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Original paper

Improved q-rung orthopair and T-spherical fuzzy sets

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Abstract

Different extensions of fuzzy sets like intuitionistic, picture, Pythagorean, and spherical have been proposed to model uncertainty. Although these extensions are able to increase the level of accuracy, imposing rigid restrictions on the grades are the main problem of them. In these types of fuzzy sets, the value of grades and also the sum of them must be in the closed unit interval of [0, 1]. The sum condition seriously restricts the eligible values for grades. q-rung orthopair and T-spherical fuzzy sets have been introduced to establish a framework to tackle the mentioned problem for two-grade and three-grade fuzzy sets, respectively. Reducing the value of grades by means of power operator is the backbone idea of the both sets. However, these fuzzy sets are suffering from two drawbacks. The first one arises from the fact that there is no automatic structure to identify a proper power. Also, information loss is the other one which affects the accuracy of the decision-making process. This problem is a damaging consequence of changing the values of the grades. This paper introduces a novel reducing strategy to improve q-rung orthopair and T-spherical fuzzy sets by tackling the mentioned drawbacks. The proposed strategy solves out the former problem by establishing an automatic framework for finding a proper power which guarantee enough reduction of the values. The automatic framework is used for reducing the value of the maximum grade. Besides, the novel strategy reduces the rest of the grades according to their distance with the the maximum grade and its reduction rate. This paper proves mathematically that the ratio between the grades before and after of the reduction process will be intact, which results in solving information loss problem. Moreover, the higher accuracy level of the novel reduction strategy in comparison with the preceding methods, q-rung orthopair and T-sipherical fuzzy sets, is shown via different examples.

Keywords: Intuitionistic fuzzy sets, non-standard fuzzy sets, picture fuzzy sets, Pythagorean fuzzy sets, q-rung orthopair fuzzy sets, reducing strategy, spherical fuzzy sets, T-spherical fuzzy sets

1 Introduction

Modeling uncertainty is the backbone idea of Fuzzy Sets (FSs). Small, tall, and cold are just samples of so-called imprecise concepts. These concepts are important due to the fact that the knowledge of an expert is conveyed by them. To the end, these ambiguous concepts must be defined, precisely for the objective of designing an intelligent expert system. Fuzzy sets are able to model the knowledge of an expert and draw inference based on the modeled knowledge, subsequently. The mentioned ability has been resulted in widespread use of fuzzy sets in different areas. Medical diagnosis [21, 23], data classification [10, 12], data clustering [11, 34], image segmentation [17, 19], and social choice [5, 16] are just few areas where fuzzy sets are used. FSs are firstly proposed by Zadeh [33]. In this research, crisp sets are extended by means of membership grades, which are in the range of 0 and 1. With the strategy of defining membership grades, FSs are able to model uncertainty. Moreover, in the proposed sets by Zadeh, non-membership grades are related to the membership ones. As an instance, if 0.6 is considered for membership grade, consequently the non-membership

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grade will be 0.4. For the purpose of improving FSs, intuitionistic Fuzzy Sets (IFSs) are proposed by Atanassov [6]. Unlike FSs, intuitionistic fuzzy sets consider membership and non-membership grades as two independent identities. These identities take values in between 0 and 1, separately. Even though the identities are independent, the sum of them must be in the closed unit interval of [0, 1], too. As an example, the membership grade can be set on 0.7 while non-membership grade is set on 0.1. Regarding to the quality of IFSs, it is set as the main infrastructure of variety of further researches like [18] and [22]. It is straightforward that considering the sum condition in IFSs results in omitting an extensive range of values between [0, 1]. For example, even though, 0.7 and 0.5 are in the acceptable interval, they are not eligible since the sum of them is not less than 1. To stretch the domain of eligible values, Pythagorean fuzzy sets (PyFSs) is presented [30]. This approach is able to extend the domain of allowable values by means of power operator. Since grades are decimal numbers, power makes them smaller. Therefore, in Pythagorean fuzzy sets, grades are decreased by squaring them. Picture fuzzy sets (PFSs) extend IFSs with adding neutral to the membership and non-membership grades [9]. In the other words, each uncertain concept must be defined by a triplet. In addition, all the grades have to be in the range of 0 and 1 while the sum of them has to be less than 1. In [20], Spherical fuzzy sets (SFSs) are proposed with the purpose of increasing the domain of eligible grades in three-grade fuzzy sets. To fulfill the mentioned purpose, Spherical fuzzy sets use squaring strategy as same as Pythagorean fuzzy sets. As an instance, the triplet (1/3,3/5,3/5) is not acceptable in picture fuzzy sets due the fact that sum of the grades is greater than 1. However, the triplet is acceptable in Spherical fuzzy sets due to the fact that the sum of squared grades is less than 1. Although squaring strategy could stretch the acceptable space of different fuzzy sets, extensive range of values are uncovered, yet. q-Rung Orthopair Fuzzy Set (q-ROF) [32], and T-Spherical Fuzzy Set (T-SFS) [20] are proposed for the objective of tackling this problem in two-grade and three-grade fuzzy sets, respectively. Since, membership grades are decimal values, therefore, increasing the power will result in smaller decimal numbers. Increasing the rate of power (q) to reduce the value of the grades is the main idea of q-ROF and T-SFS. Even though this idea works theoretically, there are two drawbacks with increasing q. The first drawback, which is a minor one, comes from the fact that there is no automatic mechanism to determine the rate of q. Consequently, this rate should be determined, manually. The second drawback, which is a major one, arises from the inherent of reducing grades by increasing q. It is obvious that the values are received from an expert. So, these grades and more importantly the ratio between them, express the knowledge. Consequently, changing the values means manipulating the knowledge of the expert. Therefore, q-ROF and T-SFS ruin the knowledge of the expert, which may lead to inaccurate decisions. However, we must be so prudent about the accuracy of decisions since in some cases manipulating the knowledge of the expert even increases the level of accuracy. The mentioned point can be clarified by the case of mistakes in transferring the knowledge where an expert can give inaccurate values. In this case, which seems inevitable in practical environment, reducing the values can result in reducing the level of mistakes. Despite some exceptional cases, changing the knowledge of an expert with no clear strategy sounds like a double edged sword because there is no guarantee for the quality of outcomes. Even though different researches have tried to increase the acceptable space of grades values, some researches ignore the rigid restriction of the eligible grades. In [24], complex fuzzy sets are introduced. The main novelty of the proposed sets is accepting the grades, which are not in the classic range of 0 and 1. Complex fuzzy sets, as its name shows, use complex numbers as membership grades. Furtheremore, the concept of complex fuzzy set is expanded in different researches and added to different fuzzy sets for intent of improving the decision-making process [1, 3, 4]. Also, Neutrosophic fuzzy sets are presented in [25]. The proposed fuzzy sets are based on neutrosophic logic, which uses a triplet to model vague information. For the first time, it adds neutral grades to the membership and non-membership grades. Moreover, considering a range of $]^{-}0,1^{+}[$ for eligible grades is another novelty of Neutrosophic fuzzy sets. Plus, some other improvements rather than increasing the acceptable space in fuzzy sets have been suggested in differe researches like [7] where different notations of fuzzy positive implicative filters and their properties are investigated or [8] and [29] where the concept of intuitionistic fuzzy sets is added to BCK and B algebras, respectively. Also, the quality of decision-making process has always been a challenging issue when it comes to fuzzy sets. Regarding this fact, the attention of many researchers have been focused on this aspect of fuzzy sets where [2, 13, 14, 15] can be cited as good references in this field.

- Contributions

Introducing a novel reducing strategy for q-rung orthopair and T-spherical fuzzy sets is the main contribution of this research. As it is mentioned earlier, respecting the sum conditions of fuzzy sets leads to restriction on acceptable grades. Consequently, the preceding fuzzy sets are not able to model the out-range values. q-ROF tries to solve the problem by means of increasing the rate of power in two-grade fuzzy sets. The same scenario happens in the case of T-SFS where there are three grads; however, these sets are suffering from inherent problems. The proposed reducing strategy is able to firstly, define a proper q according to the maximum grade, automatically. In addition, This strategy reduces the grades, simmetrically to retain the ratio between the grades. To this end, the proposed strategy assures the accuracy of decisions which is a unique feature in analogy with the preceding sets. Last but not least, the proposed

Symmetrical Reduction Strategy (SRS) is not restricted to the number of grades. In the other words, SRS is able to be adapted according to the number of grades. Because of this unique ability, SRS is able to be added to two-grade, three-grade, and even more than three grades fuzzy sets. The rest of this paper is organized as follows. Section 2 is devoted to preliminaries. Section 3 details the proposed reducing strategy. Section 4 shows fuzzy operators based on SRS. Numerical examples are presented in Section 5. Finally, the paper is concluded in section 6.

2 Preliminaries

In this section, we briefly describe the basic concepts of fuzzy sets. Intuitionistic, Pythagorean, picture, spherical, q-rung orthopair, and T-spherical sets are different extensions of fuzzy sets that will be improved by the proposed reducing strategy. In the following equations, U presents the world and x is an element of U. Also, set A is a subset of U. FSs uses (1) to define how the element x belongs to set A [33]. As it shows, FSs consider a grade in the interval of [0, 1] to model a vague concept. Also, μ_x shows membership function.

$$FSs = \{< x, \mu_x >| \ x \in U\}, \tag{1}$$
$$\mu_x \rightarrow \forall x \in U \rightarrow [0,1].$$

Intuitionistic Fuzzy Sets use two grades, namely membership and non-membership grades, for the purpose of modeling uncertain concepts. The modeling approach is detailed in (2) where ϑ_x is the non-membership grade [6]. As (2) shows, the main restriction of IFSs is the sum of the grades. According to the restriction, the fuzzy set can only use grades, which the sum of them is less than or equal to 1.

$$IFSs = \{< x, \mu_x, \vartheta_x >| \ x \in U\}, \tag{2}$$
$$0 \leq \mu_x + \vartheta_x \leq 1.$$

Pythagorean fuzzy sets are proposed with the objective of increasing the domain of acceptable grades in IFSs. The Pythagorean strategy is shown in (3) [30]. To reach this goal, PyFSs square both membership and non-membership grades. In this way, it is able to diminish the grades and consequently more grades are able to satisfy the restriction. Fig. 1 shows the difference between IFSs and PyFSs. It is plainly visible that PyFSs accept greater space than IFSs.

$$PyFSs = \{< x, \mu_x, \vartheta_x >| \ x \in U\}, \tag{3}$$
$$0 \leq \mu_x^2 + \vartheta_x^2 \leq 1.$$

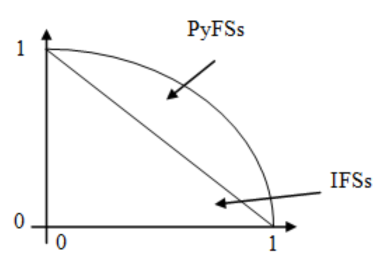

Figure 1: The acceptable space by IFSs, PyFSs [30]

It can be clearly seen that despite improvement by PyFSs, extensive range of potential values are still uncovered. q-rung orthopair fuzzy sets increase the power to cover the whole space. The theory of q-ROF is shown in (4) where $A^+(x)$ indicates the membership grade and $A^-(x)$ stands for the non-membership grade [32]. In addition, Fig. 2 illustrates the membership space of q-ROF.

$$q - ROF = \{< x, A^+, A^- >| \ x \in U\}, \tag{4}$$
$$0 \leq (A^+)^q + (A^-)^q \leq 1.$$

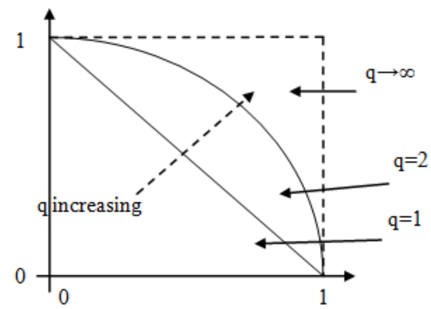

Figure 2: The acceptable space by q-ROF [32]

Picture Fuzzy Set considers neutral grade along with membership and non-membership grades to model a vague concept. The modeling strategy is shown in (5) [9]. In this equation, I_x is the neutral grade. Despite adding the third grade, the sum of grades has to be less than or equal to 1.

$$PFSs = \{< x, \mu_x, I_x, \vartheta_x >| \, x \in U\}, \tag{5}$$
$$0 \leq \mu_x + I_x + \vartheta_x \leq 1.$$

Since the mentioned restriction in (5) seriously limits the eligible values, Spherical Fuzzy Sets use squaring strategy to extend the domain of the acceptable range. The proposed model by SFSs is shown in (6) [20]. Also, Fig. 3 (a) and (b) depict the difference between acceptable space of PFSs and SFSs, respectively.

$$PFSs = \{< x, \mu_x, I_x, \vartheta_x >| \, x \in U\}, \tag{6}$$
$$0 \leq \mu_x^2 + I_x^2 + \vartheta_x^2 \leq 1.$$

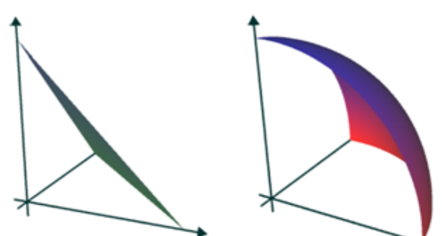

Figure 3: The acceptable space by PFSs, SFSs [26]

As like as q-ROF, T-spherical fuzzy sets are introduced to increase the range of acceptable values in three-grade fuzzy sets like picture and spherical fuzzy sets. T-SFS increases the value of power to diminish the grades for the objective of satisfying the sum condition. Mathematical aspect of T-SFS is proposed in (7) [20]. Also, Fig. 4 depicts the acceptable space of T-SFS for power of 10.

$$T - SFSs = \{< x, \mu_x, I_x, \vartheta_x >| \, x \in U\}, \tag{7}$$
$$0 \leq {\mu_x}^n + {I_x}^n + {\vartheta_x}^n \leq 1.$$

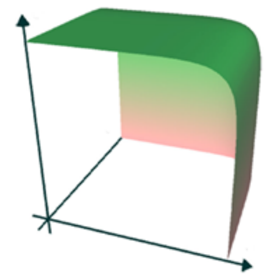

Figure 4: The acceptable space by T-SFS [26]

3 Proposed reducing strategy

Increasing the space of acceptable grades is the major purpose of q-ROF and T-SFS. Even though, these sets are attractive fuzzy sets from theoretical aspect, they are not accurate enough because they change the knowledge of an expert, artificially. Also, these fuzzy sets are not able to find a proper power, automatically. In a nutshell, SRS is introduced to improve q-ROF and T-SFS. In fact, the novel reducing strategy is proposed to fulfill the goal of increasing the space of the acceptable grades while maintaining the accuracy of decision-making process. SRS is composed of two main phases. At the first phase, SRS proposes a flexible strategy based on the maximum grade to find a suitable power, automatically. The second phase belongs to reducing the value of grades where SRS consider the distance between the grades to diminish them, symmetrically. These steps empower the proposed reducing strategy in a such way that the SRS is able to accept all the decimal grades. This ability will result in absorbing the knowledge of an expert with almost no restriction. Not only all the knowledge can be absorbed, also, the accuracy of decision-making process will be assured by SRS, too.

Moreover, SRS can be adapted according to the number of grades. Regarding to this ability, this section, at first, explains the proposed strategy according to two grades. Afterwards, an extension of SRS will be proposed with three grades. Finally, a general extension of SRS will be proposed according to n grades to clarify the adaptability of the proposed strategy.

3.1 Proposed reducing strategy with two grades

As it is explained earlier, IFSs use non-membership grade along with membership grade for the first time. Also, PyFSs and q-ROF are proposed for the objective of increasing the space of the acceptable membership and non-membership grades. The proposed reducing strategy for improving q-ROF is explained in (8) where μ_{x_i} shows the membership grade and ϑ_{x_i} represents the non-membership grade of the i^{th} member of the world U.

$$q - ROF\_SRS = \{ < x, \mu_x, \vartheta_x > | \; x \in U \}. \tag{8}$$

$$0 \leq {\mu_x}^{new} + {\vartheta_x}^{new} \leq 1.$$

As we know, sum of the grades must be less than or equal to 1. To this end, SRS changes the original values and (8) is resulted where ${\mu_x}^{new}$ is the new membership grade and ${\vartheta_x}^{new}$ is the new non-membership grade.

As it is mentioned earlier, finding a proper power is the first step of SRS. Consider q as the proper power. For the purpose of finding q, SRS starts with finding the maximum value among both membership and non-membership grades. Suppose M stands for the maximum value which is shown in (9).

$$M = maximum(\mu_{x_i}, \vartheta_{x_i}), \tag{9}$$

After finding the maximum value, all the grades in the sum condition are replaced by M^p. So, the outcome of the mentioned substitution will be as like as follows:

$M^q + M^q \leq 1,$

From then,

$2 \times M^q \leq 1,$

$M^q \leq \frac{1}{2}.$

Finally, to find the desired q, logarithm function must be used. Logarithm function has descending behavior when its base is less than 1. For example, $\log_{0.9} 0.3$ is greater than $\log_{0.9} 0.4$. Fig. 5 illustrates the plot of $y = \log_{0.9}(x)$ when x is in the closed unit interval of [0.1, 2]. It is plainly visible that the logarithm function is descending when the base is less than 1.

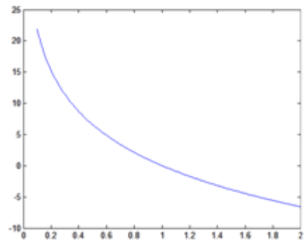

Figure 5: The plot of $y = \log_{0.9}(x)$

Since, the maximum grade is always less than 1, the logarithm with the base of M is definitely, descending. Therefore, (10) will be resulted from the trend of logarithm function.

$$\log_M M^q \leq \log_M \frac{1}{2}. \tag{10}$$

So,

$$q \geq \log_M \frac{1}{2}. \tag{11}$$

According to (11), if we set $q = \log_M \frac{1}{2}$, then, it can be taken for granted that even $M^q + M^q$ will be smaller than or equal to 1 and the sum condition is satisfied. Therefore, the proper power is found. In the other words, $\log_M \frac{1}{2}$ is an upper bound for the power.
After finding the proper power, it is time to diminish the value of the grades. One immediate way is to reduce all the values by power of q, as like as PyFSs and q-ROF. Even though this approach guarantees the satisfaction of the sum condition, it does not seem accurate enough because it changes the relationship between the values. Consider (0.2,0.9) where the membership value is 0.2 and the non-membership value is 0.9. According to the SRS, the proper power is 6.58 and consequently the new value will be 0.000025 and 0.499937. It is straightforward that not only the distance between the original values is changed, but also, the ratio between them is changed, as well. The ratio between original values, 0.2 and 0.9, is 4.5 where the ratio between decreased values is around 20. Even if we use Pythagorean fuzzy set to model the pair, the ratio between them will be changed. To this end, SRS adjust just the maximum value by power of q.
Regarding the abovementioned problem, SRS diminishes the other grades according to their distance with the maximum one and the decreasing rate of it. The decreasing rate of the maximum grade is defined as follows:

$$D_{Rate} = \frac{Maximum_{new}}{M} = \frac{M^q}{M} = M^{q-1}, \tag{12}$$

where D_{Rate} shows the decreasing rate of the maximum grade. Since, the maximum value is changed by means of decreasing rate, SRS changes the distance between the other values with the maximum one with the same rate. Suppose μ_x is a grade in U. Then,

$$Distance_{new} = Distance \times D_{Rate}, \tag{13}$$
$$Distance = M - \mu_x.$$

Therefore, the new value of the grade will be calculated as follows:

$${\mu_x}^{new} = M^q - Distance_{new}. \tag{14}$$

In this way, the grades will be decreased enough to satisfy the sum condition and more importantly, the ratio between the grades will be intact. The former claim is proved by Theorem 3.1.

Theorem 3.1. *The sum of the reduced values by SRS will always be less than or equal to 1,* $M^q + {\mu_x}^{new} \leq 1$.

Proof. We know that by setting q to $\log_M \frac{1}{2}$ the sum of $M^q + M^q$ will be less than one. So,

$$\begin{aligned} M^q + {\mu_x}^{new} &= M^q + M^q - Distance_{new} = M^q + M^q - (Distance \times D_{Rate}) = M^q + M^q - ((M - \mu) \times M^{q-1}) \\ &= M^{q-1}(M + M - M + \mu) = (M \times M^{q-1}) + (\mu \times M^{q-1}). \end{aligned}$$

Because $M^q + M^q \leq 1$ and $\mu \leq M$, then $(M \times M^{q-1}) + (\mu \times M^{q-1})$ is smaller than or equal to 1. □

The latter mentioned property of SRS is an important and unique one because it will lead to higher level of accuracy. The constant ratio between the new values is proved in the next theorem.

Theorem 3.2. *Ratio between maximum value and any grade will remain constant after decreasing process.*

Proof. We start from ${\mu_x}^{new}$:

$$\begin{aligned} {\mu_x}^{new} &= M^q - Distance_{new} = M^q - (Distance \times D_{Rate}) = M^q - (Distance \times M^{q-1}) \\ &= M^{q-1}(M - Distance) = M^{q-1}(M - (M - \mu)) = M^{q-1} \times \mu. \end{aligned}$$

If we divide both sides by M^q, then:

$$\frac{\mu_{new}}{M^q} = \frac{M^{q-1} \times \mu}{M^q},$$

So, $\frac{\mu_{new}}{M^q} = \frac{\mu}{M}$. □

Moreover, it easily can be proven that the ratio between non-maximum grades will be intact, as well. Proposition 3.3 stands for the proof.

Proposition 3.3. *The ratio between any two grades will be intact after reduction process.*

Proof. Consider a^1 and a^2 as two grades. Obviously, these grades can be membership or non-membership. By Theorem 3.2 we have:

$$a^1_{new} = M^{q-1} \times a^1, \quad \text{and} \quad a^2_{new} = M^{q-1} \times a^2.$$

Therefore,

$$\frac{a^1_{new}}{a^2_{new}} = \frac{M^{q-1} \times a^1}{M^{q-1} \times a^2} = \frac{a^1}{a^2}.$$

□

To clarify the process of SRS the example of (0.9, 0.8) is presented. As the example shows, the membership grade is 0.9 while 0.8 is set for the non-membership grade. It is straightforward that SRS satisfies the sum condition while retaining the ratio between the grades. The process is as follow:

$$M = maximum(0.9, 0, 8) = 0.9, \quad \text{and} \quad q = \log_M \frac{1}{n} = \log_{0.9} \frac{1}{2} = 6.58.$$

So, $Maximum_{new} = 0.9^{6.58} = 0.499937$. Then, $D_{Rate} = \frac{Maximum_{new}}{M} = 0.9^{5.58}$.

$$Distance_{new} = Distance \times D_{Rate} = (0.9 - 0.8) \times 0.9^{5.58} = 0.055548.$$

$$Non\_membership_{new} = Maximum_{new} - Distance_{new} = 0.499937 - 0.055548 = 0.444389.$$

And obviously,

$$0.499937 + 0.444389 \leq 1.$$

And of course,

$$\frac{0.9}{0.8} = \frac{0.499937}{0.444389} = 1.125.$$

In addition, for the objective of simplifying the process of SRS, the proper power can be set to $\lceil \log_M \frac{1}{n} \rceil$ instead of $\log_M \frac{1}{n}$. It should be emphasized that using the corner ceiling brackets dont affect the accuracy of SRS because the ratio between the grades will be intact.

The aforementioned example shows how strong and accurate SRS is. Furthermore, the next theorem proves that q-ROF along with SRS is able to model wider range of values in comparison with IFSs and PyFSs.

Theorem 3.4. *q-ROF_SRS is able to model greater space of membership grades than intuitionistic fuzzy sets and Pythagorean fuzzy sets.*

Proof. It is proved in [31] that PyFSs can model greater space than IFSs. So, if we prove that the grades space of q-ROF_SRS is greater than PyFSs, the theorem will be proved.

First of all, we show that if a pair of membership and non-membership grades is acceptable in PyFSs, then the pair will be acceptable by q-ROF_SRS, too.

Consider a and b as two membership grades where $a^2 + b^2 \leq 1$, according to PyFSs. Suppose b is the maximum grade, so, SRS can determine three different powers for b, which are $q \geq 2$, $q = 2$, and finally $q = 1$. These three cases are analyzed separately as follows:

Case I: Since b is in the interval of [0, 1], then $b > b^2 > b^3 > b^\theta$ where $\theta \geq 4$. So, if $a^2 + b^2 \leq 1$, then $b^\theta + b^\theta$ where $\theta \geq 3$ will be definitely smaller than 1. Also, according to Theorem 3.1, $a_{new} + b^\theta$ will be less than or equal to 1, too.

Case II: If the grades are acceptable in PyFSs and SRS consider 2 as the value of power, then $b^2 + b^2$ is less than or equal to one. So, according to Theorem 3.1, $a_{new} + b^2$ is acceptable in q-ROF_SRS, too.

Case III: According to the procedure of SRS

$$q \geq \log_M \frac{1}{2}.$$

We assume that b is the maximum grade. So, max is equal to b and $q \geq \log_b \frac{1}{2}$. Also, it is assumed that the considered power by SRS is 1, so, $1 \geq \log_b \frac{1}{2}$. Moreover, the following calculation can be concluded due to the fact that b is in the interval of [0, 1].

$$b^1 \leq \log_b \frac{1}{2} \to b \leq \frac{1}{2}.$$

Therefore, $b + b$ is less than or equal to 1. Again, according to Theorem 3.1, $b + a_{new}$ is definitely less than or equal to 1, too. All the aforementioned cases prove that if a pair of membership and non-membership grades is acceptable by PyFSs, it definitely will be accepted by q-ROF_SRS.

On the other hand, we prove that there are eligible pairs by q-ROF_SRS that cant be modeled by PyFSs. It can be proved by the example of (0.9, 0.8). The sum of the squared membership and non-membership grades is greater than 1 which means PyFSs is not able to model the pair. However, as it is shown earlier, this pair can be modeled by q-ROF_SRS. □

In addition, Fig. 6 clearly shows that q-ROF_SRS is able to model greater space than PyFSs and IFSs. In this figure, the area under the plot of q-ROF_SRS can be modeled by the improved q-ROF. Also, the space under PyFSs and IFSs covers eligible values for Pythagorean and intuitionistic fuzzy sets, respectively. As the figure illustrates, q-ROF_SRS is able to model the whole decimal grades. Moreover, to draw q-ROF_SRS, we set maximum=0.99 and then, q=68.97. Furthermore, this plot is drawn by MATLAB 2020.

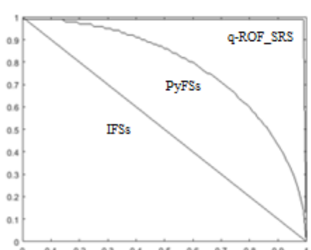

Figure 6: Comparison of the acceptable space by IFSs, PyFSs, and q-ROF_SRS.

Despite the strength of the proposed strategy, there are only two exceptional cases, which cannot be modeled by q-ROF_SRS. The exceptional cases happen when the maximum value is either 0 or 1 because logarithm function does not work on these points. The proper power must be defined manually same as q-ROF in these two cases. However, the reduction process must be done as same as SRS to guarantee the accuracy.

3.2 Proposed reducing strategy with three grades

T-SFSs along with SRS is defined in (15). Also, (16) shows how T-SFS_SRS explores the proper power in the case of three grades, where M is the maximum grade.

$$\text{T-SFS\_SRS} = \{< x, \mu_x, I_x, \vartheta_x >| \ x \in U\}. \tag{15}$$
$$0 \leq \mu_x^{new} + I_x^{new} + \vartheta_x^{new} \leq 1.$$

$$q = \log_M \frac{1}{3}. \tag{16}$$

As like as two-grade fuzzy sets, after finding the proper power, the grades should be reduced by means of decreasing rate. Equations (12), (13), and (14) are used to determines new value for grades in three-grade fuzzy sets, too. Moreover, as like as Theorem 3.1, it can be easily proved that the sum of the reduced grades will be certainly less than or equal to 1. In addition, it is straightforward that T-SFS_SRS retains the ratio between the grades after decreasing process. Additionally, Proposition 3.5 proves that T-SFS_SRS is able to accept greater space than picture and spherical fuzzy sets.

Proposition 3.5. *T-SFS_SRS models greater space than picture fuzzy set and spherical fuzzy sets.*

Proof. Since the greater space of SFSs than PFSs is proved in [20], we just need to prove that the space of T-SFS_SRS is greater than SFSs. As like as Theorem 3.4, this proof is composed of two parts. At first, we prove that all the possible membership, non-membership, and neutral grades in SFSs are eligible in T-SFS_SRS. Then, an example will be presented, which shows that T-SFS_SRS can model some grades that are inaccessible for SFSs. Suppose (a, b, c) are membership, neutral, and non-membership grades, respectively. We presume that the mentioned triplet is acceptable by SFSs, which means it satisfies the sum condition, $a^2 + b^2 + c^2 \leq 1$. SRS considers a power according to the maximum grade which has three total cases, $q > 2$, $q = 2$, and $q = 1$. The proofs for the first two cases are as like as Case I and Case II in Theorem 3.4 with an additional grade. So, we just consider case III where SRS sets $q = 1$. To determine the proper q, SRS with 3 grades uses (16). So,

$$q \geq \log_M \frac{1}{3}.$$

We presume that the neutral grade is the maximum one among the others. Consequently, $q \geq \log_b \frac{1}{3}$. Additionally, in this case, the proper power is set to 1, so, $1 \geq \log_b \frac{1}{3}$. Since, the maximum grade, b, is less than 1, we have

$$b^1 \leq b^{\log_b \frac{1}{3}} \rightarrow b \leq \frac{1}{3}.$$

Therefore, $b+b+b$ is less than or equal to 1. Since we know that a_{new} and c_{new} are smaller than b then, $a_{new}+b+c_{new}$ is less than or equal to 1, too. Therefore, it has been proved that if a triplet is acceptable for SFSs, T-SFS with the proposed reducing strategy will be able to model it, too.

For proving Proposition 3.5, we have to prove that T-SFS_SRS is able to model triplets which are not acceptable by SFSs. The difference between T-SFS_SRS and SFSs can be clarified with the example of (0.9,0.8,0.8). The mentioned triplet is not in the acceptable space of SFSs due to the fact that 0.81+0.64+0.64 is greater than 1. However, T-SFS_SRS is able to model the mentioned example as follows:

$$\begin{aligned}
&M = maximum(0.9, 0.8, 0.8) = 0.9,\\
&q = \log_M \frac{1}{n} = \log_{0.9} \frac{1}{3} = 10.52,\\
&Maximum_{new} = 0.9^{10.52} \approx 0.3300891,\\
&D_{Rate} = \frac{Maximum_{new}}{M} = 0.9^{9.52},\\
&Distance_{new} = Distance \times D_{Rate} = (0.9 - 0.8) \times 0.9^{9.52} \approx 0.0366766,\\
&Membership_{new} = Maximum_{new} - Distance_{new} \approx 0.2934125,\\
&Non-membership_{new} = Maximum_{new} - Distance_{new} = 0.2934125.
\end{aligned}$$

□

Also, Fig. 7 illustrates the difference between the space of T-SFS_SRS, SFSs, and PFS. To draw T-SFS_SRS, we set $maximum = 0.99$ and according to (16), $q = 110.31$. It is plainly visible that T-SFS_SRS dominate the other fuzzy sets which means that the improved T-SFS covers greater space.

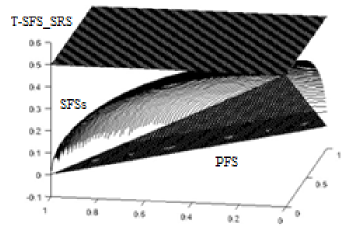

Figure 7: Comparison of the acceptable space by PFS, SFSs, and T-SFS_SRS.

3.3 Proposed reducing strategy with n grades

In this section a general state of SRS is proposed. As it is detailed earlier, fuzzy sets such as IFSs and PFSs try to model vague concepts with more than just one grade. Regarding the abilities of two and three-grade fuzzy sets, it is not out of senses if we expect new fuzzy sets with more than three grades in future. SRS is able to be adapted according to the number of grades. To show this adaptability of SRS, the proposed strategy is described based on n imaginary grades, in this section. In the other words, we assume that the n-grade fuzzy set is a future extension of the existing sets. Subsequently, n grades are used to model an ambiguous concept x. Therefore, according to (11) and (16) the proper power is defined as follows:

$$q = \log_M \frac{1}{n}, \tag{17}$$

where M is the maximum grade and n is the number of grades. Also, after reducing the maximum grade, the other ones will be reduced likewise (12), (13), and (14). Algorithm I shows how SRS works based on n imaginary grades.

Algorithm I: SRS based on n grades
Input: Original grades (g), number of grades (n)
1. Find the maximum grade (M)
2. Set the proper power (q) equal to $\log_M \frac{1}{n}$
3. $Maximum_{new} = M^q$;
4. $D_{Rate} = M^{q-1}$;
5. For i=1 to n {
6. $Distance = M - g(i)$;
7. $Distance_{new} = Distance \times D_{Rate}$;
8. $SRG(i) = Maximum_{new} - Distance_{new}$; }
Output: Symetrically Reduced Grades (SRG)

4 Fuzzy operator based on SRS

This section proposes the basic set operators, namely intersection, union, and complement for the improved q-ROF with the proposed reducing strategy; however, these operators can be adapted with more than two grades. It must be pointed that different types of operators are defined in different researches. In this section, we use [25], [31], and [32] to define the mentioned operators. It is assumed that $A = (a_1, a_2)$ and $B = (b_1, b_2)$ are two fuzzy sets. According to SRS, a decreasing rate is used to bring the sum of the membership and non-membership grades into the desired interval. So, we have $a_1^{new} + a_2^{new} \leq 1$ and $b_1^{new} + b_2^{new} \leq 1$.

The first operator is intersection. Intersection of two mentioned fuzzy sets is defined by $D = (d_1, d_2)$ where $d_1 = min(a_1^{new}, b_1^{new})$ and $d_2 = max(a_2^{new}, b_2^{new})$. It has to be assured that the outcome grades, d_1 and d_2, are in the eligible space of q-ROF_SRS and so, satisfy the pre-defined conditions. As it is mentioned earlier, the proper q is defined based on the maximum membership grade. Regarding the process of SRS with two grades, $max^q + max^q \leq 1$. Now, if we consider $max = b_2$, then $b_2^q + b_2^q \leq 1$. Due to the fact that a_1^{new},a_2^{new},b_1^{new},$b_2^{new} \leq b_2^q$, then it can be concluded that sum of each pair of a_1^{new}, a_2^{new}, b_1^{new}, and b_2^{new} will be less than 1. Therefore, from the fact that $d_1 = a_1^{new}$ or b_1^{new} and $d_2 = a_2^{new}$ or b_2^{new} and the preceding conclusion, it is resulted that the sum of d_1 and d_2 is definitely less than or equal to 1. So, the results of intersection of two q-ROF_SRS are in the desired interval and satisfy the sum condition.

Union of the two mentioned fuzzy sets, A and B, is defined by $Z = A \cup B$. Z is a fuzzy set with two grades, $Z = (z_1, z_2)$, where $z_1 = max(a_1^{new}, b_1^{new})$ and $z_2 = max(a_2^{new}, b_2^{new})$. Likewise the intersection, it is straightforward that z_1 and z_2 are within the space of q-ROF_SRS and $z_1 + z_2 \leq 1$.

Last but not least, complement operator is defined. Complement of a fuzzy set like A is shown by $\bar{A}$. If we define the fuzzy set A with two grades, $A = (a_1^{new}, a_2^{new})$, then the complement can be defined by $\bar{A} = (a_2^{new}, a_1^{new})$. Since A is a q-ROF_SRS and complement operator substitutes membership and non-membership grades, it is obvious that $\bar{A}$ is a q-ROF_SRS, too. Moreover, the following operators are derived from the basic mentioned operators:

$A - B = A \cap \bar{B} = (a_1^{new}, a_2^{new}) \cap (b_2^{new}, b_1^{new}) = min(a_1^{new}, b_2^{new}), max(a_2^{new}, b_1^{new}),$

$A \triangle B = (A - B) \cup (B - A) = [min(a_1^{new}, b_2^{new}), max(a_2^{new}, b_1^{new})] \cup [min(b_1^{new}, a_2^{new}), max(b_2^{new}, a_1^{new})],$

$A \subseteq B$ iff $a_1^{new} \leq b_1^{new}$ *and* $a_2^{new} \geq b_2^{new},$

$A = B$ iff $A \subseteq B$ *and* $B \subseteq A,$

$\overline{(\bar{A})} = \overline{(\overline{a_1^{new}, a_2^{new}})} = \overline{(a_2^{new}, a_1^{new})} = (a_1^{new}, a_2^{new}) = A.$

Since SRS is an improvement for q-ROF and T-SFS, operators of q-ROF_SRS and T-SFS_SRS are as same as the mentioned fuzzy sets. Therefore, please see references [9] and [30] for more operators.

5 Numerical examples

This section proposes three practical examples. The main purpose of the first example is to show the wider range of q-ROF_SRS in analogy with preceding fuzzy sets in a real-world problem while the second example shows the accuracy of q-ROF_SRS in medical diagnosis. Last but not least, example number three draws a comparison between q-ROF and the improved one, q-ROF_SRS in terms of accuracy.

Example 5.1. *In this example, we suppose that Table 1 is prepared by three different employers who want to hire an employee. In this table, C_1, C_2, C_3, C_4, and C_5 are five selection criteria and E_1, E_2, E_3, E_4, and E_5 represent the employers. As the table shows, each employer considers two grades for each criterion which are membership and non-membership grades. We want to decide which of the employers will hire an employee with*

$$[(C_1, 0.6, 0.7), (C_2, 0.3, 0.6), (C_3, 0.6, 0.8), (C_4, 0.1, 0.0), (C_5, 0.3, 0.4).$$

Table 1: Membership and non-membership grades of Example 5.1

	C_1	C_2	C_3	C_4	C_5
E_1	$(EC_{11}, 0.8, 0.1)$	$(EC_{12}, 0.6, 0.1)$	$(EC_{13}, 0.2, 0.8)$	$(EC_{14}, 0.7, 0.5)$	$(EC_{15}, 0.8, 0.8)$
E_2	$(EC_{21}, 0.1, 0.8)$	$(EC_{22}, 0.4, 0.4)$	$(EC_{23}, 0.6, 0.1)$	$(EC_{24}, 0.5, 0.6)$	$(EC_{25}, 0.3, 0.7)$
E_3	$(EC_{31}, 0.8, 0.1)$	$(EC_{32}, 0.8, 0.1)$	$(EC_{33}, 0.7, 0.6)$	$(EC_{34}, 0.1, 0.8)$	$(EC_{35}, 0.4, 0.4)$
E_4	$(EC_{41}, 0.4, 0.7)$	$(EC_{42}, 0.3, 0.1)$	$(EC_{43}, 0.8, 0.1)$	$(EC_{44}, 0.1, 0.4)$	$(EC_{45}, 0.3, 0.5)$
E_5	$(EC_{51}, 0.6, 0.6)$	$(EC_{52}, 0.5, 0.6)$	$(EC_{53}, 0.1, 0.7)$	$(EC_{54}, 0.4, 0.3)$	$(EC_{55}, 0.2, 0.1)$

Regarding the fact that each employer considers two grades for each criterion, the abovementioned table must be modeled by IFSs and PyFSs. Intuitionistic fuzzy set is not able to model Example 5.1 because the sum of some membership and non-membership grades are greater than 1, like EC_{33}. Also, Pythagorean fuzzy set is unable to model the example because in a case like EC_{15}, even the sum of the squared values is greater than 1. It is straightforward that q-ROF_SRS is the only choice to model Example 5.1.

According to SRS, finding the maximum grade value is the first step. In our imaginary example, the maximum grade is 0.8 and regarding to (11) the proper power is 3.11 ($q = 3.11$). Therefore, the grades in Table I are transferred according to the decreasing rate, which is $0.8^{2.11}$, and the distance with the maximum grade. Table 2 shows the transferred grades. It is straightforward that all the grades satisfy the sum condition. In addition, the ratio between the grades is maintained, as well.

Not only the grades have to be transferred, also, the abilities of the potential employee need to be transferred in the same way. Consequently, the transferred abilities are

$$[(C_1, 0.3747, 0.4371), (C_2, 0.1873, 0.3747), (C_3, 0.3747, 0.4996), (C_4, 0.0624, 0.0), (C_5, 0.1873, 0.2498)]$$

. For the objective of deciding about the potential employee, we need to measure the distance between the employees abilities and the pre-defined criteria. To this end, we use Euclidean distance which is a reliable measurement. Euclidean distance between two fuzzy sets A and B is shown in (18) [21].

$$D_E = \sqrt{\frac{1}{2}.((\mu_A^{new} - \mu_A^{new})^2 + (\vartheta_A^{new} - \vartheta_A^{new})^2 + (\pi_A^{new} - \pi_A^{new})^2)}. \tag{18}$$

Where,

$$\pi_A^{new} = 1 - \mu_A^{new} - \vartheta_A^{new}..$$
$$\pi_B^{new} = 1 - \mu_B^{new} - \vartheta_B^{new}$$

Table 2: Transferred membership grades and non-membership grades of Example 5.1

	C_1	C_2	C_3	C_4	C_5
E_1	$(0.4996, 0.0624)$	$(0.3747, 0.0624)$	$(0.1249, 0.4996)$	$(0.4371, 0.3122)$	$(0.4996, 0.4996)$
E_2	$(0.0624, 0.4996)$	$(0.2498, 0.2498)$	$(0.3747, 0.0624)$	$(0.3122, 0.3747)$	$(0.1873, 0.4371)$
E_3	$(0.4996, 0.0624)$	$(0.4996, 0.0624)$	$(0.4371, 0.3747)$	$(0.0624, 0.4996)$	$(0.2498, 0.2498)$
E_4	$(0.2498, 0.4371)$	$(0.1873, 0.0624)$	$(0.4996, 0.0624)$	$(0.0624, 0.2498)$	$(0.1873, 0.3122)$
E_5	$(0.3747, 0.3747)$	$(0.3122, 0.3747)$	$(0.0624, 0.4371)$	$(0.2498, 0.1873)$	$(0.1249, 0.624)$

The distances between the abilities of the candidate and the selection criteria of the employers are 1.9359, 1.5632, 1.3129, 1.1394, and 1.0847, respectively. Therefore, the last employer is more likely to hire the candidate because the qualities of the employee is closer than to the criteria of the last employer.

Example 5.2. [27, 28] *This example considers P_1, P_2, P_3, and P_4 as four patients. To diagnosis their diseases, total of five symptoms, S_1, S_2, S_3, S_4, and S_5 are considered, which are $Temperature$, $Headache$, $StomachPain$, $Cough$, and $ChestPain$, respectively. According to the mentioned symptoms, five diseases, D_1, D_2, D_3, D_4, and D_5, which are $ViralFever$, $Malaria$, $Typhoid$, $StomachProblems$, and $ChestProblems$ can be diagnosed.*
Tables 3 and 4 stands for patients symptoms and symptoms of diseases, respectively. As the tables show, a two-grade fuzzy set is used. The main task of the fuzzy models is to diagnose the disease of each patient. Moreover, as the tables show, the sum of all the pairs is less than or equal to 1 which means that IFSs and PyFSs are able to model such a knowledge as well as q-ROF_SRS. To diagnose the disease of each patient, the distance between the symptoms and

Table 3: Symptoms of patients in Example 5.2

	S_1	S_2	S_3	S_4	S_5
E_1	$(ES_{11}, 0.8, 0.1)$	$(ES_{12}, 0.6, 0.1)$	$(ES_{13}, 0.2, 0.8)$	$(ES_{14}, 0.6, 0.1)$	$(ES_{15}, 0.1, 0.6)$
E_2	$(ES_{21}, 0.0, 0.8)$	$(ES_{22}, 0.4, 0.4)$	$(ES_{23}, 0.6, 0.1)$	$(ES_{24}, 0.1, 0.7)$	$(ES_{25}, 0.1, 0.8)$
E_3	$(ES_{31}, 0.8, 0.1)$	$(ES_{32}, 0.8, 0.1)$	$(ES_{33}, 0.0, 0.6)$	$(ES_{34}, 0.2, 0.7)$	$(ES_{35}, 0.0, 0.5)$
E_4	$(ES_{41}, 0.6, 0.1)$	$(ES_{42}, 0.5, 0.4)$	$(ES_{43}, 0.3, 0.4)$	$(ES_{44}, 0.7, 0.2)$	$(ES_{45}, 0.3, 0.4)$

Table 4: Symptoms of diseases in Example 5.2

	S_1	S_2	S_3	S_4	S_5
D_1	$(DS_{11}, 0.4, 0.0)$	$(DS_{12}, 0.3, 0.5)$	$(DS_{13}, 0.1, 0.7)$	$(DS_{14}, 0.4, 0.3)$	$(DS_{15}, 0.1, 0.7)$
D_2	$(DS_{21}, 0.7, 0.0)$	$(DS_{22}, 0.2, 0.6)$	$(DS_{23}, 0.0, 0.9)$	$(DS_{24}, 0.7, 0.0)$	$(DS_{25}, 0.1, 0.8)$
D_3	$(DS_{31}, 0.3, 0.3)$	$(DS_{32}, 0.6, 0.1)$	$(DS_{33}, 0.2, 0.7)$	$(DS_{34}, 0.2, 0.6)$	$(DS_{35}, 0.1, 0.9)$
D_4	$(DS_{41}, 0.1, 0.7)$	$(DS_{42}, 0.2, 0.4)$	$(DS_{43}, 0.8, 0.0)$	$(DS_{44}, 0.2, 0.7)$	$(DS_{45}, 0.2, 0.7)$
D_5	$(DS_{51}, 0.1, 0.8)$	$(DS_{52}, 0.0, 0.8)$	$(DS_{53}, 0.2, 0.8)$	$(DS_{54}, 0.2, 0.8)$	$(DS_{55}, 0.8, 0.1)$

each disease has to be calculated. The closets disease will be determined as the final output. Again, Euclidean distance is used to measure the distance. Table 5 shows the results of modeling the aforementioned example with q-ROF_SRS. Since the maximum grade is 0.9, the proper power is 6.58. In addition, decreasing rate is $0.9^{5.58}$ *and the grades must be decreased by this rate.*
To find whether the proposed fuzzy sets are as accurate as the preceding sets, Example 5.2 is modeled by IFSs and PyFSs, too. Results are shown in table 6. As the results reinforce, q-ROF_SRS is as accurate as the preceding sets due to the fact that the proposed fuzzy set produces the same results as IFSs and PyFSs.

Table 5: Results of modeling Example 5.2 with q-ROF_SRS

	Euclidean Distance					2*Diagnosis
	D_1	D_2	D_3	D_4	D_5	
P_1	0.7177	0.6136	0.7189	1.3553	1.4379	D_2
P_2	1.0055	1.2953	0.7941	0.3740	1.1007	D_4
P_3	0.9447	1.1193	0.8103	1.2459	1.4410	D_3
P_4	0.6841	0.7482	0.9376	1.1552	1.3532	D_1

Table 6: Comparing the results of Example 5.2

	Euclidean Distance			
	P_1	P_2	P_3	P_4
IFSS	D_2	D_4	D_3	D_1
PYFSS	D_2	D_4	D_3	D_1
SRS	D_2	D_4	D_3	D_1

Example 5.3. *This example is proposed to test the accuracy of q-ROF_SRS in comparison with q-ROF. As like as Example 5.1, this example stands for hiring an employee with predefined qualities. Table 7 shows the selection criteria of the employers. As the table shows, there is a slight difference between the selection criteria of the employer number 4 and 5. Also, the qualifications of the candidate are considered as*

Table 7: Membership and non-membership grades of Example 5.3

	C_1	C_2	C_3	C_4	C_5
E_1	$(EC_{11}, 0.8, 0.1)$	$(EC_{12}, 0.6, 0.1)$	$(EC_{13}, 0.2, 0.8)$	$(EC_{14}, 0.7, 0.5)$	$(EC_{15}, 0.8, 0.8)$
E_2	$(EC_{21}, 0.1, 0.8)$	$(EC_{22}, 0.4, 0.4)$	$(EC_{23}, 0.6, 0.1)$	$(EC_{24}, 0.5, 0.6)$	$(EC_{25}, 0.3, 0.7)$
E_3	$(EC_{31}, 0.8, 0.1)$	$(EC_{32}, 0.8, 0.1)$	$(EC_{33}, 0.7, 0.6)$	$(EC_{34}, 0.1, 0.8)$	$(EC_{35}, 0.4, 0.4)$
E_4	$(EC_{41}, 0.6, 0.6)$	$(EC_{42}, 0.5, 0.6)$	$(EC_{43}, 0.1, 0.7)$	$(EC_{44}, 0.4, 0.3)$	$(EC_{45}, 0.2, 0.12)$
E_5	$(EC_{51}, 0.6, 0.6)$	$(EC_{52}, 0.5, 0.6)$	$(EC_{53}, 0.1, 0.7)$	$(EC_{54}, 0.4, 0.3)$	$(EC_{55}, 0.21, 0.1)$

$$[(C_1, 0.6, 0.7), (C_2, 0.3, 0.6), (C_3, 0.6, 0.8), (C_4, 0.1, 0.0), (C_5, 0.3, 0.4)].$$

If we use equation (18) to measure the distance between the qualifications and the criteria, with no change in value, the qualifications are closer to the criteria of employer number four. Therefore, we expect that the fuzzy sets produce the same outcome after changing the values and bring them into the desired interval.
Since there is no automatic framework for determining the rate of power in q-ROF, q must be determined manually. To this end, if we start from $q = 1$ *and increase this rate the proper q will be 4. So, the values are reduced by means of* $q = 4$*. According to equation (18), the distances between the qualifications of the employee and the selection criteria from the first employer to the last one are 1.4729, 1.0435, 1.2886, 0.4842, and 0.4841, respectively. Hence, the last employer would select the candidate, which is in contrast with our expectation. It can be concluded from the results that q-ROF is not accurate enough to select the closest employer in this case.*

On the other hand, in the case of solving the problem of hiring with q-ROF_SRS, since the maximum grade is 0.8, the q will be set to 3.11. It means that the grades and qualifications must be decreased by the decreasing rate of $0.8^{2.11}$ *and their distance with the maximum grade. Finally, if we calculate the distances according to equation (18) the results will be 1.9359, 1.5632, 1.3129, 1.0726, and 1.0804 for employer number one to the number five, respectively. It is obvious that the employer number four is selected as we expected. This example concludes that q-ROF_SRS is more accurate than q-rung orthopair fuzzy set.*
As it is mentioned earlier, the proper power can be set to $\lceil \log_M \frac{1}{n} \rceil$ *instead of* $\log_M \frac{1}{n}$ *for simplification purposes. In addition, it is claimed that using corner ceiling brackets will not affect the level of accuracy of q-ROF_SRS. Last but not least, Example 5.3 is solved by corner ceiling brackets to prove the point. In the case of using corner ceiling brackets, the proper power will be 4. Accordingly, decreasing rate will be* 0.8^3*. Therefore, the distances are 1.5872, 1.2817,1.0764, 0.8794, 0.8858 for employer 1 to 5, respectively. Again, employer number four is selected. Therefore, it is proved that the proposed reducing strategy is accurate.*

6 Conclusion

This paper introduces a reducing strategy to improve q-ROF and T-SFS fuzzy sets. Increasing the space of acceptable grades along with maintaining the ratio between the grades are the main goals of the proposed symmetrical reducing strategy. To reach the goals, SRS introduce a flexible framework based on power operator and the maximum grade to find a proper power for q-ROF and T-SFS sets. Then, it reduces the grades according to their distance with the maximum one to increase the level of accuracy. The proposed strategy is how strong that is able to model all the decimal grades within the closed unit interval of [0, 1]. Also, SRS retains the ratio between grades for the intent of diminishing the information loss problem. In addition, the proposed strategy is able to be adapted according to the number of grades. Regarding to the mentioned ability, SRS is added to q-ROF and T-SFS fuzzy sets. This paper has proved that the improved q-ROF and T-SFS have greater space of acceptable grades than the compared fuzzy sets. Also, practical examples have shown that q-ROF_SRS and T-SFS_SRS are highly accurate. Using an interval to define a membership grade instead of using one value is the backbone idea of interval valued fuzzy sets. Even though these types of fuzzy sets increase complexity, they increase the level of accuracy, as well. To the end, we mainly focus on introducing interval valued fuzzy sets based on q-ROF_SRS and T-SFS_SRS in our future work.

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