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An approach for damage detection of space structures using combination of second order gradient and fuzzy logic methods

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Abstract

This study presents a new method of damage detection using a combination of the second-order gradient Levenberg-Marquardt algorithm (SOGLMA) and fuzzy logic (FL) to solve the nonlinear damage detection equation for space frame structures. For damage detection in structures with a large number of degrees of freedom using the second-order gradient Levenberg-Marquardt algorithm, it is necessary to perform an iterative process of analysis and solving a set of simultaneous nonlinear equations that requires a lot of time. Therefore, the computation time and the number of iterations are reduced by using the proposed method "Combination of the second-order gradient Levenberg-Marquardt algorithm and fuzzy logic (SOGLMA-FL)". Acceleration response in sensor nodes obtained at different time steps from dynamic analysis are considered as input values for fuzzification. The output values of the proposed method after defuzzification are the damage extent of structural elements. The results show that the proposed damage detection method (SOGLMA-FL) has faster convergence, lower numbers of iteration and reduced computation time than the damage detection method (SOGLMA) for space frame structures.

Keywords: Damage detection, space structure, second-order gradient algorithm, Levenberg-Marquardt, fuzzy logic.

1 Introduction

Structural health monitoring (SHM) is essential to ensure the safety performance of the structure during its useful life. Due to the need to prevent unwanted damage, the development of methods for monitoring the health of structures is of considerable importance. All structures are affected by damage during operation that, may cause structural failures. Changes in structural responses are due to damage in structural elements. By examining these changes, the location and extent of damage in the structural elements can be identified. The basis of damage detection in most structural damage detection methods is based on the responses measured in the structure, i.e. the types of static, dynamic and time history responses. The structural damage detection equation is defined as nonlinear due to the nonlinear nature of the damage detection phenomenon. Using derivative-based damage detection methods such as conjugate gradient method, steepest reduction, Newton method, etc., the structural damage detection equation can be solved and then the extent and location of damages in the structure can be determined using it. Since the presentation of digital computers, many efforts have been made to solve structural damage detection problems using various computational techniques. Using the traditional methods such as Newton-like methods may lead to lack of convergence, slowing of convergence, numerical instabilities, time consuming calculations [39].

In 1965, Zadeh [41], introduced the foundations of the fuzzy logic theory (fuzzy sets). Soft computing methods based on artificial intelligence such as genetic algorithms, neural networks and fuzzy logic methods have had many applications in recent years in the field of science and engineering, including civil engineering, mechanics, etc. These methods can be used for data analysis, modeling and optimization. These methods are becoming more and more

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important due to their ability to extract information from data and turn it into knowledge. These methods can be learnt and generalized and can produce acceptable results from inputs not observed while learning. In recent years, a number of researchers have studied the subject of fuzzy logic and its application in solving problems related to damage detection structures. The results of these studies are as follows: Sawyer and Rao [33], studied the damage detection of structural damage using fuzzy logic and presented a new method for damage detection structures based on fuzzy logic. The method presented by them is only appropriate for damage conditions in one location and is not suitable for multiple damage conditions. Edward et al. [7], investigated the expert system of fuzzy logic for automatic damage detection by changes in the strain energy of mode shapes. The results showed that the special fuzzy system provided by them offers a reliable and accurate result in identifying various damages. Ranjan [28], proposed a health monitoring system to detect structural damage to a helicopter rotor by measuring rotational and non-rotational frequencies in a vacuum, using the first four modes of the structure. Based on the obtained results, the proposed method using non-rotational frequencies offers more sensitive indicators to identify structural damage compared to rotational frequencies. Prashant et al. [27], studied the fuzzy-genetic system for damage detection helicopter rotor beams and blades. Using changes in frequencies, a fuzzy system is generated and the base rules and membership functions are optimized by a genetic algorithm. Fuzzy-Genetic System has been proposed in this study as a method for generating automated rules in fuzzy systems for damage detection structures. Reda et al. [30], introduced a new damage detection technique using simulated data from finite element analysis of a pre-stressed concrete bridge. The acceleration obtained from the sensors distributed on the bridge has been analyzed using a neural-wavelet network to create patterns of dynamic behavior of the bridge. The obtained results show that the method shown has the ability for damage detection with high accuracy. Ettefagh et al. [8], proposed a new method for detecting the presence, location and extent of damage in structures using Autoregressive Moving Average (ARMA) modeling and fuzzy classification. The results obtained by applying their method on experimental models show the high capabilities of their method for damage detection structures with stochastic excitation. Samuel et al. [32], in a study investigated the damage detection of structures using fuzzy clustering and vibrational data. The results show that both fuzzy clustering algorithms are effective, but the GK algorithm performs slightly better than the FCM algorithm. Chandrashekhar and Ranjan [5], proposed a fuzzy logic system with a new method for damage detection in structures with uncertainty. Probabilistic analysis is performed using the Monte-Carlo model (MCS) on the finite element model of a steel beam to calculate the statistical properties of changes in the natural frequencies of the beam, which are mostly measured by noise, due to structural damage and material uncertainty (Elastic modulus). From an algorithmic point of view, their research creates different fields of probability and fuzzy logic to reduce damage detection problems. In another study, Chandrashekhar and Ranjan [4], investigated the evaluation of damage of structures with uncertainty using modal curvature and fuzzy logic. In this research, a fuzzy logic system with a new technique has been designed for damage detection in the structures using curvature damage index. Based on the numerical results obtained in this research, the fuzzy system based on damage curvature index is a powerful tool for structural damage detection.

Beena and Ranjan [3], examined the damage detection of the structures by using fuzzy cognitive maps and Hebbian learning. They used fuzzy cognitive maps to account for the uncertainty commonly found in measurement and modeling processes. Numerical results show that FCM is a powerful tool for damage detection noisy data by learning. Jiang et al. [14], proposed a new method for two-stage structural damage detection using fuzzy-neural networks (FNNs) and data integration techniques. The method proposed by them is used for structural health monitoring and damage detection, especially in cases where the measurement data is very large and has uncertainty. Mojtahedi et al. [22], in a study developed a method to monitor the health of offshore jacket platforms using the update model and fuzzy logic system (FLS). They validated their method using the various vulnerability scenarios predicted in the physical model. Their method is effective in detecting the presence or absence as well as determining the extent of damage. Parhi and Choudhury [26], investigated the detection of cracks in a cantilever beam using fuzzy logic technique with hybrid membership functions. They examined a cantilever beam with a simple crack. The results showed that fuzzy controllers can predict the crack location and relative depth of the beam with less time than computation time compared to finite element analysis.

Zheng et al. [42], proposed a new algorithm for radial basis function neural networks with applications in structural health monitoring systems. The proposed algorithm is a combination of the advantages of fuzzy logic theory, neural networks, and genetic algorithms (GA) in which the genetic algorithm plays a central role. Parhi et al. [25], studied the detection and prediction of cracks using finite element analysis (FEA) using fuzzy logic method. Comparing fuzzy and experimental results, it can be observed that fuzzy controllers can predict the position and relative crack depth of a beam with a very accurate method and with a very small error percentage in nanoseconds, this results in significant savings in computational time. Tarighat [35], studied the damage detection of concrete bridge deck using the adaptive fuzzy-neural inference system (ANFIS). He simulated several damage scenarios at different locations on the bridge deck and calculated acceleration as an expression of response at certain points. In the proposed ANFIS damage detection

model, the sampled acceleration signals are used as inputs and the most probable area of the deck that contains the damage is identified. The results show that this method can reduce the time and cost of visual inspection and be used as an intelligent real-time damage detection system in practice. Hakima and Razak [9], presented the use of ANFIS and ANNs as two different artificial intelligence techniques to identify the extent of damage in a bridge. The results showed that both ANFIS and ANNs methods provide good damage prediction, and ANFIS method has better performance than ANNs. Sethi et al. [34], proposed a new method for determining the extent and location of cracks in the cantilever beam using fuzzy logic. The fuzzy controller used by them has Bell shaped membership functions and has three input parameters including the relative deviation of the first three natural frequencies and two output parameters including the relative depth and location of the crack. Comparing the fuzzy results with the experimental results, it is observed that the fuzzy controller can predict the relative depth and location of the crack with a small percentage of error. Ranjan [29], in another study examined fuzzy cognitive maps (FCMs) for structural damage detection. He investigated damage in a cantilever beam using continuum damage mechanics and natural frequencies as indicators of damage.

Kourehli et al. [16], predicted the location of structural damage using the adaptive neural-fuzzy inference system (ANFIS). Structural damage has been determined using modal data as ANFIS inputs and five damage levels ($0\%, 10\%, 20\%, 30\%$ and 40%) as outputs. The results showed that the use of frequency data and incomplete modal data is very effective in the process of structural damage detection with ANFIS. Frequency data alone cannot accurately detect damage and have high mean square error (MSE) and low correlation coefficient (R). While incomplete modal data or a combination of frequency and incomplete modal data have a high ability of damage detection and have a low mean square error (MSE) and a high correlation coefficient (R). The results show the high accuracy of the proposed method for detecting and estimating structural damage. Oliveira and Inman [24], evaluated the performance of two simplified fuzzy ARTMAP network (SFAN) and probabilistic neural networks (PNNs) damage detection methods. In order to evaluate both methods, experiments were performed on both methods for a single-sided composite plate consisting of four pieces (lead zirconate titanate) PZT. The results show that both methods are suitable for the problem of damage growth. However, comparative analysis has shown that the SFAN method is more suitable for solving the problem of damage growth. Agarwalla et al. [1], investigated a fuzzy-genetic controller for damage detection in a cantilever metal beam under the natural vibration signature. Comparison of the analysis of the results obtained from the fuzzy-genetic controller and the analysis of the experimental results showed that the proposed method can determine the location and extent of cracks with higher accuracy and has a very good agreement with the experimental results. Sahu et al. [31], in a study of intelligent adaptive fuzzy logic system investigated beam damage detection. They designed a hybrid optimization technique to automate fuzzy knowledge-based rules using an evolution algorithm. They presented a powerful damage detection method based on differential evolution algorithm and fuzzy logic.

Hameed et al. [11], Studied Fault classification using fuzzy logic in an epicyclic gearbox with statistical features. Examination of the results indicates that their method has a high predictive accuracy. In another work, Hameed et al. [10], Performed a comparative analysis of fuzzy classifier and artificial neural network with histogram features to detect and classify a defect in the planetary gearbox. The results show that both algorithms have high accuracy in detecting defects and classifying the defect status in the planetary gearbox. Kumar et al. [17], Investigated the fault detection of multi point cutting tool using fuzzy logic and artificial neural networks. The results show that both algorithms show promising results and ensure the robustness of the signals.

The use of fuzzy logic (FL) in the problem of structural damage detection is important for the following reasons:

- Its ability in comparison with traditional and previous methods.
- Increased calculation speed.
- Simplification in modeling or implementing solution methods.

The use of fuzzy logic (FL) can help us in the area of uncertainty. If the input data from the damage detection problem (structural responses) is specified as a fuzzy set, the damage detection problem can be solved using fuzzy logic (FL). Also, the application of the number and type of fuzzy membership functions such as membership functions in the form of triangular, trapezoidal, Gaussian, etc. can affect the results. Although various methods have been used for structural damage detection by using fuzzy logic in recent years, newer and more complete methods are still needed. The results of research in the field of structural damage detection using fuzzy logic indicate that most of the methods used by researchers are more for simple structural damage detection using fuzzy logic indicate that most of the methods used by researchers are more for simple structures such as beams using frequency response changes and mode shapes of the structure and, less research has been done in the field of complex structures such as space frame structures, plates, etc. under different conditions such as the use of structural responses other than mode frequencies and shapes, multiple damages and in terms of noise effects. The technique combination of fuzzy logic and the second-order gradient

Levenberg-Marquardt optimization method based on the combination of fuzzy logic and the second-order gradient Levenberg-Marquardt optimization method based on the acceleration response of some active degrees of freedom of the structure equipped with sensor. The main purpose of this study is to present the second-order gradient Levenberg-Marquardt algorithm (SOGLMA) based on fuzzy logic concepts to solve the problem of damage detection of space frame structures. The proposed method (SOGLMA-FL) can reduce the computation time and the number of iterations in solving the damage detection problem. The limitations of the proposed method are as follows:

- The system of structural damage detection equations can be divided into three modes of over-determined, determined and under-determined. In this study, damage detection of the structures in over-determined conditions (structural responses greater than the number of damage variables (number of elements)) has been specifically examined.
- The damage is in the form of a decrease in stiffness (linear decrease in elastic modulus).
- The responses used (input data) are the acceleration response at some active degree of freedom of the structure.

2 Mathematical expression of the problem of damage detection of space frame structures

The main purpose of solving the damage detection problem is to identify the damage extent in structural elements. Eq. (1) shows the relationship between the damage vector and the response vector of the damaged structure. The problem of damage detection of space frame structures is expressed by solving the system of nonlinear equations as Eq. (1). Where the solutions are the vector of equations (Eq. (2)) and the damage of the elements are the vector of unknowns Eq. (4).

$$\mathbf{R}_{\mathrm{d}} = \mathbf{R}(\mathbf{X}). \tag{1}$$

$$\mathbf{R}_{\mathrm{d}} = \left\{ \mathrm{r_{d1}} \cdot \mathrm{r_{d2}} \cdot \ldots \cdot \mathrm{r_{dm}} \right\}. \tag{2}$$

$$\mathbf{R}(\mathbf{X}) = \left\{ \mathrm{r}_1(\mathbf{X}) \cdot \mathrm{r}_2(\mathbf{X}) \cdot \ldots \cdot \mathrm{r}_{\mathrm{m}}(\mathbf{X}) \right\}. \tag{3}$$

$$\mathbf{X} = \left\{ \mathrm{x}_1, \mathrm{x}_2 \cdot \ldots \cdot \mathrm{x_i} \cdot \ldots \cdot \mathrm{x_{ne}} \right\};\ 0 \le \mathrm{x_i} \le 1;\ \mathrm{i} = 1 \cdot 2 \cdot \ldots \cdot ne. \tag{4}$$

In this system of equations, the goal is to find the damage vector of the elements of space frame structures, i.e. the $\mathbf{X}$ vector, using the response vector of the real damaged structure ($\mathbf{R}_{\mathrm{d}}$). Where $\mathrm{x_i}$ is the damage ratio of the i-th element, ne represents the number of structural elements or unknowns of the problem, m is the number of equations, and $\mathbf{R}(\mathbf{X})$ is the response vector of the hypothetical damaged structure, which is a nonlinear function of the damage vector. The values $\mathrm{x_i} = 0$ and $\mathrm{x_i} = 1$ of the hypothetical damaged structure, which is a nonlinear function of the damage vector. The values $x_i = 0$ and $x_i = 1$ indicate the completely healthy and damaged state of the structural elements, respectively. Considering the fact that damage is a nonlinear phenomenon, solving the system of damage detection equations is directly, limited and sometimes impossible. Therefore, the problem of damage detection of space frame structures in this study is first solved by approximating and using Taylor expansion to solve the system of linear equations and then solved using the proposed method. Today, most studies are based on minimizing the difference between the response of the real and hypothetical damaged structures that results from updating the damage index at each stage of iteration. The damage index can be selected in relation to the reduction of the cross-sectional area of the element, the reduction of the moment of inertia of the element, the reduction of the elastic modulus of the element or any other parameter indicating the reduction of stiffness and damage to the element. In this study, the reduction of the elastic modulus of the elements according to Eq. (5) has been considered as the damage index x^e;

$$x^e = -\frac{E^e - E_0^e}{E_0^e} \Rightarrow E^e = E_0^e \left(1 - x^e\right). \tag{5}$$

Where E_0^e and E^e are the initial elastic modulus and the updated elastic modulus of the element, respectively. Since the damage detection expressed in this study is a kind of optimization problem, it means finding the best solution with the lowest error, in which the difference between the response of the hypothetical and real damaged structures must be reduced at each optimization step to zero. Therefore, an objective function must be defined in order to control this difference to zero. Using the first-order terms obtained from the Taylor expansion of the damage detection equation, we

can transform the nonlinear damage detection problem of Eq. (1) into an approximation of the linear system problem (Eq. (6)). Finally, by solving the system of equations in the form of an inverse problem, the structural damage vector was obtained [13].

$$\Delta \mathbf{R} \cong \frac{\partial \mathbf{R}(\boldsymbol{X})}{\partial \boldsymbol{X}} \Delta \boldsymbol{X} = \mathbf{S} \Delta \boldsymbol{X}. \tag{6}$$

$$\Delta \boldsymbol{X} \cong \mathbf{S}^{-1} \Delta \mathbf{R}. \tag{7}$$

$$\boldsymbol{S} = \left[\begin{array}{cccc} \boldsymbol{s}_1 \boldsymbol{s}_2 \ldots \boldsymbol{s}_l \ldots \boldsymbol{s}_{\mathrm{Nsensor}} \end{array} \right]^T. \tag{8}$$

In Eqs. (6) and (8) $[\mathbf{S}]_{(nt*Nsensor)*ne}$ is called the sensitivity matrix or Jacobian and indicates the degree of sensitivity of response changes $(\Delta \mathbf{R})$ to damage variation $(\Delta \mathbf{X}), \mathbf{S^{-1}}$ is the inverse of the S matrix, $[\boldsymbol{s}_l]_{nt*ne}$ is the sensitivity matrix corresponding to the l-th degree of freedom from the structure with sensor, Nsensor is the total number of degrees of freedom of the structure with sensor and nt is the number of acceleration response steps recorded in the dynamic analysis of the structure. In this research, the response of the structure is acceleration in sensor nodes, which is obtained by using dynamic analysis with nt time steps. Therefore, the number of equations is always more than unknowns ($m > ne$), so it is necessary to solve the problem of damage detection using optimization methods.

3 Second-order gradient Levenberg-Marquardt algorithm (SOGLMA)

Convergence may not be obtained in the ill-conditioned system of equations using derivative-based methods such as the steepest descent methods, conjugate gradient method, Newton method, Pseudo- Newton method, etc. due to the sensitivity of problem solving to errors such as rounding, noise, etc. during numerical calculations. When the sensitivity matrix (Jacobian) is near singular, the system of the damage detection system of equations become ill-conditioned, and the methods mentioned may not be appropriate for solving the damage detection system of equations. In such cases, methods such as the second-order gradient Levenberg-Marquardt (SOGLMA) algorithm are suitable. This algorithm is based on the second-order gradientbased numerical optimization method, i.e. the modified Levenberg method and can be expressed as the second-order gradient Levenberg-Marquardt (SOGLMA) algorithm with the following cyclic relationships [2, 19].

$$\mathbf{X}^{(\mathrm{k}+1)} = \mathbf{X}^{(\mathrm{k})} + \Delta \mathbf{X}^{(\mathrm{k})};\ \mathrm{k} = 0.1.2. \tag{9}$$

$$\mathbf{X}^{(\mathrm{k}+1)} = \mathbf{X}^{(\mathrm{k})} - \left(\boldsymbol{H}^{(k)} + \lambda_k \mathrm{diagonal} \left[\boldsymbol{H}^{(k)} \right] \right)^{-1} J^T \left(\boldsymbol{X}^{(k)} \right) \boldsymbol{e}^{(k)}. \tag{10}$$

$$\boldsymbol{H}^{(k)} \cong J^T \left(\boldsymbol{X}^{(k)} \right) J \left(\boldsymbol{X}^{(k)} \right). \tag{11}$$

$$\mathbf{X}^{(\mathrm{k}+1)} \cong \mathbf{X}^{(\mathrm{k})} - \left(J^T \left(\boldsymbol{X}^{(k)} \right) J \left(\boldsymbol{X}^{(k)} \right) + \lambda_k \, \mathrm{diagonal} \left[J^T \left(\boldsymbol{X}^{(k)} \right) J \left(\boldsymbol{X}^{(k)} \right) \right] \right)^{-1} J^T \left(\boldsymbol{X}^{(k)} \right) \boldsymbol{e}^{(k)}. \tag{12}$$

In these relationships, the index k denotes the cycle number, $\boldsymbol{X}^{(k+1)}$ is the new optimized value, $\boldsymbol{X}^{(k)}$ represents the previous optimized value, $\Delta \mathbf{X}^{(k)}$ is the small changes in the previous design, $J \left(\boldsymbol{X}^{(k)} \right)$ is the first-order derivative matrix or Jacobian matrix, $\boldsymbol{H}^{(k)}$ is the second-order derivative matrix or Hessian matrix, λ_k denotes the convergence coefficient (nonand the actual value or output error of the damage function. The second-order gradient Levenberg-Marquardt algorithm finds the steepest descent between the first-order gradient algorithm and the second-order Gauss-Newton gradient algorithm. The only drawback of this method is that its reversibility in the update process in the optimization cyclic relationships is disrupted. In order to solve this problem, the pseudo-inverse method of single value decomposition (SVD) has been used [12]. Therefore, Eq. (7) can be rewritten as follows:

$$\Delta \boldsymbol{X} \cong \mathbf{S}^{+} \Delta \mathbf{R}. \tag{13}$$

Where $\mathbf{S}^{+}$ is called the pseudo-inverse of matrix S. Therefore, the Levenberg-Marquardt algorithm (Eq. (12)) can be rewritten as Eq. (14)

$$\mathbf{X}^{(\mathrm{k}+1)} \cong \mathbf{X}^{(\mathrm{k})} - \left[\mathbf{S} \left(\mathbf{X}^{(\mathrm{k})} \right)^T \mathbf{S} \left(\mathbf{X}^{(\mathrm{k})} \right) + \lambda_k \, \mathrm{diagonal} \left(\mathbf{S} \left(\mathbf{X}^{(\mathrm{k})} \right)^T \mathbf{S} \left(\mathbf{X}^{(\mathrm{k})} \right) \right) \right]^{+} \mathbf{S} \left(\mathbf{X}^{(\mathrm{k})} \right)^T \mathbf{e}^{(\mathrm{k})} \cdot \quad \mathbf{R} \left(\mathbf{X}^{(\mathrm{k})} \right) - \boldsymbol{R}_d = \mathbf{e}^{(\mathrm{k})}. \tag{14}$$

Where $\mathbf{S} \left(\mathbf{X}^{(\mathrm{k})} \right)$ is a partial derivative of changes in responses to damage changes and is called the sensitivity matrix or Jacobian.

4 Second-order gradient Levenberg-Marquardt fuzzy method (SOGLMA-FL) for damage detection of space frame structures

A fuzzy logic system is a nonlinear cognition map of an input to output vector. Fuzzy logic theory provides a framework for nonlinear cognition mapping. The fuzzy logic system can be expressed as a linear combination of basic fuzzy functions and an approximate general function is derived from it. Using simple fuzzy rules in if-then form, fuzzy inference systems can qualitatively model human information and decision-making systems on a particular issue without the need for detailed analysis. Between the input and output of a fuzzy logic system, there is a black box containing the fuzzy inference system (FIS) (Figure 1). Among the various methods of making a black box, fuzzy logic is often the best method. Professor Lotfizadeh, the father of fuzzy logic, says: "In almost all cases, all problems can be solved without the use of fuzzy logic, but the fuzzy solution is the fastest and cheapest solution among the available solutions" [23].

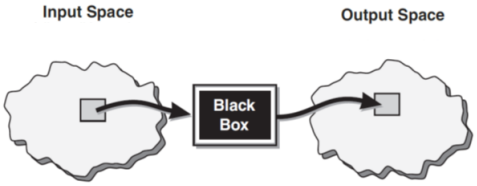


Figure 1: Input and output mechanisms in solving a problem.

A fuzzy set is a set without certain boundaries. The basic structure of a fuzzy inference system consists of three components: fuzzy rules, membership functions, and an argumentation mechanism that performs data processing operations. The most common forms of membership functions are often piecewise linear functions such as triangular or trapezoidal functions or nonlinear functions such as Gaussian. A membership function assigns the membership degree $\mu_i(x_j)$ to each member in the set, which is in the range [0, 1]. The fuzzy system designer selects the type and number of membership functions used. In this study, for input and output signals, Gaussian membership functions with the numbers of 7 has been used. In a fuzzy system, it is usually possible to implement two types of fuzzy inference system models: Mamdani fuzzy model and Sugeno fuzzy model. These two types of fuzzy models are different in terms of the method of determining the outputs (result or tally). The outputs in Mamdani fuzzy model are fuzzy, but in Sugno fuzzy model, the outputs are non-fuzzy. In this study, Mamdani fuzzy model has been used. Optimization using the second-order gradient Levenberg-Marquardt algorithm based on the rule bases of fuzzy logic controller (FLC) is more efficient for complex and nonlinear systems. Considering the slow and small step size, the training process of the steepest-descent gradient algorithm always leads to convergence [40]. However, the gradient Gauss-Newton algorithm has results with minimum error and faster response than the steepest descent gradient algorithm, but with a high probability of divergence. When the inverse of the Jacobian matrix approaches infinity, the gradient Gauss-Newton algorithm fails. Therefore, in order to take advantage of the convergence feature of the steepest-descent gradient algorithm and the fast response of the gradient Gauss-Newton algorithm, the modified Levenberg method, i.e. the second-order gradient LevenbergMarquardt algorithm, which is stable to the Jacobian matrix [21] is used. In addition, problems such as "minimizing nonlinear least squares" are controlled by the second-order gradient Levenberg-Marquardt algorithm. In general, the objective function in such problems can usually be defined as a nonlinear least squares problem in the form of the sum of the quadratic powers of the differences based on Eq. (15) [37, 36].

$$f(\boldsymbol{X}) = \frac{1}{2}\boldsymbol{e}_{\mathrm{j}}^{T}\boldsymbol{e}_{\mathrm{j}} = \frac{1}{2}\sum_{j=1}^{m}\boldsymbol{e}_j^2 \cdot \boldsymbol{e}(\boldsymbol{X}) = \boldsymbol{R}(\boldsymbol{X}) - \boldsymbol{R_d}. \tag{15}$$

In Eq. (15) $f(\boldsymbol{X})$ is the objective function, $\boldsymbol{e}_j(\boldsymbol{X})$ is the output error of the damage function and is a function in terms of R^{ne} to R and represents the remainder. Assuming $m \geq ne, \mathrm{X} = [x_1 \cdot x_2 \ldots\ldots x_{ne}]^T \in R^{ne\times 1}$ denotes damage vector, $\mathrm{x_i}$ represents the damage ratio of the i-th element, ne is the number of elements, m denotes the number of equations or number of solutions, $\mathbf{R}(\mathbf{X})$ is the response vector of the hypothetical damaged structure which is a function of the damage vector and $\mathbf{R}_{\mathrm{d}}$ is the response vector of the real damaged structure.

The proposed damage detection method, which is based on a combination of the second-order gradient LevenbergMarquardt algorithm and fuzzy logic concepts, has been used to optimize linear and nonlinear functions. In fact, the secondorder gradient Levenberg-Marquardt algorithm is used to minimize the objective function (square sum error function) $f_m\left(\boldsymbol{X}^{(k)}\right)$ based on a selective fuzzy logic system for all membership functions in the problem.

$$f_m\left(\boldsymbol{X}^{(k)}\right) = \frac{1}{2}\boldsymbol{e}_m^{(k)^T}\boldsymbol{e}_m^{(k)} = \frac{1}{2}\left\{\left(\boldsymbol{R}_m\left(\boldsymbol{X}^{(k)}\right) - \boldsymbol{R}_d\right)^T - \left(\boldsymbol{R_m}\left(\boldsymbol{X}^{(k)}\right) - \boldsymbol{R}_d\right)\right\}. \tag{16}$$

Where, $\boldsymbol{R}_m\left(\boldsymbol{X}^{(k)}\right)$ and $\boldsymbol{R}_d$ represent the calculated analytical and real values of the structural responses, respectively. $\boldsymbol{X}^{(k)}$ is the damage vector of structural elements in the k-th iteration resulting from solving a set of equations including m equations, each equation of which has an unknown number of structural elements. The objective function $f_m\left(\boldsymbol{X}^{(k)}\right)$ can be approximated using the concepts of fuzzy logic and the second-order gradient Levenberg-Marquardt algorithm. To obtain $\boldsymbol{R}_m\left(\boldsymbol{X}^{(k)}\right)$, the center of gravity method of fuzzy logic system membership functions according to Eq. (17) is used:

$$\boldsymbol{R}_m\left(\boldsymbol{X}^{(k)}\right)=\frac{\sum_{i=1}^{Rb}\boldsymbol{X}_i^{(k)}\mu_i^{(k)}\left(x_j^m\right)}{\sum_{i=1}^{Rb}\mu_i^{(k)}\left(x_j^m\right)}. \tag{17}$$

Where Rb is the number of fuzzy rule bases, $\boldsymbol{X}_i^{(k)}$ is the center of the fuzzy output membership functions corresponding to the i-th rule and $\mu_i^{(k)}\left(x_j^m\right)$ is a Gaussian membership function by which the degree of membership x_j^m is determined.

Definition 4.1. *The Gaussian membership function* $\mu_i^{(k)}\left(x_j^m\right)$ *is defined as follows:*

$$\mu_i^{(k)}\left(x_j^m\right)=\prod_{j=1}^{n}\exp\left(-\frac{1}{2}\left(\frac{x_j^m-c_j^i(k)}{\sigma_j^i(k)}\right)^2\right). \tag{18}$$

Where n *represents the number of input variables,* $c_j^i(k)$ *is the center point of the Gaussian membership function and* $\sigma_j^i(k)$ *represents the width of the Gaussian membership function. Finally, by replacing Eq.* (18) *in Eq.* (17)*, Eq.* (17) *can be rewritten as Eq.* (19)

$$\boldsymbol{R}_m\left(\boldsymbol{X}^{(k)}\right)=\frac{\sum_{i=1}^{Rb}\boldsymbol{X}_i^{(k)}\prod_{j=1}^{n}\exp\left(-\frac{1}{2}\left(\frac{x_j^m-c_j^i(k)}{\sigma_j^i(k)}\right)^2\right)}{\sum_{i=1}^{Rb}\prod_{j=1}^{n}\exp\left(-\frac{1}{2}\left(\frac{x_j^m-c_j^i(k)}{\sigma_j^i(k)}\right)^2\right)}. \tag{19}$$

The membership functions used in the fuzzy logic system are usually triangular, trapezoidal, and Gaussian. In this research, Gaussian membership functions have been used based on Eq. (18) for all inputs and outputs of the damage detection problem. Gaussian nonlinear membership functions are usually more accurate and fast than linear membership functions such as triangular and trapezoidal membership functions due to their continuity and having only two parameters $c_j^i(k)$ and $\sigma_j^i(k)$.

The second-order gradient Levenberg-Marquardt algorithm can be expressed as relation (20):

$$\boldsymbol{X}_i^{(k+1)}=\boldsymbol{X}_i^{(k)}-\left\{J^T\left(\boldsymbol{X}_i^{(k)}\right)J\left(\boldsymbol{X}_i^{(k)}\right)+\lambda_k\,\text{diagonal}\left[J^T\left(\boldsymbol{X}_i^{(k)}\right)J\left(\boldsymbol{X}_i^{(k)}\right)\right]\right\}^{-1}J^T\left(\boldsymbol{X}_i^{(k)}\right)\boldsymbol{e}_i^{(k)}. \tag{20}$$

Jacobian or first-order partial derivatives (J) are calculated based on the concepts of fuzzy logic by taking partial derivatives of the objective function, Eq. (16) with respect to the damage vector to update the second-order gradient Levenberg-Marquardt algorithm.

$$\boldsymbol{e}_m^{(k)}=\boldsymbol{R}_m\left(\boldsymbol{X}^{(k)}\right)-\boldsymbol{R}_d. \tag{21}$$

$$\frac{\partial f\left(\boldsymbol{X}^{(k)}\right)}{\partial\boldsymbol{X}_i^{(k)}}=\left(\boldsymbol{R}_m\left(\boldsymbol{X}^{(k)}\right)-\boldsymbol{R}_d\right)\frac{\partial\left(\boldsymbol{R}\left(\boldsymbol{X}^{(k)}\right)-\boldsymbol{R}_d\right)}{\partial\boldsymbol{X}_i^{(k)}}=\left(\boldsymbol{R}_m\left(\boldsymbol{X}^{(k)}\right)-\boldsymbol{R}_d\right)\frac{\partial\boldsymbol{R}\left(\boldsymbol{X}^{(k)}\right)}{\partial\boldsymbol{X}_i^{(k)}}=\boldsymbol{e}_m^{(k)}\frac{\partial\boldsymbol{R}\left(\boldsymbol{X}^{(k)}\right)}{\partial\boldsymbol{X}_i^{(k)}}. \tag{22}$$

$$\frac{\partial f\left(\boldsymbol{X}^{(k)}\right)}{\partial\boldsymbol{X}_i^{(k)}}=\boldsymbol{e}_m^{(k)}\frac{\partial}{\partial\boldsymbol{X}_i^{(k)}}\left[\frac{\sum_{i=1}^{Rb}\boldsymbol{X}_i^{(k)}\mu_i^{(k)}\left(x_j^m\right)}{\sum_{i=1}^{Rb}\mu_i^{(k)}\left(x_j^m\right)}\right]. \tag{23}$$

$$\frac{\partial f\left(\boldsymbol{X}^{(k)}\right)}{\partial\boldsymbol{X}_i^{(k)}}=\boldsymbol{e}_m^{(k)}\frac{\partial}{\partial\boldsymbol{X}_i^{(k)}}\left[\frac{\sum_{i=1}^{Rb}\boldsymbol{X}_i^{(k)}\prod_{j=1}^{n}\exp\left(-\frac{1}{2}\left(\frac{x_j^m-c_j^i(k)}{\sigma_j^i(k)}\right)^2\right)\Big]}{\sum_{i=1}^{Rb}\prod_{j=1}^{n}\exp\left(-\frac{1}{2}\left(\frac{x_j^m-c_j^i(k)}{\sigma_j^i(k)}\right)^2\right)}\right]. \tag{24}$$

$$J\left(\boldsymbol{X}_i^{(k)}\right)=\frac{\partial \boldsymbol{R}_m\left(\boldsymbol{X}^{(k)}\right)}{\partial \boldsymbol{X}_i^{(k)}}=\frac{\sum_{i=1}^{Rb}\boldsymbol{X}_i^{(k)}\mu_i^{(k)}\left(x_j^m\right)}{\sum_{i=1}^{Rb}\mu_i^{(k)}\left(x_j^m\right)}=\frac{\sum_{i=1}^{Rb}\boldsymbol{X}_i^{(k)}\prod_{j=1}^{n}\exp\left(-\frac{1}{2}\left(\frac{x_j^m-c_j^i(k)}{\sigma_j^i(k)}\right)^2\right)}{\sum_{i=1}^{Rb}\prod_{j=1}^{n}\exp\left(-\frac{1}{2}\left(\frac{x_j^m-c_j^i(k)}{\sigma_j^i(k)}\right)^2\right)}. \tag{25}$$

The proposed method in this research is to solve the damage detection equation of space frame structures. The proposed SOGLMA-FL method is based on the second-order gradient Levenberg-Marquardt algorithm(SOGLMA) and the concepts of fuzzy logic presented in the previous sections. Where for repeated updates of the damage vector at each stage of the iteration, the calculation of the damage vector changes $\Delta\boldsymbol{X}$ must be performed using the concepts of fuzzy logic. Fuzzy logic acts as a function for repeated updates of the fault vector in solving the structural damage detection problem. This function can be simulated as Eq. (26):

$$\Delta\boldsymbol{X}^{(k)}=fuz\left(\Delta\boldsymbol{R}^{(k)}\right). \tag{26}$$

Where, fuz represents the function of fuzzy logic, $\Delta\boldsymbol{R}^{(k)}$ is the vector of structural response variations in sensor nodes which is directly proportional to the $\Delta\boldsymbol{X}^{(k)}$ vector of damage extent changes of structural elements, and k represents the number of iterations. In fact, using the fuzzy concepts mentioned in the previous sections, the fuz fuzzy function for repeated updates of the damage vector at each stage of the iteration calculates the damage changes using structural responses.

In general, designing a process of a fuzzy inference system (FIS) at each stage of iteration includes the following five steps:

- Fuzzification of input variables $\left(\Delta\boldsymbol{R}^{(k)}\rightarrow\Delta\boldsymbol{R}_{fuz}^{(k)}\right)$ (Fuzzification is a process in which crisp values such as real numbers that are real-time information are changed to fuzzy rules through membership functions. (Fuzzification of input information is essential for the inference mechanism).
- Defining fuzzy membership functions (FMFs).
- Creating and defining fuzzy rule bases (FRbs) from a fuzzy model (the base rule is a set of linguistic rules, which has instructions for achieving control over good output variables).
- Process of Fuzzy Logic (PFL).
- Defuzzification of output variables $\left(\Delta\boldsymbol{X}_{fuz}^{(k)}\rightarrow\Delta\boldsymbol{X}^{(k)}\right)$ (Defuzzification is the process of inverse fuzzy by which the results of the inference mechanism become definitive output. Various techniques are used for defuzzification of such as minimum center, center of gravity, mean maximum, etc., the center of gravity method has been used in this research)

The main structure used of the fuzzy inference system (FIS) related to the proposed SOGLMA-FL method can be expressed in Figure 2.

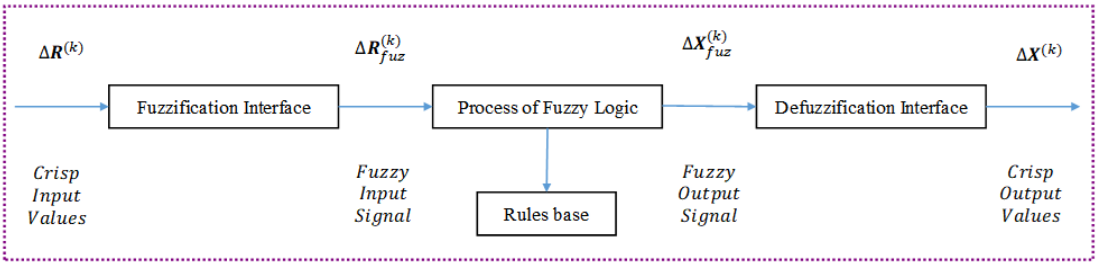


Figure 2: The main structure of fuzzy inference system (FIS) in the proposed SOGLMA-FL damage detection method.

In each iteration, the fuzzy interface first transforms and normalizes the crisp input values $\left(\Delta\boldsymbol{R}^{(k)}\right)$ to the fuzzy input signals in the sensor nodes of the structure. For normalization in each iteration, the per-unite method is used, i.e. the division of the desired quantity values into the maximum quantity value. For this purpose, the input values $\left(\Delta\boldsymbol{R}^{(k)}\right)$ in this study are transformed to fuzzy input signals $\left(\Delta\boldsymbol{R}_{fuz}^{(k)}\right)$ by Gaussian membership functions using seven

linguistic variables. The desired linguistic variables are defined as follows: Large negative (LN), medium negative (MN), small negative (SN), zero (ZR), small positive (SP), moderate positive (MP) and large positive (LP). The Gaussian membership functions used for the input signals have been presented in Table 1a and Figure 3.

Fuzzy rule bases (FRbs) are also defined using linguistic variables as follows:

Rb1: If $\Delta \boldsymbol{R}_{fuz}^{(k)}$ is LN then $\Delta \boldsymbol{X}_{fuz}^{(k)}$ is LN
Rb2: If $\Delta \boldsymbol{R}_{fuz}^{(k)}$ is MN then $\Delta \boldsymbol{X}_{fuz}^{(k)}$ is MN
Rb3: If $\Delta \boldsymbol{R}_{fuz}^{(k)}$ is SN then $\Delta \boldsymbol{X}_{fuz}^{(k)}$ is SN
Rb4: If $\Delta \boldsymbol{R}_{fuz}^{(k)}$ is ZR then $\Delta \boldsymbol{X}_{fuz}^{(k)}$ is ZR
Rb5: If $\Delta \boldsymbol{R}_{fuz}^{(k)}$ is SP then $\Delta \boldsymbol{X}_{fuz}^{(k)}$ is SP
Rb6: If $\Delta \boldsymbol{R}_{fuz}^{(k)}$ is MP then $\Delta \boldsymbol{X}_{fuz}^{(k)}$ is MP
Rb7: If $\Delta \boldsymbol{R}_{fuz}^{(k)}$ is LP then $\Delta \boldsymbol{X}_{fuz}^{(k)}$ is LP

Fuzzy rule bases are consistent with observations resulting from the correction behavior of the damage change vector that is directly consistent with the change vector of the structural responses in the nodes with sensor at each iteration step (according to Eq. (26). Fuzzy input signals $\left(\Delta \boldsymbol{R}_{fuz}^{(k)}\right)$ are sent to the process of fuzzy logic (PFL) in order to extract the fuzzy output signals ($\Delta \boldsymbol{X}_{fuz}^{(k)}$) using rule bases and linguistic variables similar to the linguistic variables defined for fuzzy input signals. Fuzzy output signals ($\Delta \boldsymbol{X}_{fuz}^{(k)}$) are defuzzified at each stage of repetition using the center of gravity method to transform crisp output values $\left(\Delta \boldsymbol{X}^{(k)}\right)$. The Gaussian membership functions used for the output signals have been presented in Table 1 b and Figure 4. Finally, the damage vector values of the structural elements $\left(\boldsymbol{X}^{(k)}\right)$ are updated at each stage of the iteration using the relationship $\boldsymbol{X}^{(k+1)} = \boldsymbol{X}^{(k)} + \Delta \boldsymbol{X}^{(k)}$. The number and type of fuzzy membership functions are effective in reducing computational time. Seven Gaussian type membership functions have been used for inputs and outputs in this research. The flowchart algorithm of the second-order gradient LevenbergMarquardt fuzzy method (SOGLMA-FL) for damage detection of space frame structures has been presented in Figure 5.

Table 1: Gaussian membership functions (MF).

a: Input Signals $\boldsymbol{\mu}(\Delta \boldsymbol{R}^{(k)})[*,**]$	b: Output Signals $\boldsymbol{\mu}(\Delta \boldsymbol{X}^{(k)})[*,**]$
$LN: \left[\frac{\Delta R_{max}^{(k)}}{3}, -\Delta R_{max}^{(k)}\right]$	$LN: \left[\frac{\Delta X_{max}^{(k)}}{3}, -\Delta X_{max}^{(k)}\right]$
$MN: \left[\frac{\Delta R_{max}^{(k)}}{6}, -\frac{\Delta R_{max}^{(k)}}{2}\right]$	$MN: \left[\frac{\Delta X_{max}^{(k)}}{6}, -\frac{\Delta X_{max}^{(k)}}{2}\right]$
$SN: \left[\frac{\Delta R_{max}^{(k)}}{12}, -\frac{\Delta R_{max}^{(k)}}{4}\right]$	$SN: \left[\frac{\Delta X_{max}^{(k)}}{12}, -\frac{\Delta X_{max}^{(k)}}{4}\right]$
$ZR: \left[\frac{\Delta R_{max}^{(k)}}{24}, 0\right]$	$ZR: \left[\frac{\Delta X_{max}^{(k)}}{24}, 0\right]$
$SP: \left[\frac{\Delta R_{max}^{(k)}}{12}, \frac{\Delta R_{max}^{(k)}}{4}\right]$	$SP: \left[\frac{\Delta X_{max}^{(k)}}{12}, \frac{\Delta X_{max}^{(k)}}{4}\right]$
$MP: \left[\frac{\Delta R_{max}^{(k)}}{6}, \frac{\Delta R_{max}^{(k)}}{2}\right]$	$MP: \left[\frac{\Delta X_{max}^{(k)}}{6}, \frac{\Delta X_{max}^{(k)}}{2}\right]$
$LP: \left[\frac{\Delta R_{max}^{(k)}}{3}, \Delta R_{max}^{(k)}\right]$	$LP: \left[\frac{\Delta X_{max}^{(k)}}{3}, \Delta X_{max}^{(k)}\right]$

*: Gaussian membership function width (σ), **: Gaussian membership function center (c)

$*$: Gaussian membership function width (σ), $**$: Gaussian membership function center (c)

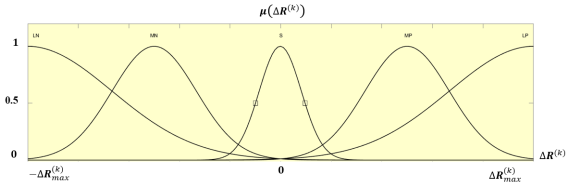


Figure 3: Gaussian membership (MF) functions of input signals ($\Delta \boldsymbol{R}^{(k)}$)

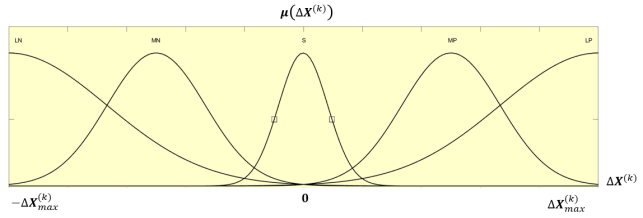


Figure 4: Gaussian membership (MF) functions of output signals ($\Delta \boldsymbol{X}^{(k)}$)

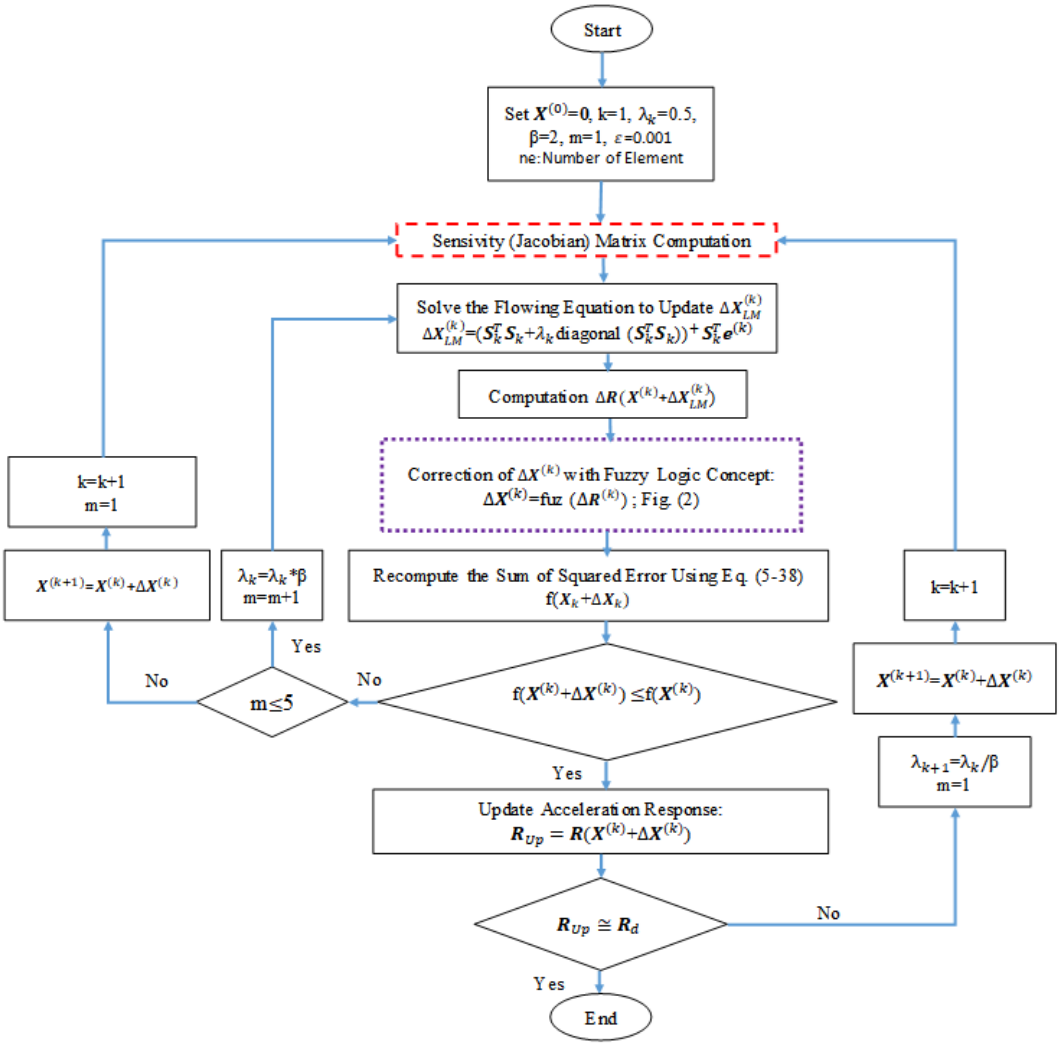


Figure 5: Flowchart of steps of the second-order gradient Levenberg-Marquardt fuzzy algorithm (SOGLMA-FL) for damage detection of space frame structures.

The steps of the proposed damage detection method called the second-order gradient Levenberg-Marquardt fuzzy algorithm (SOGLMA-FL) to detect damage in space frame structures can be described as follows:

Step 1. Proper selection of starting point and control parameters in the algorithm.

Step 2. The physical properties of a healthy structure are the starting point of the update in the finite element model.

Step 3. Choosing the right factor is important for λ_k and varies for different cases. This coefficient must be a small, nonnegative number. For all problems solved in this research, the initial value selected for λ_k is 0.5.

Step 4. Updating the sensitivity (Jacobin) matrix.

Step 5. Updating the damage index using the proposed second-order gradiant Levenberg-Marquardt algorithm.

Step 6. Calculation of the response change vector using the damage change vector update obtained from the second-order gradiant Levenberg-Marquardt algorithm. Step 7. Correction of damage change vectors using fuzzy logic concepts (FLC)

Step 8. Updating the objective function.

Step 9. If the value of the objective function decreases, it is accepted.

Step 10. If the value of the objective function exceeds the previous value, the new value is assigned to λ_k and the new $X^{(k+1)}$ is recalculated

Step 11. Using the proposed second-order gradient Levenberg-Marquardt algorithm and the new damage index, the updated value of the objective function is recalculated.

Step 12. If the value of the objective function decreases at this stage, the updated damage index is accepted.

Step 13. Frequent updates of the damage index continue to reduce the objective function for five steps.

Step 14. If the objective function does not change with the damage index update, the update process will continue from the previous state with the damage index.

Step 15. As the update continues in the new mode, the value of λ_k decreases.

5 Error and noise in data measurement

In order to bring the theoretical results closer to reality, noise is required to be applied to the data. These noises are applied to the responses in this study based on Eq. (27) [18, 38]:

$$\boldsymbol{R}_{\text{measured}} = \boldsymbol{R}_{\text{calculated}} + E_p * \boldsymbol{N}_{\text{noise}} * \sigma\left(\boldsymbol{R}_{\text{calculated}}\right). \tag{27}$$

Where, $\boldsymbol{R}_{\text{measured}}$ represents the acceleration response vector with noise, $\boldsymbol{R}_{\text{calculated}}$ is the vector of acceleration response without noise, $\sigma\left(\boldsymbol{R}_{\text{calculated}}\right)$ denotes the standard deviation of vector of acceleration response without noise, E_p represent the noise level (one percent, two percent, five percent, etc.) and $\boldsymbol{N}_{\text{noise}}$ is the Normal scatter vector with mean value of zero and standard deviation of one.

6 Numerical examples

In this section, in order to evaluate and validate the proposed damage detection method (SOGLMA-FL), two numerical examples of two- and three-dimensional truss structures under different damage detection scenarios are solved and the results with the results obtained from the basic method (SOGLMA) and Reference methods are compared. In the dynamic analysis of two-dimensional and three-dimensional models, triangular impact load according to Figure 6 and Elcentro earthquake accelerogram according to Figure 7 in the vertical direction have been used, respectively. Modeling, loading and model analysis have been done using the powerful OpenSees $^{\text{TM}}$ software [20]. One of the prominent features of this software is its high speed of analysis, which effectively helps the problem-solving process of the proposed damage detection. In all examples, using acceleration responses recorded at points with sensors, damage detection is performed by the proposed method by updating the elastic modulus parameter in the finite element model of the structure. Analysis and implementation of mathematical methods for damage detection has been done with MATLAB software. One of the advantages of the two mentioned software is their fast and easy communication, so that damage detection of structures with a computer can be done in a suitable time.

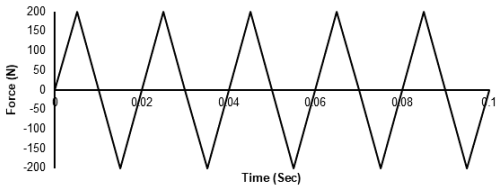


Figure 6: History of triangular impact loading used in two-dimensional truss dynamic analysis.

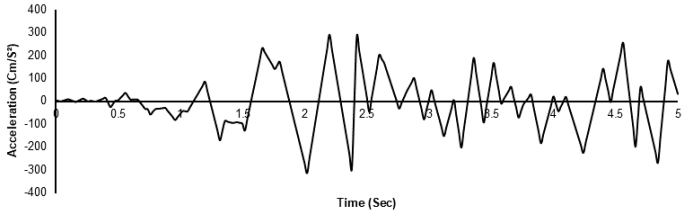


Figure 7: The first five seconds of the Centro earthquake record used in dynamic loading to analyze the timing of a three-dimensional truss.

6.1 Two-dimensional truss with 21 elements and 12 nodes

Damage detection for the 21-member steel truss has been performed as shown in Figure 8 . The finite element model of this structure consists of 12 nodes with 24 total degrees of freedom and 21 active degrees of freedom. The elastic modulus and the weight per unit volume are 200Gpa and 7800 kg/m^3, respectively. The cross section of the elements are $A_1 \ldots . A_6 = 15$ cm^2, $\mathrm{A}_7 \ldots . \mathrm{A}_{17} = 9$ cm^2 and $\mathrm{A}_{18} \ldots . \mathrm{A}_{21} = 12$ cm^2. The damage is simulated by reducing the elastic modulus to the damage ratio. In this example, it is assumed that sensors have been installed in all nodes. In this example, the gravity load in the nodes are in accordance with Table 2 and the damage cases are presented in Table 3, the damage detection of the structure under the damage scenarios in Table 4. A triangular impact load with a period of 0.02 seconds, with a time step of 0.005 seconds and for a period of one second in accordance with Eq. (28) and Figure 6 has been applied in the vertical direction to the nodes of 8-12 trusses. Comparison of the results of the

proposed method (SOGLMA-FL) with the methods used for damage detection using basic methods (SOGLMA) and Jaya algorithm (JA) [6] have been presented for different scenarios in Table 5, Figures 9, 10 and 11.

$$F_{Tri}(t) = 4E04 * \begin{cases} t & 0 \le t \le 0.005 \\ -t + 0.01 & 0.005 \le t \le 0.015 (N) \\ t - 0.02 & 0.015 \le t \le 0.02 \end{cases} \tag{28}$$

Where t is time and $F_{Tri}(t)$ is the triangular impact force at time t.

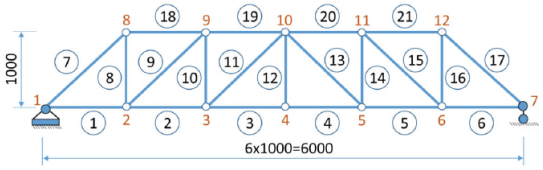
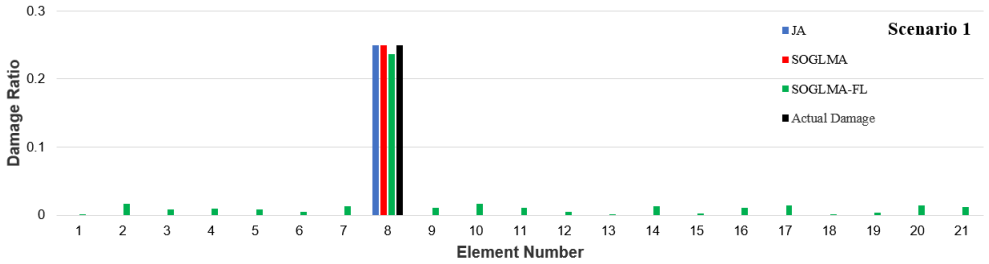


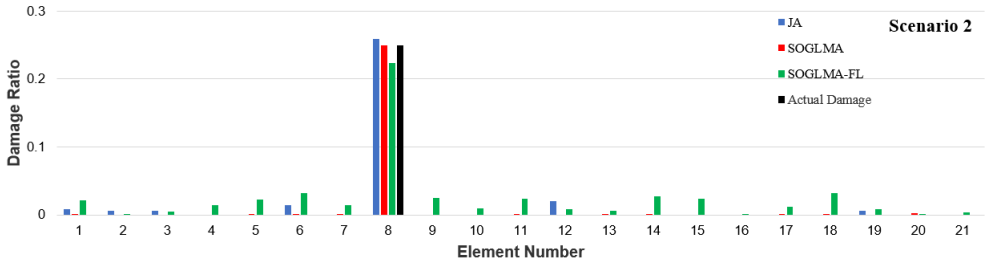
Figure 8: Two-dimensional truss with 21 elements and 12 nodes: Truss geometry, numbering of nodes and elements.

Table 2: Two-dimensional truss gravity loading mode with 21 elements.

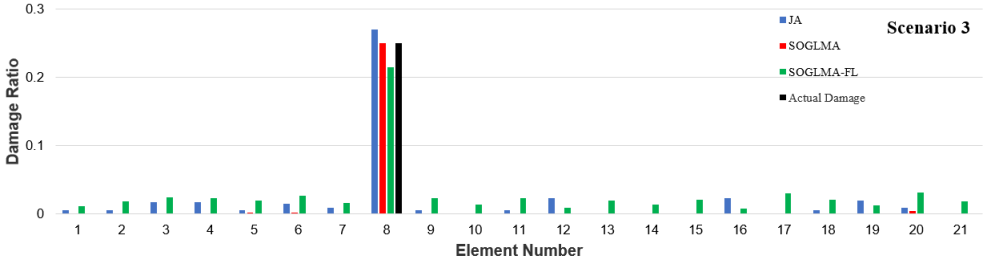
Node number	F_x (KN)	F_y (KN)
8,9,10,11,12	0.01	-1



(a) Damage scenario 1



(b) Damage scenario 2



(c) Damage scenario 3

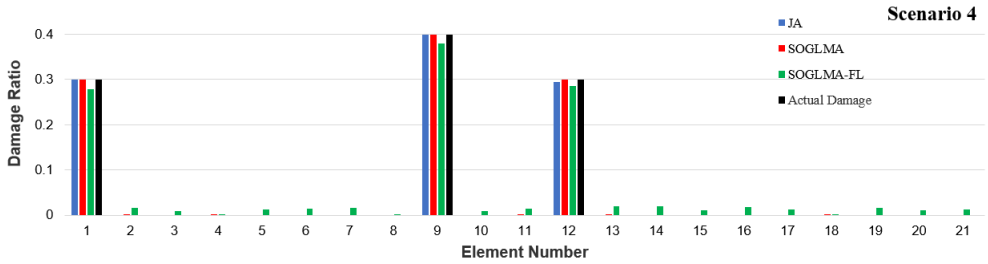
Figure 9: Comparison of damage location and extent results of the proposed method (SOGLMA-FL) with (SOGLMA) and (JA) methods for two-dimensional truss with 21 elements.

Table 3: Damage cases used for two-dimensional truss with 21 elements.

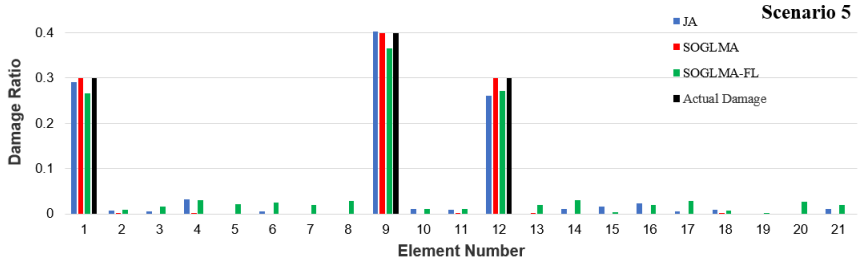
Damage case	Element number	Damage ratio (%)
1	8	25
2	1	30
	9	40
	12	30

Table 4: Different damage detection scenarios for two-dimensional truss with 21 elements.

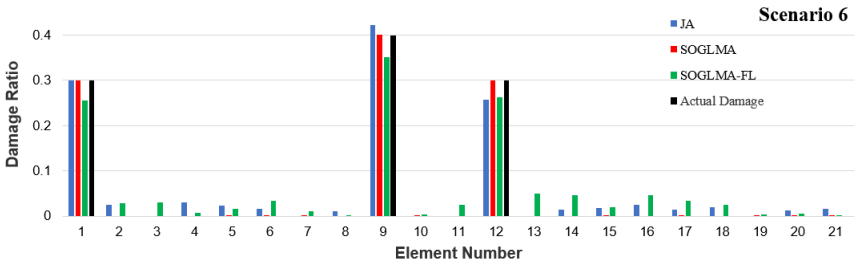
Scenario	Damage case	Noise Level (%)
1	1	0
2	1	1
3	1	2
4	2	0
5	2	1
6	2	2



(a) Damage scenario 4



(b) Damage scenario 5

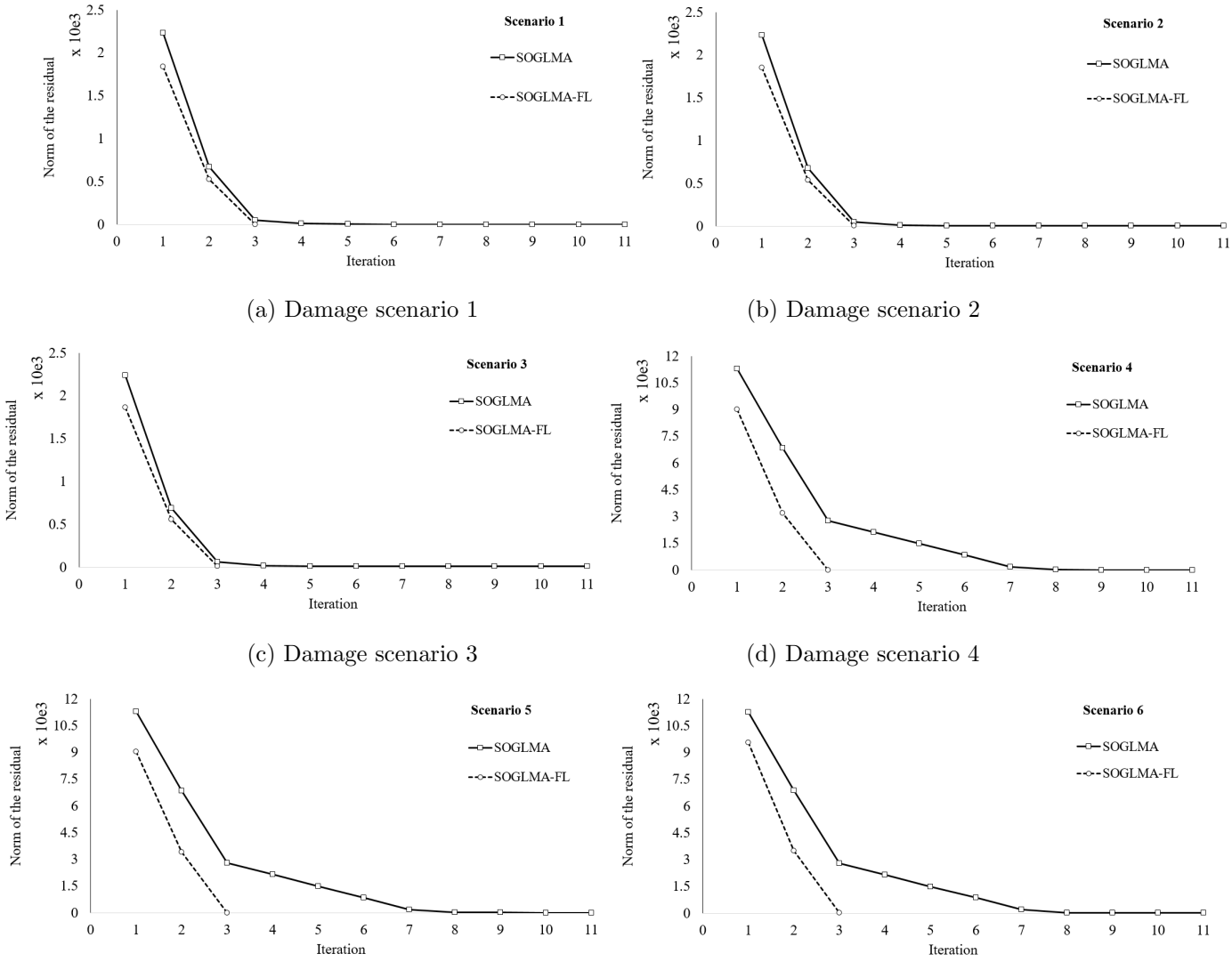


(c) Damage scenario 6

Figure 10: Comparison of damage location and extent results of the proposed method (SOGLMA-FL) with (SOGLMA) and (JA) methods for two-dimensional truss with 21 elements.

Table 5: Comparison of analysis time and number of convergence iterations of SOGLMA and SOGLMAFL methods in different damage scenarios for two-dimensional truss with 21 elements using computer with CPU.

	SOGLMA method			SOGLMA-FL method		
	CPU Time (Sec)	Number of Iteration	Error(%)	CPU Time (Sec)	Number of Iteration	Error(%)
Scenario 1	754	11	0	52	3	4.9
Scenario 2	827	11	0.11	53	3	10.1
Scenario 3	823	11	0.23	54	3	13.8
Scenario 4	774	11	0.1	57	3	5.49
Scenario 5	810	11	0.12	53	3	9.45
Scenario 6	772	11	0.32	56	3	12.87



(a) Damage scenario 1

(b) Damage scenario 2

(c) Damage scenario 3

(d) Damage scenario 4

(e) Damage scenario 5

(f) Damage scenario 6

Figure 11: Comparison of convergence diagrams (soft residual) of the proposed method (SOGLMA-FL) and the basic method (SOGLMA) of a two-dimensional truss with 21 elements.

According to Figures 9 and 10, it can be observed that the proposed method (SOGLMA-FL) has been able to identify the location or locations of damage for two-dimensional truss with 21 elements and estimate the extent of damage with an acceptable error rate for all scenarios. Using this method, the damage extent of the elements for damage scenarios are 1-6 is $4.9\%, 10.1\%, 13.8\%, 5.49\%, 9.45\%$ and 12.87%, respectively. Also, the proposed SOGLMA-FL method according to Figure 11 and Table 5 for all scenarios compared to the (SOGLMA) method has been able to identify the location and extent of damages with lower numbers of iteration and shorter time. The proposed SOGLMA-FL method converges with only three iterations in all scenarios 1-6 and has acceptable results. Although the method (SOGLMA) according to the results presented in Table 5 has a higher accuracy than the proposed method (SOGLMA-FL) but it has converged with more iterations and has more analysis time in all scenarios.

6.2 Three-dimensional truss with 52 elements and 21 nodes

The 52-member steel space frame truss has been subjected to damage detection process according to Figure 12(a,b). The finite element model of this structure consists of 21 nodes with 63 total degrees of freedom and 39 active degrees of freedom. The elastic modulus and weight per unit volume are 210Gpa and 7800 kg/m^3, respectively. The cross section of all elements is 2 cm^2. The damage is simulated by reducing the elastic modulus to the damage ratio. In this

example, the sensor placement mode is according to Figure 12(c) and Table 6, the gravity load in the nodes is according to Table 7, the damage detection of the structure is performed under the damage scenarios in Table 8 and the damage cases according to Table 9. The truss has been subjected to the dynamic load of the Elcentro Earthquake record with a time step of 0.02 seconds, in the vertical direction. Comparison of the results of the proposed method (SOGLMA-FL) with the methods used for damage detection using the basic methods (SOGLMA) and the hybrid method HRPSO (combination of particle swarm method, radial optimization and harmonic search) [15] have been presented in Table 10, Figures 13 and 14 for different scenarios.

Table 6: Sensor-placement mode of three-dimensional truss with 52 elements.

Node number of sensors
1,6,7,8,9

Table 7: Gravity loading mode for three-dimensional truss with 52 elements.

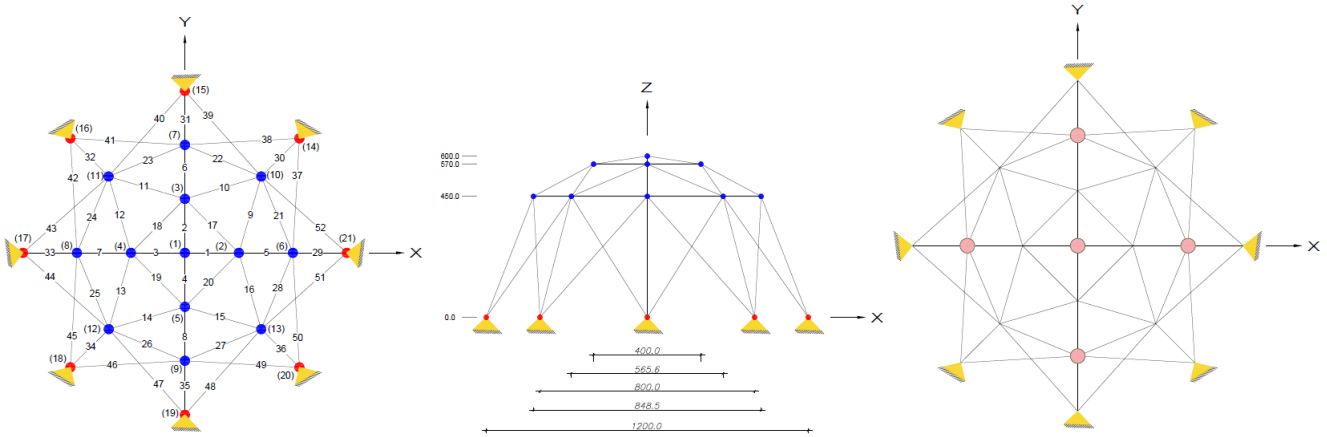
Node number	F_x (KN)	F_y (KN)	F_z (KN)
1	0	0	-10

Table 8: Different damage detection scenarios for 52-member truss.

Scenario	Damage case	Noise Level (%)
1	1	3
2	1	5
3	2	3

Table 9: Damage cases used for 52-member truss.

Damage case	Element number	Damage ratio (%)
1	13	20
	21	20
	39	15
2	12	30
	28	20
	43	25
	49	20

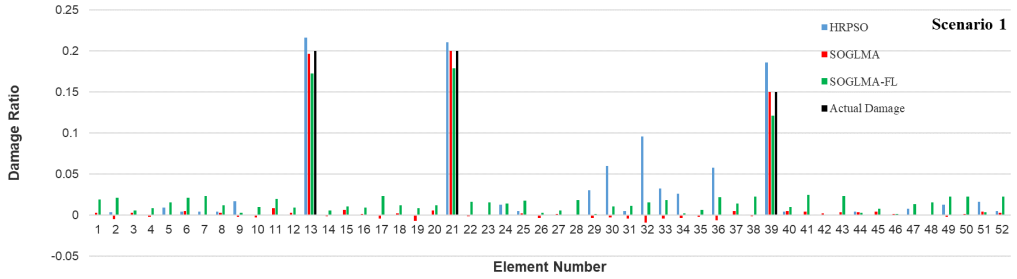


(a) Plan, numbering of nodes and elements (b) Cross section and dimensions of the truss (c) Location of sensor nodes

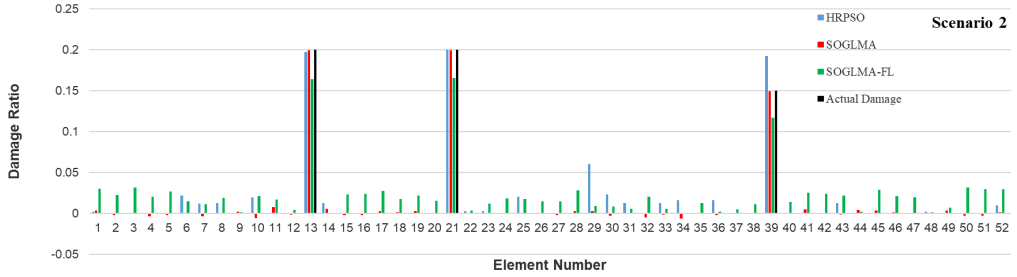
Figure 12: Three-dimensional truss geometry with 52 elements and 21 nodes.

Table 10: Comparison of analysis time and number of convergence iterations of SOGLMA and SOGLMA-FL methods in different damage scenarios for space frame truss with 52 elements.

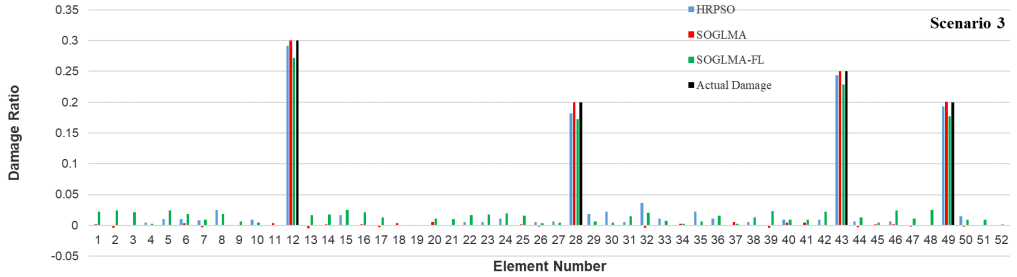
	SOGLMA method			SOGLMA-FL method		
	CPU Time (Sec)	Number of Iteration	Error(%)	CPU Time (Sec)	Number of Iteration	Error(%)
Scenario 1	3271	10	0.71	512	4	11.75
Scenario 2	4573	11	0.94	541	4	14.62
Scenario 3	4607	11	0.37	589	4	10.04



(a) Damage scenario 1

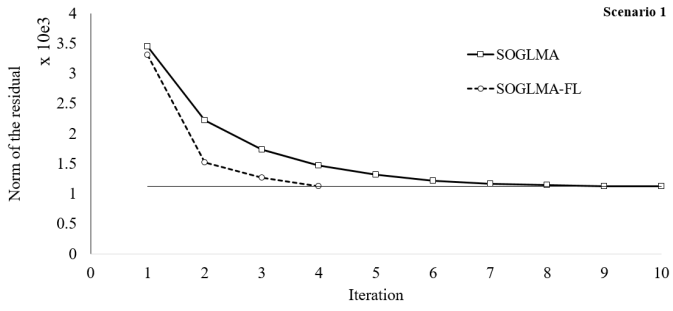


(b) Damage scenario 2

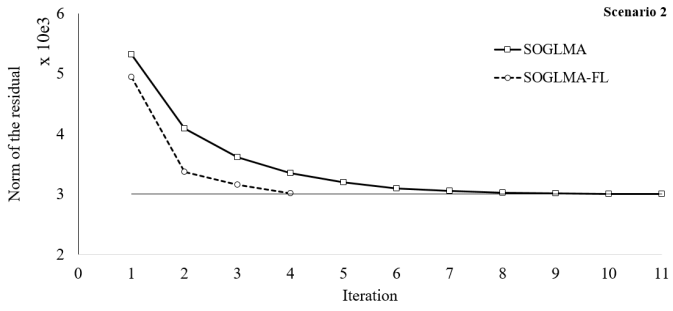


(c) Damage scenario 3

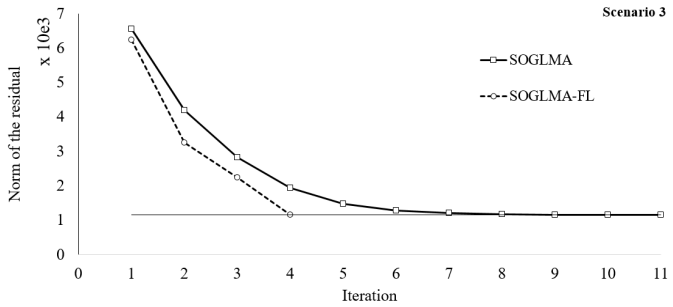
Figure 13: Comparison of the results of damage detection of the location and extent of damage of the proposed method (SOGLMA-FL) with the methods (SOGLMA) and (HRPSO) for space frame truss with 52 elements.



(a) Damage scenario 1



(b) Damage scenario 2



(c) Damage scenario 3

Figure 14: Convergence diagrams (soft residual) Proposed method (SOGLMA-FL) and base method (SOGLMA) space frame truss with 52 elements.

According to Figure 13, it can be observed that the proposed method (SOGLMA-FL) has been able to identify the location and locations of damage for the space frame truss with 52 elements and to estimate the extent of damage for all scenarios with an acceptable error rate. Using this method, the damage rate of the elements for damage scenarios 1-3 is 11.75%, 14.62% and 10.04%, respectively. Also, the proposed method (SOGLMA-FL) according to Figure 14 and Table 10 for all scenarios compared to the method (SOGLMA) has been able to identify the location and extent of damages with lower numbers of iteration and shorter time. The proposed method (SOGLMA-FL) converged in all scenarios 1 to 3 with only four iterations and shorter time. The proposed method (SOGLMA-FL) converged in all scenarios 1 to 3 with only four iterations higher accuracy than the proposed method (SOGLMA-FL) but in all scenarios, it converges with more iterations and has more higher accuracy than the proposed method (SOGLMA-FL) but in all scenarios, it converges with more iterations and has more analysis time.

7 Discussion and Conclusion

Given that damage detection of structures using gradient methods may lead to problems such as lack of convergence, slowing of convergence, numerical instabilities, timely calculations, etc., it is necessary to provide methods that can solve the problems of gradient methods. In this research, using a combination of fuzzy logic concepts and the second-order gradient Levenberg-Marquardt algorithm, called the second-order gradient Levenberg-Marquardt fuzzy method (SOGLMA-FL), a new method for damage detection of space frame structures to reduce damage detection problems with gradient methods have been proposed. The results obtained from numerical examples of the proposed method have acceptable capability in detecting the damage of the elements of two- and three-dimensional truss structures under different damage scenarios. Fuzzy inference system inputs include changes in the acceleration of sensor nodes and outputs include changes in the damage extent of structural elements at each iteration step. The results of the modeling used show the capability of the proposed method (SOGLMA-FL) to solve structural damage detection problems. The results show that the maximum error created in the two-dimensional truss in the third scenario is 13.8% and the maximum error created in the three-dimensional truss in the second scenario is 14.62%. According to the results of numerical examples of the main advantages of the proposed method (SOGLMA-FL) compared to the basic gradient method SOGLMA can be mentioned such as faster convergence, lower numbers of iterations and reduced computatione.

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