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Algorithm for multiple attribute decision-making using T-spherical fuzzy Maclaurin symmetric mean operator

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Abstract

The conception of T-spherical fuzzy set (TSFS) is a recent phenomenon in the environment of fuzzy notions that describe opinion about some certain objects under uncertainties with the help of four types of degrees known as the degree of membership (DM), degree of abstinence (DA), degree of non-membership (DNM) and degree of refusal (DR). The focus of this manuscript, to examine the interrelationship among any numbers of T-spherical fuzzy numbers (TSFNs) by the use Maclaurin symmetric mean (MSM) operators. We investigate and develop T-spherical fuzzy (TSF) MSM (TSFMSM) operator, TSF weighted MSM (TSFWMSM) operator, TSF dual MSM (TSFDMSM) operator, TSF weighted dual MSM (TSFWDMSM) operator. By using these investigated operators, some special cases of the explored operators are also developed, and their properties are examined. In addition, an algorithm for handling multi attribute decision making (MADM) problems based MSM operators in TSF setting. An illustrative example to check the applicability of the MSM operators of TSFSs is presented where the evaluation of technology commercialization is observed. To show the superiority of the newly established MSM operators, a comprehensive comparative study is designed numerically. In view of the comparative analysis, some advantages of the new MSM operators of the TSFNs are stated.

Keywords: T-spherical fuzzy set, Maclaurin symmetric mean operator, multiple attribute decision making method.

1 Introduction

To describe the belongingness of objects to some certain phenomena under uncertainty and vagueness, the theory of intuitionistic fuzzy set (IFS) was investigated by Atanassov [2], which is the extension of the fuzzy set (FS) [41]. IFS is a generalized form of FS to manage with vague data in day-to-day life issues where the DM and the DNM are denoted by m_i and d_i that makes a duplet of the form (m,d) such that $m + d \in [0, 1]$. Some more work on IFS can be found in [3, 4, 10]. Although the IFS is widely used but sometimes there exists some pair of information such as (0.7,0.6) for which 0.7+0.6=1.3¿1 which exceeds the limit 0 to 1. To handle such kind of information, a less restricted fuzzy framework of Pythagorean FS (PyFS) was developed by Yager [6]. A PyFS allows the sum of the square of the DM and DNM in the range of 0 to 1 i.e., $m^2 + d^2 \in [0, 1]$. The duplet $(0.7, 0.6)$ can now be categorized as a Pythagorean fuzzy because $(0.7)^2 + (0.6)^2 = 0.85$. To read some work on the theory of PyFS, we refer to the articles [5, 8, 26, 27] and therein references. Likewise, IFS, PyFS also faces applicability issues as some duplets (m, d) cannot be sorted by IFS or PyFS. Due to the fact, a relatively more flexible frame of q-rung orthopair FS (QROFS) was developed by Yager [40] that allows the q-power of the DM and DNM in the range of 0 to 1. This framework of QROFS removes the previously existing barriers by introducing a variable parameter $q \in Z^+$. Due to this parameter q every duplet can be regarded to

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lie in the frame of q-rung orthopair fuzzy environment. The duplet $(0.7, 0.8)$ that could be considered as intuitionistic or Pythagorean fuzzy can now be considered to lie in the frame of QROFS for $q = 3$. To read the work on QROFS, we refer to the articles [9, 20, 32, 34, 37] and therein references.

Cuong [6] observed that description of uncertain events using the DM and the DNM is not sufficient and the opinion of being remaining abstain denoted by DA and the refusal degree expressed by DR have also a key role in the description modelling of the human observation.Cuong presented the notion of picture fuzzy set (PFS) by imposing certain restriction on the DM, DNM, and DA as $m + \bar{a} + d \in [0, 1]$. A great amount of research on PFS and its applications are studied in [11, 18, 30, 33]. In numerous practical situations, the theory of PFS is not applicable,for example, when information has the form (0.5, 0.3, 0.4) for which $0.5 + 0.3 + 0.4 = 1.2 \notin [0, 1]$. Due to this fact, the theory of spherical FS (SFS) was developed by Mahmood et al. [24], which increases the range imposed on picture fuzzy triplets by allowing sum of squares of the DM, DNM, and DA by introducing the condition $m^2 + \bar{a}^2 + d^2 \in [0, 1]$. The notion of SFS significantly enlarge the range for assigning the DM, DNM, DA but still some triplets are uncategorized which leads Mahmood et al. [24] to develop the notion of TSFS that provides a limitless flexibility for the assigning of the DM, DNM and DA etc. A TSFS has an associated variable parameter $q \in Z^+$ that can categorize every triplet $(m, \bar{a}, d)$ as a TSF number (TSFN). Consider the 3-touple $(0.9, 0.8, 0.7)$ where $0.9 + 0.8 + 0.7 = 2.4 \notin [0, 1]$ and $0.9^2 + 0.8^2 + 0.7^2 = 1.94 \notin [0, 1]$. We can see that even squaring the DM, DNM, and DA cannot categorize the given triplet as picture fuzzy or spherical fuzzy however, by setting $q = 6$, we have $0.9^q + 0.8^q + 0.7^q = 0.911 \in [0, 1]$. The TSFS is a novel addition in the study of fuzzy sets and its generalization and has gotten serious attention recently. Recent studies on the aggregation operators (AOs) of the TSFSs can be seen in [12, 13, 15, 17, 25, 31, 35] while studies on the information measures of the TSFSs can be seen in [1, 19, 29]. Some studies on the TSF graphs and TSF soft sets can be viewed in [14, 38].

To find the interrelationships among any fuzzy information, the MSM operators are considered among the significant AOs. The theory of MSM operator was initiated by Maclaurin [22] as an aggregation tool for non-negative real numbers. This concept was extended to generalized MSM operators of two variables by Detemple and Robertson [7]. The concept of MSM operators was greatly utilized in various fuzzy settings to comply with information under uncertainties. MSM operators of IFSs are investigated in the problems of MADM by Qin and Liu [28]. Wei and Lu [36] extended the notion of MSM operators to Pythagorean fuzzy environment which covers a wide range of information in MADM process. By observing the restricted range of the MSM operators of the IFSs and PyFSs, Liu et al. [21] proposed Heronian mean operators for QROFSs where the ability of aggregating the uncertain information has increased sufficiently. If we observe the theory of MSM operators developed in [21, 28, 36], it discusses only two aspects of uncertain information by using the DM and DNM where the abstinence and refusal information are lost. To reduce the information loss and to incorporate and the abstinence and refusal degrees of the information into account while aggregating information, we aim to develop MSM operators in the frame of TSFS. By doing so, we incorporate the four aspects of the uncertain information with the help of the DM, DNM, DA and DR associated with a varying parameter $q \in Z^+$ that ensures a large range for the various degrees. Some key features of the manuscript are discussed below:

1) Investigation of the notion of TSFMSM operator in the frame of TSFS using the DM, DNM, DA and DR.

2) A study of the characteristics of the TSF MSM operators.

3) Investigating the superiority of the TSF MSM operators over the previously defined MSM operators of IFSs, PyFSs and QROFSs theoretically and with the help of examples.

4) Setting up a MADM procedure based on TSF MSM operators.

The layout of this paper is such that the first section is having a brief introduction of previous backgrounds and recent trends. In Section 2, we recall the idea of TSFS, and their fundamental laws followed by an introduction to the notion of MSM operator. In Section 3, we defined the concept of TSFMSM, TSFWMSM, TSFDMSM and TSFWDMSM operator. In Section 4, a MADM procedure based on stated operators is presented and is illustrated by a numerical example. Finally, the conclusion is presented in Section 5.

2 Preliminaries

In this study, we recall the idea of TSFS and their fundamental terms and notions. Moreover, the notion of MSM operators is also discussed. Throughout this article, the symbol Y represents the universal set and the triplet $(m, \bar{a}, d)$ denotes the DM, DA, and DF. Throughout this paper $i = 1, 2, \ldots, n; k = 1, 2, \ldots, n$ denote the indexing terms and $(i_1, i_2, \ldots, i_k)$ represents the k-tuple combination of $(1, 2, \ldots, n)$.

Definition 2.1. [24] *A TSFS P is elaborated by:*

$$P = \{(y, (m(y), \overline{\mathrm{a}}(y), d(y))) : \ y \in Y\},$$

with a condition that $0 \leq m^q(y) + \overline{\mathrm{a}}^q(y) + d^q(y) \leq 1, q \in Z^+$. The expression $r(y) = (1 - (m^q(y) + \overline{\mathrm{a}}^q(y) + d^q(y)))^{1/2}$ is termed as DR. We call $P = (m, \overline{\mathrm{a}}, d)$ a TSF number (TSFN).

From this definition, we observe the following special cases:

1) If $q = 2$ in T-SFS, then we get SFS.
2) If $q = 1$ in T-SFS, then we get PFS.
3) If $\overline{\mathrm{a}}(y) = 0$ in T-SFS, then we get QROFS.
4) If $\overline{\mathrm{a}}(y) = 0$ with $q = 2$ in T-SFS, then we get PyFS.
5) If $\overline{\mathrm{a}}(y) = 0$ with $q = 1$ in T-SFS, then we get IFS.

Definition 2.2. [12] *Let $P = (m, \overline{\mathrm{a}}, d)$, $P_1 = (m_1, \overline{\mathrm{a}}_1, d_1)$ and $P_2 = (m_2, \overline{\mathrm{a}}_2, d_2)$ be three TSFNs and $\gamma \geq 0$. Then*

1) $P_1 \oplus P_2 = \left(\sqrt[q]{m_1^q + m_2^q - m_1^q m_2^q}, \overline{\mathrm{a}}_1 \overline{\mathrm{a}}_2, d_1 d_2 \right)$,
2) $P_1 \otimes P_2 = \left(m_1 m_2, \sqrt[q]{(\overline{\mathrm{a}}_1)^q + (\overline{\mathrm{a}}_2)^q - (\overline{\mathrm{a}}_1)^q (\overline{\mathrm{a}}_2)^q}, \sqrt[q]{d_1^q + d_2^q - d_1^q d_2^q} \right)$,
3) $\gamma P = \left(\sqrt[q]{1 - (1 - m^q)^\gamma}, \overline{\mathrm{a}}^\gamma, d^\gamma \right)$,
4) $P^\gamma = \left(m^\gamma, \sqrt[q]{1 - (1 - \overline{\mathrm{a}}^q)^\gamma}, \sqrt[q]{1 - (1 - d^q)^\gamma} \right)$.

To compare the different TSFNs, we express the score and accuracy functions for any TSFN $P = (m, \overline{\mathrm{a}}, d)$ by:

$$\overline{\mathrm{S}}(P) = \frac{1 + m^q - \overline{\mathrm{a}}^q - d^q}{2}, \qquad ; \qquad \overline{\mathrm{E}}(P) = m^q + \overline{\mathrm{a}}^q + d^q.$$

Clearly, $\overline{\mathrm{S}}(P), \overline{\mathrm{E}}(P) \in [0, 1]$. Based on these functions, TSFN corresponds to a greater value of score is considered as larger and in case of two TSFNs have same score then they can be differentiated by using accuracy values.

Definition 2.3. [18] *Let P_j be the family of positive numbers. Then the MSM operator is defined by*

$$MSM^k(P_1, P_2, \ldots, P_n) = \left(\frac{\sum\limits_{\substack{1 \leq i_1 \leq i_2 \\ \leq, \ldots, \leq i_k \leq n}} \prod\limits_{j=1}^{k} P_{i_j}}{C_n^k} \right)^{1/k},$$

where C_n^k represented the binomial coefficients and $(i_1, i_2, \ldots, i_k)$ denote all the k-tuples. MSM operator satisfies the following conditions:

1) $MSM^k(0, 0, \ldots, 0) = 0$,
2) $MSM^k(P, P, \ldots, P) = P$,
3) $MSM^k(P_1, P_2, \ldots, P_n) \leq MSM^k(Q_1, Q_2, \ldots, Q_n)$ *if* $P_i \leq Q_i \forall i$,
4) $\min P_i \leq MSM^k(P_1, P_2, \ldots, P_n) \leq \max P_i$.

Definition 2.4. [18] *Let P_j be the family of positive numbers. Then the DMSM operator is defined by:*

$$DMSM^k(P_1, P_2, \ldots, P_n) = \frac{1}{k} \left(\prod_{\substack{1 \leq i_1 \leq i_2 \\ \leq, \ldots, \leq i_k \leq n}} \left(\sum_{j=1}^{k} P_{i_j} \right)^{\frac{1}{C_n^k}} \right),$$

where C_n^k represented the binomial coefficients.

3 Proposed operators

In this study, we combine the idea of MSM and WMSM operators with TSFSs is to investigate the idea of TSFMSM and TSFWMSM operators and discussed their different properties. Moreover, the specific cases of the newly developed study are assessed. In our further study, $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ denote the weight vector of P_j where $\omega_j \geq 0$, and $\sum_{j=1}^{n} \omega_j = 1$.

Definition 3.1. *Let $P_j = (m_j, \overline{\mathrm{a}}_j, d_j)$ be the family of TSFNs. Then the TSFMSM operator is stated as:*

$$TSFMSM^k(P_1, P_2, \ldots, P_n) = \left(\frac{\bigoplus\limits_{\substack{1 \leq i_1 \leq i_2 \\ \leq, \ldots, \leq i_k \leq n}} \bigotimes\limits_{j=1}^{k} P_{i_j}}{C_n^k} \right)^{1/k},$$

where C_n^k represented the binomial coefficients.

Theorem 3.2. *For a family of TSFNs $P_j = (m_j, \overline{\mathrm{a}}_j, d_j)$ the value obtained by TSFMSM operators, is*

$$TSFMSM^k(P_1, P_2, \ldots, P_n) = (M, A, D),$$

where

$$\begin{aligned}
M &= \left(\sqrt[q]{1 - \left(\prod_{\substack{1 \leq i_1 \leq i_2 \\ \leq, \ldots, \leq i_k \leq n}} \left(1 - \left(\prod_{j=1}^{k} m_{i_j} \right) \right) \right)^{\frac{1}{C_n^k}}} \right)^{\frac{1}{k}}, \\
A &= \sqrt[q]{1 - \left(1 - \left(\prod_{\substack{1 \leq i_1 \leq i_2 \\ \leq, \ldots, \leq i_k \leq n}} \left(1 - \left(\prod_{j=1}^{k} \left(1 - \overline{\mathrm{a}}_{i_j}^q \right) \right) \right) \right)^{\frac{1}{C_n^k}} \right)^{\frac{1}{k}}}, \\
D &= \sqrt[q]{1 - \left(1 - \left(\prod_{\substack{1 \leq i_1 \leq i_2 \\ \leq, \ldots, \leq i_k \leq n}} \left(1 - \left(\prod_{j=1}^{k} \left(1 - d_{i_j}^q \right) \right) \right) \right)^{\frac{1}{C_n^k}} \right)^{\frac{1}{k}}}.
\end{aligned}$$

Proof. The proof can be easily obtained, so we can omit here. □

This proposed operator satisfies certain properties for a family of TSFNs $p_j = (m_j, \overline{\mathrm{a}}_j, d_j)$ such as

Property 3.3. *If $P_j = P$, then $TSFMSM^k(P_1, P_2, P_3, ..., P_n) = P$.*

Property 3.4. *Let P_j and $\overline{\overline{\mathrm{P}}}_j$ be the family of TSFNs: if $m_j \geq \overline{\mathrm{m}}_j$;$\overline{\mathrm{a}}_j \leq \overline{\overline{\mathrm{a}}}_j$ and $d_j \leq \overline{\mathrm{d}}_j$, for all j, then*

$$TSFMSM^k(P_1, P_2, \ldots, P_n) \geq TSFMSM^k(\overline{\mathrm{P}}_1, \overline{\mathrm{P}}_2, \ldots, \overline{\mathrm{P}}_n).$$

Property 3.5. *Let $\overline{\mathrm{P}}_j$ be any permutation of P_j for $j = 1, 2, ..., n$, then*

$$TSFMSM^k(P_1, P_2, \ldots, P_n) = TSFMSM^k(\overline{\mathrm{P}}_1, \overline{\mathrm{P}}_2, \ldots, \overline{\mathrm{P}}_n).$$

Property 3.6. *Let P_j be the family of TSFNs with $P^- = \min(P_j) = (\min(m_j), \max(\overline{\mathrm{a}}_j), \max(d_j))$ and $P^+ = \max(P_j) = (\max(m_j), \min(\overline{\mathrm{a}}_j), \min(d_j))$. Then*

$$P^- \leq TSFMSM^k(P_1, P_2, \ldots, P_n) \leq P^+.$$

Example 3.7. *Let* $P_1 = (0.3, 0.5, 0.6)$, $P_2 = (0.4, 0.6, 0.7)$, $P_3 = (0.5, 0.4, 0.7)$, $P_4 = (0.6, 0.5, 0.6)$ *be four TSFNs. To aggregate these numbers by using the proposed TSFMSM operator, we take* $k = 2, q = 5$. *Then,*

$$\begin{aligned}
P_1 \otimes P_2 &= \begin{pmatrix} 0.3 \times 0.4, \left(0.5^5 + 0.6^5 - 0.5^5 \times 0.6^5\right)^{1/5}, \\ \left(0.6^5 + 0.7^5 - 0.6^5 \times 0.7^5\right)^{1/5} \end{pmatrix} = (0.12, 0.63, 0.74), \\
P_1 \otimes P_3 &= \begin{pmatrix} 0.3 \times 0.5, \left(0.5^5 + 0.4^5 - 0.5^5 \times 0.4^5\right)^{1/5}, \\ \left(0.6^5 + 0.7^5 - 0.6^5 \times 0.7^5\right)^{1/5} \end{pmatrix} = (0.15, 0.52, 0.74), \\
P_1 \otimes P_4 &= \begin{pmatrix} 0.3 \times 0.6, \left(0.5^5 + 0.5^5 - 0.5^5 \times 0.5^5\right)^{1/5}, \\ \left(0.6^5 + 0.6^5 - 0.6^5 \times 0.6^5\right)^{1/5} \end{pmatrix} = (0.18, 0.57, 0.68), \\
P_2 \otimes P_3 &= \begin{pmatrix} 0.4 \times 0.5, \left(0.6^5 + 0.4^5 - 0.6^5 \times 0.4^5\right)^{1/5}, \\ \left(0.7^5 + 0.7^5 - 0.7^5 \times 0.7^5\right)^{1/5} \end{pmatrix} = (0.20, 0.61, 0.79), \\
P_2 \otimes P_4 &= \begin{pmatrix} 0.4 \times 0.6, \left(0.6^5 + 0.5^5 - 0.6^5 \times 0.5^5\right)^{1/5}, \\ \left(0.7^5 + 0.6^5 - 0.7^5 \times 0.6^5\right)^{1/5} \end{pmatrix} = (0.24, 0.63, 0.74), \\
P_3 \otimes P_4 &= \begin{pmatrix} 0.5 \times 0.6, \left(0.4^5 + 0.5^5 - 0.4^5 \times 0.5^5\right)^{1/5}, \\ \left(0.7^5 + 0.6^5 - 0.7^5 \times 0.6^5\right)^{1/5} \end{pmatrix} = (0.30, 0.52, 0.74).
\end{aligned}$$

Hence, we get $TSFMSM(P_1, P_2, P_3, P_4) = (0.47, 0.51, 0.54)$.

Definition 3.8. *Let* $P_j = (m_j, \overline{\mathrm{a}}_j, d_j)$ *be the family of TSFNs. Then the TSFWMSM operator is stated as:*

$$TSFWMSM^k(P_1, P_2, \ldots, P_n) = \left(\frac{\bigoplus\limits_{\substack{1 \le i_1 \le i_2 \\ \le, \ldots, \le i_k \le n}} \left(\bigotimes\limits_{j=1}^{k} P_{i_j} \right)^{\omega_j}}{C_n^k} \right)^{\frac{1}{k}},$$

where C_n^k *represented the binomial coefficients.*

Theorem 3.9. *For a family of TSFNs* $P_j = (m_j, \overline{\mathrm{a}}_j, d_j)$ *the value obtained by TSFMSM operators, is*

$$TSFWMSM^k(P_1, P_2, \ldots, P_n) = (\mathcal{M}, \mathcal{A}, \mathcal{D}),$$

where

$$\begin{aligned}
\mathcal{M} &= \left(\sqrt[q]{1 - \left(\prod_{\substack{1 \le i_1 \le i_2 \\ \le, \ldots, \le i_k \le n}} \left(1 - \left(\prod_{j=1}^{k} m_{i_j} \right)^{\omega_j * q} \right) \right)^{\frac{1}{C_n^k}}} \right)^{\frac{1}{k}}, \\
\mathcal{A} &= \sqrt[q]{1 - \left(1 - \left(\prod_{\substack{1 \le i_1 \le i_2 \\ \le, \ldots, \le i_k \le n}} \left(1 - \left(\prod_{j=1}^{k} \left(1 - \overline{\mathrm{a}}_{i_j}^q \right) \right)^{\omega_j} \right) \right)^{\frac{1}{C_n^k}} \right)^{\frac{1}{k}}}, \\
\mathcal{D} &= \sqrt[q]{1 - \left(1 - \left(\prod_{\substack{1 \le i_1 \le i_2 \\ \le, \ldots, \le i_k \le n}} \left(1 - \left(\prod_{j=1}^{k} \left(1 - d_{i_j}^q \right) \right)^{\omega_j} \right) \right)^{\frac{1}{C_n^k}} \right)^{\frac{1}{k}}}.
\end{aligned}$$

We observe some special Cases for the proposed TSFWMSM operator as

1) For $k = 1$ the TSFWMSM operator is reduced into T-spherical fuzzy weighted averaging (TSFWA) operator.
2) For $k = 2$ the TSFWMSM operator is reduced into T-spherical fuzzy weighted Bonferroni mean (TSFWBM) operator.

Example 3.10. *Let* $P_1 = (0.3, 0.5, 0.6)$, $P_2 = (0.4, 0.6, 0.7)$, $P_3 = (0.5, 0.4, 0.7)$, $P_4 = (0.6, 0.5, 0.6)$ *be four TSFNs. To aggregate these four TSFNs, we utilize the proposed TSFWMSM operator with* $k = 2$ *and* $q = 3$. *Assume* $\omega = (0.3, 0.1, 0.4, 0.2)^T$ *be the weight vector of four TSFNs.*

$$P_1^{0.3} \otimes P_2^{0.1} = \begin{pmatrix} 0.6^{0.3} \times 0.6^{0.1}, \left(1 - \left(1 - 0.5^3\right)^{0.3} \times \left(1 - 0.6^3\right)^{0.1}\right)^{1/3}, \\ \left(1 - \left(1 - 0.8^3\right)^{0.3} \times \left(1 - 0.2^3\right)^{0.1}\right)^{1/3} \end{pmatrix} = (0.81, 0.39, 0.57),$$

$$P_1^{0.3} \otimes P_3^{0.4} = \begin{pmatrix} 0.6^{0.3} \times 0.4^{0.4}, \left(1 - \left(1 - 0.5^3\right)^{0.3} \times \left(1 - 0.4^3\right)^{0.3}\right)^{1/3}, \\ \left(1 - \left(1 - 0.8^3\right)^{0.3} \times \left(1 - 0.3^3\right)^{0.4}\right)^{1/3} \end{pmatrix} = (0.71, 0.40, 0.58),$$

$$P_1^{0.3} \otimes P_4^{0.2} = \begin{pmatrix} 0.6^{0.3} \times 0.5^{0.2}, \left(1 - \left(1 - 0.5^3\right)^{0.3} \times \left(1 - 0.7^3\right)^{0.2}\right)^{1/3}, \\ \left(1 - \left(1 - 0.8^3\right)^{0.3} \times \left(1 - 0.4^3\right)^{0.2}\right)^{1/3} \end{pmatrix} = (0.74, 0.48, 0.58),$$

$$P_2^{0.1} \otimes P_3^{0.4} = \begin{pmatrix} 0.6^{0.1} \times 0.4^{0.4}, \left(1 - \left(1 - 0.6^3\right)^{0.1} \times \left(1 - 0.4^3\right)^{0.4}\right)^{1/3}, \\ \left(1 - \left(1 - 0.2^3\right)^{0.1} \times \left(1 - 0.3^3\right)^{0.4}\right)^{1/3} \end{pmatrix} = (0.65, 0.36, 0.22),$$

$$P_2^{0.1} \otimes P_4^{0.2} = \begin{pmatrix} 0.6^{0.1} \times 0.5^{0.2}, \left(1 - \left(1 - 0.6^3\right)^{0.1} \times \left(1 - 0.7^3\right)^{0.2}\right)^{1/3}, \\ \left(1 - \left(1 - 0.2^3\right)^{0.1} \times \left(1 - 0.4^3\right)^{0.2}\right)^{1/3} \end{pmatrix} = (0.82, 0.46, 0.24),$$

$$P_3^{0.4} \otimes P_4^{0.2} = \begin{pmatrix} 0.4^{0.4} \times 0.5^{0.2}, \left(1 - \left(1 - 0.4^3\right)^{0.4} \times \left(1 - 0.7^3\right)^{0.2}\right)^{1/3}, \\ \left(1 - \left(1 - 0.3^3\right)^{0.4} \times \left(1 - 0.4^3\right)^{0.2}\right)^{1/3} \end{pmatrix} = (0.60, 0.47, 0.28).$$

Thus, we get $TSFWMSM^k(P_1, P_2, \ldots, P_n) = (0.39, 0.046, 0.30)$.

Next, we combine the idea of DMSM and WDMSM operators with TSFSs and proposed the concept of TSFDMSM and TSFWDMSM operators. Some of their desirable properties are also discussed.

Definition 3.11. *Let* $P_j = (m_j, \bar{a}_j, d_j)$, $j = 1, 2, ..., n$ *be the family of* n *TSFNs.Then the TSFDMSM is stated as*

$$TSFDMSM^k(P_1, P_2, \ldots, P_n) = \frac{1}{k} \left(\bigoplus_{\substack{1 \le i_1 \le i_2 \\ \le, \ldots, \le i_k \le n}} \left(\bigotimes_{j=1}^{k} P_{i_j} \right)^{\frac{1}{C_n^k}} \right),$$

where C_n^k *represented the binomial coefficients.*

Theorem 3.12. *For a family of TSFNs* $P_j = (m_j, \bar{a}_j, d_j)$ *the value obtained by TSFMSM operators, is given as*

$$TSFDMSM^k(P_1, P_2, \ldots, P_n) = (M, A, D),$$

where

$$M = \sqrt[q]{1-\left(1-\left(\prod_{\substack{1\le i_1\le i_2\\ \le,\ldots,\le i_k\le n}}\left(1-\left(\prod_{j=1}^{k}\left(1-m_{i_j}^{q}\right)\right)\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}},$$

$$A = \left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le i_2\\ \le,\ldots,\le i_k\le n}}\left(1-\left(\prod_{j=1}^{k}\overline{\mathrm{a}}_{i_j}\right)^{q}\right)\right)^{\frac{1}{C_n^k}}}\right)^{\frac{1}{k}},$$

$$D = \left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le i_2\\ \le,\ldots,\le i_k\le n}}\left(1-\left(\prod_{j=1}^{k}d_{i_j}^{q}\right)\right)^{\frac{1}{C_n^k}}\right)}\right)^{\frac{1}{k}}.$$

Definition 3.13. *Let $P_j=(m_j,\overline{\mathrm{a}}_j,d_j)$, $j=1,2,...,n$ be the family of n TSFNs. Then the TSFWDMSM is stated as*

$$TSFWDMSM^{k}(P_1,P_2,\ldots,P_n)=\frac{1}{k}\left(\bigoplus_{\substack{1\le i_1\le i_2\\ \le,\ldots,\le i_k\le n}}\left(\bigotimes_{j=1}^{k}\left(\omega_{i_j}\otimes P_{i_j}\right)\right)^{\frac{1}{C_n^k}}\right),$$

where C_n^k represented the binomial coefficients.

Theorem 3.14. *For a family of n TSFNs $P_j=(m_j,\overline{\mathrm{a}}_j,d_j)$ the value obtained by TSFMSM operators, is defined as*

$$TSFWDMSM^{k}(P_1,P_2,\ldots,P_n)=(M,A,D),$$

where

$$M = \sqrt[q]{1-\left(1-\left(\prod_{\substack{1\le i_1\le i_2\\ \le,\ldots,\le i_k\le n}}\left(1-\left(\prod_{j=1}^{k}\left(1-m_{i_j}^{q}\right)\right)^{\omega_j}\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}},$$

$$A = \left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le i_2\\ \le,\ldots,\le i_k\le n}}\left(1-\left(\prod_{j=1}^{k}\left(\overline{\mathrm{a}}_{i_j}\right)^{\omega_j q}\right)\right)\right)^{\frac{1}{C_n^k}}}\right)^{\frac{1}{k}},$$

$$D = \left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le i_2\\ \le,\ldots,\le i_k\le n}}\left(1-\left(\prod_{j=1}^{n}\left(d_{i_j}\right)^{\omega_j q}\right)\right)\right)^{\frac{1}{C_n^k}}}\right)^{\frac{1}{k}}.$$

Further, we observe some of the special cases from the proposed operators as follows.

1) For $\overline{\mathrm{a}}_{i_j}=0$, the TSFMSM operator is reduced into MSM operators of QROFSs.
2) For $q=2$,the TSFMSM operator is reduced into MSM operators of Spherical fuzzy environment.
3) For $q=2$ and $\overline{\mathrm{a}}_{i_j}=0$, the TSFMSM operator is reduced into MSM operators of Pythagorean fuzzy environment.
4) For $q=1$, the TSFMSM operator is reduced into MSM operators of Picture fuzzy set.
5) For $q=1$ and $\overline{\mathrm{a}}_{i_j}=0$, the TSFMSM operator is reduced into MSM operators of intuitionistic fuzzy environment.
6) For $q=1$ and $\overline{\mathrm{a}}_{i_j}=d_{i_j}=0$, the TSFMSM operator is reduced into MSM operators of Fuzzy set environment.

4 Proposed MADM procedure using proposed operators

In this investigation work, we develop a MADM procedure by using the idea of TSFMS, TSFWMSM, TSFDMSM, and TSFWDMSM operator based on TSFNs. To resolve the above types of issues, we choose the family of alternative and their attributes whose expressions are summarized as $A = A_1, A_2, ..., A_m$ denote family of alternatives and $G = G_1, G_2, ..., G_n$ denote the family of attributes.

Moreover, the terms $\omega = \omega_1, \omega_2, ..., \omega_n$ represent the weight vector for the attributes with a rule that is $\omega_j \in [0, 1], j = 1, 2, ..., n$ and $\sum_{j=1}^{n}(\omega_j) = 1$.
An expert has hired to evaluate each alternative and provide their ratings in terms of the TSFNs $\Phi_{ij} = (m_{ij}, \overline{\mathrm{a}}_{ij}, d_{i_j})$ for $i = 1(1)m; j = 1(1)n$. The following are the steps stated in the proposed algorithm to find the finest alternative(s).

Step 1: The collective information is arranged in the form of the decision matrix $\hat{\mathcal{D}} = [\Phi_{ij}]_{m\times n}$ as

$$\hat{\mathcal{D}} = \begin{pmatrix} \Phi_{11} & \Phi_{12} & \dots & \Phi_{1n} \\ \Phi_{21} & \Phi_{22} & \dots & \Phi_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_{m1} & \Phi_{m2} & \dots & \Phi_{mn} \end{pmatrix}, \tag{1}$$

Step 2: If there is any cost type attributes in $\hat{\mathcal{D}} = [\Phi_{ij}]_{m\times n}$ then we can normalize its rating to the benefit type by using the Eq. (2).
Such ratings are denoted by $\hat{\mathcal{R}} = [t_{ij}]_{m\times n}$

$$t_{ij} = (\overline{\mathrm{m}}_{ij}, \overline{\overline{\mathrm{a}}_{\mathrm{ij}}}_{ij}, \overline{\mathrm{d}}_{ij}) = \begin{cases} (m_{ij}, \overline{\mathrm{a}}_{ij}, d_{ij}), & \text{for benefit types} \\ (d_{ij}, \overline{\mathrm{a}}_{ij}, m_{ij}), & \text{for cost types} \end{cases} \tag{2}$$

Step 3: Aggregate the normalized ratings $\hat{\mathcal{R}} = (t_{ij})$ for each alternative into the collective value $t_i = (\overline{\mathrm{m}}_i, \overline{\overline{\mathrm{a}}_{\mathrm{i}}}_{ij}, \overline{\mathrm{d}}_i)$ by using either of the proposed TSFMSM, TSFWMSM, TSFDMSM, and TSFWDMSM operators.

Step 4: Compute the score value $t_i = (\overline{\mathrm{m}}_i, \overline{\overline{\mathrm{a}}_{\mathrm{i}}}_{ij}, \overline{\mathrm{d}}_i)$ as

$$\overline{\mathrm{S}}(t_i) = \frac{1 + \overline{\mathrm{m}}_i^q - \overline{\overline{\mathrm{a}}_{\mathrm{i}}}^q - \overline{\mathrm{d}}_i^q}{2}. \tag{3}$$

If for any two indices i_1 and i_2, $\overline{\mathrm{S}}(t_1) = \overline{\mathrm{S}}(t_2)$ then we compute the accuracy degree as

$$\overline{\mathrm{E}}(t_i) = \overline{\mathrm{m}}_i^q + \overline{\overline{\mathrm{a}}_{\mathrm{i}}}^q + \overline{\mathrm{d}}_i^q. \tag{4}$$

Step 5: Rank the alternatives based on the score values and select the finest alternatives.

To demonstrate the above discussed steps, we present an example as follows:

4.1 Illustrative example

A technology commercialization-based example is discussed in Ref. [21]. We adapt the same example to apply the MSM operators proposed by in this paper where the information is based on TSFNs.
We resolve a MADM problem for the selection of most optimum technology enterprise among a lest of enterprises, denoted by A_1, A_2, A_3, A_4, A_5. The selection of optimum enterprise is carried out by observing some attributes defined as:

- G_1: Technical achievement.
- G_2: Potential market and market risk.
- G_3: Financial benefits.
- G_4: Development of science and employment creation.

For this we consider the weight vector such as $\omega_j = (0.3, 0.1, 0.4, 0.2)^T$. Now, we present a stepwise computation of the above discussed MADM problem using the algorithm discussed earlier in this section.

Step 1: All the technology enterprises are analysed by a panel of decision makers who gave their opinion using TSFNs to describe such enterprises in terms of the DM, DA, DNM and the DR.
The summarized ratings are given in Eq. (5).

$$\hat{\mathcal{D}} = \begin{pmatrix} A_1 & (0.9, 0.7, 0.6) & (0.91, 0.71, 0.61) & (0.92, 0.72, 0.62) & (0.93, 0.73, 0.63) \\ A_2 & (0.8, 0.6, 0.7) & (0.81, 0.61, 0.71) & (0.82, 0.62, 0.72) & (0.83, 0.63, 0.73) \\ A_3 & (0.85, 0.5, 0.8) & (0.86, 0.51, 0.81) & (0.87, 0.52, 0.82) & (0.88, 0.53, 0.83) \\ A_4 & (0.8, 0.2, 0.7) & (0.81, 0.21, 0.71) & (0.82, 0.22, 0.72) & (0.83, 0.23, 0.73) \\ A_5 & (0.9, 0.5, 0.5) & (0.91, 0.51, 0.51) & (0.92, 0.52, 0.52) & (0.93, 0.53, 0.53) \end{pmatrix}, \tag{5}$$

Step 2: Since all the attributes G_j's are of benefit types, so the normalized matrix $\hat{\mathcal{R}} = [t_{ij}]_{m\times n}$ is same as $\hat{\mathcal{D}}$.

Step 3: With weight $\omega_j = (0.3, 0.1, 0.4, 0.2)^T$ and utilize the stated operators with $k = 2, q = 10$, we aggregate the values of each alternatives and summarized their results in Table 1.

Table 1: Aggregated values with different operators

	Used Operator			
Alternative	TSFMSM	TSFWMSM	TSFDMSM	TSFWDMSM
A_1	$(0.9437, 0.9855, 0.979)$	$(0.9102, 0.9799, 0.9735)$	$(0.9972, 0.8718, 0.8378)$	$(0.9922, 0.8425, 0.81)$
A_2	$(0.9052, 0.979, 0.9855)$	$(0.8738, 0.9735, 0.9799)$	$(0.9914, 0.8378, 0.8718)$	$(0.9858, 0.81, 0.8425)$
A_3	$(0.9232, 0.9715, 0.9914)$	$(0.8905, 0.9661, 0.9858)$	$(0.9943, 0.8001, 0.9052)$	$(0.9889, 0.7738, 0.8738)$
A_4	$(0.9052, 0.9351, 0.9855)$	$(0.8738, 0.9299, 0.9799)$	$(0.9914, 0.6363, 0.8718)$	$(0.9858, 0.6153, 0.8425)$
A_5	$(0.9437, 0.9715, 0.9715)$	$(0.9102, 0.9661, 0.9661)$	$(0.9972, 0.8001, 0.8001)$	$(0.9922, 0.7738, 0.7738)$

Step 4: By Eq. (3), we obtain the score values summarized in Table 2

Table 2: Score values of the alternatives

Alternative	TSFMSM	TSFWMSM	TSFDMSM	TSFWDMSM
A_1	-0.055	-0.095	0.7405	0.7841
A_2	-0.147	-0.161	0.7155	0.7576
A_3	-0.105	-0.13	0.7039	0.7539
A_4	-0.008	-0.028	0.8086	0.8264
A_5	0.0255	-0.019	0.8532	0.8667

Step 5: The final ranking order of the given alternatives is recorded as

$$\begin{aligned} \text{TSFMSM} &: A_5 \geq A_4 \geq A_1 \geq A_3 \geq A_2, \\ \text{TSFWMSM} &: A_5 \geq A_4 \geq A_1 \geq A_3 \geq A_2, \\ \text{TSFDMSM} &: A_5 \geq A_4 \geq A_1 \geq A_2 \geq A_3, \\ \text{TSFWDMSM} &: A_5 \geq A_4 \geq A_1 \geq A_2 \geq A_3, \end{aligned}$$

and hence compute that A_5 is the best alternative.

4.2 Impact of q on the decision process

Further, for different value of parameter q, we check the influence and stability of the proposed work, described in Table 3.
Ullah et al [35] suggested that varying q may result in the variation of results and one may get stability value of q. It is considered as those value of q after which the results did not change.

Table 3: Variation in score values with large value of q

Parameter q	Operators	Score Values	Ranking order
20	TSFMSM	0.0009, -0.093, -0.052, 0.103, 0.1378	$A_5 \geq A_4 \geq A_1 \geq A_3 \geq A_2$
	TSFWMSM	-0.019, -0.086, -0.056, 0.1001, 0.111	$A_5 \geq A_4 \geq A_1 \geq A_3 \geq A_2$
	TSFDMSM	0.8823, 0.8369, 0.8224, 0.8616, 0.9409	$A_5 \geq A_4 \geq A_1 \geq A_2 \geq A_3$
	TSFWDMSM	0.8797, 0.8369, 0.8308, 0.8552, 0.9231	$A_5 \geq A_4 \geq A_1 \geq A_2 \geq A_3$
40	TSFMSM	0.1229, 0.045, 0.0607, 0.2584, 0.2972	$A_5 \geq A_4 \geq A_1 \geq A_3 \geq A_2$
	TSFWMSM	0.1174, 0.060, 0.0689, 0.263, 0.2832	$A_5 \geq A_4 \geq A_1 \geq A_3 \geq A_2$
	TSFDMSM	0.9089, 0.8338, 0.8468, 0.8357, 0.9193	$A_5 \geq A_1 \geq A_3 \geq A_4 \geq A_2$
	TSFWDMSM	0.8911, 0.8197, 0.8371, 0.8212, 0.8987	$A_5 \geq A_1 \geq A_3 \geq A_2 \geq A_4$
60	TSFMSM	0.2214, 0.1668, 0.2417, 0.3063, 0.5658	$A_5 \geq A_4 \geq A_3 \geq A_1 \geq A_2$
	TSFWMSM	0.2211, 0.1808, 0.2484, 0.3134, 0.5487	$A_5 \geq A_4 \geq A_3 \geq A_1 \geq A_2$
	TSFDMSM	0.8826, 0.7846, 0.8218, 0.7847, 0.8843	$A_5 \geq A_1 \geq A_3 \geq A_4 \geq A_2$
	TSFWDMSM	0.8642, 0.771, 0.8084, 0.7711, 0.8654	$A_5 \geq A_1 \geq A_3 \geq A_2 \geq A_4$
80	TSFMSM	0.3866, 0.3514, 0.2747, 0.3514, 0.5388	$A_5 \geq A_1 \geq A_2 \geq A_4 \geq A_3$
	TSFWMSM	0.3839, 0.3579, 0.2831, 0.3579, 0.5287	$A_5 \geq A_1 \geq A_2 \geq A_4 \geq A_3$
	TSFDMSM	0.8517, 0.7374, 0.7872, 0.7374, 0.8519	$A_5 \geq A_1 \geq A_3 \geq A_4 \geq A_2$
	TSFWDMSM	0.8345, 0.7258, 0.7739, 0.7258, 0.8347	$A_5 \geq A_1 \geq A_3 \geq A_2 \geq A_4$
100	TSFMSM	0.4097, 0.3879, 0.3082, 0.3879, 0.5229	$A_5 \geq A_1 \geq A_2 \geq A_4 \geq A_3$
	TSFWMSM	0.4088, 0.3927, 0.3164, 0.3927, 0.517	$A_5 \geq A_4 \geq A_3 \geq A_1 \geq A_2$
	TSFDMSM	0.8223, 0.6973, 0.7528, 0.6973, 0.8223	$A_5 \geq A_1 \geq A_3 \geq A_4 \geq A_2$
	TSFWDMSM	0.8066, 0.6877, 0.7407, 0.6877, 0.8066	$A_5 \geq A_1 \geq A_3 \geq A_2 \geq A_4$

4.3 Comparative analysis

Next, we analyze the performance of the proposed approach with some of the existing studies. For this, we present a comprehensive comparison of the presented approach with that of interactive AOs based on TSFNs by Garg et al. [12], Liu et al. [19] and Ullah et al. [35]. The computed results are summarized in Table 4. From this table, we conclude that the finest alterative remains the same with the existing studies, however; the current approach has advantage over the previous one as MSM operator has the ability of interrelate information which the previous theories fail to interrelate. The graphical representation of such results is shown in Figure 1.

5 Conclusions

In this paper, we examined the conception of MSM operators in the environment of TSFSs as only TSF environment provides flexibility in the assigning of the degrees of memberships and reduces information loss. Our main achievements are as follows:

1) We developed TSFMSM operators and TSFDMSM operators and investigated their characteristics.

2) We support the presented theory by examples.

3) The limitation of the previous work is observed and pointed out in view of some remarks where the generalization of the current work is proved.

4) The newly develop operators are utilized in MADM problem to show their effectiveness and versatility.

5) The results obtained using TSFMSM operators are compared with previous study.

We conclude two main advantages of our proposed work which are discussed as follows:

Table 4: Comparative study with the existing approaches

Method	Score Values	Ranking Values
Garg et al. [12]	-0.057,-0.149,-0.107,-0.019,0.0161	$A_5 \geq A_4 \geq A_1 \geq A_3 \geq A_2$
Garg et al. [12]	-0.01,-0.166,-0.136,-0.043,-0.031	$A_5 \geq A_4 \geq A_1 \geq A_3 \geq A_2$
Garg et al. [12]	0.7142,0.6917,0.6804,0.7947,0.83	$A_5 \geq A_4 \geq A_1 \geq A_2 \geq A_3$
Garg et al. [12]	0.7665,0.7416,0.7383,0.8176,0.851	$A_5 \geq A_4 \geq A_1 \geq A_2 \geq A_3$
Liu et al. [19]	-0.059,-0.149,-0.107,-0.003,0.0071	$A_5 \geq A_4 \geq A_1 \geq A_3 \geq A_2$
Liu et al. [19]	-0.105,-0.17,-0.14,-0.057,-0.042	$A_5 \geq A_4 \geq A_1 \geq A_3 \geq A_2$
Liu et al. [19]	0.6835,0.6638,0.6526,0.7768,0.8018	$A_5 \geq A_4 \geq A_1 \geq A_2 \geq A_3$
Liu et al. [19]	0.7461,0.723,0.7198,0.8061,0.8318	$A_5 \geq A_4 \geq A_1 \geq A_2 \geq A_3$
Ullah et al. [35]	-0.051,-0.145,-0.102,-0.0036,0.0353	$A_5 \geq A_4 \geq A_1 \geq A_3 \geq A_2$
Ullah et al. [35]	-0.089,-0.155,-0.124,-0.014,-0.007	$A_5 \geq A_4 \geq A_1 \geq A_3 \geq A_2$
Ullah et al. [35]	0.7634,0.7361,0.724,0.8197,0.8722	$A_5 \geq A_4 \geq A_1 \geq A_2 \geq A_3$
Ullah et al. [35]	0.7996,0.7715,0.7672,0.8333,0.8795	$A_5 \geq A_4 \geq A_1 \geq A_2 \geq A_3$
TSFMSM Operator	-0.055,-0.147,-0.105,-0.008,0.0255	$A_5 \geq A_4 \geq A_1 \geq A_3 \geq A_2$
TSFWMSM Operator	-0.095,-0.161,-0.13,-0.028,-0.019	$A_5 \geq A_4 \geq A_1 \geq A_3 \geq A_2$
TSFDMSM Operator	0.7405,0.7155,0.7039,0.8086,0.3532	$A_5 \geq A_4 \geq A_1 \geq A_2 \geq A_3$
TSFWDMSM Operator	0.7841,0.7576,0.7539,0.8264,0.8667	$A_5 \geq A_4 \geq A_1 \geq A_2 \geq A_3$

1) The use of fours degrees i.e., the DM, DA, DNM and DR greatly reduces information loss, which is very often in the frames of IFSs, PyFSs, QROFSs.

2) The use of variable parameter gives us flexible boundaries in assigning the DM, DNM, and DA etc. unlike PFSs, IFSs, SFSs, and PyFSs.

Our future work aims to investigate the idea of MSM operators in the context of IVTSFSs [25] and complex TSFSs [16, 23] for MADM problems.

Compliance with ethical standards

Conflict of interest: The authors declare that they have no conflict of interest.
Ethical approval: This article does not contain any studies with human participants or animals performed by any of the authors.

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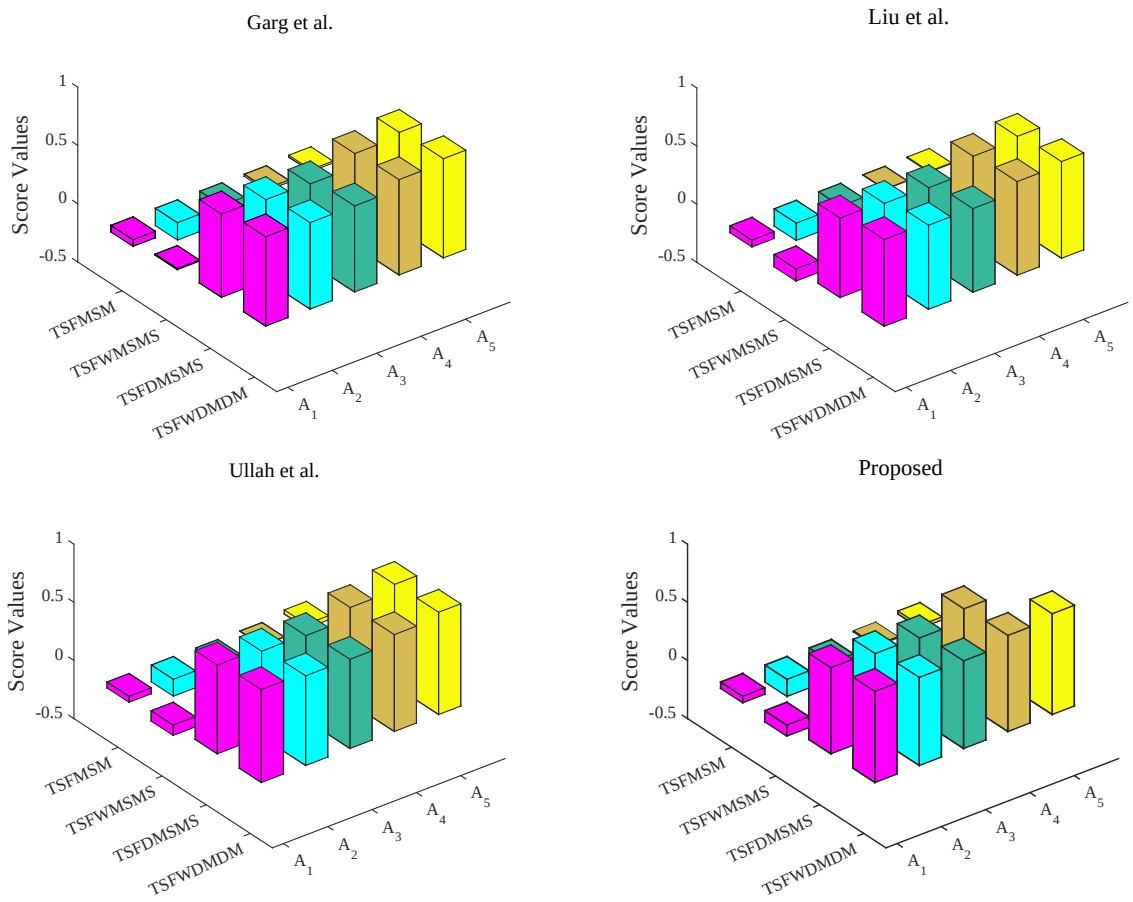


Figure 1: Graphical representation of the results

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