Rank the alternatives and select the optimum alternative which has the maximum score value.

The method explained above is shown with the frame diagram (Figure 1) as follows:

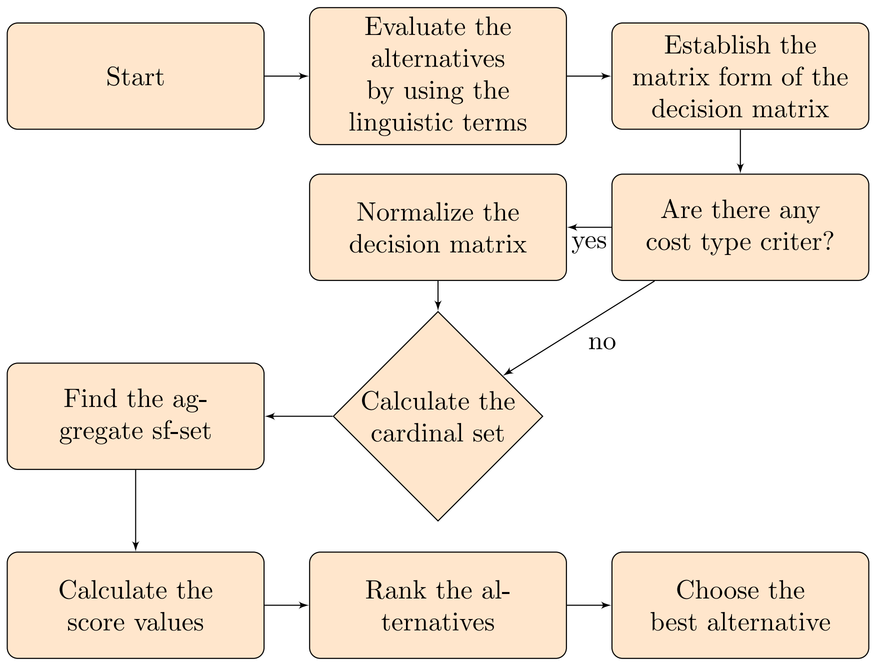

Figure 1: Flow chart of the proposed method

Example 4.5. *Suppose that a lawyer company chooses a trainee. There are four candidates applied for this position. Let us show the candidates as u_1, u_2, u_3, u_4 with the set of alternatives $U = \{u_1, u_2, u_3, u_4\}$. The committee will select a person in these candidates by evaluating them according to education (e_1), computer knowledge (e_2), good speaking (e_3) and age (e_4) criteria. After some interviews with the candidates, the committee evaluates the candidates from the restrict view according to a chosen subset $A = \{e_1, e_2, e_3\}$ of E.*

Step I: *The committee establish the sfs-set Δ_A over U by filling the decision and criteria evaluation matrices using the linguistic terms given in Table 1 and shown in Table 2.*

Table 2: sfs-set Δ_A established by commitee

Δ_A	e_1	e_2	e_3
u_1	AMI	LI	HI
u_2	VLI	SMI	AMI
u_3	ALI	AMI	EI
u_4	SLI	AMI	VLI

From here, we can show this sfs-set as matrix form as follows:

$$M_{\Delta_A} = \begin{pmatrix} <0.9, 0.1, 0.1> & <0.3, 0.7, 0.3> & <0.7, 0.3, 0.3> & <0, 0, 1> \\ <0.2, 0.8, 0.2> & <0.6, 0.4, 0.4> & <0.9, 0.1, 0.1> & <0, 0, 1> \\ <0.1, 0.9, 0.1> & <0.9, 0.1, 0.1> & <0.5, 0.5, 0.5> & <0, 0, 1> \\ <0.4, 0.6, 0.4> & <0.9, 0.1, 0.1> & <0.2, 0.8, 0.2> & <0, 0, 1> \end{pmatrix}.$$

Step II: *Since all criteria are benefit types criteria, this step is skipped.*

Step III: *The cardinal set $c\Delta_A$ of Δ_A is calculated as follows:*

$$c\Delta_A = \{(e_1, < 0.4, 0.6, 0.2 >), (e_2, < 0.675, 0.325, 0.225 >), (e_3, < 0.575, 0.425, 0.275 >)\}.$$

Step IV: *The aggregate sf-set Δ_A^* of Δ_A is found by using Eq.(9) as follows:*

$$\begin{aligned} M_{\Delta_A^*} &= \frac{1}{4}\left[\begin{pmatrix} <0.9,0.1,0.1> & <0.3,0.7,0.3> & <0.7,0.3,0.3> & <0,0,1> \\ <0.2,0.8,0.2> & <0.6,0.4,0.4> & <0.9,0.1,0.1> & <0,0,1> \\ <0.1,0.9,0.1> & <0.9,0.1,0.1> & <0.5,0.5,0.5> & <0,0,1> \\ <0.4,0.6,0.4> & <0.9,0.1,0.1> & <0.2,0.8,0.2> & <0,0,1> \end{pmatrix} \odot \begin{pmatrix} <0.4,0.6,0.2> \\ <0.675,0.325,0.225> \\ <0.575,0.425,0.275> \\ <0,1,1> \end{pmatrix}\right] \\ &= \begin{pmatrix} <0.2924,0.2042,0.4254> \\ <0.3419,0.2269,0.4377> \\ <0.3579,0.2471,0.4170> \\ <0.3427,0.2511,0.4362> \end{pmatrix}. \end{aligned}$$

Step V: *If we calculate the score values of the aggregate sf-set Δ_A^*, then we have that $SC(< 0.2924, 0.2042, 0.4254 >) = 0.5542$, $SC(< 0.3419, 0.2269, 0.4377 >) = 0.5591$, $SC(< 0.3579, 0.2471, 0.4170 >) = 0.5646$, $SC(< 0.3427, 0.2511, 0.4362 >) = 0.5518$.*

Step VI: *Since the ranking order of score values is that $SC(u_3) > SC(u_2) > SC(u_1) > SC(u_4)$, the best alternative is u_3. So, u_3 is the best candidate for the company.*

5 Comparative study

In this section, we analyze the authenticity of results with the help of the comparative studies by using a real-time data set and the existing methods given in [1, 16, 30, 42].

5.1 By using a real-time data set

The WISDM Smartphone and Smartwatch Activity and Biometrics Dataset, which is taken in UCI Machine Learning Repository data sets, includes activity data from 51 participants from an activity recognition project. Each participant performed each of the 18 activities for 3 minutes and the sensor data (accelerometer and gyroscope for smartphone and smartwatch) was recorded at a rate of 20 Hz.

In this project, they considered the accelerometer and gyroscope sensor on both the smartphone and smartwatch and determined which combination of sensors (phone-accel, phone-gyro, watch-accel, and watch-gyro, phone:phone-accel+phone-gyro, watch:watch-accel+watch-gyro, accels:phone-accel+watch-accel, gyros:phone-gyro+watch-gyro, all: phone-accel+phone-gyro+watch-accel+watch-gyro) performs best. For this aim, 18 diverse activities of daily living (walking, jogging, stairs, sitting, standing, kicking, dribbling, catch, typing, writing, clapping, teeth, folding, pasta, soup, sandwich, chips, drinking) were evaluated for their biometric efficacy and, unlike most other studies, biometric identification was evaluated in addition to biometric authentication. The detailed authentication results for the Random Forest algorithm, where decisions are based on a single 10-second example, are provided in Table 3 [38].

Results are provided for each of the eighteen activities and each of the nine sensor combinations. The relative value of each of the nine sensor configurations can be determined by comparing the values in the different columns.

Let $U = \{u_1, u_2, ..., u_9\}$ and $A = \{e_1, e_2, ..., e_{18}\}$ where $u_1 = phone - accel$, $u_2 = phone - gyro$, $u_3 = watch - accel$, $u_4 = watch - gyro$, $u_5 = phone$, $u_6 = watch$,$u_7 = accels$, $u_8 = gyros$, $u_9 = all$, $e_1 = walking$, $e_2 = jogging$, $e_3 = stairs$,$e_4 = sitting$,$e_5 = standing$,$e_6 = kicking$,$e_7 = dribbling$,$e_8 = catch$, $e_9 = typing$,$e_{10} = writing$,$e_{11} = clapping$,$e_{12} = teeth$,$e_{13} = folding$, $e_{14} = pasta$,$e_{15} = soup$,$e_{16} = sandwich$,$e_{17} = chips$ and $e_{18} = drinking$.

We now, apply the steps to solve this decision-making problem as follows:

Step I: The values in Table 3 are transformed to the fuzzy value by using the formula 5.2 given in [32] and the decision matrix is obtained as in the Table 4.

Step II: Since all criteria are benefit types criteria, this step is skipped.

Table 3: Authentication EER using a one 10-s example

Activity	*Phone accel*	*Phone gyro*	*Watch accel*	*Watch gyro*	Phone	Watch	Accels	Gyros	All
Walking	11.2	11.3	17.5	18.8	9.3	16.1	12.6	10.2	7.9
Jogging	11.5	13.2	18.1	19.3	10.3	15.1	11.3	13.8	9.8
Stairs	12.3	16.4	24.3	26.1	11.8	21.6	13.9	16.5	13.5
Sitting	13.6	26.3	21.8	33.4	12.8	22.3	10.7	27.2	13
Standing	14.7	26	22.6	33.3	15.6	23	11.9	27.9	15.4
Kicking	12.5	18.5	21.8	26.7	11.5	21.1	13.8	16.7	14
Dribbling	12.2	19.9	18.9	21	12.7	17.9	11.2	15.7	12
Catch	10.8	20.3	20.6	20.8	13.4	16.7	12.1	17.2	12.2
Typing	11.5	19.4	16.8	26.2	11.3	18	10.4	19	8.7
Writing	13.3	19.4	15.3	27.1	12.3	15.6	11.2	18.5	10.8
Clapping	11.3	20.5	15.8	20.8	11.7	19.2	9.7	14.6	10.6
Teeth	11.8	19.7	18.6	22.7	12.7	17.2	11.4	19.9	12.2
Folding	11.4	16.6	19.6	24.7	12.3	17.1	8.3	17	10.9
Pasta	12.4	23	18.4	28.8	14.4	20.4	12.3	22.6	10.9
Soup	9.6	22.4	17.6	24.6	10.1	17.5	8.6	21.7	9.8
Sandwich	11.4	22.6	24.1	30.2	10.4	22.1	10.1	23.6	12.3
Chips	12.3	23.3	19.2	29.5	11.7	20.3	11.3	20.4	10.2
Drinking	12	24.2	20	30.1	12.9	20.1	11.8	19.7	12.4

Table 4: sfs-set Δ_A of the fuzzy authentication EER using a one 10-s example

Δ_A	e_1	e_2	e_3	e_4	e_5	e_6
u_1	$<0.7058,0,0>$	$<0.6966,0,0>$	$<0.6569,0,0>$	$<0.5982,0,0>$	$<0.5487,0,0>$	$<0.6479,0,0>$
u_2	$<0.7014,0,0>$	$<0.6163,0,0>$	$<0.4737,0,0>$	$<0.1464,0,0>$	$<0.1529,0,0>$	$<0.3865,0,0>$
u_3	$<0.4271,0,0>$	$<0.4025,0,0>$	$<0.1939,0,0>$	$<0.2671,0,0>$	$<0.2420,0,0>$	$<0.2671,0,0>$
u_4	$<0.3746,0,0>$	$<0.3553,0,0>$	$<0.1507,0,0>$	$<0.0451,0,0>$	$<0.0459,0,0>$	$<0.1380,0,0>$
u_5	$<0.7864,0,0>$	$<0.7448,0,0>$	$<0.6792,0,0>$	$<0.6344,0,0>$	$<0.5086,0,0>$	$<0.6926,0,0>$
u_6	$<0.4867,0,0>$	$<0.5308,0,0>$	$<0.2736,0,0>$	$<0.2512,0,0>$	$<0.2301,0,0>$	$<0.2903,0,0>$
u_7	$<0.6434,0,0>$	$<0.7014,0,0>$	$<0.5847,0,0>$	$<0.7276,0,0>$	$<0.6748,0,0>$	$<0.5892,0,0>$
u_8	$<0.7490,0,0>$	$<0.5892,0,0>$	$<0.4694,0,0>$	$<0.1281,0,0>$	$<0.1151,0,0>$	$<0.4608,0,0>$
u_9	$<0.8408,0,0>$	$<0.7658,0,0>$	$<0.6028,0,0>$	$<0.6253,0,0>$	$<0.5175,0,0>$	$<0.5802,0,0>$
	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}
u_1	$<0.6614,0,0>$	$<0.7233,0,0>$	$<0.6926,0,0>$	$<0.6118,0,0>$	$<0.7014,0,0>$	$<0.6792,0,0>$
u_2	$<0.3329,0,0>$	$<0.3183,0,0>$	$<0.3515,0,0>$	$<0.3515,0,0>$	$<0.3112,0,0>$	$<0.3403,0,0>$
u_3	$<0.3707,0,0>$	$<0.3077,0,0>$	$<0.4566,0,0>$	$<0.5219,0,0>$	$<0.4999,0,0>$	$<0.3825,0,0>$
u_4	$<0.2938,0,0>$	$<0.3007,0,0>$	$<0.1486,0,0>$	$<0.1300,0,0>$	$<0.3007,0,0>$	$<0.2390,0,0>$
u_5	$<0.6389,0,0>$	$<0.6073,0,0>$	$<0.7014,0,0>$	$<0.6569,0,0>$	$<0.6837,0,0>$	$<0.6389,0,0>$
u_6	$<0.4106,0,0>$	$<0.4608,0,0>$	$<0.4066,0,0>$	$<0.5086,0,0>$	$<0.3592,0,0>$	$<0.4396,0,0>$
u_7	$<0.7058,0,0>$	$<0.6658,0,0>$	$<0.7405,0,0>$	$<0.7058,0,0>$	$<0.7700,0,0>$	$<0.6970,0,0>$
u_8	$<0.5042,0,0>$	$<0.4396,0,0>$	$<0.3669,0,0>$	$<0.3865,0,0>$	$<0.5532,0,0>$	$<0.3329,0,0>$
u_9	$<0.6703,0,0>$	$<0.6614,0,0>$	$<0.8104,0,0>$	$<0.7233,0,0>$	$<0.7319,0,0>$	$<0.6614,0,0>$
	e_{13}	e_{14}	e_{15}	e_{16}	e_{17}	e_{18}
u_1	$<0.6970,0,0>$	$<0.6524,0,0>$	$<0.7741,0,0>$	$<0.6970,0,0>$	$<0.6569,0,0>$	$<0.6703,0,0>$
u_2	$<0.4651,0,0>$	$<0.2301,0,0>$	$<0.2481,0,0>$	$<0.2420,0,0>$	$<0.2213,0,0>$	$<0.1966,0,0>$
u_3	$<0.3440,0,0>$	$<0.3905,0,0>$	$<0.4230,0,0>$	$<0.1992,0,0>$	$<0.3592,0,0>$	$<0.3292,0,0>$
u_4	$<0.1837,0,0>$	$<0.0999,0,0>$	$<0.1862,0,0>$	$<0.0794,0,0>$	$<0.0892,0,0>$	$<0.0807,0,0>$
u_5	$<0.6569,0,0>$	$<0.5621,0,0>$	$<0.7532,0,0>$	$<0.7405,0,0>$	$<0.6837,0,0>$	$<0.6299,0,0>$
u_6	$<0.4439,0,0>$	$<0.3147,0,0>$	$<0.4271,0,0>$	$<0.2575,0,0>$	$<0.3183,0,0>$	$<0.3255,0,0>$
u_7	$<0.8258,0,0>$	$<0.6569,0,0>$	$<0.8143,0,0>$	$<0.7532,0,0>$	$<0.7014,0,0>$	$<0.6792,0,0>$
u_8	$<0.4481,0,0>$	$<0.2420,0,0>$	$<0.2704,0,0>$	$<0.2129,0,0>$	$<0.3147,0,0>$	$<0.3403,0,0>$
u_9	$<0.7189,0,0>$	$<0.7189,0,0>$	$<0.7658,0,0>$	$<0.6569,0,0>$	$<0.7490,0,0>$	$<0.6524,0,0>$

Step III: The cardinal set $c\Delta_A$ of Δ_A is calculated as follows:

$$\begin{aligned} c\Delta_A = \{ & (e_1,<0.6350,0,0>),(e_2,<0.5999,0,0>),(e_3,<0.4353,0,0>),(e_4,<0.3804,0,0>),(e_5,<0.3373,0,0>) \\ & ,(e_6,<0.4503,0,0>),(e_7,<0.5098,0,0>),(e_8,<0.4983,0,0>),(e_9,<0.5194,0,0>),(e_{10},<0.5107,0,0>) \\ & ,(e_{11},<0.5457,0,0>),(e_{12},<0.4901,0,0>),(e_{13},<0.5315,0,0>),(e_{14},<0.4297,0,0>) \\ & ,(e_{15},<0.5180,0,0>),(e_{16},<0.4265,0,0>),(e_{17},<0.4549,0,0>),(e_{18},<0.4338,0,0>)\}. \end{aligned}$$

Step IV: The aggregate sf-set Δ_A^* of Δ_A is found by using Eq.(9) as follows:

$$M_{\Delta_A^*} = \begin{pmatrix} <0.3342,0,0> \\ <0.2013,0,0> \\ <0.1879,0,0> \\ <0.1139,0,0> \\ <0.3360,0,0> \\ <0.2000,0,0> \\ <0.3480,0,0> \\ <0.2253,0,0> \\ <0.3524,0,0> \end{pmatrix}.$$

Step V: We obtain the score values of the aggregate sf-set Δ_A^* as $SC(<0.3342,0,0>) = 0.7781$, $SC(<0.2013,0,0>) = 0.7338$, $SC(<0.1879,0,0>) = 0.7293$, $SC(<0.1139,0,0>) = 0.7046$, $SC(<0.3360,0,0>) = 0.7787$, $SC(<0.2000,0,0>) = 0.7333$, $SC(<0.3480,0,0>) = 0.7827$, $SC(<0.2253,0,0>) = 0.7418$ and $SC(<0.3524,0,0>) = 0.7841$.

Step VI: Since the ranking order of score values is that $SC(u_9) > SC(u_7) > SC(u_5) > SC(u_1) > SC(u_8) > SC(u_2) > SC(u_6) > SC(u_3) > SC(u_4)$, *all* is the best overall configuration. The next best sensor configuration is *accels*. Therefore, one can see easily that the proposed method gives the same result as the method in [38].

5.2 With the existing methods

In this subsection, we consider the different multi-criteria decision-making problems in the picture fuzzy soft and spherical fuzzy soft environments given in [1, 16, 30, 42] and solve these problems with the presented approach. Then, we compare the results under these approaches with the proposed method in this study to show the validity and reliability of this method.

Now, let us take the multi-criteria decision-making problem given in [30] which is related to that a group of associates wants to start a paper producing factory in someplace. Suppose that there are seven plots under consideration as $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$. The associates must take a decision according to criteria set $A = \{e_1, e_2, e_3, e_4, e_5\}$ where e_1 stands for availability of water, e_2 stands for availability of raw materials, e_3 stands for waste management, e_4 stands for transportation facility and e_5 stands for low population density near the plot.

Step I: Consider that the sfs-set Δ_A describes "the suitability of different plots" given in [30] as in the Table 5.

Table 5: Rating values of sfs-set Δ_A

Δ_A	e_1	e_2	e_3	e_4	e_5
u_1	$<0.8,0.1,0.4>$	$<0.5,0.2,0.6>$	$<0.7,0.15,0.3>$	$<0.6,0.25,0.31>$	$<0.55,0.12,0.5>$
u_2	$<0.9,0.1,0.3>$	$<0.7,0.2,0.45>$	$<0.55,0.03,0.12>$	$<0.65,0.15,0.56>$	$<0.8,0.17,0.3>$
u_3	$<0.52,0.02,0.7>$	$<0.7,0.12,0.31>$	$<0.45,0.15,0.5>$	$<0.6,0.2,0.4>$	$<0.75,0.05,0.25>$
u_4	$<0.7,0.05,0.4>$	$<0.45,0.1,0.55>$	$<0.56,0.15,0.49>$	$<0.25,0.05,0.8>$	$<0.51,0.1,0.25>$
u_5	$<0.7,0.1,0.4>$	$<0.3,0.2,0.7>$	$<0.47,0.25,0.56>$	$<0.31,0.15,0.6>$	$<0.8,0.2,0.4>$
u_6	$<0.55,0.25,0.65>$	$<0.81,0.15,0.3>$	$<0.1,0.2,0.8>$	$<0.6,0.05,0.3>$	$<0.65,0.1,0.5>$
u_7	$<0.92,0.1,0.3>$	$<0.2,0.25,0.6>$	$<0.35,0.2,0.7>$	$<0.8,0.12,0.4>$	$<0.75,0.01,0.3>$

Step II: Since all criteria are benefit types criteria, this step is skipped.

Step III: The cardinal set $c\Delta_A$ of Δ_A is calculated as follows:

$$\begin{aligned} c\Delta_A &= \{(e_1,<0.7271,0.0939,0.45>),(e_2,<0.5229,0.1743,0.5014>),(e_3,<0.4543,0.1614,0.4957>) \\ &\quad ,(e_4,<0.5443,0.1386,0.4814>),(e_5,<0.6871,0.1071,0.3571>)\} \end{aligned}$$

Step IV: The aggregate sf-set Δ_A^* of Δ_A is found by using Eq.(9) as follows:

$$M_{\Delta_A^*} = \frac{1}{5}\left[\begin{pmatrix} <0.8,0.1,0.4> & <0.5,0.2,0.6> & <0.7,0.15,0.3> & <0.6,0.25,0.31> & <0.55,0.12,0.5> \\ <0.9,0.1,0.3> & <0.7,0.2,0.45> & <0.55,0.03,0.12> & <0.65,0.15,0.56> & <0.8,0.17,0.3> \\ <0.52,0.02,0.7> & <0.7,0.12,0.31> & <0.45,0.15,0.5> & <0.6,0.2,0.4> & <0.75,0.05,0.25> \\ <0.7,0.05,0.4> & <0.45,0.1,0.55> & <0.56,0.15,0.49> & <0.25,0.05,0.8> & <0.51,0.1,0.25> \\ <0.7,0.1,0.4> & <0.3,0.2,0.7> & <0.47,0.25,0.56> & <0.31,0.15,0.6> & <0.8,0.2,0.4> \\ <0.55,0.25,0.65> & <0.81,0.15,0.3> & <0.1,0.2,0.8> & <0.6,0.05,0.3> & <0.65,0.1,0.5> \\ <0.92,0.1,0.3> & <0.2,0.25,0.6> & <0.35,0.2,0.7> & <0.8,0.12,0.4> & <0.75,0.01,0.3> \end{pmatrix}\right.$$

$$\left.\odot\begin{pmatrix} <0.7271,0.0939,0.45> \\ <0.5229,0.1743,0.5014> \\ <0.4543,0.1614,0.4957> \\ <0.5443,0.1386,0.4814> \\ <0.6871,0.1071,0.3571> \end{pmatrix}\right] = \begin{pmatrix} <0.3970,0.0204,0.5964> \\ <0.4721,0.0143,0.5548> \\ <0.3760,0.0107,0.5930> \\ <0.3292,0.0108,0.6234> \\ <0.3755,0.0226,0.6495> \\ <0.3637,0.0171,0.6479> \\ <0.4531,0.0119,0.6036> \end{pmatrix}.$$

Step V: If we calculate the score values of the aggregate sf-set Δ_A^*, then we have

$$\begin{aligned} &SC(<0.3970,0.0204,0.5964>) = 0.0387, \quad SC(<0.4721,0.0143,0.5548>) = 0.1201, \\ &SC(<0.3760,0.0107,0.5930>) = 0.0241, \quad SC(<0.3292,0.0108,0.6234>) = -0.0213, \\ &SC(<0.3755,0.0226,0.6495>) = 0.0001, \quad SC(<0.3637,0.0171,0.6479>) = -0.0078, \\ &SC(<0.4531,0.0119,0.6036>) = 0.0837. \end{aligned}$$

Step VI: Since the ranking order of score values is that $SC(u_2) > SC(u_7) > SC(u_1) > SC(u_3) > SC(u_5) > SC(u_6) > SC(u_4)$, the best alternative is u_2. When the results are compared with the method given in [30], we see that the best and the after alternative are the same.

To observe the results with the proposed method and the other methods in the papers [1, 16, 42], we follow the same procedures and show the results in Table 6.

From the comparative study with the existing methods shown in Table 6, we conclude the following results:

- The proposed method gives the same results for the best choice of the multi-criteria decision-making problems with different numbers of alternatives and criteria given in the papers [1, 16, 30, 42].
- The graphes, shown in Fig 2, are drawn between alternatives versus ranking measures to represent the outcomes of both methods given by Guleria-Bajaj [16] and Ahmmad et al. [1] with the presented method in this study, respectively, that depicts the relationship and ranking of alternatives. The graph implies that the not only the best alternative are same but the ranking lists are also same.

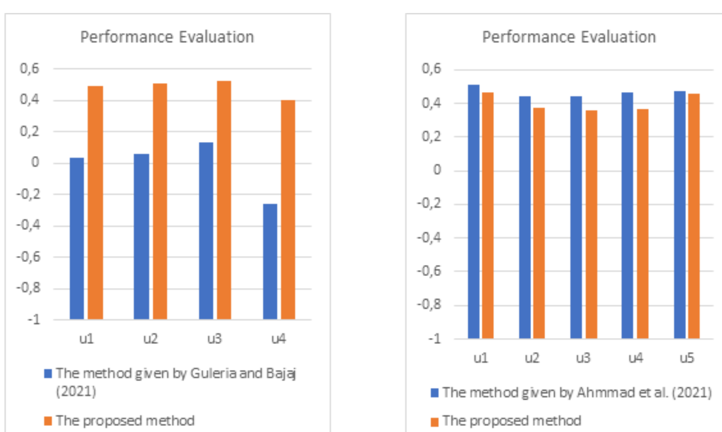

Figure 2: Performance Evaluation

As a consequence of comparison, the proposed method gives reliable and appropriate results.

Table 6: Comparison the proposed method with the existing methods

Methods given by	*Alternatives/ Attributes*	Decision Matrices	*Results by existing methods*	*Results by proposed method*
Perveen et al.[30]	$U=\{u_1,...,u_7\}$ $A=\{e_1,...,e_5\}$	$\begin{pmatrix} <0.8,0.1,0.4> & <0.5,0.2,0.6> & <0.7,0.15,0.3> & <0.6,0.25,0.31> & <0.55,0.12,0.5> \\ <0.9,0.1,0.3> & <0.7,0.2,0.45> & <0.55,0.03,0.12> & <0.65,0.15,0.56> & <0.8,0.17,0.3> \\ <0.52,0.02,0.7> & <0.7,0.12,0.31> & <0.45,0.15,0.5> & <0.6,0.2,0.4> & <0.75,0.05,0.25> \\ <0.7,0.05,0.4> & <0.45,0.1,0.55> & <0.56,0.15,0.49> & <0.25,0.05,0.8> & <0.51,0.1,0.25> \\ <0.7,0.1,0.4> & <0.3,0.2,0.7> & <0.47,0.25,0.56> & <0.31,0.15,0.6> & <0.8,0.2,0.4> \\ <0.55,0.25,0.65> & <0.81,0.15,0.3> & <0.1,0.2,0.8> & <0.6,0.05,0.3> & <0.65,0.1,0.5> \\ <0.92,0.1,0.3> & <0.2,0.25,0.6> & <0.35,0.2,0.7> & <0.8,0.12,0.4> & <0.75,0.01,0.3> \end{pmatrix}$	$u_2 > u_7 > \{u_1, u_3, u_4, u_6\} > u_5$	$u_2 > u_7 > u_1 > u_3 > u_5 > u_6 > u_4$
Yang et al.[42]	$U=\{u_1,...,u_6\}$ $A=\{e_1,...,e_4\}$	$\begin{pmatrix} <0.31,0.22,0.41> & <0.54,0.21,0.15> & <0.6,0.14,0.26> & <0.38,0.21,0.4> \\ <0.12,0.41,0.33> & <0.81,0.11,0.02> & <0.26,0.51,0.2> & <0.65,0.15,0.18> \\ <0.23,0.52,0.21> & <0.13,0.48,0.37> & <0.72,0.15,0.03> & <0.29,0.58,0.12> \\ <0.45,0.09,0.36> & <0.23,0.59,0.18> & <0.32,0.49,0.15> & <0.14,0.32,0.45> \\ <0.57,0.3,0.05> & <0.6,0.23,0.14> & <0.81,0.11,0.06> & <0.43,0.18,0.35> \\ <0.44,0.4,0.13> & <0.42,0.36,022> & <0.43,0.27,0.13> & <0.35,0.29,0.34> \end{pmatrix}$	$u_5 > u_2 > \{u_1, u_3\} > \{u_4, u_6\}$	$u_5 > u_1 > u_2 > u_3 > u_6 > u_4$
Guleria and Bajaj [16]	$U=\{u_1,...,u_4\}$ $A=\{e_1,...,e_4\}$	$\begin{pmatrix} <0.6,0.2,0.2> & <0.5,0.3,0.2> & <0.5,0.2,0.3> & <0.2,0.3,0.4> \\ <0.4,0.4,0.3> & <0.6,0.3,0.1> & <0.5,0.3,0.2> & <0.7,0.1,0.2> \\ <0.2,0.5,0.4> & <0.6,0.3,0.2> & <0.7,0.2,0.2> & <0.5,0.3,0.3> \\ <0.6,0.2,0.3> & <0.2,0.2,0.6> & <0.2,0.3,0.6> & <0.4,0.2,0.4> \end{pmatrix}$	$u_3 > u_2 > u_1 > u_4$	$u_3 > u_2 > u_1 > u_4$
Ahmmad et al.[1]	$U=\{u_1,...,u_5\}$ $A=\{e_1,...,e_5\}$	$\begin{pmatrix} <0.7,0.1,0.2> & <0.7,0.1,0.4> & <0.4,0.4,0.4> & <0.5,0.6,0.1> & <0.4,0.4,0.4> \\ <0.4,0.3,0.4> & <0.5,0.6,0.4> & <0.5,0.4,0.6> & <0.4,0.5,0.3> & <0.2,0.6,0.5> \\ <0.5,0.5,0.3> & <0.5,0.5,0.7> & <0.2,0.7,0.3> & <0.1,0.7,0.5> & <0.3,0.4,0.5> \\ <0.3,0.3,0.6> & <0.5,0.5,0.5> & <0.2,0.8,0.1> & <0.3,0.6,0.2> & <0.5,0.6,0.3> \\ <0.6,0.1,0.4> & <0.4,0.3,0.6> & <0.9,0.2,0.2> & <0.5,0.5,0.4> & <0.4,0.7,0.4> \end{pmatrix}$	$u_1 > u_5 > u_4 > u_2 > u_6 > u_3$	$u_1 > u_5 > u_2 > u_4 > u_6 > u_3$

6 Conclusion

In this paper, the soft set theory is extended by considering spherical fuzziness to handle the uncertainty and to make them more functional for solving multi-criteria decision-making problems. We first introduced the new spherical fuzzy soft set theory with their operations, then we represented a spherical fuzzy soft aggregation operator to use them for constructing an algorithm to apply to the multi-criteria decision-making procedure. Finally giving a numerical example to demonstrate the applicability of this method, we compare the results of this method with the methods given in [1, 16, 30, 42]. But it can be remarked as the proposed theory is unable to capture two-dimensional information so it does not carry any information about the two-dimensional data especially physical problems. For future work, we aim to extend the sfs-sets theory to the complex sfs-sets theory and apply this to the problems contains two-dimensional information. Also, we plan to study different kinds of aggregation operators to solve the related problems to the (complex) spherical fuzzy soft environment.

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