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On the global stabilization of perturbed nonlinear fuzzy control systems

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Abstract

In this paper, we deal with the global practical exponential stabilization of a class of perturbed Takagi-Sugeno fuzzy control systems. The terms of perturbations are supposed uniformly bounded by some known functions and in certain cases not necessarily smooth. We prove that the solution of the closed-loop system with a linear fuzzy controller converge to a neighborhood of the origin. We use common quadratic Lyapunov function and parallel distributed compensation controller techniques to study the asymptotic behavior of the solutions of fuzzy system. Numerical simulations are given to validate the proposed approach.

Keywords: Takagi-Sugeno fuzzy systems, practical exponential stabilization, perturbations, PDC controller, quadratic Lyapunov function, linear matrix inequalities.

1 Introduction

In recent years, Takagi-Sugeno (T-S) fuzzy models [19] have become a useful tool to deal with a class of nonlinear systems. The models can be described by a set of if-then rules which gives local linear approximations of an underlying system. One of the most important problems while designing fuzzy controller is to derive the desired fuzzy rule base. Trial and error has been a natural choice to design fuzzy controller in this case. The selection of fuzzy if-then rules often relies on a substantial amount of heuristic observations to express proper strategy. Obviously it is difficult for human experts to examine all the input data from a complex system to find the number of proper rules for a fuzzy system. To cope with this difficulty, much research effort has been devoted to develop alternative design methods. The stability analysis and control design for T-S fuzzy systems keep attracting researchers for decades However, the use of a (CQLF) often leads to overly conservative results because a common Lyapunov matrix should be found for all subsystems [14, 15, 16, 17, 18, 19, 21]. For the stability analysis of T-S fuzzy systems, many researchers have presented the conventional quadratic Lyapunov function satisfying the stability conditions of all subsystems [9, 8, 18, 22]. The Lyapunov stability theory is the main approach for these kinds of problems, among them, the simplest approaches consists in looking for a common quadratic Lyapunov function (CQLF) by using the concept of the parallel distributed compensation (PDC) technique [15, 18, 21] to design a stabilizing controller. However, another important issue in stability analysis of nonlinear systems is to study the behavior of the solutions in practical sense, it means that the trajectories converge to a small neighborhood of the origin. Stability is one of the most important concepts concerning the properties of control systems. In practical engineering systems, external disturbances tend to introduce oscillation, even causes instability. In these cases, the systems are usually not stable in the sense of Lyapunov stability, but sometimes their performance may be acceptable in practice just because they oscillate sufficiently near a mathematically unstable course. Furthermore, in some cases, though a system is stable, it can not be acceptable in practice engineering just because the attraction domain is not large enough. To deal with these situation, the concept of practical stability [1, 2, 3, 10, 11, 4, 5, 12, 7, 6, 13], which is derived from the so called, finite time stability, is more useful. In these studies, the origin was not supposed to be necessarily an equilibrium point of the system. So, we can no longer expect to design a controller that guarantee the stability of the origin as an equilibrium point. A new approach for the stability

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analysis is proposed. The approach allows the computation of the bound which characterizes the main novelty of this work relies on the fact that the proposed approach for stability analysis allows for the computation of the bound which characterize the exponential rate of convergence of the solutions under some assumptions on the perturbed term. In this paper, we study the stabilization problem of a class of Takagi-Sugeno fuzzy systems in presence of external disturbances (perturbations). The common quadratic Lyapunov function and parallel distributed compensation controller are used to show the convergence to a small ball for the solutions of the uncertain T-S fuzzy systems, even when the origin is not an equilibrium point, provided that the uncertainties are supposed uniformly bounded by known integrable functions. Moreover, we give some new sufficient conditions to solve this problem when the uncertainties are not smooth. An example is given to prove the validity of the main result for inverted pendulum system.

2 Fuzzy control systems

Consider a class of the continuous-time T-S fuzzy control system which can be described by the following fuzzy rules,

$$\text{Rule } i: \text{ If } z_1(t) \text{ is } M_{i1} \text{ and } z_2(t) \text{ is } M_{i2} \text{ ... and } z_p(t) \text{ is } M_{ip}, \text{ then}$$

$$\dot{x}(t) = A_i x(t) + B_i u(t),\ i = 1, 2, ..., r,$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, $A_i \in \mathbb{R}^{n\times n}$ and $B_i \in \mathbb{R}^{n\times m}$ are the system matrix input matrix, $i = 1, ..., r$ is the number of fuzzy rules, M_{ij} are the inputs fuzzy sets, $z(t) = [z_1(t), ..., z_p(t)]^T$ are measurable variables, i.e., the premise variables. Using weighted average defuzzifiers, the aggregated fuzzy model is given by

$$\dot{x}(t) = \frac{\sum_{i=1}^{r} w_i(z)\big(A_i x(t) + B_i u(t)\big)}{\sum_{i=1}^{r} w_i(z)},$$

where

$$w_i(z) = \prod_{j=1}^{r} M_{ij}(z_j).$$

Let $\mu_i(z)$ be the membership functions that belong to class $\mathcal{C}^1$, i,e., they are continuous differentiable and defined as

$$\mu_i(z) = \frac{w_i(z)}{\sum_{i=1}^{r} w_i(z)}.$$

Then, the fuzzy system has the state-space form

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(z)\big(A_i x(t) + B_i u(t)\big), \tag{1}$$

where $\mu_i(z) \geq 0$ for $i = 1, 2, ..., r$ and $\sum_{i=1}^{r} \mu_i(z) = 1$.

Many published results, concerning the control of the fuzzy system, are based on the PDC principle. The design of the fuzzy controller shares the same antecedent as the fuzzy system and employs a linear state feedback control in the consequent part. For each local dynamics the controller is defined as

$$\text{Rule } i: \text{ If } z_1(t) \text{ is } M_{i1} \text{ and } z_2(t) \text{ is } M_{i2} \text{ ... and } z_p(t) \text{ is } M_{ip}, \text{ then}$$

$$u(t) = -K_i x(t),\ i = 1, 2, ..., r, \tag{2}$$

where K_i is the local state feedback gain. Consequently, the defuzzified result is

$$u(t) = -\sum_{i=1}^{r} \mu_i(z) K_i x(t). \tag{3}$$

The system (2.1) in closed-loop with the fuzzy controller (2.3) yields the following fuzzy system,

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(z)\mu_j(z)\big(A_i - B_i K_j\big)x(t). \tag{4}$$

A sufficient condition for the stability is deduced using Lyapunov's direct method. Suppose that a common positive definite matrix P exists, so that the following conditions are satisfied:

$$(A_i - B_i K_i)^T P + P(A_i - B_i K_i) < 0, \; i = 1, 2, ..., r,$$

and

$$\frac{1}{2}(A_i - B_i K_j + A_j - B_j K_i)^T P + \frac{1}{2}P(A_i - B_i K_j + A_j - B_j K_i) < 0, \; 1 \le i < j \le r.$$

When these conditions are satisfied, the fuzzy system (2.4) is then asymptotically stable. The design work can be transformed into a convex problem [18], which is efficiently solved by linear matrix inequalities optimization. If the solution is feasible, meaning that the stabilization constraints are met, then local state feedback gains are obtained.

3 Control of perturbed fuzzy systems

Motivated by the results of the above section concerning the control of fuzzy-model, we will extend the T-S fuzzy system with the presence of external disturbances. Consider the following T-S fuzzy uncertain model,

$$\text{Rule } i: \text{ If } z_1(t) \text{ is } M_{i1} \text{ and } z_2(t) \text{ is } M_{i2} \text{ ... and } z_p(t) \text{ is } M_{ip}, \text{ then}$$

$$\dot{x}(t) = A_i x(t) + B_i u(t) + \phi_i(t, x(t)), \; i = 1, 2, ..., r. \tag{5}$$

$\phi_i : \mathbb{R}^+ \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$ are continuous functions which represent the perturbations terms for $i = 1, 2, ..., r$.

We shall suppose the following assumption required for the stabilization problem.

$(\mathcal{H}_1)$ The pairs (A_i, B_i), $i = 1, ..., r$, are controllable, that is each nominal local model is controllable.
The fuzzy system is then inferred to be

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(z)\Big(A_i x(t) + B_i u(t) + \phi_i(t, x(t))\Big). \tag{6}$$

As is mentioned in the above section, μ_i are the membership functions that belong to class $\mathcal{C}^1$, i.e., they are continuously differentiable, which ensures the existence and uniqueness of solutions, and are subject to the following conditions: $0 \le \mu_i(z) \le 1$ and $\sum_{i=1}^{r} \mu_i(z) = 1$.
The functions ϕ_i, $i = 1, 2, ..., r$, represent the uncertain external disturbances of each fuzzy subsystem and are time-varying satisfying the following condition.

$(\mathcal{H}_2)$ Assume that,

$$\|\phi_i(t, x(t))\| \le \kappa_i(t), \quad i = 1, 2, ..., r, \tag{7}$$

for all $t \ge 0$ and $x \in \mathbb{R}^n$, where κ_i are known nonnegative continuous functions for $i = 1, ..., r$, with

$$\Big(\int_0^{+\infty} \kappa(t)^2 \, dt\Big)^{\frac{1}{2}} \le \tilde{\kappa} < +\infty,$$

where

$$\kappa(t) := (\sum_{i=1}^{r} \kappa_i(t)^2)^{\frac{1}{2}},$$

$\tilde{\kappa}$ is a nonnegative constant.
The fuzzy control rule is defined as above and we will consider the fuzzy uncertain system (3.2). Therefore, the closed-loop system with respect the fuzzy control (2.2) – (2.3) is given by

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(z)\mu_j(z)\big(A_i - B_iK_j\big)x(t) + \sum_{i=1}^{r} \mu_i(z)\phi_i(t, x(t)). \tag{8}$$

Thus,

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i^2(z)G_{ii}x(t) + 2\sum_{i<j}^{r} \mu_i(z)\mu_j(z)G_{ij}x(t) + \sum_{i=1}^{r} \mu_i(z)\phi_i(t, x(t)),$$

where

$$G_{ii} = A_i - B_iK_i$$

and

$$G_{ij} = \frac{1}{2}(A_i - B_iK_j + A_j - B_jK_i).$$

The controller synthesis initially considers the stability of the local fuzzy dynamics. That is, the stable feedback gains are determined for every subsystem. Suppose that there exist positive symmetric and definite matrices P, Q_i, and Q_{ij} $(i < j)$, and some matrices K_i, $i = 1, ..., r$, such that the following inequalities [16] hold,

$$G_{ii}^T P + PG_{ii} < -Q_i, \; i = 1, 2, ..., r, \tag{9}$$

and

$$G_{ij}^T P + PG_{ij} < -Q_{ij}, \; 1 \le i < j \le r. \tag{10}$$

Based on this assumption, each nominal local model is controllable and a suitable feedback gain can be obtained.

As a first step, we need to recall what is meant by uniformly ultimately bounded and uniform global practical exponential stability of dynamic systems [1, 2, 3, 10]. Consider a system described by

$$\dot{x} = F(t, x), \tag{11}$$

with $t \in \mathbb{R}_+$ is the time and $x \in \mathbb{R}^n$ is the state.

Definition 3.1. *[10] The system (11) is said uniformly ultimately bounded if there exists $R > 0$, such that for all $R_1 > 0$, there exists a $T = T(R_1) > 0$ such that*

$$\|x(t_0)\| \le R_1 \Rightarrow \|x(t)\| \le R \quad \textit{for all} \quad t \ge t_0 + T \;\; \textit{and} \;\; t_0 \ge 0.$$

Definition 3.2. *[1] The system (11) is said uniformly globally practically exponentially stable, if there exists a ball*

$$\mathcal{B}_r = \{x \in \mathbb{R}^n \;/\; \|x\| \le r\},$$

such that $\mathcal{B}_r$ is uniformly globally practically exponentially stable, it means that, there exists $r > 0$ such that, for all $\varepsilon > r$, there exists $\epsilon = \epsilon(\varepsilon) > 0$ such that, for all $t_0 \ge 0$, $\|x(t_0)\| \le \epsilon$, we have

$$\|x(t)\| \le \eta\|x(t_0)\|e^{-\upsilon(t-t_0)} + r, \quad \textit{for all} \;\; t \ge t_0,$$

with $\eta > 0$, $\upsilon > 0$.
In the case where $t = t_0$, the solution of (3.7) with respect the initial condition $(0, x(0))$, satisfies:

$$\|x(t)\| \le \gamma\|x(0)\|e^{-\upsilon t} + r, \quad \textit{for all} \;\; t \ge 0, \tag{12}$$

with $\eta > 0$, $\upsilon > 0$.

The goal of this paper is to find some conditions on the functions $\kappa_i(t)$ such that the fuzzy system (3.4) is globally uniformly practically exponentially stable. If $\kappa_i(t) = 0$ for all $i = 1, ..., r$, the fuzzy uncertain system (3.4) has an equilibrium point at the origin. In this case, we can analyze the stability of the closed loop system behavior for the origin as an equilibrium point. If $\kappa_i(t) \neq 0$, for some $i = 1, ..., r$, then the origin will not be an equilibrium point of the fuzzy uncertain system (3.4). In this case, we can study the uniform ultimate boundedness of the solutions of the fuzzy uncertain system, also, the convergence of the solutions toward a neighborhood of the origin. Note that, while the proposed fuzzy system is actually a model of a nonlinear system, the perturbation is originated from the fuzzy modelling error.

Now, one can state the following theorem.

Theorem 3.3. *Suppose that the assumptions* $(\mathcal{H}_1)$, $(\mathcal{H}_2)$ *hold and there exist a common positive definite matrix* P *and some feedback gain matrices* K_i, $i = 1, ..., r$, *such that the stability conditions* $(3.5) - (3.6)$ *are satisfied, then the fuzzy closed-loop system* (3.4) *with the control laws* $(2.2) - (2.3)$ *is guaranteed to be globally uniformly practically exponentially stable.*

Proof. Consider the Lyapunov function candidate $V(t, x) = x^T Px$. It's derivative with respect to time is given by,

$$\dot{V}(t,x) = \sum_{i=1}^{r} \mu_i^2(z) x^T (G_{ii}^T P + PG_{ii})x + 2\sum_{i<j}^{r} \mu_i(z)\mu_j(z) x^T (G_{ij}^T P + PG_{ij})x + 2x^T P \sum_{i=1}^{r} \mu_i(z)\phi_i(t, x(t)).$$

The first two terms on the right-hand side constitute the derivative of the Lyapunov function $V(x)$ with respect the nominal system, while the third term is the effect of the perturbation. On the one hand, we have

$$x^T (G_{ii}^T P + PG_{ii})x \leq -\lambda_{min}(Q_i)\|x\|^2, \ i = 1, 2, ..., r,$$

and

$$x^T (G_{ij}^T P + PG_{ij})x \leq -\lambda_{min}(Q_{ij})\|x\|^2, \ 1 \leq i < j \leq r.$$

It follows that,

$$\dot{V}(t,x) \leq -\sum_{i=1}^{r} \mu_i^2(z)\lambda_{min}(Q_i)\|x\|^2 - 2\sum_{i<j}^{r} \mu_i(z)\mu_j(z)\lambda_{min}(Q_{ij})\|x\|^2 + 2x^T P \sum_{i=1}^{r} \mu_i(z)\phi_i(t, x(t)).$$

Thus,

$$\dot{V}(t,x) \leq -\big(\sum_{i=1}^{r} \mu_i^2(z)\lambda_{min}(Q_i) + 2\sum_{i<j}^{r} \mu_i(z)\mu_j(z)\lambda_{min}(Q_{ij})\big)\|x\|^2 + 2x^T P \sum_{i=1}^{r} \mu_i(z)\phi_i(t, x(t)).$$

Then, one gets

$$\dot{V}(t,x) \leq -\lambda_0\|x\|^2 \sum_{i=1}^{r}\sum_{i=1}^{r} \mu_i(z)\mu_j(z) + 2x^T P \sum_{i=1}^{r} \mu_i(z)\phi_i(t, x(t)),$$

where $\lambda_0 = \inf\{(\lambda_{\min}(Q_i); i = 1, ..., r); (\lambda_{\min}(Q_{ij}); 1 \leq i < j \leq r)\}$, $\lambda_{\min(\max)}$ denotes the smallest (largest) eigenvalue of the matrix.
Since,

$$\sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(z)\mu_j(z) = 1,$$

then, we have

$$\dot{V}(t,x) \leq -\lambda_0\|x\|^2 + 2x^T P \sum_{i=1}^{r} \mu_i(z)\phi_i(t, x(t)).$$

Taking into account the assumption $(\mathcal{H}_2)$,, we have

$$\|\sum_{i=1}^{r} \mu_i\phi_i(t, x(t))\| \leq \sum_{i=1}^{r} \mu_i(z)\kappa_i(t).$$

Taking into account the above expressions, it follows that

$$\dot{V}(t,x) \leq -\lambda_0 \|x\|^2 + 2\|x\|\|P\| \sum_{i=1}^{r} \mu_i(z)\kappa_i(t).$$

Thus, by using the Cauchy-Schwartz inequality, one has

$$\dot{V}(t,x) \leq -\lambda_0 \|x\|^2 + 2\|x\|\|P\| \big((\sum_{i=1}^{r} \mu_i^2(z))^{\frac{1}{2}} (\sum_{i=1}^{r} \kappa_i(t)^2)^{\frac{1}{2}} \big).$$

It follows that,

$$\dot{V}(t,x) \leq -\lambda_0 \|x\|^2 + 2\|P\|\kappa(t)\|x\|.$$

Since,

$$\lambda_{min}(P)\|x\|^2 \leq V(t,x) = x^T P x \leq \lambda_{max}(P)\|x\|^2,$$

then, by taking $\|P\| = \lambda_{max}(P)$, yields

$$\dot{V}(t,x) \leq -\frac{\lambda_0}{\lambda_{max}(P)} V(t,x) + 2\frac{\lambda_{max}(P)}{\lambda_{min}^{\frac{1}{2}}(P)} \kappa(t) V(t,x)^{\frac{1}{2}}.$$

Let,

$$\eta = \frac{\lambda_0}{\lambda_{max}(P)} > 0,$$

and

$$\rho(t) = 2\frac{\lambda_{max}(P)}{\lambda_{min}^{\frac{1}{2}}(P)} \kappa(t).$$

With the previous notations, it follows that

$$\dot{V}(t,x) \leq -\eta V(t,x) + \rho(t) V(t,x)^{\frac{1}{2}}.$$

In the last expression, we make the following change of variable, $w(t) = V(t,x)^{\frac{1}{2}}$. The derivative with respect to time is given by

$$\dot{w}(t) = \frac{\dot{V}(t,x)}{2V(t,x)^{\frac{1}{2}}}.$$

This implies that,

$$\dot{w}(t) \leq -\frac{1}{2}\eta w(t) + \frac{1}{2}\rho(t).$$

Thus,

$$w(t) \leq w(0)e^{-\frac{1}{2}\eta t} + \frac{1}{2}e^{-\frac{1}{2}\eta t}. \int_0^t \rho(s) e^{\frac{1}{2}\eta s} ds.$$

It follows that,

$$\lambda_{min}^{\frac{1}{2}}(P)\|x(t)\| \leq \lambda_{max}^{\frac{1}{2}}(P)\|x(0)\| e^{-\frac{1}{2}\eta t} + \frac{1}{2}e^{-\frac{1}{2}\eta t}.\big((\int_0^t \rho(s)^2 ds)^{\frac{1}{2}}.(\int_0^t (e^{\frac{1}{2}\eta s})^2 ds)^{\frac{1}{2}} \big).$$

So,

$$\lambda_{min}^{\frac{1}{2}}(P)\|x(t)\| \leq \lambda_{max}^{\frac{1}{2}}(P)\|x(0)\| e^{-\frac{1}{2}\eta t} + \frac{1}{2}e^{-\frac{1}{2}\eta t}.\big((\int_0^{+\infty} \rho(s)^2 ds)^{\frac{1}{2}}.(\int_0^t e^{\eta s} ds)^{\frac{1}{2}} \big).$$

One gets,

$$\lambda_{min}^{\frac{1}{2}}(P)\|x(t)\| \leq \lambda_{max}^{\frac{1}{2}}(P)\|x(0)\| e^{-\frac{1}{2}\eta t} + 2\tilde{\kappa}\big(\frac{\lambda_{max}(P)}{\lambda_{min}^{\frac{1}{2}}(P)}\big)^2 e^{-\frac{1}{2}\eta t}.(\frac{1}{\eta}(e^{\frac{1}{\eta t}} - 1))^{\frac{1}{2}}.$$

Hence,

$$\|x(t)\| \leq \frac{\lambda_{max}^{\frac{1}{2}}(P)}{\lambda_{min}^{\frac{1}{2}}(P)} \|x(0)\| e^{-\frac{1}{2}\eta t} + 2\tilde{\kappa}\frac{\lambda_{max}^2(P)}{\lambda_{min}^{\frac{3}{2}}(P)} e^{-\frac{1}{2}\eta t}.(\frac{1}{\eta}e^{\frac{1}{\eta t}})^{\frac{1}{2}}.$$

Then,

$$\|x(t)\| \leq \frac{\lambda_{max}^{\frac{1}{2}}(P)}{\lambda_{min}^{\frac{1}{2}}(P)}\|x(0)\|e^{-\frac{1}{2}\eta t} + 2\frac{\lambda_{max}^{2}(P)}{\lambda_{min}^{\frac{3}{2}}(P)}\frac{\tilde{\kappa}}{\eta^{\frac{1}{2}}}.$$

Therefore, $\mathcal{B}_r$, with $r = 2\frac{\lambda_{max}^{2}(P)}{\lambda_{min}^{\frac{3}{2}}(P)}\frac{\tilde{\kappa}}{\eta^{\frac{1}{2}}}$, is globally uniformly practically exponentially stable. □

Remark 3.4. *Note that the last inequality implies that system* (3.4) *is uniformly ultimately bounded in the sense of Definition 3.1.*

Remark 3.5. *In this paper, the origin of the considered systems may not be an equilibrium point. Indeed, the desired system may be unstable and yet the system may oscillate sufficiently near this state that its performance is acceptable, thus the notion of practical stability is more suitable in several situations than traditional stability. One also desires that the state approaches the origin (or some sufficiently small neighborhood of it) in a sufficiently fast manner. To this end, we have used the fuzzy PDC controller to show the exponential practical stability of the considered closed-loop fuzzy perturbed systems.*

A simple extension can be given if we suppose the following assumption instead of $(\mathcal{H}_2)$.
$(\mathcal{A}_2)$ Assume that,

$$\|\phi_i(t,x(t))\| \leq \alpha_i\|x\| + \kappa_i(t), \quad i = 1,2,...,r, \tag{13}$$

for all $t \geq 0$ and $x \in \mathbb{R}^n$, where $\alpha_i > 0$, κ_i are known nonnegative continuous functions for $i = 1,...,r$, with

$$\left(\int_0^{+\infty} \kappa(t)^2\, dt\right)^{\frac{1}{2}} \leq \tilde{\kappa} < +\infty,$$

where

$$\kappa(t) := \left(\sum_{i=1}^{r} \kappa_i(t)^2\right)^{\frac{1}{2}},$$

$\tilde{\kappa}$ is a nonnegative constant.
Let

$$\alpha = \left(\sum_{i=1}^{r} \alpha_i^2\right)^{\frac{1}{2}},$$

and suppose that,

$$\alpha < \frac{\inf\{(\lambda_{\min}(Q_i), (\lambda_{\min}(Q_{ij})\}}{2\|P\|},$$

for $i = 1,...,r;\ 1 \leq i < j \leq r$.

Corollary 3.6. *Suppose that the assumptions* $(\mathcal{H}_1)$, $(\mathcal{A}_2)$ *hold and there exist a common positive definite matrix* P *and some feedback gain matrices* K_i, $i = 1,...,r$, *such that the stability conditions* $(3.5) - (3.6)$ *are satisfied, then the fuzzy closed-loop system* (3.4) *with the control laws* $(2.2) - (2.3)$ *is guaranteed to be globally uniformly practically exponentially stable.*

Proof. Using the same argument as the above proof, we consider the Lyapunov function candidate $V(t,x) = x^T P x$. It's derivative with respect to time is given by,

$$\dot{V}(t,x) = \sum_{i=1}^{r} \mu_i^2(z)x^T(G_{ii}^T P + PG_{ii})x + 2\sum_{i<j}^{r} \mu_i(z)\mu_j(z)x^T(G_{ij}^T P + PG_{ij})x + 2x^T P \sum_{i=1}^{r} \mu_i(z)\phi_i(t,x(t)).$$

Taking into account the assumption $(\mathcal{A}_2)$, we get,

$$\dot{V}(t,x) \leq -\lambda_0\|x\|^2 + 2\|x\|\|P\| \sum_{i=1}^{r} \mu_i(z)(\alpha_i\|x\| + \kappa_i(t)),$$

where $\lambda_0 = \inf\{(\lambda_{\min}(Q_i); i = 1,...,r); (\lambda_{\min}(Q_{ij}); 1 \leq i < j \leq r)\}$, $\lambda_{\min(\max)}$ denotes the smallest (largest) eigenvalue of the matrix.

Thus, by using the Cauchy-Schwartz inequality, one has

$$\dot{V}(t,x) \leq -\lambda_0\|x\|^2 + 2\|x\|^2\|P\|\big((\sum_{i=1}^{r}\mu_i^2(z))^{\frac{1}{2}}(\sum_{i=1}^{r}\alpha_i^2)^{\frac{1}{2}}\big)+2\|x\|\|P\|\big((\sum_{i=1}^{r}\mu_i^2(z))^{\frac{1}{2}}(\sum_{i=1}^{r}\kappa_i(t)^2)^{\frac{1}{2}}\big).$$

It follows that,

$$\dot{V}(t,x) \leq -(\lambda_0 - 2\alpha\|P\|)\|x\|^2 + 2\|P\|\kappa(t)\|x\|.$$

By taking $\|P\| = \lambda_{max}(P)$ and $\tilde{\lambda}_0 = \lambda_0 - 2\alpha\|P\|$, yields

$$\dot{V}(t,x) \leq -\frac{\tilde{\lambda}_0}{\lambda_{max}(P)}V(t,x) + 2\frac{\lambda_{max}(P)}{\lambda_{min}^{\frac{1}{2}}(P)}\kappa(t)V(t,x)^{\frac{1}{2}}.$$

Let,

$$\tilde{\eta} = \frac{\tilde{\lambda}_0}{\lambda_{max}(P)} > 0,$$

and

$$\rho(t) = 2\frac{\lambda_{max}(P)}{\lambda_{min}^{\frac{1}{2}}(P)}\kappa(t).$$

With the previous notations, it follows that

$$\dot{V}(t,x) \leq -\tilde{\eta}V(t,x) + \rho(t)V(t,x)^{\frac{1}{2}}.$$

A simple computation shows that, $\mathcal{B}_{\tilde{r}}$, with $\tilde{r} = 2\frac{\lambda_{max}^2(P)}{\lambda_{min}^{\frac{3}{2}}(P)}\frac{\tilde{\kappa}}{\tilde{\eta}^{\frac{1}{2}}}$, is globally uniformly practically exponentially stable. □

Remark 3.7. *One can check here that if we suppose that there exists a constant* $\tilde{\kappa} > 0$ *such that* $\kappa(t) \leq \tilde{\kappa}$, *then* $\mathcal{B}_r$, *with* $r = \frac{\lambda_{max}(P)}{\lambda_{min}(P)}\frac{\tilde{\kappa}}{\eta}$ *is globally uniformly practically exponentially stable.*

Motivated by the above result, the design principle can be extended to the T-S fuzzy system with non-smooth uncertainties. One can consider the following T-S fuzzy uncertain model,

Rule i : If $z_1(t)$ is M_{i1} and $z_2(t)$ is M_{i2} ... and $z_p(t)$ is M_{ip}, then

$$\dot{x}(t) = A_ix(t) + B_iu(t) + \varphi_i(t,x(t)),\ i = 1,2,...,r. \tag{14}$$

$(\mathcal{H}_2')$ Assume that,

$$\|\varphi_i(t,x(t))\| \leq \zeta_i(t)\|x\|^p,\ \ i = 1,2,...,r, \tag{15}$$

for all $t \geq 0$ and $x \in \mathbb{R}^n$, where $p \in]0,1[$, ζ_i are known nonnegative continuous functions for $i = 1,...,r$, with with

$$(\int_0^{+\infty}\zeta(t)^2\,dt)^{\frac{1}{2}} \leq \tilde{\zeta} < +\infty,$$

where

$$\zeta(t) := (\sum_{i=1}^{r}\zeta_i(t)^2)^{\frac{1}{2}},$$

for a certain nonnegative constant $\tilde{\zeta}$.

We have the following theorem.

Theorem 3.8. *Suppose that the assumptions* $(\mathcal{H}_1)$, $(\mathcal{H}_2')$ *hold and there exist a common positive definite matrix* P *and some feedback gain matrices* K_i, $i = 1,...,r$, *such that the stability conditions* $(3.5)-(3.6)$ *are satisfied, then the fuzzy closed-loop system* (3.4) *with the control laws* $(2.2)-(2.3)$ *is guaranteed to be globally uniformly practically exponentially stable.*

Proof. Consider the Lyapunov function candidate $V(t,x) = x^T P x$. It's derivative with respect to time is given by,

$$\dot{V}(t,x) = \sum_{i=1}^{r} \mu_i^2(z) x^T (G_{ii}^T P + P G_{ii}) x + 2 \sum_{i<j}^{r} \mu_i(z)\mu_j(z) x^T (G_{ij}^T P + P G_{ij}) x + 2x^T P \sum_{i=1}^{r} \mu_i(z)\phi_i(t, x(t)).$$

The first two terms on the right-hand side constitute the derivative of the Lyapunov function $V(x)$ with respect the nominal system, while the third term is the effect of the perturbation. On the one hand, we have

$$x^T (G_{ii}^T P + P G_{ii}) x \leq -\lambda_{min}(Q_i)\|x\|^2, \ i = 1, 2, ..., r,$$

and

$$x^T (G_{ij}^T P + P G_{ij}) x \leq -\lambda_{min}(Q_{ij})\|x\|^2, \ 1 \leq i < j \leq r.$$

It follows that,

$$\dot{V}(t,x) \leq -\sum_{i=1}^{r} \mu_i^2 \lambda_{min}(Q_i)\|x\|^2 - 2\sum_{i<j}^{r} \mu_i\mu_j \lambda_{min}(Q_{ij})\|x\|^2 + 2x^T P \sum_{i=1}^{r} \mu_i \varphi_i(t, x(t)).$$

Thus,

$$\dot{V}(t,x) \leq -\big(\sum_{i=1}^{r} \mu_i^2(z)\lambda_{min}(Q_i) + 2\sum_{i<j}^{r} \mu_i(z)\mu_j(z)\lambda_{min}(Q_{ij})\big)\|x\|^2 + 2x^T P \sum_{i=1}^{r} \mu_i(z)\varphi_i(t, x(t)).$$

Then, one gets

$$\dot{V}(t,x) \leq -\lambda_0\|x\|^2 \sum_{i=1}^{r}\sum_{i=1}^{r} \mu_i(z)\mu_j(z) + 2x^T P \sum_{i=1}^{r} \mu_i(z)\varphi_i(t, x(t)).$$

Since,

$$\sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(z)\mu_j(z) = 1,$$

then, we have

$$\dot{V}(t,x) \leq -\lambda_0\|x\|^2 + 2x^T P \sum_{i=1}^{r} \mu_i(z)\varphi_i(t, x(t)).$$

Taking into account the assumption $(\mathcal{H}_2')$, we have

$$\|\sum_{i=1}^{r} \mu_i \varphi_i(t, x(t))\| \leq \sum_{i=1}^{r} \mu_i(z)\zeta_i(t)\|x\|^p.$$

Taking into account the above expressions, it follows that

$$\dot{V}(t,x) \leq -\lambda_0\|x\|^2 + 2\|x\|\|P\| \sum_{i=1}^{r} \mu_i(z)\zeta_i(t)\|x\|^p.$$

Thus, by using the Cauchy-Schwartz inequality, one has

$$\dot{V}(t,x) \leq -\lambda_0\|x\|^2 + 2\|x\|^{p+1}\|P\|\big((\sum_{i=1}^{r} \mu_i^2(z))^{\frac{1}{2}}(\sum_{i=1}^{r} \zeta_i(t)^2)^{\frac{1}{2}}\big).$$

It follows that,

$$\dot{V}(t,x) \leq -\lambda_0\|x\|^2 + 2\|P\|\zeta(t)\|x\|^{p+1}.$$

Since,

$$\lambda_{min}(P)\|x\|^2 \leq V(t,x) = x^T P x \leq \lambda_{max}(P)\|x\|^2,$$

then, by taking $\|P\| = \lambda_{max}(P)$, yields

$$\dot{V}(t,x) \leq -\frac{\lambda_0}{\lambda_{max}(P)} V(t,x) + 2\frac{\lambda_{max}(P)}{\lambda_{min}^{\frac{p+1}{2}}(P)}\zeta(t)V(t,x)^{\frac{p+1}{2}}.$$

Let,

$$\eta = \frac{\lambda_0}{\lambda_{max}(P)} > 0,$$

and

$$\tilde{\rho}(t) = 2\frac{\lambda_{max}(P)}{\lambda_{min}^{\frac{p+1}{2}}(P)}\zeta(t).$$

With the previous notations, it follows that

$$\dot{V}(t,x) \leq -\eta V(t,x) + \tilde{\rho}(t)V(t,x)^{\frac{p+1}{2}}.$$

In the last expression, we make the following change of variable, $\tilde{w}(t) = V(t,x)^{\frac{1-p}{2}}$. The derivative with respect to time is given by

$$\dot{\tilde{w}}(t) = \frac{1-p}{2}\dot{V}(t,x)V(t,x)^{-\frac{(1+p)}{2}}.$$

Since,

$$\frac{1-p}{2}\dot{V}(t,x)V(t,x)^{-\frac{(1+p)}{2}} \leq -\eta\frac{1-p}{2}V(t,x)^{\frac{1-p}{2}} + \frac{1-p}{2}\tilde{\rho}(t),$$

Then,

$$\dot{\tilde{w}}(t) \leq -\eta\frac{1-p}{2}\tilde{w}(t) + \frac{1-p}{2}\tilde{\rho}(t).$$

Thus,

$$\tilde{w}(t) \leq \tilde{w}(0)e^{-\eta\frac{1-p}{2}t} + \frac{1-p}{2}e^{-\eta\frac{1-p}{2}t}.\int_0^t \tilde{\rho}(s)e^{\eta\frac{1-p}{2}s}ds.$$

It follows that,

$$\lambda_{min}^{\frac{1-p}{2}}(P)\|x(t)\|^{1-p} \leq \lambda_{max}^{\frac{1-p}{2}}(P)\|x(0)\|^{1-p}e^{-\eta\frac{1-p}{2}t} + \frac{1-p}{2}e^{-\eta\frac{1-p}{2}t}.\int_0^t \tilde{\rho}(s)e^{\eta\frac{1-p}{2}s}ds.$$

So,

$$\lambda_{min}^{\frac{1-p}{2}}(P)\|x(t)\|^{1-p} \leq \lambda_{max}^{\frac{1-p}{2}}(P)\|x(0)\|^{1-p}e^{-\eta\frac{1-p}{2}t} + \frac{1-p}{2}e^{-\eta\frac{1-p}{2}t}.((\int_0^{+\infty} \tilde{\rho}(s)^2ds)^{\frac{1}{2}}.(\int_0^t e^{2\eta\frac{1-p}{2}s}ds)^{\frac{1}{2}}).$$

Thus,

$$\lambda_{min}^{\frac{1-p}{2}}(P)\|x(t)\|^{1-p} \leq \lambda_{max}^{\frac{1-p}{2}}(P)\|x(0)\|^{1-p}e^{-\eta\frac{1-p}{2}t} + 2\frac{1-p}{2}e^{-\eta\frac{1-p}{2}t}\frac{\lambda_{max}^2(P)}{\lambda_{min}^{p+1}(P)}\tilde{\zeta}.\frac{1}{(1-p)\eta}(e^{\eta(1-p)t}-1)^{\frac{1}{2}}.$$

Then,

$$\|x(t)\|^{1-p} \leq \frac{\lambda_{max}^{\frac{1-p}{2}}(P)}{\lambda_{min}^{\frac{1-p}{2}}(P)}\|x(0)\|^{1-p}e^{-\eta\frac{1-p}{2}t} + \frac{1}{\eta}\tilde{\zeta}\frac{\lambda_{max}^2(P)}{\lambda_{min}^{\frac{p+3}{2}}(P)}.$$

Hence,

$$\|x(t)\| \leq \frac{\lambda_{max}^{\frac{1}{2}}(P)}{\lambda_{min}^{\frac{1}{2}}(P)}\|x(0)\|e^{-\eta\frac{1}{2}t} + \Big(\frac{1}{\eta}\tilde{\zeta}\frac{\lambda_{max}^2(P)}{\lambda_{min}^{\frac{p+3}{2}}(P)}\Big)^{\frac{1}{1-p}}.$$

Therefore, $\mathcal{B}_{\bar{r}}$, with $\bar{r} = \Big(\frac{1}{\eta}\tilde{\zeta}\frac{\lambda_{max}^2(P)}{\lambda_{min}^{\frac{p+3}{2}}(P)}\Big)^{\frac{1}{1-p}}$, is globally uniformly practically exponentially stable. □

A simple extension can be given if we suppose the following assumption instead of $(\mathcal{H}_2')$.
$(\mathcal{A}_2')$ Assume that,

$$\|\varphi_i(t,x(t))\| \leq \alpha_i\|x\| + \zeta_i(t)\|x\|^p, \;\; i=1,2,...,r, \tag{16}$$

for all $t \geq 0$ and $x \in \mathbb{R}^n$, where $p \in]0,1[$, $\alpha_i > 0$, ζ_i are known nonnegative continuous functions for $i=1,...,r$, with with

$$(\int_0^{+\infty} \zeta(t)^2\,dt)^{\frac{1}{2}} \leq \tilde{\zeta} < +\infty,$$

where

$$\zeta(t) := (\sum_{i=1}^{r} \zeta_i(t)^2)^{\frac{1}{2}},$$

for a certain nonnegative constant $\tilde{\zeta}$.
Let

$$\alpha = (\sum_{i=1}^{r} \alpha_i^2)^{\frac{1}{2}},$$

and suppose that,

$$\alpha < \frac{\inf\{(\lambda_{\min}(Q_i), (\lambda_{\min}(Q_{ij})\}}{2\|P\|},$$

for $i = 1, ..., r;\ 1 \leq i < j \leq r$.

Corollary 3.9. *Suppose that the assumptions* $(\mathcal{H}_1)$, $(\mathcal{A}_2')$ *hold and there exist a common positive definite matrix* P *and some feedback gain matrices* K_i, $i = 1, ..., r$, *such that the stability conditions* $(3.5) - (3.6)$ *are satisfied, then the fuzzy closed-loop system* (3.4) *with the control laws* $(2.2) - (2.3)$ *is guaranteed to be globally uniformly practically exponentially stable.*

Proof. Using the same argument as the above proof and the idea used in Corollary 3.2, taking into account the assumption $(\mathcal{A}_2')$, it suffices to take the same Lyapunov function where the derivative with respect time gives:

$$\dot{V}(t, x) \leq -\tilde{\lambda}_0\|x\|^2 + 2\|x\|\|P\| \sum_{i=1}^{r} \mu_i(z)\zeta_i(t)\|x\|^p.$$

Therefore, $\mathcal{B}_{r^*}$, with $r^* = \left(\frac{1}{\tilde{\eta}}\tilde{\zeta}\frac{\lambda_{max}^2(P)}{\lambda_{min}^{\frac{p+3}{2}}(P)}\right)^{\frac{1}{1-p}}$, is globally uniformly practically exponentially stable. □

Remark 3.10. *Note that, if we suppose that there exists a constant* $\tilde{\zeta} > 0$ *such that* $\zeta(t) \leq \tilde{\zeta}$, *then* $\mathcal{B}_r$, *with* $r = \left(\frac{\lambda_{max}(P)}{\lambda_{min}(P)}\frac{\tilde{\zeta}}{\eta}\right)^{\frac{1}{1-p}}$ *is globally uniformly practically exponentially stable.*

Example 3.11. *A control problem of balancing an inverted pendulum on a cart is considered in this study [20]. The state equations of the inverted pendulum are given by,*

$$\dot{x}_1(t) = x_2(t), \tag{17}$$

$$\dot{x}_2(t) = \frac{gsin(x_1(t)) - amlx_2^2 sin(2x_1(t))/2 - acos(x_1(t))u(t)}{4l/3 - amlcos^2(x_1(t))}, \tag{18}$$

where $x_1(t)$ *denotes the angle (in radians) of the pendulum from the vertical and* $x_2(t)$ *is the angular velocity;* $g = 9.8$ m/s^2 *is the gravity constant,* m *is the mass (kg) of the pendulum,* M *is the mass (kg) of the cart,* $2l$ *is the length of the pendulum, and* u *is the force applied to the cart (in newtons); and* $a = l/(m + M)$. *We choose* $m = 2kg$, $M = 8kg$, $2l = 1m$ *in the simulations.*
We suppose that $x_1(t) \in [-88^\circ \ \ 88^\circ]$, *then we approximate the system by the following two-rule fuzzy model:*
Rule 1: If x_1 *is about* 0 *then*

$$\dot{x}(t) = A_1 x(t) + B_1 u(t).$$

Rule 2: If x_1 *is about* $\pm\frac{\pi}{2}$ *then*

$$\dot{x}(t) = A_2 x(t) + B_2 u(t),$$

where $x(t) = [x_1(t) \quad x_2(t)]^T$ *as well as the matrices:*

$$A_1 = \begin{bmatrix} 0 & 1 \\ \dfrac{g}{4l/3 - aml} & 0 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0 \\ -\dfrac{a}{4l/3 - aml} \end{bmatrix},$$

and

$$A_2 = \begin{bmatrix} 0 & 1 \\ \dfrac{2g}{\pi(4l/3 - aml\beta^2)} & 0 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0 \\ -\dfrac{a\beta}{4l/3 - aml\beta^2} \end{bmatrix},$$

with $\beta = cos(88°)$.
Membership functions for rules 1 *and* 2 *are shown in Figure 1.*

Using an LMI optimisation algorithm, we obtain:

$$P = \begin{bmatrix} 0.0521 & 0.0362 \\ 0.0362 & 0.0521 \end{bmatrix},$$

the following feedback gains:

$$K_1 = \begin{bmatrix} -115.7705 & -17.3927 \end{bmatrix},$$

and

$$K_2 = \begin{bmatrix} -80.2120 & -17.4054 \end{bmatrix}.$$

and the following positive definite matrices:

$$Q_1 = \begin{bmatrix} 0.1136 & 0.1112 \\ 0.1112 & 0.1236 \end{bmatrix}, \ Q_2 = \begin{bmatrix} 0.1136 & 0.1112 \\ 0.1112 & 0.1236 \end{bmatrix}, \ \text{and } Q_{12} = \begin{bmatrix} 0.1517 & 0.1498 \\ 0.1498 & 0.1679 \end{bmatrix}.$$

It can be easily shown that the following stability conditions are satisfied:

$$G_{ii}^T P + PG_{ii} < -Q_i, \ i = 1, 2,$$

and

$$G_{12}^T P + PG_{12} < -Q_{12}.$$

Then, we have

$$\lambda_{min}(P) = 0.0159, \quad \lambda_{max}(P) = \|P\| = 0.0883,$$

and

$$\lambda_0 = \inf\{(\lambda_{min}(Q_i); i = 1, 2), (\lambda_{min}(Q_{12}))\} = 0.0073.$$

The resulting PDC control law is as follows:

Rule 1: If x_1 *is about* 0, *then*

$$u(t) = -K_1 x(t).$$

Rule 2: If x_1 *is about* $\pm\frac{\pi}{2}$, *then*

$$u(t) = -K_2 x(t).$$

That is,

$$u(t) = -\mu_1(x_1(t))K_1 x(t) - \mu_2(x_1(t))K_2 x(t).$$

The membership values of rules 1 and 2 are μ_1 *and* μ_2, *respectively* ($\mu_1 + \mu_2 = 1$). *This nonlinear control law guarantees the stability of the fuzzy control system (fuzzy model + PDC control).*
Now, we introduce external disturbances and we approximate the system by the following two-rule fuzzy model:
Rule 1: If x_1 *is about* 0 *then*

$$\dot{x}(t) = A_1 x(t) + B_1 u(t) + \varphi_1(t, x(t)).$$

Rule 2: If x_1 *is about* $\pm\frac{\pi}{2}$ *then*

$$\dot{x}(t) = A_2 x(t) + B_2 u(t) + \varphi_2(t, x(t)),$$

where

$$\varphi_1(t, x(t)) = \varphi_2(t, x(t)) = \begin{bmatrix} kx_2 + \frac{1}{\sqrt{1+t^2}}\sqrt{|x_1|} \\ 0 \end{bmatrix}.$$

Remark that, we are in the situation where the solutions exist and are unique outside a small ball centered at the origin. The only problem is arround the origin given that the function $\sqrt{|x_1|}$ *is not Lipschitzian in zero.*
We can see that

$$\|\varphi_1(t, x)\| = \|\varphi_2(t, x)\| \leq k\|x\| + \frac{1}{\sqrt{1+t^2}}\sqrt{|x_1|}, \ k > 0, \ \text{for all } \ t \geq 0.$$

Therefore, we can choose

$$\alpha_1 = \alpha_2 = k \leq \frac{\tilde{\lambda}_0}{2\lambda_{max}(P},$$

and

$$\zeta_1(t) = \zeta_2(t) = \frac{1}{\sqrt{1+t^2}}.$$

Figures 1, *and* 2, *show that the resulting closed-loop system with uncertainties is globally uniformly practically exponentially stable with initial conditions* $x_1 = 2$ *and* $x_2 = 2$. *Thus, the trajectories of system the* (17) *with uncertainties are bounded and approach a sufficiently neighborhood of the origin.*
For the component x_1, *we have found:*

$$\textit{rise time} = 15.5277 \quad \textit{and} \quad \textit{settling time} = 73.7025.$$

For the component x_2 *we have found:*

$$\textit{rise time} = 23.4427 \quad \textit{and} \quad \textit{settling time} = 162.4598.$$

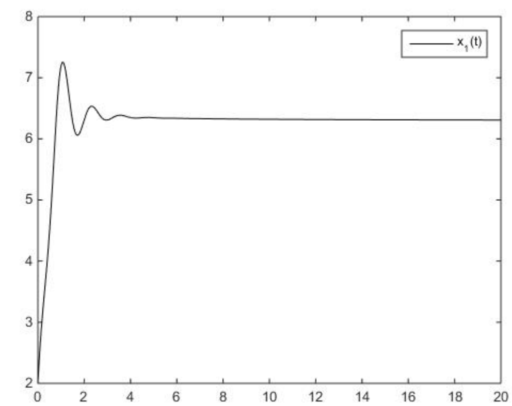

Figure 1: Time evolution of the state $x_1(t)$ of system (17) with uncertainties.

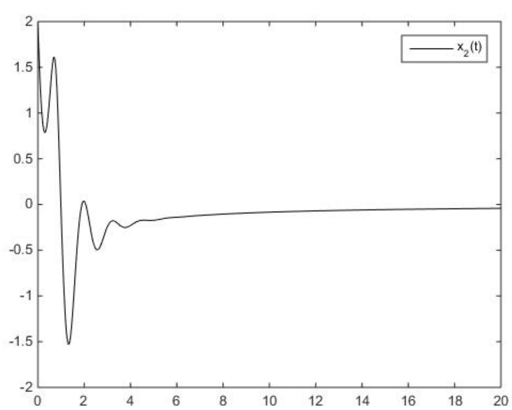

Figure 2: Time evolution of the state $x_2(t)$ of system (17) with uncertainties.

4 Conclusions

In this paper, we have studied the global uniform practical exponential stability for some classes of perturbed fuzzy systems. The asymptotic behaviors of the solutions are in term of convergence toward a small neighborhood of the origin. The perturbations are supposed uniformly bounded by some known functions where the nominal systems are linear. The effectiveness of the proposed theory is illustrated by computer simulation of the inverted pendulum system.

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References

[1] A. Ben Abdallah, I. Ellouze, M. A. Hammami, *Practical stability of nonlinear time-varying cascade systems*, Journal of Dynamical and Control Systems, **15** (2009), 45-62.

[2] A. Ben Abdallah, I. Ellouze, M. A. Hammami, *Practical exponential stability of perturbed triangular systems and a separation principle*, Asian Journal of Control, **13** (2011), 445-448.

[3] H. Ben Zina, M. Bouattour, M. Chaabane, *Robust Takagi-Sugeno sensor fault tolerant control strategy for nonlinear system*, Iranian Journal of Fuzzy Systems, **6** (2019), 177-189.

[4] T. Caraballo, M. A. Hammami, L. Mchiri, *Practical stability of stochastic delay evolution equations*, Acta Applicandae Mathematicae, **142** (2016), 91-105.

[5] T. Caraballo, M. A. Hammami, L. Mchiri, *Practical exponential stability of impulsive stochastic functional differential equations*, Systems and Control Letters, **109** (2017), 43-48.

[6] N. Hadj Taieb, *Stability an analysis for time-varying nonlinear systems*, International Journal of Control, **9** (2022), 1497-1506.

[7] N. Hadj Taieb, M. A Hammami, *Some new results on the global uniform asymptotic stability of time-varying dynamical systems*, IMA Journal of Mathematical and Control Information, **32** (2017), 1-22.

[8] N. Hadj Taieb, M. A. Hammami, F. Delmotte, *Stabilization of a certain class of fuzzy control systems with uncertainties*, Archives of Control Sciences, **3** (2017), 453-481.

[9] N. Hadj Taieb, M. A. Hammami, F. Delmotte, *A separation principle for Takagi-Sugeno control fuzzy systems*, Archives of Control Sciences, **29** (2019), 227-245.

[10] B. Hamed, I. Ellouze, M. A. Hammami, *Practical uniform stability of nonlinear differential delay equations*, Mediterranean Journal of Mathematics, **6** (2010), 139-150.

[11] B. Hamed, M. A. Hammami, *Practical stabilisation of a class of uncertain time-varying nonlinear delay systems*, Journal of Control Theory and Applications, **7** (2009), 175-180.

[12] M. A. Hammami, M. De La Sen, *On the uniform exponential stability of time-varying systems subject to discrete time-varying delays and nonlinear delayed perturbations*, Mathematical Problems in Engineering, **2015**(1) (2015), 12 pages. DOI:10.1155/2015/641268.

[13] A. Hamzaoui, N. Hadj Taieb, M. A. Hammami, *Practical partial stability of time-varying systems*, Discrete and Continuous Dynamical Systems-B, **7** (2022), 3585-3603.

[14] D. Huang, S. K. Nguang, *Robust H_∞ static output feedback control of fuzzy systems: An LMI approach*, IEEE Transactions on Systems, Man, and Cybernetics, Part B, **36** (2006), 216-222.

[15] H. J. Lee, J. B. Park, G. Chen, *Robust fuzzy control of nonlinear systems with parameter uncertainties*, IEEE Transactions on Fuzzy Systems, **9** (2001), 369-379.

[16] X. D. Liu, Q. L. Zhang, *New approach to H_∞ controller designs based on fuzzy observers for T-S fuzzy systems via LMI*, Automatica, **39** (2003), 1571-1582.

[17] X. Nian, J. Feng, *Guaranteed-cost control of a linear uncertain system with multiple time-varying delays: An LMI approach*, IEE Proceedings D (Control Theory and Applications), **150** (2003), 17-22.

[18] J. Park, J. Kim, D. Park, *LMI-based design of stabilizing fuzzy controllers for nonlinear systems described by Takagi-Sugeno fuzzy model*, Fuzzy Sets and Systems, **122** (2003), 73-82.

[19] T. Takagi, M. Sugeno, *Fuzzy identification of systems and its applications to modeling and control*, IEEE Transactions on Systems, Man, and Cybernetics, Part B, **15** (1985), 116-132.

[20] S. Tong, T. Wang, H. X. Li, *Fuzzy robust tracking control for uncertain nonlinear systems*, International Journal of Approximate Reasoning, **30** (2002), 73-90.

[21] C. S. Tseng, B. S.Chen, H. J. Uang, *Fuzzy tracking control design for nonlinear dynamics systems via T-S fuzzy model*, IEEE Transactions on Fuzzy Systems, **9** (2022), 381-392.

[22] J. M. Zhang, R. H. Li, P. A. Zhang, *Stability analysis and systtematic design of fuzzy control systems*, Fuzzy Sets and Systems, **120** (2001), 65-72.