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Picture m-polar fuzzy soft sets and their application in decision-making problems

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Abstract

The aim of this paper is to introduce a new multiple attribute decision-making model named picture m-polar fuzzy soft sets, which is a combination of the soft sets and picture m-polar fuzzy set. Some operations and properties of the new model, including subset, equality, union, intersection, and complement are discussed. Further, the basic six definitions, three theorems, and six examples on picture m-polar fuzzy soft sets are explained. Lastly, we construct a new methodology to extend the TOPSIS to picture m-polar fuzzy soft sets (i.e., an application of picture m-polar fuzzy soft sets) in which capable of different objects recognizing belonging to the same family is constructed and illustrated its applicability via a numerical example.

Keywords: Picture m-polar fuzzy soft sets, multi-person TOPSIS technique, decision-making.

1 Introduction

Many areas (e.g., social sciences, physics, medicine, and computer sciences) deal with uncertain data which necessitates the introduction of fuzzy sets [25], intuitionistic fuzzy sets [4], Pythagorean fuzzy sets [24], and other mathematical tools. Molodtsov [22] presented a novel approach (termed soft set theory) that plays a very significant role in different fields. Therefore, several researchers are developing some novel notions and methods for soft set theory. For example, Maji et al. [20] introduced some notions and operations on soft sets. Further, Maji et al. [21] gave an application on soft set to solve the soft decision-making. Maji et al. [19] proposed the notion of fuzzy soft sets, followed by studies on inverse fuzzy soft sets [13], Fermatean fuzzy soft sets [23], interval-valued intuitionistic fuzzy soft sets [11], interval-valued picture fuzzy soft sets [14], interval-valued neutrosophic soft sets [8], generalized belief interval-valued soft sets [6], and generalized picture fuzzy soft sets [18]. In addition, several expansion models of soft sets are very quickly developed, for example, the possibility fuzzy soft sets [3], possibility Pythagorean fuzzy soft sets [10], possibility m-polar fuzzy soft sets [15], possibility neutrosophic soft sets [12], and possibility multi-fuzzy soft sets [27].

There is some problem with intuitionistic fuzzy sets [4] and picture fuzzy sets [7] with information occurs (i.e., which cannot be elaborated well using the existing approaches), while the picture m-polar fuzzy soft sets are used to resolve the uncertain and more variant data, as the motivation and objectives of this article, we will present the concept of picture m-polar fuzzy soft sets, as a combination of the two above-mentioned models. Furthermore, we study the properties and operations of picture m-polar fuzzy soft sets, including complement, restricted union, and extended intersection. At the end, we also explore a TOPSIS application under the picture m-polar fuzzy soft sets framework.

To smooth our discussion, first in Section 2 we extant some fundamentals related to this article. In Section 3, the concept of picture m-polar fuzzy sets and their basic operations are defined. In Section 4, the new methodology to extend the TOPSIS to picture m-polar fuzzy soft sets is described. The article is then ended with a review and further work outlook.

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2 Preliminaries

We give a short survey of concepts of bipolar fuzzy sets, m-polar fuzzy sets, and picture fuzzy sets as indicated below.

2.1 Bipolar fuzzy sets, m-polar fuzzy sets, picture fuzzy sets, and picture m-polar fuzzy sets

Definition 2.1. [26] *Suppose that* $\mathbb{X}$ *(i.e.,* $\mathbb{X} = \{t_1, t_2, ..., t_r | r = 1, 2, ..., n\}, n \in N$ *(Natural Numbers)) be a set of elements. A bipolar fuzzy set on* $\mathbb{X}$ *is a pair* (α, β)*, where*

$$\alpha : \mathbb{X} \to [0, 1] \text{ and } \beta : \mathbb{X} \to [-1, 0],$$

are two mappings. We note that Bipolar fuzzy sets are an extension of fuzzy sets whose membership degree range is $[-1, 1]$.

Definition 2.2. [5] *An m-polar fuzzy set on* $\mathbb{X}$ *(i.e.,* $\mathbb{X} = \{t_1, t_2, ..., t_r\}$ *is a set of elements), is a mapping*

$$\alpha : \mathbb{X} \to [0, 1]^m.$$

Example 2.3. *Put* $m = 3$ *in Definition 2.2 and let* $\mathbb{X}$ *(i.e.,* $\mathbb{X} = \{t_1, t_2, t_3\}$*) is defined by*

$$\alpha(t_1) = \langle 0.35, 0.41, 0.69\rangle,\ \alpha(t_2) = \langle 0.41, 0.51, 0.8\rangle,\ \text{and } \alpha(t_3) = \langle 0.5, 0.54, 0.3\rangle.$$

Definition 2.4. [7] *A picture fuzzy set* $\chi = (\alpha, \beta, \gamma)$ *on* $\mathbb{X}$ *(i.e.,* $\mathbb{X} = \{t_1, t_2, ..., t_r\}$ *is a set of elements), is a mapping*

$$(\alpha, \beta, \gamma) : \mathbb{X} \longrightarrow [0, 1] \times [0, 1] \times [0, 1],$$

where $\alpha : \mathbb{X} \longrightarrow [0, 1]$ *(i.e., the degree of positive membership),* $\beta : \mathbb{X} \longrightarrow [0, 1]$ *(i.e., the degree of neutral-membership), and* $\gamma : \mathbb{X} \longrightarrow [0, 1]$ *(i.e., the degree of negative-membership) are fuzzy sets over* $\mathbb{X}$ *such that* $\alpha(t_r) + \beta(t_r) + \gamma(t_r) \leq 1$ $(\forall t_r \in \mathbb{X},\ r = 1, 2, ..., n)$

or

$$\chi = \left\{ \frac{\big(\alpha_\chi(t_r), \beta_\chi(t_r), \gamma_\chi(t_r)\big)}{t_r} \ \middle|\ t_r \in \mathbb{X},\ \alpha_\chi(t_r) + \beta_\chi(t_r) + \gamma_\chi(t_r) \leq 1 \right\}.$$

Definition 2.5. [7] *Let*

$$\chi = \left\{ \frac{\big(\alpha_\chi(t_r), \beta_\chi(t_r), \gamma_\chi(t_r)\big)}{t_r} \ \middle|\ t_r \in \mathbb{X},\ \alpha_\chi(t_r) + \beta_\chi(t_r) + \gamma_\chi(t_r) \leq 1 \right\},$$

and

$$\lambda = \left\{ \frac{\big(\zeta_\lambda(t_r), \eta_\lambda(t_r), \varepsilon_\lambda(t_r)\big)}{t_r} \ \middle|\ t_r \in \mathbb{X},\ \zeta_\lambda(t_r) + \eta_\lambda(t_r) + \varepsilon_\lambda(t_r) \leq 1 \right\},$$

are picture fuzzy sets. The following five operations (i.e., complement, inclusion, equality, union, and intersection) are defined by

(1) $\chi^c = \left\{ \frac{\big(\gamma_\chi(t_r), \beta_\chi(t_r), \alpha_\chi(t_r)\big)}{t_r} \ \middle|\ t_r \in \mathbb{X},\ \gamma_\chi(t_r) + \beta_\chi(t_r) + \alpha_\chi(t_r) \leq 1 \right\}.$

(2) $\chi \subseteq \lambda \Longleftrightarrow \alpha_\chi(t_r) \leq \zeta_\lambda(t_r), \beta_\chi(t_r) \leq \eta_\lambda(t_r)$ *and* $\varepsilon_\lambda(t_r) \geq \gamma_\chi(t_r)$ $(\forall t_r \in \mathbb{X})$.

(3) $\chi = \lambda \Longleftrightarrow \chi \subseteq \lambda$ *and* $\chi \supseteq \lambda$.

(4) $\chi \cup \lambda = \left\{ \frac{\big(\alpha_\chi(t_r) \vee \zeta_\lambda(t_r), \beta_\chi(t_r) \wedge \eta_\lambda(t_r), \gamma_\chi(t_r) \wedge \varepsilon_\lambda(t_r)\big)}{t_r} \ \middle|\ t_r \in \mathbb{X} \right\}.$

(5) $\chi \cap \lambda = \left\{ \frac{\big(\alpha_\chi(t_r) \wedge \zeta_\lambda(t_r), \beta_\chi(t_r) \wedge \eta_\lambda(t_r), \gamma_\chi(t_r) \vee \varepsilon_\lambda(t_r)\big)}{t_r} \ \middle|\ t_r \in \mathbb{X} \right\}.$

Definition 2.6. [1] *A picture m-polar fuzzy set* χ *on* $\mathbb{X}$ *(i.e.,* $\mathbb{X} = \{t_1, t_2, ..., t_r\}$ *is a set of elements), is a mapping*

$$\chi : \mathbb{X} \longrightarrow [0, 1]^m \times [0, 1]^m \times [0, 1]^m.$$

It can be written as

$$\chi = \left\{ \frac{t_r}{(p_k \circ \alpha_\chi(t_r), p_k \circ \beta_\chi(t_r), p_k \circ \gamma_\chi(t_r))} \middle| \; t_r \in \mathbb{X},\ k = 1, 2, ..., m \right\},$$

where $(p_k \circ \alpha_\chi(t_r), p_k \circ \beta_\chi(t_r), p_k \circ \gamma_\chi(t_r)) : [0,1] \times [0,1] \times [0,1] \longrightarrow [0,1] \times [0,1] \times [0,1]$ *is the kth projection mapping.* $p_k \circ \alpha_\chi$ *the kth positive membership value,* $p_k \circ \beta_\chi$ *is the kth neutral membership value, and* $p_k \circ \gamma_\chi$ *is the kth negative membership value of* χ *such that* $p_k \circ \alpha_\chi(t_r) + p_k \circ \beta_\chi(t_r) + p_k \circ \gamma_\chi(t_r) \leq \boldsymbol{1}$ $(\forall t_r \in \mathbb{X},\ r = 1, 2, ..., n)$.

3 Picture m-polar fuzzy soft sets

In this section, we will propose a novel concept of picture m-polar fuzzy soft sets (for short, $(\mathbb{P}_p)_m^{\mathcal{IS}}$) and introduce some notions, propositions, and examples as indicated below.

Definition 3.1. *Let* $\mathbb{X} = \{t_1, t_2, ..., t_r\}$ *be a set of elements and* $\vartheta = \{\vartheta_1, \vartheta_2, ..., \vartheta_s\}$ *be a set of parameters (i.e.,* $r, s \in \mathbb{N}$, $\mathbb{N}$ *is a set of natural numbers). A picture m-polar fuzzy soft sets over* $\mathbb{X}$ *is defining as*

$$\chi : \vartheta \longrightarrow (\mathbb{P}_p)_m^{\mathcal{I}},$$

or

$$\begin{aligned}
\chi_\vartheta = \Bigg\{ \Bigg(\vartheta_1, \Bigg\{ & \frac{t_1}{(p_1 \circ \alpha_{\chi_{\vartheta_1}}(t_1), p_1 \circ \beta_{\chi_{\vartheta_1}}(t_1), p_1 \circ \gamma_{\chi_{\vartheta_1}}(t_1))}, \frac{t_2}{(p_2 \circ \alpha_{\chi_{\vartheta_1}}(t_2), p_2 \circ \beta_{\chi_{\vartheta_1}}(t_2), p_2 \circ \gamma_{\chi_{\vartheta_1}}(t_2))}, ..., \\
& \frac{t_r}{(p_m \circ \alpha_{\chi_{\vartheta_1}}(t_r), p_m \circ \beta_{\chi_{\vartheta_1}}(t_r), p_m \circ \gamma_{\chi_{\vartheta_1}}(t_r))} \Bigg\} \Bigg), \Bigg(\vartheta_2, \Bigg\{ \frac{t_1}{(p_1 \circ \alpha_{\chi_{\vartheta_2}}(t_1), p_1 \circ \beta_{\chi_{\vartheta_2}}(t_1), p_1 \circ \gamma_{\chi_{\vartheta_2}}(t_1))}, \\
& \frac{t_2}{(p_2 \circ \alpha_{\chi_{\vartheta_2}}(t_2), p_2 \circ \beta_{\chi_{\vartheta_2}}(t_2), p_2 \circ \gamma_{\chi_{\vartheta_2}}(t_2))}, ..., \frac{t_r}{(p_m \circ \alpha_{\chi_{\vartheta_2}}(t_r), p_m \circ \beta_{\chi_{\vartheta_2}}(t_r), p_m \circ \gamma_{\chi_{\vartheta_2}}(t_r))} \Bigg\} \Bigg), ..., \\
& \Bigg(\vartheta_s, \Bigg\{ \frac{t_1}{(p_1 \circ \alpha_{\chi_{\vartheta_s}}(t_1), p_1 \circ \beta_{\chi_{\vartheta_s}}(t_1), p_1 \circ \gamma_{\chi_{\vartheta_s}}(t_1))}, \frac{t_2}{(p_2 \circ \alpha_{\chi_{\vartheta_s}}(t_2), p_2 \circ \beta_{\chi_{\vartheta_s}}(t_2), p_2 \circ \gamma_{\chi_{\vartheta_s}}(t_2))}, ..., \\
& \frac{t_r}{(p_m \circ \alpha_{\chi_{\vartheta_s}}(t_r), p_m \circ \beta_{\chi_{\vartheta_s}}(t_r), p_m \circ \gamma_{\chi_{\vartheta_s}}(t_r))} \Bigg\} \Bigg) \Bigg| \; t_r \in \mathbb{X},\ \vartheta_s \in \vartheta \Bigg\} \\
= \Bigg\{ \Bigg(\vartheta_s, & \Bigg\{ \frac{t_r}{(p_k \circ \alpha_{\chi_{\vartheta_s}}(t_r), p_k \circ \beta_{\chi_{\vartheta_s}}(t_r), p_k \circ \gamma_{\chi_{\vartheta_s}}(t_r))} \Bigg\} \Bigg) \Bigg| \; t_r \in \mathbb{X},\ \vartheta_s \in \vartheta,\ k = 1, 2, ..., m \Bigg\},
\end{aligned}$$

where $p_k \circ \alpha_{\chi_{\vartheta_s}}(t_r) + p_k \circ \beta_{\chi_{\vartheta_s}}(t_r) + p_k \circ \gamma_{\chi_{\vartheta_s}}(t_r) \leq 1$ $(\forall t_r \in \mathbb{X},\ \vartheta_s \in \vartheta,\ k = 1, 2, ..., m)$.

The tabular formation of χ_ϑ *is showing by the following Table 1:*

Table 1: Tabular formation of χ_ϑ.

χ_ϑ	ϑ_1	ϑ_2	$\cdots$	ϑ_s
t_1	$(p_k \circ \alpha_{\chi_{\vartheta_1}}(t_1), p_k \circ \beta_{\chi_{\vartheta_1}}(t_1), p_k \circ \gamma_{\chi_{\vartheta_1}}(t_1))$	$(p_k \circ \alpha_{\chi_{\vartheta_2}}(t_1), p_k \circ \beta_{\chi_{\vartheta_2}}(t_1), p_k \circ \gamma_{\chi_{\vartheta_2}}(t_1))$	$\cdots$	$(p_k \circ \alpha_{\chi_{\vartheta_s}}(t_1), p_k \circ \beta_{\chi_{\vartheta_s}}(t_1), p_k \circ \gamma_{\chi_{\vartheta_s}}(t_1))$
t_2	$(p_k \circ \alpha_{\chi_{\vartheta_1}}(t_2), p_k \circ \beta_{\chi_{\vartheta_1}}(t_2), p_k \circ \gamma_{\chi_{\vartheta_1}}(t_2))$	$(p_k \circ \alpha_{\chi_{\vartheta_2}}(t_2), p_k \circ \beta_{\chi_{\vartheta_2}}(t_2), p_k \circ \gamma_{\chi_{\vartheta_2}}(t_2))$	$\cdots$	$(p_k \circ \alpha_{\chi_{\vartheta_s}}(t_2), p_k \circ \beta_{\chi_{\vartheta_s}}(t_2), p_k \circ \gamma_{\chi_{\vartheta_s}}(t_2))$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
t_r	$(p_k \circ \alpha_{\chi_{\vartheta_1}}(t_r), p_k \circ \beta_{\chi_{\vartheta_1}}(t_r), p_k \circ \gamma_{\chi_{\vartheta_1}}(t_r))$	$(p_k \circ \alpha_{\chi_{\vartheta_2}}(t_r), p_k \circ \beta_{\chi_{\vartheta_s}}(t_r), p_k \circ \gamma_{\chi_{\vartheta_2}}(t_r))$	$\cdots$	$(p_k \circ \alpha_{\chi_{\vartheta_s}}(t_r), p_k \circ \beta_{\chi_{\vartheta_s}}(t_r), p_k \circ \gamma_{\chi_{\vartheta_s}}(t_r))$

and in matrix form as

$$\chi_\vartheta = \begin{pmatrix}
(p_k \circ \alpha_{\chi_{\vartheta_1}}(t_1), p_k \circ \beta_{\chi_{\vartheta_1}}(t_1), p_k \circ \gamma_{\chi_{\vartheta_1}}(t_1)) & (p_k \circ \alpha_{\chi_{\vartheta_2}}(t_1), p_k \circ \beta_{\chi_{\vartheta_2}}(t_1), p_k \circ \gamma_{\chi_{\vartheta_2}}(t_1)) & \cdots & (p_k \circ \alpha_{\chi_{\vartheta_s}}(t_1), p_k \circ \beta_{\chi_{\vartheta_s}}(t_1), p_k \circ \gamma_{\chi_{\vartheta_s}}(t_1)) \\
(p_k \circ \alpha_{\chi_{\vartheta_1}}(t_2), p_k \circ \beta_{\chi_{\vartheta_1}}(t_2), p_k \circ \gamma_{\chi_{\vartheta_1}}(t_2)) & (p_k \circ \alpha_{\chi_{\vartheta_2}}(t_2), p_k \circ \beta_{\chi_{\vartheta_2}}(t_2), p_k \circ \gamma_{\chi_{\vartheta_2}}(t_2)) & \cdots & (p_k \circ \alpha_{\chi_{\vartheta_s}}(t_2), p_k \circ \beta_{\chi_{\vartheta_s}}(t_2), p_k \circ \gamma_{\chi_{\vartheta_s}}(t_2)) \\
\vdots & \vdots & \ddots & \vdots \\
(p_k \circ \alpha_{\chi_{\vartheta_1}}(t_r), p_k \circ \beta_{\chi_{\vartheta_1}}(t_r), p_k \circ \gamma_{\chi_{\vartheta_1}}(t_r)) & (p_k \circ \alpha_{\chi_{\vartheta_2}}(t_r), p_k \circ \beta_{\chi_{\vartheta_s}}(t_r), p_k \circ \gamma_{\chi_{\vartheta_2}}(t_r)) & \cdots & (p_k \circ \alpha_{\chi_{\vartheta_s}}(t_r), p_k \circ \beta_{\chi_{\vartheta_s}}(t_r), p_k \circ \gamma_{\chi_{\vartheta_s}}(t_r))
\end{pmatrix}.$$

Example 3.2. *Assume that* $\mathbb{X} = \{t_1, t_2, t_3\}$ *be three of elements and* $\vartheta = \{\vartheta_1, \vartheta_2, \vartheta_3\}$ *be three of parameters. Then*

$$\chi_\vartheta = \left\{ \left(\vartheta_1, \left\{ \frac{t_1}{(0.3, 0.2, 0.4), (0.2, 0.5, 0.1)}, \frac{t_2}{(0.5, 0.2, 0.1), (0.3, 0.3, 0.1)}, \frac{t_3}{(0.4, 0.4, 0.2), (0.1, 0.2, 0.1)} \right\} \right),$$

$$\left(\vartheta_2, \left\{\frac{t_1}{(0.3,0.3,0.1),(0.7,0.2,0.0)}, \frac{t_2}{(0.8,0.1,0.1),(0.6,0.2,0.1)}, \frac{t_3}{(0.3,0.2,0.4),(0.2,0.1,0.1)}\right\}\right),$$
$$\left(\vartheta_3, \left\{\frac{t_1}{(0.2,0.2,0.4),(0.4,0.1,0.4)}, \frac{t_2}{(0.1,0.2,0.1),(0.5,0.2,0.1)}, \frac{t_3}{(0.2,0.6,0.1),(0.1,0.2,0.3)}\right\}\right)\Bigg\}.$$

Then, $\chi_\vartheta \in (\mathbb{P}_p)_2^{\mathcal{IS}}$.

In the following Table 2, we can represent χ_ϑ *as*

Table 2: Picture m-polar fuzzy soft sets χ_ϑ.

χ_ϑ	ϑ_1	ϑ_2	ϑ_3
t_1	{(0.3 ,0.2,0.4),(0.2 ,0.5,0.1)}	{(0.3 ,0.3,0.1),(0.7 ,0.2,0.0)}	{(0.2 ,0.2,0.4),(0.4 ,0.1,0.4)}
t_2	{(0.5 ,0.2,0.1),(0.3 ,0.3,0.1)}	{(0.8 ,0.1,0.1),(0.6 ,0.2,0.1)}	{(0.1 ,0.2,0.1),(0.5 ,0.2,0.1)}
t_3	{(0.4 ,0.4,0.2),(0.1 ,0.2,0.1)}	{(0.3 ,0.2,0.4),(0.2 ,0.1,0.1)}	{(0.2 ,0.6,0.1),(0.1 ,0.2,0.3)}

In the following matrix, we also can represent χ_ϑ *as*

$$\chi_\vartheta = \begin{pmatrix} \{(0.3,0.2,0.4),(0.2,0.5,0.1)\} & \{(0.3,0.3,0.1),(0.7,0.2,0.0)\} & \{(0.2,0.2,0.4),(0.4,0.1,0.4)\} \\ \{(0.5,0.2,0.1),(0.3,0.3,0.1)\} & \{(0.8,0.1,0.1),(0.6,0.2,0.1)\} & \{(0.1,0.2,0.1),(0.5,0.2,0.1)\} \\ \{(0.4,0.4,0.2),(0.1,0.2,0.1)\} & \{(0.3,0.2,0.4),(0.2,0.1,0.1)\} & \{(0.2,0.6,0.1),(0.1,0.2,0.3)\} \end{pmatrix}.$$

Definition 3.3. *Let* $\chi_\vartheta, \lambda_\omega \in (\mathbb{P}_p)_m^{\mathcal{IS}}$ *on a set* $\mathbb{X}$, *where*

$$\chi_\vartheta = \left\{\left(\vartheta_s, \left\{\frac{t_r}{(p_k \circ \alpha_{\chi_{\vartheta_s}}(t_r), p_k \circ \beta_{\chi_{\vartheta_s}}(t_r), p_k \circ \gamma_{\chi_{\vartheta_s}}(t_r))}\right\}\right) \middle| \; t_r \in \mathbb{X},\ \vartheta_s \in \vartheta,\ k = 1,2,...,m\right\}$$

and

$$\lambda_\omega = \left\{\left(\omega_s, \left\{\frac{t_r}{(p_k \circ \zeta_{\lambda_{\omega_s}}(t_r), p_k \circ \eta_{\lambda_{\omega_s}}(t_r), p_k \circ \varepsilon_{\lambda_{\omega_s}}(t_r))}\right\}\right) \middle| \; t_r \in \mathbb{X},\ \omega_s \in \omega,\ k = 1,2,...,m\right\}.$$

Thus, $\chi_\vartheta \sqsubseteq \lambda_\omega$ *(i.e.,* χ_ϑ *is a picture m-polar fuzzy soft subset of* λ_ω*) if* $\vartheta_s \subseteq \omega_s$ *and* $p_k \circ \alpha_{\chi_{\vartheta_s}}(t_r) \leq p_k \circ \zeta_{\lambda_{\omega_s}}(t_r)$, $p_k \circ \beta_{\chi_{\vartheta_s}}(t_r) \leq p_k \circ \eta_{\lambda_{\omega_s}}(t_r)$, *and* $p_k \circ \gamma_{\chi_{\vartheta_s}}(t_r) \geq p_k \circ \varepsilon_{\lambda_{\omega_s}}(t_r)$ $(\forall t_r \in \mathbb{X},\ \vartheta_s \in \vartheta,\ \omega_s \in \omega,\ k = 1,2,...,m)$.

Example 3.4. *(Continued from Example 3.2). Suppose that* λ_ω *is defined by*

$$\lambda_\omega = \left\{\left(\omega_1, \left\{\frac{t_1}{(0.3,0.2,0.2),(0.2,0.5,0.0)}, \frac{t_2}{(0.5,0.2,0.0),(0.3,0.3,0.1)}, \frac{t_3}{(0.4,0.4,0.1),(0.1,0.2,0.0)}\right\}\right),$$
$$\left(\omega_2, \left\{\frac{t_1}{(0.3,0.3,0.0),(0.7,0.2,0.0)}, \frac{t_2}{(0.8,0.1,0.1),(0.6,0.2,0.1)}, \frac{t_3}{(0.3,0.2,0.3),(0.2,0.1,0.1)}\right\}\right),$$
$$\left(\omega_3, \left\{\frac{t_1}{(0.2,0.2,0.2),(0.4,0.1,0.2)}, \frac{t_2}{(0.1,0.2,0.0),(0.5,0.2,0.0)}, \frac{t_3}{(0.2,0.6,0.0),(0.1,0.2,0.2)}\right\}\right)\Bigg\}.$$

Hence, $\chi_\vartheta \sqsubseteq \lambda_\omega$ $(\forall t_r \in \mathbb{X}, r = 1,2,3)$.

Definition 3.5. *Let* $\chi_\vartheta, \lambda_\omega \in (\mathbb{P}_p)_m^{\mathcal{IS}}$ *on a set* $\mathbb{X}$, *where*

$$\chi_\vartheta = \left\{\left(\vartheta_s, \left\{\frac{t_r}{(p_k \circ \alpha_{\chi_{\vartheta_s}}(t_r), p_k \circ \beta_{\chi_{\vartheta_s}}(t_r), p_k \circ \gamma_{\chi_{\vartheta_s}}(t_r))}\right\}\right) \middle| \; t_r \in \mathbb{X},\ \vartheta_s \in \vartheta,\ k = 1,2,...,m\right\},$$

and

$$\lambda_\omega = \left\{\left(\omega_s, \left\{\frac{t_r}{(p_k \circ \zeta_{\lambda_{\omega_s}}(t_r), p_k \circ \eta_{\lambda_{\omega_s}}(t_r), p_k \circ \varepsilon_{\lambda_{\omega_s}}(t_r))}\right\}\right) \middle| \; t_r \in \mathbb{X},\ \omega_s \in \omega,\ k = 1,2,...,m\right\}.$$

Thus, $\chi_\vartheta = \lambda_\omega$ *(i.e.,* χ_ϑ *is a picture m-polar fuzzy soft equal of* λ_ω*) if* $\chi_\vartheta \sqsubseteq \lambda_\omega$ *and* $\chi_\vartheta \sqsupseteq \lambda_\omega$.

Definition 3.6. *Let* $\chi_\vartheta \in (\mathbb{P}_p)_m^{\mathcal{IS}}$ *on a set* $\mathbb{X}$, *where*

$$\chi_\vartheta = \left\{ \left(\vartheta_s, \left\{ \frac{t_r}{(p_k \circ \alpha_{\chi_{\vartheta_s}}(t_r), p_k \circ \beta_{\chi_{\vartheta_s}}(t_r), p_k \circ \gamma_{\chi_{\vartheta_s}}(t_r))} \right\} \right) \middle| \ t_r \in \mathbb{X},\ \vartheta_s \in \vartheta,\ k = 1, 2, ..., m \right\}.$$

Thus,

(1) χ_ϑ *is called a null picture m-polar fuzzy soft set (denoted by* $\hat{\emptyset}_\vartheta$*), is defined as*

$$\hat{\emptyset}_\vartheta = \left\{ \left(\vartheta_s, \left\{ \frac{t_r}{(0,0,1)} \right\} \right) \middle| \ t_r \in \mathbb{X},\ \vartheta_s \in \vartheta \right\},$$

where $(p_k \circ \alpha_{\chi_{\vartheta_s}}(t_r), p_k \circ \beta_{\chi_{\vartheta_s}}(t_r), p_k \circ \gamma_{\chi_{\vartheta_s}}(t_r)) = (0,0,1)$ $(\forall k = 1, 2, ..., m)$. *The tabular formation of* $\hat{\emptyset}_\vartheta$ *is shown by the following Table 3:*

Table 3: Null picture m-polar fuzzy soft set $\hat{\emptyset}_\vartheta$.

$\hat{\emptyset}_\vartheta$	ϑ_1	ϑ_2	$\cdots$	ϑ_s
t_1	$\{(0,0,1),(0,0,1),...,(0,0,1)\}$	$\{(0,0,1),(0,0,1),...,(0,0,1)\}$	$\cdots$	$\{(0,0,1),(0,0,1),...,(0,0,1)\}$
t_2	$\{(0,0,1),(0,0,1),...,(0,0,1)\}$	$\{(0,0,1),(0,0,1),...,(0,0,1)\}$	$\cdots$	$\{(0,0,1),(0,0,1),...,(0,0,1)\}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
t_r	$\{(0,0,1),(0,0,1),...,(0,0,1)\}$	$\{(0,0,1),(0,0,1),...,(0,0,1)\}$	$\cdots$	$\{(0,0,1),(0,0,1),...,(0,0,1)\}$

(2) χ_ϑ *is called absolute picture m-polar fuzzy soft set (denoted by* $\hat{\chi}_\vartheta$*), is defined as*

$$\hat{\chi}_\vartheta = \left\{ \left(\vartheta_s, \left\{ \frac{t_r}{(1,0,0)} \right\} \right) \middle| \ t_r \in \mathbb{X},\ \vartheta_s \in \vartheta \right\},$$

where $(p_k \circ \alpha_{\chi_{\vartheta_s}}(t_r), p_k \circ \beta_{\chi_{\vartheta_s}}(t_r), p_k \circ \gamma_{\chi_{\vartheta_s}}(t_r)) = (1,0,0)$ $(\forall k = 1, 2, ..., m)$. *The tabular formation of* $\hat{\chi}_\vartheta$ *is shown by the following Table 4:*

Table 4: Absolute picture m-polar fuzzy soft set $\hat{\chi}_\vartheta$.

$\hat{\chi}_\vartheta$	ϑ_1	ϑ_2	$\cdots$	ϑ_s
t_1	$\{(1,0,0),(1,0,0),...,(1,0,0)\}$	$\{(1,0,0),(1,0,0),...,(1,0,0)\}$	$\cdots$	$\{(1,0,0),(1,0,0),...,(1,0,0)\}$
t_2	$\{(1,0,0),(1,0,0),...,(1,0,0)\}$	$\{(1,0,0),(1,0,0),...,(1,0,0)\}$	$\cdots$	$\{(1,0,0),(1,0,0),...,(1,0,0)\}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
t_r	$\{(1,0,0),(1,0,0),...,(1,0,0)\}$	$\{(1,0,0),(1,0,0),...,(1,0,0)\}$	$\cdots$	$\{(1,0,0),(1,0,0),...,(1,0,0)\}$

Definition 3.7. *Let* $\chi_\vartheta, \lambda_\omega \in (\mathbb{P}_p)_m^{\mathcal{IS}}$ *on a set* $\mathbb{X}$, *where*

$$\chi_\vartheta = \left\{ \left(\vartheta_s, \left\{ \frac{t_r}{(p_k \circ \alpha_{\chi_{\vartheta_s}}(t_r), p_k \circ \beta_{\chi_{\vartheta_s}}(t_r), p_k \circ \gamma_{\chi_{\vartheta_s}}(t_r))} \right\} \right) \middle| \ t_r \in \mathbb{X},\ \vartheta_s \in \vartheta,\ k = 1, 2, ..., m \right\},$$

and

$$\lambda_\omega = \left\{ \left(\omega_s, \left\{ \frac{t_r}{(p_k \circ \zeta_{\lambda_{\omega_s}}(t_r), p_k \circ \eta_{\lambda_{\omega_s}}(t_r), p_k \circ \varepsilon_{\lambda_{\omega_s}}(t_r))} \right\} \right) \middle| \ t_r \in \mathbb{X},\ \omega_s \in \omega,\ k = 1, 2, ..., m \right\}.$$

(1) *The union* $\chi_\vartheta \sqcup \lambda_\omega$, *is defined by* $(\forall t_r \in \mathbb{X},\ \vartheta_s \in \vartheta,\ \omega_s \in \omega,\ k = 1, 2, ..., m)$:

$$\chi_\vartheta \sqcup \lambda_\omega = \left\{ \left(\vartheta_s \cup \omega_s, \left\{ \frac{t_r}{(p_k \circ \alpha_{\chi_{\vartheta_s}}(t_r) \vee p_k \circ \zeta_{\lambda_{\omega_s}}(t_r), p_k \circ \beta_{\chi_{\vartheta_s}}(t_r) \wedge p_k \circ \eta_{\lambda_{\omega_s}}, p_k \circ \gamma_{\chi_{\vartheta_s}}(t_r)) \wedge p_k \circ \varepsilon_{\lambda_{\omega_s}}} \right\} \right) \right\}.$$

(2) *The intersection* $\chi_\vartheta \sqcap \lambda_\omega$, *is defined by* $(\forall t_r \in \mathbb{X},\ \vartheta_s \in \vartheta,\ \omega_s \in \omega,\ k = 1, 2, ..., m)$:

$$\chi_\vartheta \sqcap \lambda_\omega = \left\{ \left(\vartheta_s \cap \omega_s, \left\{ \frac{t_r}{(p_k \circ \alpha_{\chi_{\vartheta_s}}(t_r) \wedge p_k \circ \zeta_{\lambda_{\omega_s}}(t_r), p_k \circ \beta_{\chi_{\vartheta_s}}(t_r) \wedge p_k \circ \eta_{\lambda_{\omega_s}}, p_k \circ \gamma_{\chi_{\vartheta_s}}(t_r)) \vee p_k \circ \varepsilon_{\lambda_{\omega_s}}} \right\} \right) \right\}.$$

Example 3.8. *(Continued from Examples 3.2 and 3.4). The union and intersection are computing, respectively, as follows:*

$$\chi_\vartheta \sqcup \lambda_\omega = \left\{ \left(\vartheta_1 \cup \omega_1, \left\{ \frac{t_1}{(0.3,0.2,0.2),(0.2,0.5,0.0)}, \frac{t_2}{(0.5,0.2,0.0),(0.3,0.3,0.1)}, \frac{t_3}{(0.4,0.4,0.1),(0.1,0.2,0.0)} \right\} \right), \right.$$
$$\left(\vartheta_2 \cup \omega_2, \left\{ \frac{t_1}{(0.3,0.3,0.0),(0.7,0.2,0.0)}, \frac{t_2}{(0.8,0.1,0.1),(0.6,0.2,0.1)}, \frac{t_3}{(0.3,0.2,0.3),(0.2,0.1,0.1)} \right\} \right),$$
$$\left. \left(\vartheta_3 \cup \omega_3, \left\{ \frac{t_1}{(0.2,0.2,0.2),(0.4,0.1,0.2)}, \frac{t_2}{(0.1,0.2,0.0),(0.5,0.2,0.0)}, \frac{t_3}{(0.2,0.6,0.0),(0.1,0.2,0.2)} \right\} \right) \right\}.$$

and

$$\chi_\vartheta \sqcap \lambda_\omega = \left\{ \left(\vartheta_1 \cap \omega_1, \left\{ \frac{t_1}{(0.3,0.2,0.4),(0.2,0.5,0.1)}, \frac{t_2}{(0.5,0.2,0.1),(0.3,0.3,0.1)}, \frac{t_3}{(0.4,0.4,0.2),(0.1,0.2,0.1)} \right\} \right), \right.$$
$$\left(\vartheta_2 \cap \omega_2, \left\{ \frac{t_1}{(0.3,0.3,0.1),(0.7,0.2,0.0)}, \frac{t_2}{(0.8,0.1,0.1),(0.6,0.2,0.1)}, \frac{t_3}{(0.3,0.2,0.4),(0.2,0.1,0.1)} \right\} \right),$$
$$\left. \left(\vartheta_3 \cap \omega_3, \left\{ \frac{t_1}{(0.2,0.2,0.4),(0.4,0.1,0.4)}, \frac{t_2}{(0.1,0.2,0.1),(0.5,0.2,0.1)}, \frac{t_3}{(0.2,0.6,0.1),(0.1,0.2,0.3)} \right\} \right) \right\}.$$

Theorem 3.9. *Suppose $\chi_\vartheta, \lambda_\omega, \rho_\tau \in (\mathbb{P}_p)_m^{\mathcal{IS}}$ on a set $\mathbb{X}$, $\hat{\emptyset}_\vartheta$ is a null picture m-polar fuzzy soft set, and $\hat{\chi}_\vartheta$ is absolute picture m-polar fuzzy soft set. Then the following properties are holding:*

(1) $\chi_\vartheta \sqcup \hat{\emptyset}_\vartheta \sqsubseteq \chi_\vartheta$,
(2) $\chi_\vartheta \sqcap \hat{\chi}_\vartheta \sqsubseteq \chi_\vartheta$,
(3) $\chi_\vartheta \sqcup \hat{\chi}_\vartheta = \hat{\chi}_\vartheta$,
(4) $\chi_\vartheta \sqcap \hat{\emptyset}_\vartheta = \hat{\emptyset}_\vartheta$,
(5) $\chi_\vartheta \sqcup \chi_\vartheta = \chi_\vartheta$,
(6) $\chi_\vartheta \sqcap \chi_\vartheta = \chi_\vartheta$,
(7) $\chi_\vartheta \sqcup \lambda_\omega = \lambda_\omega \sqcup \chi_\vartheta$,
(8) $\chi_\vartheta \sqcap \lambda_\omega = \lambda_\omega \sqcap \chi_\vartheta$,
(9) $[\chi_\vartheta \sqcup \lambda_\omega] \sqcup \rho_\tau = \chi_\vartheta \sqcup [\lambda_\omega \sqcup \rho_\tau]$,
(10) $[\chi_\vartheta \sqcap \lambda_\omega] \sqcap \rho_\tau = \chi_\vartheta \sqcap [\lambda_\omega \sqcap \rho_\tau]$,
(11) $\chi_\vartheta \sqcup [\lambda_\omega \sqcap \rho_\tau] = [\chi_\vartheta \sqcup \lambda_\omega] \sqcap [\chi_\vartheta \sqcup \rho_\tau]$,
(12) $\chi_\vartheta \sqcap [\lambda_\omega \sqcup \rho_\tau] = [\chi_\vartheta \sqcap \lambda_\omega] \sqcup [\chi_\vartheta \sqcap \rho_\tau]$.

Proof. We only prove (1), (3), (5) and (7), (9), and (11). For each $t_r \in \mathbb{X},\ \vartheta_s \in \vartheta,\ \omega_s \in \omega,\ \tau_s \in \tau,\ k = 1, 2, ..., m$, we have

(1) $\chi_\vartheta \sqcup \hat{\emptyset}_\vartheta = \left\{ \left(\vartheta_s, \left\{ \frac{t_r}{(p_k \circ \alpha_{\chi_{\vartheta_s}}(t_r), p_k \circ \beta_{\chi_{\vartheta_s}}(t_r), p_k \circ \gamma_{\chi_{\vartheta_s}}(t_r))} \right\} \right) \right\} \sqcup \left\{ \left(\vartheta_s, \left\{ \frac{t_r}{(0,0,1)} \right\} \right) \right\}$
$= \left\{ \left(\vartheta_s, \left\{ \frac{t_r}{(p_k \circ \alpha_{\chi_{\vartheta_s}}(t_r) \vee 0, p_k \circ \beta_{\chi_{\vartheta_s}}(t_r) \wedge 0, p_k \circ \gamma_{\chi_{\vartheta_s}}(t_r)) \wedge 1} \right\} \right) \right\}$
$= \left\{ \left(\vartheta_s, \left\{ \frac{t_r}{(p_k \circ \alpha_{\chi_{\vartheta_s}}(t_r), 0, p_k \circ \gamma_{\chi_{\vartheta_s}}(t_r))} \right\} \right) \right\}$
$\sqsubseteq \left\{ \left(\vartheta_s, \left\{ \frac{t_r}{(p_k \circ \alpha_{\chi_{\vartheta_s}}(t_r), p_k \circ \beta_{\chi_{\vartheta_s}}(t_r), p_k \circ \gamma_{\chi_{\vartheta_s}}(t_r))} \right\} \right) \right\}$
$= \chi_\vartheta.$

(3) $\chi_\vartheta \sqcup \hat{\chi}_\vartheta = \left\{ \left(\vartheta_s, \left\{ \frac{t_r}{(p_k \circ \alpha_{\chi_{\vartheta_s}}(t_r), p_k \circ \beta_{\chi_{\vartheta_s}}(t_r), p_k \circ \gamma_{\chi_{\vartheta_s}}(t_r))} \right\} \right) \right\} \sqcup \left\{ \left(\vartheta_s, \left\{ \frac{t_r}{(1,0,0)} \right\} \right) \right\}$
$= \left\{ \left(\vartheta_s, \left\{ \frac{t_r}{(p_k \circ \alpha_{\chi_{\vartheta_s}}(t_r) \vee 1, p_k \circ \beta_{\chi_{\vartheta_s}}(t_r) \wedge 0, p_k \circ \gamma_{\chi_{\vartheta_s}}(t_r)) \wedge 0} \right\} \right) \right\}$
$= \left\{ \left(\vartheta_s, \left\{ \frac{t_r}{(1,0,0)} \right\} \right) \right\}$
$= \hat{\chi}_\vartheta.$

(5) $\chi_\vartheta \sqcup \chi_\vartheta = \left\{ \left(\vartheta_s, \left\{ \frac{t_r}{(p_k \circ \alpha_{\chi_{\vartheta_s}}(t_r), p_k \circ \beta_{\chi_{\vartheta_s}}(t_r), p_k \circ \gamma_{\chi_{\vartheta_s}}(t_r))} \right\} \right) \right\} \sqcup \left\{ \left(\vartheta_s, \left\{ \frac{t_r}{(p_k \circ \alpha_{\chi_{\vartheta_s}}(t_r), p_k \circ \beta_{\chi_{\vartheta_s}}(t_r), p_k \circ \gamma_{\chi_{\vartheta_s}}(t_r))} \right\} \right) \right\}$

$$= \left\{\left(\vartheta_s, \left\{\frac{t_r}{(p_k\circ\alpha_{\chi_{\vartheta_s}}(t_r)\vee p_k\circ\alpha_{\chi_{\vartheta_s}}(t_r), p_k\circ\beta_{\chi_{\vartheta_s}}(t_r)\wedge p_k\circ\beta_{\chi_{\vartheta_s}}(t_r), p_k\circ\gamma_{\chi_{\vartheta_s}}(t_r))\wedge p_k\circ\gamma_{\chi_{\vartheta_s}}(t_r))}\right\}\right)\right\}$$
$$= \left\{\left(\vartheta_s, \left\{\frac{t_r}{(p_k\circ\alpha_{\chi_{\vartheta_s}}(t_r), p_k\circ\beta_{\chi_{\vartheta_s}}(t_r), p_k\circ\gamma_{\chi_{\vartheta_s}}(t_r))}\right\}\right)\right\}$$
$$= \chi_\vartheta.$$

(7) $\chi_\vartheta \sqcup \lambda_\omega = \left\{\left(\vartheta_s, \left\{\frac{t_r}{(p_k\circ\alpha_{\chi_{\vartheta_s}}(t_r), p_k\circ\beta_{\chi_{\vartheta_s}}(t_r), p_k\circ\gamma_{\chi_{\vartheta_s}}(t_r))}\right\}\right)\right\} \sqcup \left\{\left(\omega_s, \left\{\frac{t_r}{(p_k\circ\zeta_{\lambda_{\omega_s}}(t_r), p_k\circ\eta_{\lambda_{\omega_s}}(t_r), p_k\circ\varepsilon_{\lambda_{\omega_s}}(t_r))}\right\}\right)\right\}$

$$= \left\{\left(\vartheta_s \cup \omega_s, \left\{\frac{t_r}{(p_k\circ\alpha_{\chi_{\vartheta_s}}(t_r)\vee p_k\circ\zeta_{\lambda_{\omega_s}}(t_r), p_k\circ\beta_{\chi_{\vartheta_s}}(t_r)\wedge p_k\circ\eta_{\lambda_{\omega_s}}, p_k\circ\gamma_{\chi_{\vartheta_s}}(t_r))\wedge p_k\circ\varepsilon_{\lambda_{\omega_s}}}\right\}\right)\right\}$$
$$= \left\{\left(\omega_s \cup \vartheta_s, \left\{\frac{t_r}{(p_k\circ\zeta_{\lambda_{\omega_s}}(t_r)\vee p_k\circ\alpha_{\chi_{\vartheta_s}}(t_r), p_k\circ\eta_{\lambda_{\omega_s}}\wedge p_k\circ\beta_{\chi_{\vartheta_s}}(t_r), p_k\circ\varepsilon_{\lambda_{\omega_s}}\wedge p_k\circ\gamma_{\chi_{\vartheta_s}}(t_r))}\right\}\right)\right\}$$
$$= \left\{\left(\omega_s, \left\{\frac{t_r}{(p_k\circ\zeta_{\lambda_{\omega_s}}(t_r), p_k\circ\eta_{\lambda_{\omega_s}}(t_r), p_k\circ\varepsilon_{\lambda_{\omega_s}}(t_r))}\right\}\right)\right\} \sqcup \left\{\left(\vartheta_s, \left\{\frac{t_r}{(p_k\circ\alpha_{\chi_{\vartheta_s}}(t_r), p_k\circ\beta_{\chi_{\vartheta_s}}(t_r), p_k\circ\gamma_{\chi_{\vartheta_s}}(t_r))}\right\}\right)\right\}$$
$$= \lambda_\omega \sqcup \chi_\vartheta.$$

(9) $[\chi_\vartheta \sqcup \lambda_\omega] \sqcup \rho_\tau = \left[\left\{\left(\vartheta_s, \left\{\frac{t_r}{(p_k\circ\alpha_{\chi_{\vartheta_s}}(t_r), p_k\circ\beta_{\chi_{\vartheta_s}}(t_r), p_k\circ\gamma_{\chi_{\vartheta_s}}(t_r))}\right\}\right)\right\} \sqcup \left\{\left(\omega_s, \left\{\frac{t_r}{(p_k\circ\zeta_{\lambda_{\omega_s}}(t_r), p_k\circ\eta_{\lambda_{\omega_s}}(t_r), p_k\circ\varepsilon_{\lambda_{\omega_s}}(t_r))}\right\}\right)\right\}\right]$

$$\sqcup \left\{\left(\tau_s, \left\{\frac{t_r}{(p_k\circ\pi_{\rho_{\tau_s}}(t_r), p_k\circ\iota_{\rho_{\tau_s}}(t_r), p_k\circ\kappa_{\rho_{\tau_s}}(t_r))}\right\}\right)\right\}$$
$$= \left\{\left(\vartheta_s \cup \omega_s, \left\{\frac{t_r}{(p_k\circ\alpha_{\chi_{\vartheta_s}}(t_r)\vee p_k\circ\zeta_{\lambda_{\omega_s}}(t_r), p_k\circ\beta_{\chi_{\vartheta_s}}(t_r)\wedge p_k\circ\eta_{\lambda_{\omega_s}}, p_k\circ\gamma_{\chi_{\vartheta_s}}(t_r))\wedge p_k\circ\varepsilon_{\lambda_{\omega_s}}}\right\}\right)\right\}$$
$$\sqcup \left\{\left(\tau_s, \left\{\frac{t_r}{(p_k\circ\pi_{\rho_{\tau_s}}(t_r), p_k\circ\iota_{\rho_{\tau_s}}(t_r), p_k\circ\kappa_{\rho_{\tau_s}}(t_r))}\right\}\right)\right\}$$
$$= \left\{\left(\vartheta_s \cup \omega_s \cup \tau_s, \left\{\frac{t_r}{(p_k\circ\alpha_{\chi_{\vartheta_s}}(t_r)\vee p_k\circ\zeta_{\lambda_{\omega_s}}(t_r)\vee p_k\circ\pi_{\rho_{\tau_s}}(t_r), p_k\circ\beta_{\chi_{\vartheta_s}}(t_r)\wedge p_k\circ\eta_{\lambda_{\omega_s}}\wedge p_k\circ\iota_{\rho_{\tau_s}}(t_r), p_k\circ\gamma_{\chi_{\vartheta_s}}(t_r))\wedge p_k\circ\varepsilon_{\lambda_{\omega_s}}\wedge p_k\circ\kappa_{\rho_{\tau_s}}(t_r))}\right\}\right)\right\}$$
$$= \left\{\left(\vartheta_s, \left\{\frac{t_r}{(p_k\circ\alpha_{\chi_{\vartheta_s}}(t_r), p_k\circ\beta_{\chi_{\vartheta_s}}(t_r), p_k\circ\gamma_{\chi_{\vartheta_s}}(t_r))}\right\}\right)\right\}$$
$$\sqcup \left\{\left(\omega_s \cup \tau_s, \left\{\frac{t_r}{(p_k\circ\zeta_{\lambda_{\omega_s}}(t_r)\vee p_k\circ\pi_{\rho_{\tau_s}}(t_r), p_k\circ\eta_{\lambda_{\omega_s}}\wedge p_k\circ\iota_{\rho_{\tau_s}}(t_r), p_k\circ\varepsilon_{\lambda_{\omega_s}}\wedge p_k\circ\kappa_{\rho_{\tau_s}}(t_r))}\right\}\right)\right\}$$
$$= \left\{\left(\vartheta_s, \left\{\frac{t_r}{(p_k\circ\alpha_{\chi_{\vartheta_s}}(t_r), p_k\circ\beta_{\chi_{\vartheta_s}}(t_r), p_k\circ\gamma_{\chi_{\vartheta_s}}(t_r))}\right\}\right)\right\}$$
$$\sqcup \left[\left\{\left(\omega_s, \left\{\frac{t_r}{(p_k\circ\zeta_{\lambda_{\omega_s}}(t_r), p_k\circ\eta_{\lambda_{\omega_s}}(t_r), p_k\circ\varepsilon_{\lambda_{\omega_s}}(t_r))}\right\}\right)\right\} \sqcup \left\{\left(\tau_s, \left\{\frac{t_r}{(p_k\circ\pi_{\rho_{\tau_s}}(t_r), p_k\circ\iota_{\rho_{\tau_s}}(t_r), p_k\circ\kappa_{\rho_{\tau_s}}(t_r))}\right\}\right)\right\}\right]$$
$$= \chi_\vartheta \sqcup (\lambda_\omega \sqcup \rho_\tau).$$

(11) $\chi_\vartheta \sqcup [\lambda_\omega \sqcap \rho_\tau] = \left\{\left(\vartheta_s, \left\{\frac{t_r}{(p_k\circ\alpha_{\chi_{\vartheta_s}}(t_r), p_k\circ\beta_{\chi_{\vartheta_s}}(t_r), p_k\circ\gamma_{\chi_{\vartheta_s}}(t_r))}\right\}\right)\right\}$

$$\sqcup \left[\left\{\left(\omega_s, \left\{\frac{t_r}{(p_k\circ\zeta_{\lambda_{\omega_s}}(t_r), p_k\circ\eta_{\lambda_{\omega_s}}(t_r), p_k\circ\varepsilon_{\lambda_{\omega_s}}(t_r))}\right\}\right)\right\} \sqcap \left\{\left(\tau_s, \left\{\frac{t_r}{(p_k\circ\pi_{\rho_{\tau_s}}(t_r), p_k\circ\iota_{\rho_{\tau_s}}(t_r), p_k\circ\kappa_{\rho_{\tau_s}}(t_r))}\right\}\right)\right\}\right]$$
$$= \left\{\left(\vartheta_s, \left\{\frac{t_r}{(p_k\circ\alpha_{\chi_{\vartheta_s}}(t_r), p_k\circ\beta_{\chi_{\vartheta_s}}(t_r), p_k\circ\gamma_{\chi_{\vartheta_s}}(t_r))}\right\}\right)\right\}$$
$$\sqcup \left\{\left(\omega_s \cap \tau_s, \left\{\frac{t_r}{(p_k\circ\zeta_{\lambda_{\omega_s}}(t_r)\wedge p_k\circ\pi_{\rho_{\tau_s}}(t_r), p_k\circ\eta_{\lambda_{\omega_s}}\wedge p_k\circ\iota_{\rho_{\tau_s}}(t_r), p_k\circ\varepsilon_{\lambda_{\omega_s}}\vee p_k\circ\kappa_{\rho_{\tau_s}}(t_r))}\right\}\right)\right\}$$
$$= \left\{\left(\vartheta_s \cup (\omega_s \cap \tau_s), \left\{\frac{t_r}{(p_k\circ\alpha_{\chi_{\vartheta_s}}(t_r)\vee[p_k\circ\zeta_{\lambda_{\omega_s}}(t_r)\wedge p_k\circ\pi_{\rho_{\tau_s}}(t_r)], p_k\circ\beta_{\chi_{\vartheta_s}}(t_r)\wedge[p_k\circ\eta_{\lambda_{\omega_s}}\wedge p_k\circ\iota_{\rho_{\tau_s}}(t_r)], p_k\circ\gamma_{\chi_{\vartheta_s}}(t_r)\wedge[p_k\circ\varepsilon_{\lambda_{\omega_s}}\vee p_k\circ\kappa_{\rho_{\tau_s}}(t_r)])}\right\}\right)\right\}$$
$$= \left\{\left((\vartheta_s \cup \omega_s) \cap (\vartheta_s \cup \tau_s), \left\{\frac{t_r}{[p_k\circ\alpha_{\chi_{\vartheta_s}}(t_r)\vee p_k\circ\zeta_{\lambda_{\omega_s}}(t_r)]\wedge[p_k\circ\alpha_{\chi_{\vartheta_s}}(t_r)\vee p_k\circ\pi_{\rho_{\tau_s}}(t_r)], [p_k\circ\beta_{\chi_{\vartheta_s}}(t_r)\wedge p_k\circ\eta_{\lambda_{\omega_s}}]\wedge[p_k\circ\beta_{\chi_{\vartheta_s}}(t_r)\wedge p_k\circ\iota_{\rho_{\tau_s}}(t_r)], [p_k\circ\gamma_{\chi_{\vartheta_s}}(t_r)\wedge p_k\circ\varepsilon_{\lambda_{\omega_s}}]\vee[p_k\circ\gamma_{\chi_{\vartheta_s}}(t_r)\wedge p_k\circ\kappa_{\rho_{\tau_s}}(t_r)])}\right\}\right)\right\}$$
$$= [\chi_\vartheta \sqcup \lambda_\omega] \sqcap [\chi_\vartheta \sqcup \rho_\tau].$$ □

Definition 3.10. *Let $\chi_\vartheta \in (\mathbb{P}_p)_m^{\mathcal{IS}}$ on a set $\mathbb{X}$, where*

$$\chi_\vartheta = \left\{\left(\vartheta_s, \left\{\frac{t_r}{(p_k\circ\alpha_{\chi_{\vartheta_s}}(t_r), p_k\circ\beta_{\chi_{\vartheta_s}}(t_r), p_k\circ\gamma_{\chi_{\vartheta_s}}(t_r))}\right\}\right)\middle|\ t_r \in \mathbb{X},\ \vartheta_s \in \vartheta,\ k = 1, 2, ..., m\right\}.$$

The complement χ_ϑ^c of χ_ϑ, is defined as

$$\chi_\vartheta^c = \left\{\left(\vartheta_s, \left\{\frac{t_r}{(p_k\circ\gamma_{\chi_{\vartheta_s}}(t_r), p_k\circ\beta_{\chi_{\vartheta_s}}(t_r), p_k\circ\alpha_{\chi_{\vartheta_s}}(t_r))}\right\}\right)\middle|\ t_r \in \mathbb{X},\ \vartheta_s \in \vartheta,\ k = 1, 2, ..., m\right\}.$$

Example 3.11. *(Continued from Example 3.2). The complement χ_ϑ^c of χ_ϑ, is calculated by*

$$\chi_\vartheta^c = \left\{ \left(\vartheta_1, \left\{ \frac{t_1}{(0.4,0.2,0.3),(0.1,0.5,0.2)}, \frac{t_2}{(0.1,0.2,0.5),(0.1,0.3,0.3}, \frac{t_3}{(0.2,0.4,0.4),(0.1,0.2,0.1)} \right\} \right), \right.$$
$$\left(\vartheta_2, \left\{ \frac{t_1}{(0.1,0.3,0.3),(0.0,0.2,0.7)}, \frac{t_2}{(0.1,0.1,0.8),(0.1,0.2,0.6)}, \frac{t_3}{(0.4,0.2,0.3),(0.1,0.1,0.2)} \right\} \right),$$
$$\left. \left(\vartheta_3, \left\{ \frac{t_1}{(0.4,0.2,0.2),(0.4,0.1,0.4)}, \frac{t_2}{(0.1,0.2,0.1),(0.1,0.2,0.5)}, \frac{t_3}{(0.1,0.6,0.2),(0.3,0.2,0.1)} \right\} \right) \right\}.$$

Theorem 3.12. *Suppose $\chi_\vartheta \in (\mathbb{P}_p)_m^{\mathcal{IS}}$ on a set $\mathbb{X}$, $\hat{\emptyset}_\vartheta$ is a null picture m-polar fuzzy soft set, and $\hat{\chi}_\vartheta$ is absolute picture m-polar fuzzy soft set. Then the following properties are holding:*

(1) $\hat{\emptyset}_\vartheta^c = \hat{\chi}_\vartheta$,
(2) $\hat{\chi}_\vartheta^c = \hat{\emptyset}_\vartheta$,
(3) $(\chi_\vartheta^c)^c = \chi_\vartheta$.

Proof. (1) $\hat{\emptyset}_\vartheta^c = \left(\left\{ \left(\vartheta_s, \left\{ \frac{t_r}{(0,0,1)} \right\} \right) \middle| \; t_r \in \mathbb{X},\ \vartheta_s \in \vartheta \right\} \right)^c = \left\{ \left(\vartheta_s, \left\{ \frac{t_r}{(1,0,0)} \right\} \right) \middle| \; t_r \in \mathbb{X},\ \vartheta_s \in \vartheta \right\} = \hat{\chi}_\vartheta.$

(2) Similar to (1).

(3) $(\chi_\vartheta^c)^c = \left(\left\{ \left(\vartheta_s, \left\{ \frac{t_r}{(p_k \circ \gamma_{\chi_{\vartheta_s}}(t_r), p_k \circ \beta_{\chi_{\vartheta_s}}(t_r), p_k \circ \alpha_{\chi_{\vartheta_s}}(t_r))} \right\} \right) \middle| \; t_r \in \mathbb{X},\ \vartheta_s \in \vartheta,\ k = 1,2,...,m \right\} \right)^c$
$= \left\{ \left(\vartheta_s, \left\{ \frac{t_r}{(p_k \circ \alpha_{\chi_{\vartheta_s}}(t_r), p_k \circ \beta_{\chi_{\vartheta_s}}(t_r), p_k \circ \gamma_{\chi_{\vartheta_s}}(t_r))} \right\} \right) \middle| \; t_r \in \mathbb{X},\ \vartheta_s \in \vartheta,\ k = 1,2,...,m \right\}$
$= \chi_\vartheta.$ □

By the following Example 3.13, we will show the equalities $\chi_\vartheta \sqcup \chi_\vartheta^c = \hat{\chi}_\vartheta$ and $\chi_\vartheta \sqcap \chi_\vartheta^c = \hat{\emptyset}_\vartheta$ do not hold.

Example 3.13. *(Continued from Examples 3.2 and 3.11). Then $\chi_\vartheta \sqcup \chi_\vartheta^c$ and $\chi_\vartheta \sqcap \chi_\vartheta^c$ are calculated by*

$$\chi_\vartheta \sqcup \chi_\vartheta^c = \left\{ \left(\vartheta_1, \left\{ \frac{t_1}{(0.4,0.2,0.3),(0.1,0.5,0.2)}, \frac{t_2}{(0.1,0.2,0.5),(0.1,0.3,0.3}, \frac{t_3}{(0.2,0.4,0.4),(0.1,0.2,0.1)} \right\} \right), \right.$$
$$\left(\vartheta_2, \left\{ \frac{t_1}{(0.1,0.3,0.3),(0.0,0.2,0.7)}, \frac{t_2}{(0.1,0.1,0.8),(0.1,0.2,0.6)}, \frac{t_3}{(0.4,0.2,0.3),(0.1,0.1,0.2)} \right\} \right),$$
$$\left. \left(\vartheta_3, \left\{ \frac{t_1}{(0.4,0.2,0.2),(0.4,0.1,0.4)}, \frac{t_2}{(0.1,0.2,0.1),(0.1,0.2,0.5)}, \frac{t_3}{(0.1,0.6,0.2),(0.3,0.2,0.1)} \right\} \right) \right\},$$

and

$$\chi_\vartheta \sqcap \chi_\vartheta^c = \left\{ \left(\vartheta_1, \left\{ \frac{t_1}{(0.3,0.2,0.4),(0.2,0.5,0.1)}, \frac{t_2}{(0.5,0.2,0.1),(0.3,0.3,0.1)}, \frac{t_3}{(0.4,0.4,0.2),(0.1,0.2,0.1)} \right\} \right), \right.$$
$$\left(\vartheta_2, \left\{ \frac{t_1}{(0.3,0.3,0.1),(0.7,0.2,0.0)}, \frac{t_2}{(0.8,0.1,0.1),(0.6,0.2,0.1)}, \frac{t_3}{(0.3,0.2,0.4),(0.2,0.1,0.1)} \right\} \right),$$
$$\left. \left(\vartheta_3, \left\{ \frac{t_1}{(0.2,0.2,0.4),(0.4,0.1,0.4)}, \frac{t_2}{(0.1,0.2,0.1),(0.5,0.2,0.1)}, \frac{t_3}{(0.2,0.6,0.1),(0.1,0.2,0.3)} \right\} \right) \right\}.$$

This show $\chi_\vartheta \sqcup \chi_\vartheta^c \neq \hat{\chi}_\vartheta$ and $\chi_\vartheta \sqcap \chi_\vartheta^c \neq \hat{\emptyset}_\vartheta$.

Theorem 3.14. *Suppose $\chi_\vartheta, \lambda_\omega \in (\mathbb{P}_p)_m^{\mathcal{IS}}$ on a set $\mathbb{X}$. Then the following properties are holding:*

(1) $(\chi_\vartheta \sqcup \lambda_\omega)^c = \chi_\vartheta^c \sqcap \lambda_\omega^c$,
(2) $(\chi_\vartheta \sqcap \lambda_\omega)^c = \chi_\vartheta^c \sqcup \lambda_\omega^c$.

Proof. Clear. □

4 New methodology to extend the TOPSIS to picture m-polar fuzzy soft sets

Now, we will propose an Algorithm of the multi-decision maker multi-criteria decision-making of picture m-polar fuzzy soft sets as follows:

Algorithm Determination the optimal decision based on picture m-polar fuzzy soft sets.

First step: Assume that there is a group of the decision-maker of n persons, they evaluate the ratings of alternatives Z_r $(r = 1, 2, ..., p)$ concerning criteria C_s $(s = 1, 2, ..., q)$ in picture value form and the information about criterion weights is known.

Second step: Create a picture m-polar fuzzy soft sets decision (i.e., a multi-criteria group decision-making problem) which can be represented in matrix form as

$$\mathcal{M}_{p\times q} = \begin{pmatrix} \vartheta_{11} & \vartheta_{12} & \cdots, & \vartheta_{1q} \\ \vartheta_{21} & \vartheta_{22} & \cdots, & \vartheta_{2q} \\ \vdots & \vdots & \ddots & \vdots \\ \vartheta_{p1} & \vartheta_{p2} & \cdots, & \vartheta_{pq} \end{pmatrix}.$$

where p (i.e., the number of alternatives), q (i.e., the number of criteria), and ϑ_{rs} $\forall$ r $(r = 1, 2, ..., p)$, $\forall$ s $(s = 1, 2, ..., q)$; represents the ratings of r-alternatives concering the s-criteria in the picture value. The multiple data of n persons of decision-maker group for rating ϑ_{rs} can be expressed as

$$\vartheta_{rs} = (p_k \circ \alpha_{rs}, p_k \circ \beta_{rs}, p_k \circ \gamma_{rs}) \ (k = 1, 2, .., m).$$

Third step: Convert the picture m-polar fuzzy soft value data of alternatives into picture m-polar fuzzy soft n-single-value data by using the following formula

$$p_k \circ \epsilon_{rs} = \frac{p_k \circ \alpha_{rs} + p_k \circ \beta_{rs} + p_k \circ \gamma_{rs}}{3}. \tag{1}$$

and we obtain rating of alternatives as

$$\hat{\epsilon}_{rs} = (p_1 \circ \epsilon_{rs}, p_2 \circ \epsilon_{rs}, ..., p_m \circ \epsilon_{rs}) \ (k = 1, 2, ..., m),$$

and the weights of criterion is expressed in information as

$$\hat{w}_s = (p_1 \circ w_s, p_2 \circ w_s, ..., p_m \circ w_s) \ (k = 1, 2, ..., m; s = 1, 2, .., q).$$

Fourth step: By normalization (i.e., the normalization process is to preserve the property of picture m-polar fuzzy soft that the ranges of membership of elements is [0, 1]) we get the normalized picture m-polar fuzzy soft to picture m-polar fuzzy decision matrix (i.e., fuzzy numbers belong to [0,1]), denoted by $\hat{E}$, is defined by

$$\hat{E} = (\mathcal{E}_{rs})_{p\times q},$$

where

$$\mathcal{E}_{rs} = \left(p_k \circ \hat{\mathcal{E}}_{rs} = \frac{p_k \circ \epsilon_{rs}}{\sqrt{\sum_{r=1}^{p} (p_k \circ \epsilon_{rs})^2}} \right) \ (k = 1, 2, ..., m; s = 1, 2, .., q). \tag{2}$$

Fifth step: Construct the weighted normalized picture m-polar fuzzy soft decision matrix, denoted by $\hat{F}$, is defined by

$$\hat{F} = (f_{rs})_{p\times q},$$

such that

$$f_{rs} = \mathcal{E}_{rs}(.)\hat{w}_s = (p_k \circ \mathcal{E}_{rs}(.) p_k \circ \hat{w}_s) \ \ (k = 1, 2, ..., m; s = 1, 2, .., q), \tag{3}$$

where the elements f_{rs} $(r = 1, 2, ..., p; s = 1, 2, ..., q)$ are normaliz the given m values to belong their ranges to closed interval $[0, 1]$.

Sixth step: Determine the m-fuzzy positive ideal solution (denoted by $\mathcal{A}^*$) and m-fuzzy negative ideal solution (denoted by $\mathcal{A}^-$), are defined by

$$\mathcal{A}^* = (f_1^*, f_2^*, ..., f_q^*) \text{ and } \mathcal{A}^- = (f_1^-, f_2^-, ..., f_q^-),$$

where

$$f_s^* = p_k \circ \hat{f}_s^* = \begin{cases} (1,1,...,1), & s \in \mathcal{B} \\ (0,0,...,0), & s \in \mathcal{C} \end{cases}, \qquad f_s^- = p_k \circ \hat{f}_s^- = \begin{cases} (0,0,...,0), & s \in \mathcal{B} \\ (1,1,...,1), & s \in \mathcal{C} \end{cases},$$

for all $k = 1, 2, ..., m; s = 1, 2, ..., q$ and $\mathcal{B}$ denotes the benefit criteria and $\mathcal{C}$ denotes the cost criteria.

Seventh step: Computing every alternative from $\mathcal{A}^*$ and $\mathcal{A}^-$, respectively, as follows:

$$\mathcal{S}_r^* = d(f_{rs}, f_s^*) = \sqrt{\frac{1}{m}\left\{\sum_{k=1}^{m}\sum_{s=1}^{q}(p_k \circ \hat{f}_{rs} - p_k \circ \hat{f}_s^*)^2\right\}}, \tag{4}$$

and

$$\mathcal{S}_r^- = d(f_{rs}, f_s^-) = \sqrt{\frac{1}{m}\left\{\sum_{k=1}^{m}\sum_{s=1}^{q}(p_k \circ \hat{f}_{rs} - p_k \circ \hat{f}_s^-)^2\right\}}. \tag{5}$$

Eighth step: Compute the closeness coefficient (denoted by, $\overline{Cf_r}$) of each alternative Z_r $(r = 1, 2, ..., p)$ (i.e., as alternative Z_r is near to $\mathcal{A}^*$ and far from $\mathcal{A}^-$ as closeness coefficient $\overline{Cf_r}$ go closer to 1), is defined by

$$\overline{Cf_r} = \frac{\mathcal{S}_r^-}{\mathcal{S}_r^- + \mathcal{S}_r^*}. \tag{6}$$

Ninth step: According to the $\overline{Cf_r}$, we can give the rank to all alternatives and select the best one of them that is nearer to 1.

In the following example, we will explain and apply the above nine steps of the above Algorithm as:

Example 4.1. *Let us consider there is an investment company, which wants to invest some money in the best alternative. The company made a committee of three members for the selection of best alternative (i.e., Z_r $(r = 1, 2, 3, 4)$) among a panel of four possible alternatives to invest the money, where Z_1 is 'a medicine company', Z_2 is 'an electronics company', Z_3 is 'a food company', and Z_4 is 'a telecommunication company'. The committee should make a decision concerning the following four criteria (i.e., C_s $(s = 1, 2, 3, 4)$): C_1 is 'the environmental impact analysis', C_2 is 'the risk analysis', C_3 is 'the social-political impact analysis', and C_4 is 'the growth analysis'. The data of picture 3-polar fuzzy soft decision matrix (in the range (0,10)) and 3-polar weights by the committee of three decision-makers p_k $(k = 1, 2, 3)$ evaluate all four possible alternatives (i.e, Z_1, Z_2, Z_3, Z_4) in first and second steps of above the Algorithm are given by*

$$\mathcal{M} = \left(\begin{array}{c|cccc} & C_1 & C_2 & C_3 & C_4 \\ \hline Z_1 & ((5,7),(4,7),(3,6)) & ((2,3),(5,8),(4,7)) & ((1,4),(2,7),(5,6)) & ((5,7),(4,5),(3,8)) \\ Z_2 & ((3,5),(5,7),(1,4)) & ((2,3),(6,8),(3,6)) & ((7,9),(1,5),(5,6)) & ((4,8),(3,4),(2,7)) \\ Z_3 & ((1,4),(2,7),(5,6)) & ((4,6),(2,9),(1,6)) & ((3,8),(2,8),(5,9)) & ((3,4),(2,6),(3,5)) \\ Z_4 & ((3,5),(2,6),(4,8)) & ((2,4),(3,5),(6,7)) & ((5,6),(3,9),(7,9)) & ((2,6),(3,4),(4,7)) \\ \hat{w} & (0.35,0.26,0.38) & (0.42,0.31,0.25) & (0.26,0.45,0.15) & (0.20,0.36,0.24) \end{array}\right).$$

By Equation (1) in third step of the above Algorithm, we obtain the following 3-polar decision matrix as follows:

$$\mathcal{M}' = \left(\begin{array}{c|cccc} & C_1 & C_2 & C_3 & C_4 \\ \hline Z_1 & (6,5.5,4.5) & (2.5,6.5,5.5) & (2.5,4.5,5.5) & (6,4.5,5.5) \\ Z_2 & (4,6,2.5) & (2.5,7,4.5) & (8,3,5.5) & (6,3.5,4.5) \\ Z_3 & (2.5,4.5,5.5) & (5,5.5,3.5) & (5.5,5,7) & (3.5,4,4) \\ Z_4 & (4,4,6) & (3,4,6.5) & (5.5,6,8) & (4,3.5,5.5) \end{array}\right).$$

By Equation (2) in fourth step of the above Algorithm, we normalized the 3-polar decision matrix to get the normalized picture 3-polar fuzzy soft decision matrix as follows:

$$\mathcal{M}'' = \left(\begin{array}{c|cccc} & C_1 & C_2 & C_3 & C_4 \\ \hline Z_1 & (0.69, 0.54, 0.47) & (0.36, 0.55, 0.53) & (0.21, 0.47, 0.41) & (0.59, 0.57, 0.55) \\ Z_2 & (0.46, 0.59, 0.26) & (0.36, 0.59, 0.43) & (0.69, 0.31, 0.41) & (0.59, 0.44, 0.45) \\ Z_3 & (0.29, 0.44, 0.57) & (0.73, 0.46, 0.34) & (0.48, 0.32, 0.53) & (0.34, 0.51, 0.40) \\ Z_4 & (0.46, 0.39, 0.62) & (0.43, 0.34, 0.63) & (0.48, 0.63, 0.60) & (0.39, 0.44, 0.55) \end{array}\right).$$

By Equation (3) in fifth step of the above Algorithm, we obtain the weighted normalized picture 3-polar fuzzy soft decision matrix as follows:

$$\mathcal{M}''' = \left(\begin{array}{c|cccc} & C_1 & C_2 & C_3 & C_4 \\ \hline Z_1 & (0.24, 0.14, 0.17) & (0.15, 0.17, 0.13) & (0.05, 0.21, 0.06) & (0.11, 0.20, 0.14) \\ Z_2 & (0.16, 0.15, 0.09) & (0.15, 0.18, 0.10) & (0.17, 0.13, 0.06) & (0.11, 0.15, 0.10) \\ Z_3 & (0.10, 0.11, 0.21) & (0.30, 0.14, 0.08) & (0.12, 0.14, 0.07) & (0.06, 0.18, 0.09) \\ Z_4 & (0.16, 0.10, 0.23) & (0.12, 0.10, 0.15) & (0.12, 0.28, 0.09) & (0.07, 0.15, 0.13) \end{array}\right).$$

As C_1 and C_2 (resp., C_3 and C_4) are cost criteria (resp., are benefit criteria). Then the 3-fuzzy positive ideal solution $\mathcal{A}^*$ and 3-fuzzy negative ideal solution $\mathcal{A}^-$ in sixth step of the above Algorithm are given as follows:

$$\mathcal{A}^* = ((0,0,0),(0,0,0),(1,1,1),(1,1,1)),$$

$$\mathcal{A}^- = ((1,1,1),(1,1,1),(0,0,0),(0,0,0)).$$

In seventh step of the above Algorithm, by computing separation measure $(\mathcal{S}_r^*, \mathcal{S}_r^-)$ between each attribute with $\mathcal{A}^*$ and $\mathcal{A}^-$ by using Equations (4) and (5), respectively, we get the following results as follows $(r = 1, 2, 3, 4)$:

$$\mathcal{S}_1^*(Z_1, \mathcal{A}^*) = 1.25,\ \mathcal{S}_2^*(Z_2, \mathcal{A}^*) = 1.26,\ \mathcal{S}_2^*(Z_3, \mathcal{A}^*) = 1.28,\ \mathcal{S}_2^*(Z_4, \mathcal{A}^*) = 1.23,$$

and

$$\mathcal{S}_1^-(Z_1, \mathcal{A}^-) = 1.19,\ \mathcal{S}_2^-(Z_2, \mathcal{A}^-) = 1.23,\ \mathcal{S}_2^-(Z_3, \mathcal{A}^-) = 1.20,\ \mathcal{S}_2^-(Z_4, \mathcal{A}^-) = 1.23.$$

In eighth step of the above Algorithm, by computing closeness coefficient $\overline{Cf_r}$ of each alternative Z_r $(r = 1, 2, 3, 4)$ and by Equation (6), we get the following results as follows:

$$\overline{Cf_r}(Z_1) = 0.48,\ \overline{Cf_r}(Z_2) = 0.49,\ \overline{Cf_r}(Z_3) = 0.48,\ \overline{Cf_r}(Z_4) = 0.5.$$

Finally, as $\overline{Cf_r}(Z_4) = 0.5$ is highest among others, which is the best ideal solution we got using multi-person TOPSIS method, so by the result obtained with the help of decision-making committee of the company is to invest money in a medicine company to get the best results in earning money.

5 Conclusions

We have initiated the model of picture m-polar fuzzy soft sets and its application. We have studied the several structure operations of the picture m-polar fuzzy soft sets and also discussed the basic properties of the picture m-polar fuzzy soft sets. We have constructed an Algorithm to extend the TOPSIS to picture m-polar fuzzy soft sets and illustrated its applicability through a numerical example. In the future, we expect that this group will be of great importance to researchers in more than one field interested in the decision-making process and problem-solving. Further, we will provide a real application with a real data set or we will apply the approach (i.e., Algorithm (selection of best alternative)) to lung cancer disease [16] and coronary artery disease [9]. In addition, we will describe more the detail in order to clarify if the method converges or diverges from standard approaches such as the possibility m-polar fuzzy soft sets [15], possibility neutrosophic soft sets [12], possibility multi-fuzzy soft sets [19], spherical m-polar fuzzy sets [2], and Pythagorean fuzzy sets [17].

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