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Distributed adaptive iterative learning exact consensus for nonlinear strict-feedback multi-agent systems with unknown control directions

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Abstract

This paper investigates a novel distributed fuzzy adaptive iterative learning control protocol of exact consensus for second-order unknown nonlinear strict-feedback multi-agent systems with partially unknown time-varying control directions. Fuzzy logic system estimates the unknown nonlinear dynamics of each agent. Based on Nussbaum function, the control protocols of all follower agents with partially unknown virtual and actual control directions are designed by the backstepping method. The proposed protocol guarantees that the output of every follower agent could exactly track the desired output trajectory on a finite time interval. As an extension, the formation control results can also be solved. Finally, two simulation examples prove that the proposed control scheme is effective and rigorous.

Keywords: Strict-feedback multi-agent systems, unknown control directions, fuzzy logic system, iterative learning control, backstepping design.

1 Introduction

Multi-agent systems (MASs) have been widely applied in various advanced fields in recent decades, such as formation control of aircraft, sensor network [12], multi-manipulator cooperative control and so on. In the exploration of MASs, we can conclude that consensus problem [20] is the most basic and important problem. The consensus results of MASs include asymptotic consensus [31], exponential consensus[25], finite-time/fixed-time [11, 21] or prescribed finite-time consensus [35], exactly consensus [10] over finite time interval, among which the learning consensus of MASs still needs further research.

It is well known that iterative learning control(ILC) is the effective method to settle the accurate tracking control over a finite time interval. Therefore, it is very important to along ILC method to consider the consensus tracking of MASs with repetitive environment and dynamics [30]. The fuzzy ILC algorithm was firstly proposed by Ahn and Chen [1] to achieve formation control results for the nonlinear MASs in the process of repeated operations. From then on, many works of MASs were studied via ILC. The consensus problem of linear MASs from the perspective of two-dimensional systems was studied in [14], which designed a fully distributed adaptive iterative learning control (AILC) protocol based on observer. Due to the complexity and practicality of nonlinear MASs, many scholars began to learn nonlinear MASs, where the unknown nonlinear dynamics was approximated by fuzzy logic system [17, 18, 34] or neural network [8, 22, 27]. The fuzzy AILC consensus scheme was proposed in [18] for first-order and second-order unknown nonlinear MAS. The leader-following consensus of heterogeneous MASs was addressed by ILC in [8]. The authors in [27] proposed a distributed disturbance-compensation to achieve the consensus tracking of nonlinear MASs based on model free AILC algorithm. Strict-feedback structure dynamics plays an important role in system modeling, which can describe complex high-order nonlinear systems well. Backstepping technique is a basic framework of control protocol in strict-feedback MASs. In [29], an optimized leader-following consensus control scheme was raised for the nonlinear strict-feedback MASs by referring to the control idea of optimized backstepping technique. Based on command filters,

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[7] avoided the explosion of complexity phenomenon in traditional backstepping technique and investigated an adaptive finite-time consensus control method for nonlinear strict-feedback MASs that contain unknown parameters. Combining graph theory and neural network technology, [33] developed an adaptive neural containment protocol for stochastic nonlinear strict-feedback MASs within the traditional backstepping framework. In addition, the works [2, 5, 9] all used ILC method to study the consensus of single system with strict-feedback structure, while we found that few works on strict-feedback MASs. To sum up, it can be noted from the above works that there are few researches simultaneously using backstepping and ILC method for strict-feedback MASs, which is an interesting subject and a motivation of this paper.

The subjects of unknown control directions are of great practical significance. Scholars in [13, 30] assumed that their control directions were already known, but actually, it is often impossible to get certain control directions [24]. The control directions stand for the symbols of control input signals, which will determine the movement directions of all agents [19]. An effective method to work out such problem is Nussbaum-gain method, which was first proposed by [23]. Then [4] introduced a new lemma to deal with the adaptive consensus problem of linearly parameterized MASs with unknown identical control directions. References [3, 15, 16] all adopted the Nussbaum lemma in [4]. The work [15] designed AILC protocol for first-order parameterized MASs with unknown control gains. The works [3, 16] responded to the consensus control algorithms for second-order nonlinear MASs with unknown control gains, and [3] also considered the state constraints of all agents. However, the lemma of [4] had to satisfy a condition with global information, namely that the control protocols given by [3, 4, 15, 16] were not completely distributed. Besides, [4] required all control directions to be identical, which had certain limitations. To improve the above defects, [6] proposed a new Nussbaum-type function to solve the situation of unknown and different directions, which can make the control scheme completely distributed. Then based on [6, 32] proposed a novel ILC algorithm based neural network for leaderless consensus of the second-order time-varying uncertain nonlinear MASs. As of now, the problem of uncertain control directions for ILC consensus of strict-feedback MASs is almost no research until now. Thus it is a great subject to further explore, and this is another motivation of our paper.

Based on above enlightenments, the purpose of this paper is to use AILC method to study the consensus of second-order unknown nonlinear strict-feedback MASs, where the control directions are partially unknown and their signs can be different. We use fuzzy logic system to approximate the uncertain nonlinear dynamics of each agent. In addition, based on the Nussbaum lemma in [6], the new fuzzy AILC algorithm is designed by adopting backstepping technique. In the case of external disturbance, our algorithm proves that with the increase of iterations, all agents can accurately track our desired output trajectory on $[0, T]$. All in all, the creative points of this paper are described as follows.

(1). Compared with the studies of ILC consensus for MASs with unknown control directions in [15, 16, 32], it is the first time for us to discuss the consensus problem of strict-feedback MASs through fuzzy AILC method, where the directions of virtual control and actual control are partially unknown with different signs.

(2). In [3, 4, 15, 16], their control directions were all unknown, but the signs of directions were the same. And [2, 5, 7, 9, 29, 33] did not take into account that the directions of virtual control are also partially unknown. Differently, we consider all the directions of virtual control and actual control that are partially unknown and their signs can be different, and we obtain exact consensus results finally.

(3). Different from the incompletely distributed protocols [4, 3, 15, 16], our control protocol does not need to rely on any global information, but takes advantage of the distributed nature of systems. Besides, unknown bounded external disturbances are also considered. Finally, it is proved that the output of every agent could reach exact consensus on finite time interval $[0, T]$.

The structure of this paper is organized as follows. Section 2 covers some of necessary preliminaries. Section 3 provides main results. The validity of the results in Section 4 is verified by numerical examples. Finally, conclusions are drawn in Section 5.

2 Preliminaries and problem formulation

2.1 Graph theory and notations

Let $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ denote an undirected graph, the set of nodes $\mathcal{V} = \{v_1, \cdots, v_n\}$ and the set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. The weighted adjacency matrix of the graph $\mathcal{G}$ is noted by $\mathcal{A} = [a_{ij}] \in \Re^{n\times n}$, where $a_{ii} = 0$ for node i and $a_{ij} = 1$ if $(v_i, v_j) \in \mathcal{E}$, or else $a_{ij} = 0$. Define $d_i = \sum_{j=1}^{n} a_{ij}$ as the in-degree of node i and the in-degree matrix $\mathcal{D} = diag\{d_1, \cdots, d_n\} \in \Re^{n\times n}$. $\mathcal{L} = \mathcal{D} - \mathcal{A}$ denotes the Laplacian matrix of graph $\mathcal{G}$. If there is a path between any two vertices, the graph $\mathcal{G}$ is connected. In this work, we will study the augmented graph $\bar{\mathcal{G}}$ consisting of one leader and n agents. $\mathcal{B} = diag\{b_1, \cdots, b_n\}$ represents the connection weighted matrix between all agents and the leader, where

$b_i = 1$ if ith agent could obtain information of the leader, $b_i = 0$ or else. The graph $\bar{\mathcal{G}}$ is connected if at least one agent in $\mathcal{G}$ could get the information of the leader. Further, the symmetric matrix $H = \mathcal{L} + \mathcal{B}$ associated with $\bar{\mathcal{G}}$ is positive definite if $\bar{\mathcal{G}}$ is connected.

In this paper, $|\cdot|$, $\|\cdot\|$ respectively mean the absolute of a real number and the Euclidean norm of a vector; matrix $Q > 0$ implies that Q is positive definite; θ^{T} equals the transpose of the vector θ.

2.2 Problem description

Consider the second-order strict-feedback MASs composed of n agents and one leader, and the dynamics of the ith agent is

$$\begin{aligned} \dot{x}_{i,1}^k &= f_{i,1}\left(x_{i,1}^k\right) + q_i\left(t\right) x_{i,2}^k + \sigma_{i,1}^k\left(t\right), \\ \dot{x}_{i,2}^k &= f_{i,2}\left(x_{i,1}^k, x_{i,2}^k\right) + p_i\left(t\right) u_i^k + \sigma_{i,2}^k\left(t\right), \\ y_i^k &= x_{i,1}^k, \end{aligned} \tag{1}$$

where $x_{i,1}^k, x_{i,2}^k \in \Re, i = 1, \cdots, n$ denote the states of ith agent at the kth iteration, $u_i^k, y_i^k \in \Re$ are the input and output. The continuous nonlinear functions $f_{i,1}\left(x_{i,1}^k\right), f_{i,2}\left(x_{i,1}^k, x_{i,2}^k\right)$ are unknown. It is worth noting that $q_i\left(t\right), p_i\left(t\right) \in \Re$ are control gains, which are time-varying continuous satisfying $|q_i\left(t\right)| \in [q_{\min}, q_{\max}], |p_i\left(t\right)| \in [p_{\min}, p_{\max}]$ with the unknown positive constants $p_{\min}, p_{\max}$ and $q_{\min}, q_{\max}$. $\sigma_{i,1}^k\left(t\right), \sigma_{i,2}^k\left(t\right) \in \Re$ are unknown continuous and bounded, meanwhile, $\left|\sigma_{i,1}^k\left(t\right)\right| \leq \sigma_{i,1}^*, \left|\sigma_{i,2}^k\left(t\right)\right| \leq \sigma_{i,2}^*$ with unknown positive constants $\sigma_{i,1}^*$ and $\sigma_{i,2}^*$. Here we assume that control gains of $N\left(1 \leq N \leq n\right)$ agents are unknown and have identical signs, while the remaining $n - N$ agents have known control gains with different signs. For convenience, $f_{i,1}\left(x_{i,1}^k\right), f_{i,2}\left(x_{i,1}^k, x_{i,2}^k\right)$ are short as $f_{i,1}^k, f_{i,2}^k$.

The trajectory of the leader $y_r\left(t\right)$ is totally unknown by some of the agents. Give the consensus error for the ith agent at the kth iteration as

$$\delta_i^k\left(t\right) = y_i^k\left(t\right) - y_r\left(t\right). \tag{2}$$

Our goal is to find an appropriate control protocol sequence $\left\{u_i^k\left(t\right), i = 1, \cdots, n, k \in Z^+\right\}$ to ensure that when the number of iterations increases to infinity, the outputs of all agents would track the leader's output on $[0, T]$, that is $\lim\limits_{k\to\infty} \delta_i^k\left(t\right) = 0$ for all $t \in [0, T]$.

Remark 2.1. *On the one hand, ILC method was used to solve the consensus of the single strict-feedback systemin* [15, 16, 32]*, without considering the unknown control directions. On the other hand,* [2, 5, 6, 9] *studied the consensus of MASs with unknown control directions through ILC method, rather than the more complex strict-feedback MASs. Overall, it is the first time for us to consider the consensus of unknown nonlinear strict-feedback MASs through fuzzy AILC method, with the virtual and actual control directions are different and partially unknown.*

We now propose the following assumptions and technical lemmas to facilitate the discussion of main results.

Assumption 1. The leader's trajectory satisfies $y_r\left(0\right) = y_r\left(T\right), \dot{y}_r\left(0\right) = \dot{y}_r\left(T\right)$ and the alignment initial conditions $x_{i,1}^k\left(0\right) = x_{i,1}^{k-1}\left(T\right), x_{i,2}^k\left(0\right) = x_{i,2}^{k-1}\left(T\right)$.

Remark 2.2. *Here $y_r\left(0\right) = y_r\left(T\right), \dot{y}_r\left(0\right) = \dot{y}_r\left(T\right)$ means that $y_r\left(t\right)$ is spatially closed, that is, in each iteration, the leader's trajectory and its derivative will end at the beginning of the next iteration* [15, 16, 32]*. In addition, we choose alignment conditions as the initial conditions of ILC, implying that the system will currently run from where it stopped last iteration.*

Assumption 2. The communication topology graph $\bar{\mathcal{G}}$ of leader-following MASs(1) is connected.

Lemma 2.3. [32] *Define $\bar{V}\left(t\right)$, $\mathcal{N}\left(\varsigma_i\right), \varsigma_i\left(t\right), i = 1, \cdots, n$ as the smooth functions on $[0, t_\tau)$ with $\bar{V}\left(t\right) > 0$ and bounded initial value $\varsigma_i\left(0\right)$. If there has*

$$\bar{V}\left(t\right) \leq \sum_{i=1}^{n} \int_0^t \left(p_i\left(\tau\right) \mathcal{N}\left(\varsigma_i\left(\tau\right)\right) + 1\right) \dot{\varsigma}_i\left(\tau\right) d\tau + \phi_0, \tag{3}$$

where ϕ_0 denotes a constant, $\mathcal{N}\left(\varsigma_i\right)$ stands for the Nussbaum-type piecewise function, which is defined as

$$\mathcal{N}\left(\varsigma_i\right) = \begin{cases} \mathcal{N}^{\tilde{1}}, & \text{if the control gain is unknown,} \\ \mathcal{N}^{\tilde{2}}, & \text{if the control gain is known,} \end{cases} \tag{4}$$

$\mathcal{N}^{\tilde{1}}\left(\varsigma_i\right) = -\alpha \exp\left(\frac{\varsigma_i^2}{2}\right)\left(\varsigma_i^2 + 2\right) \sin\left(\varsigma_i\right), \mathcal{N}^{\tilde{2}}\left(\varsigma_i\right) = -\exp\left(\frac{\varsigma_i^2}{2}\right) \varsigma_i$ *and $\alpha > 0$, then $\varsigma_i, \bar{V}\left(t\right)$ and $\sum\limits_{i=1}^{n} \int_0^t \left(p_i\left(\tau\right) \mathcal{N}\left(\varsigma_i\left(\tau\right)\right) + 1\right) \dot{\varsigma}_i\left(\tau\right) d\tau$ are bounded on $[0, t_\tau)$.*

Remark 2.4. *In* [3, 4, 15, 16]*, the authors only considered one special case that the control gains of all agents were unknown and had identical signs, i.e $N = n$ in MASs(1). Furthermore,* [2, 5, 7, 9, 29, 33] *did not even consider that the gains of virtual control are also unknown with the existence of virtual control input disturbance. Notice that all control gains in this paper are time-varying with directions partially unknown and unequal. Lemma 2.3 is used to deal with this general case.*

Remark 2.5. *. The Nussbaum lemma in* [3] *was difficult to guarantee the control protocol completely distributed since the Nussbaum lemma was connected with the number of follower agents. In this paper, the piecewise Nussbaum function in Lemma 2.3 can solve this deficiency.*

Lemma 2.6. [32]*. For any $\omega > 0$ and $\beta \in \Re$, there is always the following inequality*

$$0 \leq |\beta| - \beta \tanh\left(\frac{\beta}{\omega}\right) \leq \bar{\delta}\omega, \bar{\delta} = 0.2785, \tag{5}$$

where $\tanh(\kappa) = \frac{e^{\kappa}-e^{-\kappa}}{e^{\kappa}+e^{-\kappa}}$.

Lemma 2.7. [32]*. Define a convergent series sequence Δ_k as $\Delta_k = a/k^l, a > 0, l \geq 2$, where $k, l \in Z_+, a \in \Re$ are designed constants. Then we obtain*

$$\lim_{k\to\infty} \sum_{i=1}^{k} \Delta_i \leq 2a. \tag{6}$$

In kth iteration, the neighborhood consensus error of ith agent is

$$e_i^k(t) = \sum_{j=1}^{n} a_{ij}\left(y_i^k(t) - y_j^k(t)\right) + b_i\left(y_i^k(t) - y_r(t)\right). \tag{7}$$

Then (7) updates

$$e^k(t) = (L+B)\left(y^k(t) - 1_n y_r(t)\right) = H\delta^k(t), \tag{8}$$

where $\delta^k(t) = \left[\delta_1^k(t), \cdots, \delta_n^k(t)\right]^{\mathrm{T}}$, $\delta_i^k(t) = y_i^k(t) - y_r(t), e^k(t) = \left[e_1^k(t), \cdots, e_n^k(t)\right]^{\mathrm{T}}, 1_n = [1, \cdots, 1]^{\mathrm{T}}$.

Lemma 2.8. [15] *If the topology graph $\bar{\mathcal{G}}$ is connected, then it follows (8) that*

$$\left\|\delta^k\right\| \leq \left\|e^k\right\| \Big/ \bar{\sigma}(H), \tag{9}$$

and $\bar{\sigma}(H) > 0$ denotes the minimum singular value of H. Furthermore, $\lim_{k\to\infty}\left\|e^k\right\| = 0$ could guarantee $\lim_{k\to\infty}\left\|\delta^k\right\| = 0$. Based on Assumption 1, we can infer that $\delta_i^k(0) = \delta_i^{k-1}(T)$ and further $e_i^k(0) = e_i^{k-1}(T)$ for each agent.

Lemma 2.9. [28] *If the continuous function $f(x)$ is defined on a compact set Ω. Then there exists a fuzzy logic system $\varpi^{\mathrm{T}}\varphi(x)$ such that*

$$\sup_{x\in\Omega}\left|f(x) - \varpi^{\mathrm{T}}\varphi(x)\right| \leq \varepsilon, \tag{10}$$

with a constant $\varepsilon > 0$, where $\varpi = [\omega_1, \cdots, \omega_m]^{\mathrm{T}}$ is ideal constant weight vector, $\varphi(x) = [\varphi_1(x), \cdots, \varphi_m(x)]^{\mathrm{T}} / \sum_{j=1}^{m} \varphi_j(x)$ means basis function vector with $\varphi_j(x)$ being chosen as Gaussian function and the fuzzy rules number m.

3 Main results

3.1 Backstepping design of controller

For achieving our control objective of MASs (1), the following coordinate transformations will be designed as

$$z_{i,1}^k = e_i^k, \tag{11}$$

$$z_{i,2}^k = x_{i,2}^k - \alpha_{i,1}, \tag{12}$$

where $\alpha_{i,1}$ is virtual control signal.

Step 1: From (8) and (11), one has $Z_1^k = H\delta^k$, where $Z_1^k = \left[z_{1,1}^k, \cdots, z_{n,1}^k\right]^{\mathrm{T}}$. Let composite energy function E_1^k in kth iteration define as

$$E_1^k = \frac{1}{2}\left(Z_1^k\right)^{\mathrm{T}} H^{-1} Z_1^k + \frac{c_{1,i}}{2r_{i1}} \sum_{i=1}^{n} \left(\tilde{\bar{\varepsilon}}_{i,1}^k\right)^2 + \sum_{i=1}^{n} \int_t^T \frac{1}{2\gamma_{i1}} \left(\tilde{\varpi}_{i,1}^{k-1}\right)^{\mathrm{T}} \tilde{\varpi}_{i,1}^{k-1} d\tau + \sum_{i=1}^{n} \int_0^t \frac{1}{2\gamma_{i1}} \left(\tilde{\varpi}_{i,1}^k\right)^{\mathrm{T}} \tilde{\varpi}_{i,1}^k d\tau, \tag{13}$$

where the symmetric matrix H^{-1} satisfies $H^{-1} > 0$. $c_{1,i}, r_{i1}, \gamma_{i1}$ are the positive constants to be designed; $\tilde{\bar{\varepsilon}}_{i,1}^k = \bar{\varepsilon}_{i,1} - \hat{\bar{\varepsilon}}_{i,1}^k$ and $\tilde{\varpi}_{i,1}^k = \varpi_{i,1} - \hat{\varpi}_{i,1}^k$ denote unknown variables which will be specified in the following control design process, and $\hat{\bar{\varepsilon}}_{i,1}^k, \hat{\varpi}_{i,1}^k$ represent the estimation of $\bar{\varepsilon}_{i,1}, \varpi_{i,1}$, respectively, and $\tilde{\varpi}_{i,1}^k = \left[\tilde{\omega}_{i,11}^k, \cdots, \tilde{\omega}_{i,1m}^k\right]^{\mathrm{T}}$.

Recall (1), the derivative of E_1^k is

$$\begin{aligned}
\dot{E}_1^k &= \sum_{i=1}^{n} z_{i,1}^k \left(\dot{y}_i^k - \dot{y}_r\right) + \sum_{i=1}^{n} \frac{1}{2\gamma_{i1}} \left(\tilde{\varpi}_{i,1}^k\right)^{\mathrm{T}} \tilde{\varpi}_{i,1}^k(t) + \frac{c_{1,i}}{r_{i,1}} \sum_{i=1}^{n} \tilde{\bar{\varepsilon}}_{i,1}^k \dot{\tilde{\bar{\varepsilon}}}_{i,1}^k - \sum_{i=1}^{n} \frac{1}{2\gamma_{i1}} \left(\tilde{\varpi}_{i,1}^{k-1}\right)^{\mathrm{T}} \tilde{\varpi}_{i,1}^{k-1}(t) \\
&= \sum_{i=1}^{n} z_{i,1}^k \left(f_{i,1}^k + q_i(t)\, x_{i,2}^k + \sigma_{i,1}^k - \dot{y}_r\right) + \frac{c_{1,i}}{r_{i,1}} \sum_{i=1}^{n} \tilde{\bar{\varepsilon}}_{i,1}^k \dot{\tilde{\bar{\varepsilon}}}_{i,1}^k - \sum_{i=1}^{n} \frac{1}{2\gamma_{i1}} \left(\tilde{\varpi}_{i,1}^{k-1}\right)^{\mathrm{T}} \tilde{\varpi}_{i,1}^{k-1}(t) + \sum_{i=1}^{n} \frac{1}{2\gamma_{i1}} \left(\tilde{\varpi}_{i,1}^k\right)^{\mathrm{T}} \tilde{\varpi}_{i,1}^k(t).
\end{aligned} \tag{14}$$

Let $\bar{f}_{i,1}^k = f_{i,1}^k - \dot{y}_r$. Since $\bar{f}_{i,1}^k$ is unknown, it can not be directly used in controller design. Based on Lemma 2.9, a fuzzy logic system $\varpi_{i,1}^{\mathrm{T}} \varphi_{i,1}(\chi_{i,1})$ such that

$$\bar{f}_{i,1}^k = \varpi_{i,1}^{\mathrm{T}} \varphi_{i,1}(\chi_{i,1}) + \varepsilon_{i,1}(\chi_{i,1}), |\varepsilon_{i,1}(\chi_{i,1})| \le \varepsilon_{i,1}^*, \tag{15}$$

where $\varepsilon_{i,1}^* > 0$ is given as constant, $\varepsilon_{i,1}(\chi_{i,1})$ denotes the error in the approximation and $\chi_{i,1} = \left[x_{i,1}^k, \dot{y}_r\right]^{\mathrm{T}}$.

The virtual control signal $\alpha_{i,1}$ and the auxiliary variable $\zeta_i^k(t)$ are designed as follows:

$$\alpha_{i,1} = \mathcal{N}\left(\zeta_i^k\right) \bar{\alpha}_{i,1}, \tag{16}$$

$$\bar{\alpha}_{i,1} = c_{1,i} z_{i,1}^k + \hat{\bar{f}}_{i,1}^k + \hat{\bar{\varepsilon}}_{i,1}^k \tanh\left(\frac{\hat{\bar{\varepsilon}}_{i,1}^k z_{i,1}^k}{\Delta_k}\right), \tag{17}$$

$$\dot{\zeta}_i^k(t) = z_{i,1}^k \bar{\alpha}_{i,1}, \zeta_i^k(0) = \zeta_i^{k-1}(T), \zeta_i^0(0) = 0. \tag{18}$$

From (16-18), (14) becomes

$$\begin{aligned}
\dot{E}_1^k = &\sum_{i=1}^{n} \left(q_i(t)\mathcal{N}\left(\zeta_i^k\right) + 1\right)\dot{\zeta}_i^k - \sum_{i=1}^{n} c_{1,i}\left(z_{i,1}^k\right)^2 + \sum_{i=1}^{n} q_i(t)\, z_{i,1}^k z_{i,2}^k + \sum_{i=1}^{n} z_{i,1}^k \sigma_{i,1}^k(t) \\
&+ \sum_{i=1}^{n} z_{i,1}^k \left(\left(\tilde{\varpi}_{i,1}^k\right)^{\mathrm{T}} \varphi_{i,1}^k + \varepsilon_{i,1}^k\right) - \sum_{i=1}^{n} z_{i,1}^k \left(\hat{\bar{\varepsilon}}_{i,1}^k \tanh\left(\frac{\hat{\bar{\varepsilon}}_{i,1}^k z_{i,1}^k}{\Delta_k}\right)\right) + \frac{c_{1,i}}{r_{i,1}} \sum_{i=1}^{n} \tilde{\bar{\varepsilon}}_{i,1}^k \dot{\tilde{\bar{\varepsilon}}}_{i,1}^k \\
&- \sum_{i=1}^{n} \frac{1}{2\gamma_{i1}} \left(\tilde{\varpi}_{i,1}^{k-1}\right)^{\mathrm{T}} \tilde{\varpi}_{i,1}^{k-1}(t) + \sum_{i=1}^{n} \frac{1}{2\gamma_{i1}} \left(\tilde{\varpi}_{i,1}^k\right)^{\mathrm{T}} \tilde{\varpi}_{i,1}^k(t).
\end{aligned} \tag{19}$$

In (19), one has $\sum_{i=1}^{n} z_{i,1}^k \varepsilon_{i,1}^k + \sum_{i=1}^{n} z_{i,1}^k \sigma_{i,1}^k(t) \le \sum_{i=1}^{n} \left|z_{i,1}^k\right| \left(\varepsilon_{i,1}^* + \sigma_{i,1}^*\right)$. Let $\bar{\varepsilon}_{i,1} = \varepsilon_{i,1}^* + \sigma_{i,1}^*$, (19) becomes

$$\begin{aligned}
\dot{E}_1^k \le &\sum_{i=1}^{n} \left(q_i(t)\mathcal{N}\left(\zeta_i^k\right) + 1\right)\dot{\zeta}_i^k - \sum_{i=1}^{n} c_{1,i}\left(z_{i,1}^k\right)^2 + \sum_{i=1}^{n} q_i(t)\, z_{i,1}^k z_{i,2}^k + \sum_{i=1}^{n} z_{i,1}^k \left(\tilde{\varpi}_{i,1}^k\right)^{\mathrm{T}} \varphi_{i,1}^k \\
&+ \sum_{i=1}^{n} \left|z_{i,1}^k\right| \bar{\varepsilon}_{i,1} - \sum_{i=1}^{n} z_{i,1}^k \left(\hat{\bar{\varepsilon}}_{i,1}^k \tanh\left(\frac{\hat{\bar{\varepsilon}}_{i,1}^k z_{i,1}^k}{\Delta_k}\right)\right) + \frac{c_{1,i}}{r_{i,1}} \sum_{i=1}^{n} \tilde{\bar{\varepsilon}}_{i,1}^k \dot{\tilde{\bar{\varepsilon}}}_{i,1}^k \\
&- \sum_{i=1}^{n} \frac{1}{2\gamma_{i1}} \left(\tilde{\varpi}_{i,1}^{k-1}\right)^{\mathrm{T}} \tilde{\varpi}_{i,1}^{k-1}(t) + \sum_{i=1}^{n} \frac{1}{2\gamma_{i1}} \left(\tilde{\varpi}_{i,1}^k\right)^{\mathrm{T}} \tilde{\varpi}_{i,1}^k(t).
\end{aligned} \tag{20}$$

The parameter adaptive laws of $\hat{\varpi}_{i,1}^k, \hat{\bar{\varepsilon}}_{i,1}^k$ are designed as

$$\begin{aligned}
&\hat{\varpi}_{i,1}^k(t) = sat\left(\hat{\varpi}_{i,1*}^k(t), \bar{\varpi}_{i,1}\right), \\
&\hat{\varpi}_{i,1*}^k(t) = \begin{cases} sat\left(\hat{\varpi}_{i,1*}^{k-1}(t), \bar{\varpi}_{i,1}\right) + \gamma_{i1}\varphi_{i,1}^k z_{i,1}^k, & k \ge 1, \\ \gamma_{i0}(t)\,\varphi_{i,1}^0 z_{i,1}^0, & k = 0, \\ 0, & k = -1, \end{cases}
\end{aligned} \tag{21}$$

$$\dot{\hat{\bar{\varepsilon}}}_{i,1}^k(t) = \frac{r_{i1}}{c_{1,i}} \left|z_{i,1}^k\right|, \hat{\bar{\varepsilon}}_{i,1}^k(0) = \hat{\bar{\varepsilon}}_{i,1}^{k-1}(T), \hat{\bar{\varepsilon}}_{i,1}^0(0) \ge 0, \tag{22}$$

where $sat(\cdot)$ is called the saturation function[32] and the strictly increasing continuous function $\gamma_{i0}(t)$ satisfies $\gamma_{i0}(0)=0, \gamma_{i0}(T)=\gamma_{i1}$. $\bar{\varpi}_{i,1}(t)=[\bar{\varpi}_{i,11},\cdots,\bar{\varpi}_{i,1m}]^{\mathrm{T}}$ with saturation upper bound $\bar{\varpi}_{i,1j}>0, j=1,\cdots,m$ being arbitrarily chosen.

Remark 3.1. *It's worth noting that* $\hat{\bar{\varepsilon}}_{i,1}^{k}\tanh\left(\frac{\hat{\bar{\varepsilon}}_{i}^{k}z_{i,1}^{k}}{\Delta_k}\right)$ *in (17) is used to avoid sign function item sign* $\left(z_{i,1}^{k}\right)\hat{\bar{\varepsilon}}_{i,1}^{k}$*, which theoretically overcomes chattering. Because of* $r_{i1},c_{1,i}>0,\hat{\bar{\varepsilon}}_{i,1}^{0}(0)\geq 0$, $\hat{\bar{\varepsilon}}_{i,1}^{k}(t)\geq 0$ *can be ensured, which implies that the chatter phenomenon of adaptive parameter* $\hat{\bar{\varepsilon}}_{i,1}^{k}(t)$ *can also be effectively avoided.*

Lemma 3.2. *Virtual control protocols (16-18) and updated laws (21-22) guarantee that*

$$\dot{\zeta}_{i}^{k}(0)=\dot{\zeta}_{i}^{k-1}(T),\hat{\varpi}_{i,1}^{k}(0)=\hat{\varpi}_{i,1}^{k-1}(T). \tag{23}$$

Proof. First, we will prove $\hat{\varpi}_{i,1}^{k}(0)=\hat{\varpi}_{i,1}^{k-1}(T)$ based on mathematical induction. When $k=0$, $\hat{\varpi}_{i,1}^{0}(0)=\hat{\varpi}_{i,1}^{-1}(T)=0$ from (21). Next, suppose $\hat{\varpi}_{i,1}^{l}(0)=\hat{\varpi}_{i,1}^{l-1}(T)$ for all $l=1,\cdots,k-1$. When $l=k$, from (21) there are

$$\begin{aligned}\hat{\varpi}_{i,1}^{k}(0)&=sat\left(\hat{\varpi}_{i,1*}^{k}(0)\right)\\&=sat\left(sat\left(\hat{\varpi}_{i,1*}^{k-1}(0)\right)+\gamma_{i1}\varphi_{i,1}\left(x_{i,1}^{k}(0),\dot{y}_r(0)\right)z_{i,1}^{k}(0)\right)\\&=sat\left(\hat{\varpi}_{i,1}^{k-1}(0)+\gamma_{i1}\varphi_{i,1}\left(x_{i,1}^{k}(0),\dot{y}_r(0)\right)z_{i,1}^{k}(0)\right),\end{aligned} \tag{24}$$

and

$$\begin{aligned}\hat{\varpi}_{i,1}^{k-1}(T)&=sat\left(\hat{\varpi}_{i,1*}^{k-1}(T)\right)\\&=sat\left(sat\left(\hat{\varpi}_{i,1*}^{k-2}(T)\right)+\gamma_{i1}\varphi_{i,1}\left(x_{i,1}^{k-1}(T),\dot{y}_r(T)\right)z_{i,1}^{k-1}(T)\right)\\&=sat\left(\hat{\varpi}_{i,1}^{k-2}(T)+\gamma_{i1}\varphi_{i,1}\left(x_{i,1}^{k-1}(T),\dot{y}_r(T)\right)z_{i,1}^{k-1}(T)\right)\\&=\hat{\varpi}_{i,1}^{k}(0).\end{aligned} \tag{25}$$

Thus, we get that $\hat{\varpi}_{i,1}^{k}(0)=\hat{\varpi}_{i,1}^{k-1}(T)$. From Assumption 1, (18) and $\hat{\varpi}_{i,1}^{k}(0)=\hat{\varpi}_{i,1}^{k-1}(T)$, it is true for $\dot{\zeta}_{i}^{k}(0)=\dot{\zeta}_{i}^{k-1}(T)$. Notice that

$$-\sum_{i=1}^{n}\frac{1}{2\gamma_{i1}}\left(\tilde{\varpi}_{i,1}^{k-1}\right)^{\mathrm{T}}\tilde{\varpi}_{i,1}^{k-1}(t)+\sum_{i=1}^{n}\frac{1}{2\gamma_{i1}}\left(\tilde{\varpi}_{i,1}^{k}\right)^{\mathrm{T}}\tilde{\varpi}_{i,1}^{k}(t)\leq\sum_{i=1}^{n}\frac{1}{\gamma_{i1}}\left(\tilde{\varpi}_{i,1}^{k}(t)\right)^{\mathrm{T}}\left(\hat{\varpi}_{i,1}^{k-1}-\hat{\varpi}_{i,1}^{k}\right). \tag{26}$$

Substitute (21-22) into (20),

$$\begin{aligned}\dot{E}_1^k\leq&\sum_{i=1}^{n}\left(q_i(t)\mathcal{N}\left(\zeta_i^k\right)+1\right)\dot{\zeta}_i^k-\sum_{i=1}^{n}c_{1,i}\left(z_{i,1}^{k}\right)^2+\sum_{i=1}^{n}q_i(t)z_{i,1}^{k}z_{i,2}^{k}+\sum_{i=1}^{n}\left|z_{i,1}^{k}\right|\hat{\bar{\varepsilon}}_{i,1}^{k}\\&-\sum_{i=1}^{n}z_{i,1}^{k}\left(\hat{\bar{\varepsilon}}_{i,1}^{k}\tanh\left(\frac{\hat{\bar{\varepsilon}}_{i,1}^{k}z_{i,1}^{k}}{\Delta_k}\right)\right)+\sum_{i=1}^{n}\frac{1}{\gamma_{i1}}\left(\tilde{\varpi}_{i,1}^{k}(t)\right)^{\mathrm{T}}\left(\hat{\varpi}_{i,1*}^{k}-\hat{\varpi}_{i,1}^{k}\right).\end{aligned} \tag{27}$$

From Lemma 2.6, it has

$$\dot{E}_1^k\leq\sum_{i=1}^{n}\left(q_i(t)\mathcal{N}\left(\zeta_i^k\right)+1\right)\dot{\zeta}_i^k-\sum_{i=1}^{n}c_{1,i}\left(z_{i,1}^{k}\right)^2+\sum_{i=1}^{n}q_i(t)z_{i,1}^{k}z_{i,2}^{k}+n\bar{\delta}\Delta_k, \tag{28}$$

which is followed by $\sum_{i=1}^{n}\frac{1}{\gamma_{i1}}\left(\tilde{\varpi}_{i,1}^{k}\right)^{\mathrm{T}}\left(\hat{\varpi}_{i,1*}^{k}-\tilde{\varpi}_{i,1}^{k}\right)\leq 0$.

Step 2: Recall that $z_{i,2}^{k}=x_{i,2}^{k}-\alpha_{i,1}$, we can calculate that

$$\dot{z}_{i,2}^{k}=f_{i,2}^{k}+p_i(t)u_i^k+\sigma_{i,2}^{k}-\dot{\alpha}_{i,1}, \tag{29}$$

where $\dot{\alpha}_{i,1}=\frac{\partial\alpha_{i,1}}{\partial x_{i,1}^{k}}\dot{x}_{i,1}^{k}+\frac{\partial\alpha_{i,1}}{\partial y_r}\dot{y}_r$. □

Remark 3.3. *Under Assumption 1, the leader's output trajectory satisfies the spatial closure condition, that is* $y_r(0)=y_r(T)$*. Besides, because of* $\dot{y}_r(0)=\dot{y}_r(T)$ *and the alignment conditions of* $x_{i,1}^{k}(t),x_{i,2}^{k}(t)$*, we can obviously get* $z_{i,1}^{k}(0)=z_{i,1}^{k-1}(T),z_{i,2}^{k}(0)=z_{i,2}^{k-1}(T)$.

The composite energy function in kth iteration is expressed by

$$E^k(t) = E_1^k(t) + \frac{1}{2}\left(Z_2^k\right)^{\mathrm{T}} Z_2^k + \sum_{i=1}^{n}\int_t^T \frac{1}{2\gamma_{i2}}\left(\tilde{\varpi}_{i,2}^{k-1}\right)^{\mathrm{T}}\tilde{\varpi}_{i,2}^{k-1}d\tau + \sum_{i=1}^{n}\int_0^t \frac{1}{2\gamma_{i2}}\left(\tilde{\varpi}_{i,2}^{k}\right)^{\mathrm{T}}\tilde{\varpi}_{i,2}^{k}d\tau + \frac{c_{2,i}}{2r_{i2}}\sum_{i=1}^{n}\left(\tilde{\bar{\varepsilon}}_{i,2}^{k}\right)^2, \quad (30)$$

where $t \in [0,T], c_{2,i}, r_{i2}, \gamma_{i2} > 0$ are constants being designed, $\tilde{\bar{\varepsilon}}_{i,2}^k = \bar{\varepsilon}_{i,2} - \hat{\bar{\varepsilon}}_{i,2}^k$ and $\tilde{\varpi}_{i,2}^k = \varpi_{i,2} - \hat{\varpi}_{i,2}^k$ denote unknown variables which will be specified in the following control design process, and $\hat{\bar{\varepsilon}}_{i,2}^k, \hat{\varpi}_{i,2}^k$ represent the estimation of $\bar{\varepsilon}_{i,2}, \varpi_{i,2}$, respectively, and $\tilde{\varpi}_{i,2}^k = \left[\tilde{\omega}_{i,21}^k, \cdots, \tilde{\omega}_{i,2m}^k\right]^{\mathrm{T}}$.

From (28-29), we could obtain

$$\begin{aligned}\dot{E}^k \le & \sum_{i=1}^{n}\left(q_i(t)\mathcal{N}\left(\zeta_i^k\right)+1\right)\dot{\zeta}_i^k - \sum_{i=1}^{n} c_{1,i}\left(z_{i,1}^k\right)^2 + \frac{c_{2,i}}{r_{i2}}\sum_{i=1}^{n}\tilde{\bar{\varepsilon}}_{i,2}^k\dot{\tilde{\bar{\varepsilon}}}_{i,2}^k + \sum_{i=1}^{n} z_{i,2}^k\left(f_{i,2}^k + q_i(t)z_{i,1}^k + p_i(t)u_i^k + \sigma_{i,2}^k - \dot{\alpha}_{i,1}\right)\\ & + n\bar{\delta}\Delta_k - \sum_{i=1}^{n}\frac{1}{2\gamma_{i2}}\left(\tilde{\varpi}_{i,2}^{k-1}\right)^{\mathrm{T}}\tilde{\varpi}_{i,2}^{k-1}(t) + \sum_{i=1}^{n}\frac{1}{2\gamma_{i2}}\left(\tilde{\varpi}_{i,2}^{k}\right)^{\mathrm{T}}\tilde{\varpi}_{i,2}^{k}(t). \end{aligned} \quad (31)$$

Let $\bar{f}_{i,2}^k = f_{i,2}^k + q_i(t)z_{i,1}^k$. Since $\bar{f}_{i,2}^k$ is unknown, it can not be directly used in controller design. Based on Lemma 2.9, a fuzzy logic system $\varpi_{i,2}^{\mathrm{T}}\varphi_{i,2}(\chi_{i,2})$ such that

$$\bar{f}_{i,2}^k = \varpi_{i,2}^{\mathrm{T}}\varphi_{i,2}(\chi_{i,2}) + \varepsilon_{i,2}(\chi_{i,2}), |\varepsilon_{i,2}(\chi_{i,2})| \le \varepsilon_{i,2}^*, \quad (32)$$

where $\varepsilon_{i,2}^* > 0$ is given as constant, $\varepsilon_{i,2}(\chi_{i,2})$ denotes the error in the approximation and $\chi_{i,2} = \left[x_{i,1}^k, x_{i,2}^k, y_r\right]^{\mathrm{T}}$.

The control input u_i^k and the auxiliary variable $\varsigma_i^k(t)$ are designed as

$$u_i^k = \mathcal{N}\left(\varsigma_i^k\right)\bar{u}_i^k, \quad (33)$$

$$\bar{u}_i^k(t) = c_{2,i}z_{i,2}^k + \hat{\bar{f}}_{i,2}^k + \hat{\bar{\varepsilon}}_{i,2}^k\tanh\left(\frac{\hat{\bar{\varepsilon}}_{i,2}^k z_{i,2}^k}{\Delta_k}\right) - \dot{\alpha}_{i,1}, \quad (34)$$

$$\dot{\varsigma}_i^k(t) = z_{i,2}^k\bar{u}_i^k(t), \varsigma_i^k(0) = \varsigma_i^{k-1}(T), \varsigma_i^0(0) = 0. \quad (35)$$

From (33-35), (31) becomes

$$\begin{aligned}\dot{E}^k \le & -\sum_{i=1}^{n} c_{1,i}\left(z_{i,1}^k\right)^2 - \sum_{i=1}^{n} c_{2,i}\left(z_{i,2}^k\right)^2 - \sum_{i=1}^{n}\left(q_i(t)\mathcal{N}\left(\zeta_i^k\right)+1\right)\dot{\zeta}_i^k + \sum_{i=1}^{n}\left(p_i(t)\mathcal{N}\left(\varsigma_i^k\right)+1\right)\dot{\varsigma}_i^k\\ & + \sum_{i=1}^{n} z_{i,2}^k\left(\tilde{\varpi}_{i,2}^k\right)^{\mathrm{T}}\varphi_{i,2}^k + \sum_{i=1}^{n} z_{i,2}^k\varepsilon_{i,2}^k + \sum_{i=1}^{n} z_{i,2}^k\sigma_{i,2}^k - \sum_{i=1}^{n} z_{i,2}^k\hat{\bar{\varepsilon}}_{i,2}^k\tanh\left(\frac{z_{i,2}^k\hat{\bar{\varepsilon}}_{i,2}^k}{\Delta_k}\right) + n\bar{\delta}\Delta_k\\ & + \frac{c_{2,i}}{r_{i2}}\sum_{i=1}^{n}\tilde{\bar{\varepsilon}}_{i,2}^k\dot{\tilde{\bar{\varepsilon}}}_{i,2}^k - \sum_{i=1}^{n}\frac{1}{2\gamma_{i2}}\left(\tilde{\varpi}_{i,2}^{k-1}\right)^{\mathrm{T}}\tilde{\varpi}_{i,2}^{k-1}(t) + \sum_{i=1}^{n}\frac{1}{2\gamma_{i2}}\left(\tilde{\varpi}_{i,2}^{k}\right)^{\mathrm{T}}\tilde{\varpi}_{i,2}^{k}(t). \end{aligned} \quad (36)$$

In (36), there is $\sum_{i=1}^{n} z_{i,2}^k\varepsilon_{i,2}^k + \sum_{i=1}^{n} z_{i,2}^k\sigma_{i,2}^k \le \sum_{i=1}^{n}\left|z_{i,2}^k\right|\left(\varepsilon_{i,2}^* + \sigma_{i,2}^*\right)$. Let $\bar{\varepsilon}_{i,2} = \varepsilon_{i,2}^* + \sigma_{i,2}^*$, (36) will be

$$\begin{aligned}\dot{E}^k \le & -\sum_{i=1}^{n} c_{1,i}\left(z_{i,1}^k\right)^2 - \sum_{i=1}^{n} c_{2,i}\left(z_{i,2}^k\right)^2 - \sum_{i=1}^{n}\left(q_i(t)\mathcal{N}\left(\zeta_i^k\right)+1\right)\dot{\zeta}_i^k + \sum_{i=1}^{n}\left(p_i(t)\mathcal{N}\left(\varsigma_i^k\right)+1\right)\dot{\varsigma}_i^k\\ & + \sum_{i=1}^{n} z_{i,2}^k\left(\tilde{\varpi}_{i,2}^k\right)^{\mathrm{T}}\varphi_{i,2}^k + \sum_{i=1}^{n}\left|z_{i,2}^k\right|\bar{\varepsilon}_{i,2} - \sum_{i=1}^{n} z_{i,2}^k\hat{\bar{\varepsilon}}_{i,2}^k\tanh\left(\frac{z_{i,2}^k\hat{\bar{\varepsilon}}_{i,2}^k}{\Delta_k}\right) + n\bar{\delta}\Delta_k\\ & + \frac{c_{2,i}}{r_{i2}}\sum_{i=1}^{n}\tilde{\bar{\varepsilon}}_{i,2}^k\dot{\tilde{\bar{\varepsilon}}}_{i,2}^k - \sum_{i=1}^{n}\frac{1}{2\gamma_{i2}}\left(\tilde{\varpi}_{i,2}^{k-1}\right)^{\mathrm{T}}\tilde{\varpi}_{i,2}^{k-1}(t) + \sum_{i=1}^{n}\frac{1}{2\gamma_{i2}}\left(\tilde{\varpi}_{i,2}^{k}\right)^{\mathrm{T}}\tilde{\varpi}_{i,2}^{k}(t). \end{aligned} \quad (37)$$

Design the following updating laws for $\hat{\varpi}_i^k$ and $\hat{\bar{\varepsilon}}_i^k$

$$\begin{aligned}&\hat{\varpi}_{i,2}^k(t) = sat\left(\hat{\varpi}_{i,2*}^k(t), \bar{\varpi}_{i,2}\right),\\ &\hat{\varpi}_{i,2*}^k(t) = \begin{cases} sat\left(\hat{\varpi}_{i,2*}^{k-1}(t), \bar{\varpi}_{i,2}\right) + \gamma_{i2}\varphi_{i,2}^k z_{i,2}^k, & k \ge 1,\\ \bar{\gamma}_{i0}(t)\varphi_{i,2}^0 z_{i,2}^0, & k = 0,\\ 0, & k = -1, \end{cases} \end{aligned} \quad (38)$$

$$\dot{\hat{\bar{\varepsilon}}}_{i,2}^k(t) = \frac{r_{i2}}{c_{2,i}}\left|z_{i,2}^k\right|, \hat{\bar{\varepsilon}}_{i,2}^k(0) = \hat{\bar{\varepsilon}}_{i,2}^{k-1}(T), \hat{\bar{\varepsilon}}_{i,2}^0(0) \ge 0, \quad (39)$$

where the strictly increasing continuous function $\bar{\gamma}_{i0}(t)$ satisfies $\bar{\gamma}_{i0}(0)=0, \bar{\gamma}_{i0}(T)=\gamma_{i2}$. $\bar{\varpi}_{i,2}(t)=[\bar{\varpi}_{i,21},\cdots,\bar{\varpi}_{i,2m}]^{\mathrm{T}}$ with saturation upper bound $\bar{\varpi}_{i,2j}>0, j=1,\cdots,m$ being arbitrarily chosen. Because of $r_{i2}, c_{2,i}>0, \hat{\bar{\varepsilon}}^0_{i,2}(0)\geq 0$, $\hat{\bar{\varepsilon}}^k_{i,2}(t)\geq 0$ can be ensure.

Similar to Lemma 3.2, we can conclude that control protocols (33-35) and updated laws (38-39) guarantee that

$$\dot{\varsigma}^k_i(0)=\dot{\varsigma}^{k-1}_i(T), \hat{\varpi}^k_{i,2}(0)=\hat{\varpi}^{k-1}_{i,2}(T). \tag{40}$$

Similar to (26), substitute (38-39) into (37)

$$\begin{aligned}\dot{E}^k \leq & -\sum_{i=1}^n c_{1,i}\big(z^k_{i,1}\big)^2-\sum_{i=1}^n c_{2,i}\big(z^k_{i,2}\big)^2+\sum_{i=1}^n\big(q_i(t)\mathcal{N}\big(\zeta^k_i\big)+1\big)\dot{\zeta}^k_i+n\bar{\delta}\Delta_k\\ &+\sum_{i=1}^n\big|z^k_{i,2}\big|\hat{\bar{\varepsilon}}^k_{i,2}-\sum_{i=1}^n z^k_{i,2}\hat{\bar{\varepsilon}}^k_{i,2}\tanh\left(\frac{z^k_{i,2}\hat{\bar{\varepsilon}}^k_{i,2}}{\Delta_k}\right)+\sum_{i=1}^n\big(p_i(t)\mathcal{N}\big(\varsigma^k_i\big)+1\big)\dot{\varsigma}^k_i\\ &+\sum_{i=1}^n\frac{1}{\gamma_{i2}}\big(\tilde{\varpi}^k_{i,2}\big)^{\mathrm{T}}\big(\hat{\varpi}^k_{i,2*}-\hat{\varpi}^k_{i,2}\big).\end{aligned} \tag{41}$$

From Lemma 2.6, (41) is updated to

$$\dot{E}^k(t)\leq -\sum_{i=1}^n c_{1,i}\big(z^k_{i,1}\big)^2-\sum_{i=1}^n c_{2,i}\big(z^k_{i,2}\big)^2+\sum_{i=1}^n\big(p_i(t)\mathcal{N}\big(\varsigma^k_i\big)+1\big)\dot{\varsigma}^k_i+\sum_{i=1}^n\big(q_i(t)\mathcal{N}\big(\zeta^k_i\big)+1\big)\dot{\zeta}^k_i+2n\bar{\delta}\Delta_k, \tag{42}$$

which is followed by $\sum\limits_{i=1}^n\frac{1}{\gamma_{i2}}\big(\tilde{\varpi}^k_{i,2}\big)^{\mathrm{T}}\big(\hat{\varpi}^k_{i,2*}-\hat{\varpi}^k_{i,2}\big)\leq 0$.

3.2 Stability analysis

Theorem 3.4. *For second-order unknown nonlinear strict-feedback MASs (1), under Assumptions 1 and 2, the designed AILC protocols (16-18), (33-35) and the parameter updated laws (21-22), (38-39) guarantee that all agents including the leader will achieve output cooperation consensus in finite time as the iteration number k approaches infinity, i.e., $\lim\limits_{k\to\infty}\delta^k_i=0, i=1,2,\cdots,n$. In addition, the signals of the closed-loop MASs are all bounded.*

Proof. Calculate the difference of $E^k(t)$ in the kth iteration as

$$\begin{aligned}\Delta E^k=&E^k(t)-E^{k-1}(t)\\ =&\sum_{j=1}^2\sum_{i=1}^n\frac{c_{i,j}}{2r_{ij}}\left(\big(\tilde{\bar{\varepsilon}}^k_{i,j}\big)^2-\big(\tilde{\bar{\varepsilon}}^{k-1}_{i,j}\big)^2\right)+\sum_{j=1}^2\sum_{i=1}^n\int_0^t\frac{1}{2\gamma_{ij}}\left(\big(\tilde{\varpi}^k_{i,j}\big)^{\mathrm{T}}\tilde{\varpi}^k_{i,j}-\big(\tilde{\varpi}^{k-1}_{i,j}\big)^{\mathrm{T}}\tilde{\varpi}^{k-1}_{i,j}\right)d\tau\\ &+\frac{1}{2}\big(Z^k_1\big)^{\mathrm{T}}H^{-1}Z^k_1-\frac{1}{2}\big(Z^{k-1}_1\big)^{\mathrm{T}}H^{-1}Z^{k-1}_1+\frac{1}{2}\big(Z^k_2\big)^{\mathrm{T}}Z^k_2-\frac{1}{2}\big(Z^{k-1}_2\big)^{\mathrm{T}}Z^{k-1}_2\\ &+\sum_{j=1}^2\sum_{i=1}^n\int_t^T\frac{1}{2\gamma_{ij}}\left(\big(\tilde{\varpi}^{k-1}_{i,j}\big)^{\mathrm{T}}\tilde{\varpi}^{k-1}_{i,j}-\big(\tilde{\varpi}^{k-2}_{i,j}\big)^{\mathrm{T}}\tilde{\varpi}^{k-2}_{i,j}\right)d\tau,\end{aligned} \tag{43}$$

where

$$\begin{aligned}&\sum_{j=1}^2\sum_{i=1}^n\frac{c_{i,j}}{2r_{ij}}\left(\big(\tilde{\bar{\varepsilon}}^k_{i,j}\big)^2-\big(\tilde{\bar{\varepsilon}}^{k-1}_{i,j}\big)^2\right)\\ &=\sum_{j=1}^2\sum_{i=1}^n\frac{c_{i,j}}{2r_{ij}}\big(\tilde{\bar{\varepsilon}}^k_{i,j}(0)\big)^2+\sum_{j=1}^2\sum_{i=1}^n\int_0^t\frac{c_{i,j}}{r_{ij}}\tilde{\bar{\varepsilon}}^k_{i,j}\dot{\tilde{\bar{\varepsilon}}}^k_{i,j}d\tau-\sum_{j=1}^2\sum_{i=1}^n\frac{c_{i,j}}{2r_{ij}}\big(\tilde{\bar{\varepsilon}}^{k-1}_{i,j}\big)^2\\ &=\sum_{j=1}^2\sum_{i=1}^n\frac{c_{i,j}}{2r_{ij}}\big(\tilde{\bar{\varepsilon}}^k_{i,j}(0)\big)^2-\sum_{j=1}^2\sum_{i=1}^n\int_0^t\big|z^k_{i,j}\big|\tilde{\bar{\varepsilon}}^k_{i,j}d\tau-\sum_{j=1}^2\sum_{i=1}^n\frac{c_{i,j}}{2r_{ij}}\big(\tilde{\bar{\varepsilon}}^{k-1}_{i,j}\big)^2,\end{aligned} \tag{44}$$

and

$$\sum_{j=1}^2\sum_{i=1}^n\int_0^t\frac{1}{2\gamma_{ij}}\left(\big(\tilde{\varpi}^k_{i,j}\big)^{\mathrm{T}}\tilde{\varpi}^k_{i,j}-\big(\tilde{\varpi}^{k-1}_{i,j}\big)^{\mathrm{T}}\tilde{\varpi}^{k-1}_{i,j}\right)\leq\sum_{j=1}^2\sum_{i=1}^n\int_0^t\frac{1}{\gamma_{ij}}\big(\tilde{\varpi}^k_{i,j}\big)^{\mathrm{T}}\big(\hat{\varpi}^{k-1}_{i,j}-\hat{\varpi}^k_{i,j}\big)d\tau. \tag{45}$$

Besides,

$$\begin{aligned}
&\frac{1}{2}\left(Z_1^k\right)^{\mathrm{T}}H^{-1}Z_1^k-\frac{1}{2}\left(Z_1^{k-1}\right)^{\mathrm{T}}H^{-1}Z_1^{k-1}+\frac{1}{2}\left(Z_2^k\right)^{\mathrm{T}}Z_2^k-\frac{1}{2}\left(Z_2^{k-1}\right)^{\mathrm{T}}Z_2^{k-1}\\
&=\frac{1}{2}\left(Z_1^k\left(0\right)\right)^{\mathrm{T}}H^{-1}Z_1^k\left(0\right)+\frac{1}{2}\left(Z_2^k\left(0\right)\right)^{\mathrm{T}}Z_2^k\left(0\right)-\frac{1}{2}\left(Z_1^{k-1}\right)^{\mathrm{T}}H^{-1}Z_1^{k-1}-\frac{1}{2}\left(Z_2^{k-1}\right)^{\mathrm{T}}Z_2^{k-1}\\
&+\int_0^t\left(Z_1^k\right)^{\mathrm{T}}H^{-1}\dot{Z}_1^k\left(\tau\right)d\tau+\int_0^t\left(Z_2^k\right)^{\mathrm{T}}\dot{Z}_2^k\left(\tau\right)d\tau\\
&\leq\frac{1}{2}\left(Z_1^k\left(0\right)\right)^{\mathrm{T}}H^{-1}Z_1^k\left(0\right)+\frac{1}{2}\left(Z_2^k\left(0\right)\right)^{\mathrm{T}}Z_2^k\left(0\right)-\frac{1}{2}\left(Z_1^{k-1}\right)^{\mathrm{T}}H^{-1}Z_1^{k-1}-\frac{1}{2}\left(Z_2^{k-1}\right)^{\mathrm{T}}Z_2^{k-1}\\
&-\sum_{j=1}^{2}\sum_{i=1}^{n}\int_0^t c_{j,i}\left(z_{i,j}^k\right)^2d\tau+\sum_{i=1}^{n}\int_0^t\left(p_i\left(\tau\right)\mathcal{N}\left(\varsigma_i^k\right)+1\right)\dot{\varsigma}_i^k d\tau+\sum_{j=1}^{2}\sum_{i=1}^{n}\int_0^t\left|z_{i,j}^k\right|\bar{\varepsilon}_{i,j}d\tau\\
&-\sum_{j=1}^{2}\sum_{i=1}^{n}\int_0^t\hat{\bar{\varepsilon}}_{i,j}^k z_{i,j}^k\tanh\left(\frac{\hat{\bar{\varepsilon}}_{i,j}^k z_{i,j}^k}{\Delta_k}\right)d\tau+\sum_{i=1}^{n}\int_0^t\left(q_i\left(\tau\right)\mathcal{N}\left(\zeta_i^k\right)+1\right)\dot{\zeta}_i^k d\tau\\
&+\sum_{j=1}^{2}\sum_{i=1}^{n}\int_0^t\frac{1}{\gamma_{ij}}\left(\tilde{\varpi}_{i,j}^k\right)^{\mathrm{T}}\left(\hat{\varpi}_{i,j*}^k-sat\left(\hat{\varpi}_{i,j*}^{k-1}\right)\right)d\tau.
\end{aligned}\tag{46}$$

On the basis of (44-46), we have

$$\begin{aligned}
\Delta E^k\leq&\frac{1}{2}\left(Z_1^k\left(0\right)\right)^{\mathrm{T}}H^{-1}Z_1^k\left(0\right)+\frac{1}{2}\left(Z_2^k\left(0\right)\right)^{\mathrm{T}}Z_2^k\left(0\right)-\frac{1}{2}\left(Z_1^{k-1}\right)^{\mathrm{T}}H^{-1}Z_1^{k-1}-\frac{1}{2}\left(Z_2^{k-1}\right)^{\mathrm{T}}Z_2^{k-1}+2nT\bar{\delta}\Delta_k\\
&-\sum_{j=1}^{2}\sum_{i=1}^{n}\int_0^t c_{j,i}\left(z_{i,j}^k\right)^2d\tau+\sum_{i=1}^{n}\int_0^t\left(p_i\left(\tau\right)\mathcal{N}\left(\varsigma_i^k\right)+1\right)\dot{\varsigma}_i^k d\tau+\sum_{i=1}^{n}\int_0^t\left(q_i\left(\tau\right)\mathcal{N}\left(\zeta_i^k\right)+1\right)\dot{\zeta}_i^k d\tau\\
&+\sum_{j=1}^{2}\sum_{i=1}^{n}\int_t^T\frac{1}{2\gamma_{ij}}\left(\left(\tilde{\varpi}_{i,j}^{k-1}\right)^{\mathrm{T}}\tilde{\varpi}_{i,j}^{k-1}-\left(\tilde{\varpi}_{i,j}^{k-2}\right)^{\mathrm{T}}\tilde{\varpi}_{i,j}^{k-2}\right)d\tau+\sum_{j=1}^{2}\sum_{i=1}^{n}\frac{c_{i,j}}{2r_{ij}}\left(\left(\tilde{\bar{\varepsilon}}_{i,j}^k\left(0\right)\right)^2-\left(\tilde{\bar{\varepsilon}}_{i,j}^{k-1}\right)^2\right),
\end{aligned}\tag{47}$$

which is followed by $\sum\limits_{j=1}^{2}\sum\limits_{i=1}^{n}\int_0^t\frac{1}{\gamma_{ij}}\left(\tilde{\varpi}_{i,j}^k\right)^{\mathrm{T}}\left(\hat{\varpi}_{i,j*}^k-\hat{\varpi}_{i,j}^k\right)d\tau\leq 0$.

Recall $Z_1^k\left(0\right)=Z_1^{k-1}\left(T\right),Z_2^k\left(0\right)=Z_2^{k-1}\left(T\right),\tilde{\bar{\varepsilon}}_{i,j}^k\left(0\right)=\tilde{\bar{\varepsilon}}_{i,j}^{k-1}\left(T\right)$, when $t=T$, we have

$$\Delta E^k\left(T\right)\leq-\sum_{j=1}^{2}\sum_{i=1}^{n}\int_0^t c_{j,i}\left(z_{i,j}^k\right)^2d\tau+\sum_{i=1}^{n}\int_0^t\left(p_i\left(\tau\right)\mathcal{N}\left(\varsigma_i^k\right)+1\right)\dot{\varsigma}_i^k d\tau+\sum_{i=1}^{n}\int_0^t\left(q_i\left(\tau\right)\mathcal{N}\left(\zeta_i^k\right)+1\right)\dot{\zeta}_i^k d\tau+2nT\bar{\delta}\Delta_k.\tag{48}$$

Repeatedly applying (48), one has

$$\begin{aligned}
E^k\left(T\right)&=E^0\left(T\right)+\sum_{l=1}^{k}\Delta E^l\left(T\right)\\
&\leq E^0\left(T\right)-\sum_{l=1}^{k}\sum_{j=1}^{2}\sum_{i=1}^{n}\int_0^T c_{j,i}\left(z_{i,j}^l\right)^2d\tau+\sum_{l=1}^{k}\sum_{i=1}^{n}\int_0^T\left(p_i\left(\tau\right)\mathcal{N}\left(\varsigma_i^l\right)+1\right)\dot{\varsigma}_i^l d\tau\\
&+\sum_{l=1}^{k}\sum_{i=1}^{n}\int_0^T\left(q_i\left(\tau\right)\mathcal{N}\left(\zeta_i^l\right)+1\right)\dot{\zeta}_i^l d\tau+2nT\bar{\delta}\sum_{l=1}^{k}\Delta_l.
\end{aligned}\tag{49}$$

According to Lemma 2.7, there is $\lim\limits_{k\to\infty}2nT\bar{\delta}\sum\limits_{l=1}^{k}\Delta_l\leq 4nT\bar{\delta}a$, it means that $2nT\bar{\delta}\sum\limits_{l=1}^{k}\Delta_l$ is bounded. Thus there exits a

positive constant S_0 such that $2nT\bar{\delta}\sum_{l=1}^{k}\Delta_l$, and (47) becomes

$$E^k(T) \le E^0(T) - \sum_{l=1}^{k}\sum_{j=1}^{2}\sum_{i=1}^{n}\int_0^T c_{j,i}\left(z_{i,j}^l\right)^2 d\tau + \sum_{l=1}^{k}\sum_{i=1}^{n}\int_0^T \left(p_i(\tau)\mathcal{N}\left(\varsigma_i^l\right)+1\right)\dot{\varsigma}_i^l d\tau + \sum_{l=1}^{k}\sum_{i=1}^{n}\int_0^T \left(q_i(\tau)\mathcal{N}\left(\zeta_i^l\right)+1\right)\dot{\zeta}_i^l d\tau + S_0. \tag{50}$$

For $t\in[0,T]$, we define a new function satisfying $\varsigma_i(t+(k-1)T)=\varsigma_i^k(t)$ and $\dot{\varsigma}_i(t+(k-1)T)\triangleq\dot{\varsigma}_i^k(t)$. From (35) and (40), it is not hard to see that $\dot{\varsigma}_i(t)$ is continuous, $\varsigma_i(t)$ is in C^1 for $\forall t\in[0,kT]$. Thus

$$\begin{aligned}\sum_{l=1}^{k}\sum_{i=1}^{n}\int_0^T \left(p_i(\tau)\mathcal{N}\left(\varsigma_i^l\right)+1\right)\dot{\varsigma}_i^l d\tau &= \sum_{i=1}^{n}\int_0^T \left(p_i(\tau)\mathcal{N}\left(\varsigma_i^1\right)+1\right)\dot{\varsigma}_i^1 d\tau + \sum_{i=1}^{n}\int_0^T \left(p_i(\tau)\mathcal{N}\left(\varsigma_i^2\right)+1\right)\dot{\varsigma}_i^2 d\tau \\ &\quad + \cdots + \sum_{i=1}^{n}\int_0^T \left(p_i(\tau)\mathcal{N}\left(\varsigma_i^k\right)+1\right)\dot{\varsigma}_i^k d\tau \\ &= \sum_{i=1}^{n}\int_0^T \left(p_i(\tau)\mathcal{N}(\varsigma_i)+1\right)\dot{\varsigma}_i d\tau + \sum_{i=1}^{n}\int_T^{2T} \left(p_i(\tau)\mathcal{N}(\varsigma_i)+1\right)\dot{\varsigma}_i d\tau \\ &\quad + \cdots + \sum_{i=1}^{n}\int_{(k-1)T}^{kT} \left(p_i(\tau)\mathcal{N}(\varsigma_i)+1\right)\dot{\varsigma}_i d\tau \\ &= \sum_{i=1}^{n}\int_0^{kT} \left(p_i(\tau)\mathcal{N}(\varsigma_i)+1\right)\dot{\varsigma}_i d\tau. \end{aligned} \tag{51}$$

Similarly, define $\zeta_i(t+(k-1)T)=\zeta_i^k(t)$ and $\dot{\zeta}_i(t+(k-1)T)\triangleq\dot{\zeta}_i^k(t)$, then

$$\sum_{l=1}^{k}\sum_{i=1}^{n}\int_0^T \left(q_i(\tau)\mathcal{N}\left(\zeta_i^l\right)+1\right)\dot{\zeta}_i^l d\tau = \sum_{i=1}^{n}\int_0^{kT} \left(q_i(\tau)\mathcal{N}(\zeta_i)+1\right)\dot{\zeta}_i d\tau. \tag{52}$$

Denote $E^k(\tau)=V(\tau+(k-1)T)$, according to (50-52), then

$$V(kT) \le E^0(T) - \sum_{l=1}^{k}\sum_{j=1}^{2}\sum_{i=1}^{n}\int_0^T c_{j,i}\left(z_{i,j}^l\right)^2 d\tau + \sum_{i=1}^{n}\int_0^{kT} \left(p_i(\tau)\mathcal{N}(\varsigma_i)+1\right)\dot{\varsigma}_i d\tau + \sum_{i=1}^{n}\int_0^{kT} \left(q_i(\tau)\mathcal{N}(\zeta_i)+1\right)\dot{\zeta}_i d\tau + S_0. \tag{53}$$

From (42), there is

$$\begin{aligned}\dot{E}^k(t) &\le -\sum_{i=1}^{n} c_{1,i}\left(z_{i,1}^k\right)^2 - \sum_{i=1}^{n} c_{2,i}\left(z_{i,2}^k\right)^2 + \sum_{i=1}^{n}\left(p_i(\tau)\mathcal{N}\left(\varsigma_i^k\right)+1\right)\dot{\varsigma}_i^k \\ &\quad + \sum_{i=1}^{n}\left(q_i(t)\mathcal{N}\left(\zeta_i^k\right)+1\right)\dot{\zeta}_i^k + 2n\bar{\delta}\Delta_k \\ &\le \sum_{i=1}^{n}\left(p_i(\tau)\mathcal{N}\left(\varsigma_i^k\right)+1\right)\dot{\varsigma}_i^k + \sum_{i=1}^{n}\left(q_i(t)\mathcal{N}\left(\zeta_i^k\right)+1\right)\dot{\zeta}_i^k + 2n\bar{\delta}\Delta_k. \end{aligned} \tag{54}$$

Because of (53-54), for $\forall t \in [0,T]$

$$\begin{aligned}
V(kT+t) &= V(kT) + \int_0^t \dot{E}^{k+1}(\tau)\, d\tau \\
&\leq E^0(T) - \sum_{l=1}^{k}\sum_{j=1}^{2}\sum_{i=1}^{n}\int_0^t c_{j,i}\left(z_{i,j}^l\right)^2 d\tau + \sum_{i=1}^{n}\int_0^{kT}\left(p_i(\tau)\mathcal{N}(\varsigma_i)+1\right)\dot{\varsigma}_i d\tau \\
&+ \sum_{i=1}^{n}\int_{kT}^{kT+t}\left(p_i(\tau)\mathcal{N}(\varsigma_i)+1\right)\dot{\varsigma}_i d\tau + \sum_{i=1}^{n}\int_0^{kT}\left(q_i(\tau)\mathcal{N}(\zeta_i)+1\right)\dot{\zeta}_i d\tau \\
&+ \sum_{i=1}^{n}\int_{kT}^{kT+t}\left(q_i(\tau)\mathcal{N}(\zeta_i)+1\right)\dot{\zeta}_i d\tau + \int_0^t 2n\bar{\delta}\Delta_{k+1} d\tau + S_0 \\
&\leq E^0(T) - \sum_{l=1}^{k}\sum_{j=1}^{2}\sum_{i=1}^{n}\int_0^T c_{j,i}\left(z_{i,j}^l\right)^2 d\tau + \sum_{i=1}^{n}\int_0^{kT+t}\left(p_i(\tau)\mathcal{N}(\varsigma_i)+1\right)\dot{\varsigma}_i d\tau \\
&+ \sum_{i=1}^{n}\int_0^{kT+t}\left(q_i(\tau)\mathcal{N}(\zeta_i)+1\right)\dot{\zeta}_i d\tau + S_1,
\end{aligned} \tag{55}$$

where $S_0 + 2nT\bar{\delta}\Delta_{k+1} \leq S_1$.

With Lemma 2.3, $\sum_{i=1}^{n}\int_0^{kT+t}\left(p_i(\tau)\mathcal{N}(\varsigma_i)+1\right)\dot{\varsigma}_i d\tau$ and $\sum_{i=1}^{n}\int_0^{kT+t}\left(q_i(\tau)\mathcal{N}(\zeta_i)+1\right)\dot{\zeta}_i d\tau$ are both bounded. In other words, there exits a constant $M>0$ so that

$$\sum_{i=1}^{n}\int_0^{kT+t}\left(p_i(\tau)\mathcal{N}(\varsigma_i)+1\right)\dot{\varsigma}_i d\tau + \sum_{i=1}^{n}\int_0^{kT+t}\left(q_i(\tau)\mathcal{N}(\zeta_i)+1\right)\dot{\zeta}_i d\tau \leq M. \tag{56}$$

From (55-56), one has

$$\begin{aligned}
\sum_{l=1}^{k}\sum_{j=1}^{2}\sum_{i=1}^{n}\int_0^T c_{j,i}\left(z_{i,j}^l\right)^2 d\tau &\leq E^0(T) - V(kT+t) + \sum_{i=1}^{n}\int_0^{kT+t}\left(p_i(\tau)\mathcal{N}(\varsigma_i)+1\right)\dot{\varsigma}_i d\tau \\
&+ \sum_{i=1}^{n}\int_0^{kT+t}\left(q_i(\tau)\mathcal{N}(\zeta_i)+1\right)\dot{\zeta}_i d\tau + S_1 \\
&\leq E^0(T) + M + S_1.
\end{aligned} \tag{57}$$

From (57), we can conclude the finiteness of $\sum_{l=1}^{k}\sum_{j=1}^{2}\sum_{i=1}^{n}\int_0^T c_{j,i}\left(z_{i,j}^l\right)^2 d\tau$ in kth iteration if $E^0(T)$ is finite. Next we will confirm the finiteness of $E^0(t)$.

It is noted that local Lipschitz continuous functions ς_i^k, ζ_i^k may exhibit finite escape time phenomena. Relying on proposed protocols (16-18), (33-35) and parameter updated laws (21-22), (38-39), the right-hand side of (29) is continuous for all arguments. In terms of the existence theorem of differential equation, here exists a solution in $[0,T_1) \subset [0,T]$ with $T_1 > 0$. Thus, $E^0(t)$ over $[0,T_1]$ could be thought of as finite, so we just have to prove that $E^0(t)$ is bounded on small interval $(T_1,T]$.

With the definition of $E^k(t)$ in (30), $\dot{E}^0(t)$ will be

$$\begin{aligned}
\dot{E}^0(t) \leq{}& \left(Z_1^0\right)^{\mathrm{T}} H^{-1}\dot{Z}_1^0(t) + \left(Z_2^0\right)^{\mathrm{T}}\dot{Z}_2^0(t) + \sum_{j=1}^{2}\sum_{i=1}^{n}\frac{c_{j,i}}{r_{ij}}\tilde{\varepsilon}_{i,j}^0\dot{\tilde{\varepsilon}}_{i,j}^0 \\
&+ \sum_{j=1}^{2}\sum_{i=1}^{n}\frac{1}{2\gamma_{ij}}\left(\left(\tilde{\varpi}_{i,j}^0\right)^{\mathrm{T}}\tilde{\varpi}_{i,j}^0 - \left(\tilde{\varpi}_{i,j}^{-1}\right)^{\mathrm{T}}\tilde{\varpi}_{i,j}^{-1}\right),
\end{aligned} \tag{58}$$

where

$$\begin{aligned}
&\sum_{i=1}^{n}\frac{1}{2\gamma_{i1}}\left(\left(\tilde{\varpi}_{i,1}^{0}\right)^{\mathrm{T}}\tilde{\varpi}_{i,1}^{0}-\left(\tilde{\varpi}_{i,1}^{-1}\right)^{\mathrm{T}}\tilde{\varpi}_{i,1}^{-1}\right)\leq\sum_{i=1}^{n}\frac{1}{2\gamma_{i0}}\left(\tilde{\varpi}_{i,1}^{0}\right)^{\mathrm{T}}\tilde{\varpi}_{i,1}^{0}\\
&=\sum_{i=1}^{n}\frac{1}{2\gamma_{i0}}\left[\left(\tilde{\varpi}_{i,1}^{0}\right)^{\mathrm{T}}\tilde{\varpi}_{i,1}^{0}-\left(\tilde{\varpi}_{i,1}^{-1}\right)^{\mathrm{T}}\tilde{\varpi}_{i,1}^{-1}+\left(\varpi_{i,1}\right)^{\mathrm{T}}\varpi_{i,1}\right]\\
&\leq\sum_{i=1}^{n}\frac{1}{\gamma_{i0}}\left(\tilde{\varpi}_{i,1}^{0}\right)^{\mathrm{T}}\left(\hat{\varpi}_{i,1}^{-1}-\hat{\varpi}_{i,1}^{0}\right)+\sum_{i=1}^{n}\frac{1}{2\gamma_{i0}}\left(\varpi_{i,1}\right)^{\mathrm{T}}\varpi_{i,1}\\
&=-\sum_{i=1}^{n}\left(\tilde{\varpi}_{i,1}^{0}\right)^{\mathrm{T}}\varphi_{i,1}^{0}z_{i,1}^{0}+\sum_{i=1}^{n}\frac{1}{2\gamma_{i0}}\left(\varpi_{i,1}\right)^{\mathrm{T}}\varpi_{i,1},
\end{aligned}\tag{59}$$

similarly,

$$\sum_{i=1}^{n}\frac{1}{2\gamma_{i2}}\left(\left(\tilde{\varpi}_{i,2}^{0}\right)^{\mathrm{T}}\tilde{\varpi}_{i,2}^{0}-\left(\tilde{\varpi}_{i,2}^{-1}\right)^{\mathrm{T}}\tilde{\varpi}_{i,2}^{-1}\right)\leq-\sum_{i=1}^{n}\left(\tilde{\varpi}_{i,2}^{0}\right)^{\mathrm{T}}\varphi_{i,2}^{0}z_{i,2}^{0}+\sum_{i=1}^{n}\frac{1}{2\bar{\gamma}_{i0}}\left(\varpi_{i,2}\right)^{\mathrm{T}}\varpi_{i,2}.\tag{60}$$

Therefore,

$$\begin{aligned}
\dot{E}^{0}\leq&-\sum_{j=1}^{2}\sum_{i=1}^{n}c_{j,i}\left(z_{i,j}^{0}\right)^{2}+\sum_{i=1}^{n}\left(p_{i}\left(t\right)\mathcal{N}\left(\varsigma_{i}^{0}\right)+1\right)\dot{\varsigma}_{i}^{0}+\sum_{i=1}^{n}\frac{1}{2\gamma_{i0}}\left(\varpi_{i,1}\right)^{\mathrm{T}}\varpi_{i,1}\\
&+\sum_{i=1}^{n}\left(q_{i}\left(t\right)\mathcal{N}\left(\zeta_{i}^{0}\right)+1\right)\dot{\zeta}_{i}^{0}+\sum_{i=1}^{n}\frac{1}{2\bar{\gamma}_{i0}}\left(\varpi_{i,2}\right)^{\mathrm{T}}\varpi_{i,2}+2n\bar{\delta}\Delta_{0}.
\end{aligned}\tag{61}$$

By integrating (61) from T_1 to t, it updates as

$$\begin{aligned}
E^{0}\left(t\right)\leq&E^{0}\left(T_{1}\right)+\sum_{i=1}^{n}\int_{T_{1}}^{t}\left(p_{i}\left(\tau\right)\mathcal{N}\left(\varsigma_{i}^{0}\right)+1\right)\dot{\varsigma}_{i}^{0}d\tau+\sum_{i=1}^{n}\int_{T_{1}}^{t}\frac{1}{2\gamma_{i0}}\left(\varpi_{i,1}\right)^{\mathrm{T}}\varpi_{i,1}d\tau\\
&+\sum_{i=1}^{n}\int_{T_{1}}^{t}\left(q_{i}\left(\tau\right)\mathcal{N}\left(\zeta_{i}^{0}\right)+1\right)\dot{\zeta}_{i}^{0}d\tau+\sum_{i=1}^{n}\int_{T_{1}}^{t}\frac{1}{2\bar{\gamma}_{i0}}\left(\varpi_{i,2}\right)^{\mathrm{T}}\varpi_{i,2}d\tau+\left(T-T_{1}\right)2n\bar{\delta}\Delta_{0}.
\end{aligned}\tag{62}$$

$\varpi_{i,1},\varpi_{i,2}$ are bounded over $[0,T]$ because they are optimal weight vectors, and $E^{0}\left(0\right)$, $\left(T-T_{1}\right)2n\bar{\delta}\Delta_{0}$ are bounded as well. So $E^{0}\left(t\right)$, $\sum\limits_{i=1}^{n}\int_{T_{1}}^{t}\left(p_{i}\left(\tau\right)\mathcal{N}\left(\varsigma_{i}^{0}\right)+1\right)\dot{\varsigma}_{i}^{0}d\tau$ and $\sum\limits_{i=1}^{n}\int_{T_{1}}^{t}\left(q_{i}\left(\tau\right)\mathcal{N}\left(\zeta_{i}^{0}\right)+1\right)\dot{\zeta}_{i}^{0}d\tau$ are bounded on $(T_1,T]$ in the light of Lemma 2.3. Clearly, the boundedness of $E^{0}\left(t\right)$ indicates that $E^{0}\left(T\right)$ is finite. Under (57), it could be obtained that $\sum\limits_{l=1}^{k}\sum\limits_{i=1}^{n}\int_{0}^{T}c_{1,i}\left(z_{i,1}^{l}\right)^{2}d\tau$, $\sum\limits_{l=1}^{k}\sum\limits_{i=1}^{n}\int_{0}^{T}c_{2,i}\left(z_{i,2}^{l}\right)^{2}d\tau$ are convergent to 0. Therefore, the uniform boundedness of $E^{k}\left(t\right)$ can be acquired from (55). Thus, $z_{i,1}^{k},z_{i,2}^{k},\hat{\bar{\varepsilon}}_{i}^{k}$ are uniformly bounded. By Barbalat's like lemma [26], $\lim\limits_{k\to\infty}z_{i,1}^{k}=0$, $\lim\limits_{k\to\infty}z_{i,2}^{k}=0$. Consequently, we can obtain $\lim\limits_{k\to\infty}\delta_{i}^{k}=0$ relying on Lemma 2.8. When the iteration number k approaches infinity, all followers will accurately track the leader in finite time. The boundedness of $x_{i,1}^{k}\left(t\right)$ is inferred from the convergence property of $\delta_{i}^{k}\left(t\right)$. And the boundedness of $E^{k}\left(T\right)$ guarantees the boundedness of $\hat{\varpi}_{i}^{k}$. Furthermore, from (33), $u_{i}^{k},\bar{u}_{i}^{k}$ are bounded. □

Remark 3.5. *It is crucial that we use Lemma 2.3 in the proof of Theorem 3.4, and the piecewise continuous Nussbaum-type function in Lemma 2.3 is not depending on global information. Summarily, the designed learning protocols (16-18) and (33-35) are completely distributed based on Lemma 2.3 under the condition of unknown control directions, which enables each agent to accurately track the desired output trajectory on* $[0,T]$.

3.3 Formation control scheme for MASs

This section will explore formation consensus of second-order MASs (1). Initially, we denote the position error of ith follower in the kth iteration as

$$\bar{y}_{i}^{k}\left(t\right)=y_{i}^{k}\left(t\right)-\bar{\Delta}_{i},\tag{63}$$

where $\bar{\Delta}_i$ represents the expected relative state between every follower and their leader. Our formation control target is to propose the ILC scheme which makes each follower keeps the desired distance from the leader as the iteration number k approaches infinity for $t \in [0, T]$.

Similar to (2), the formation consensus error is

$$\widehat{\delta}_i^k(t) = \bar{y}_i^k(t) - y_r(t). \tag{64}$$

We further transform the formation problem into the consensus tracking problem studied in Theorem 3.4. Next, we need to prove $\lim_{k\to\infty} \left\| \widehat{\delta}^k \right\| = 0$.

Express the formation error of ith follower as

$$\widehat{e}_i^k(t) = \sum_{j=1}^{n} a_{ij} \left(\bar{y}_i^k(t) - \bar{y}_j^k(t) \right) + b_i \left(\bar{y}_i^k(t) - y_r(t) \right). \tag{65}$$

Theorem 3.6. *For second-order unknown nonlinear strict-feedback MASs (1), under Assumptions 1 and 2, the designed AILC protocols (16-18), (33-35) and the parameter updated laws (21-22), (38-39) guarantee that all agents including the leader will achieve desired formation effect in finite time as the iteration number k approaches infinity, i.e.,* $\lim_{k\to\infty} \left\| \widehat{\delta}^k \right\| = 0$. *In addition, the signals of the closed-loop MASs are all bounded.*

Proof. Similar to Theorem 3.4, we omit it for brevity. □

4 Simulations

Here we will give two simulation experiments to prove the practicability of the novel distributed AILC algorithm proposed by us. Example 4.1 is provided to show consensus results through the communication structure as Fig.1(a) with ten followers and a leader. Example 4.2 is used to prove the formation control results through the communication structure as Fig.5(a) with four followers and a leader.

In Fig.1(a), it is assumed that u_i^k of $1, ..., 5$ agents have unknown positive control gains, while $6, ..., 10$ agents have negative known control gains, and the virtual control gains of $x_{i,2}^k$ of agents $3, 6, 9$ are unknown, other agents have known gains. In Fig.5(a), u_i^k of $1, 2$ agents have unknown positive control gains, while $3, 4$ agents have negative known control gains, and the virtual control gain of $x_{i,2}^k$ of 2th agent is unknown, other agents have known gains. Obviously, it is shown that only 1th agent could receive the leader's information in Fig.1(a) and Fig.5(a).

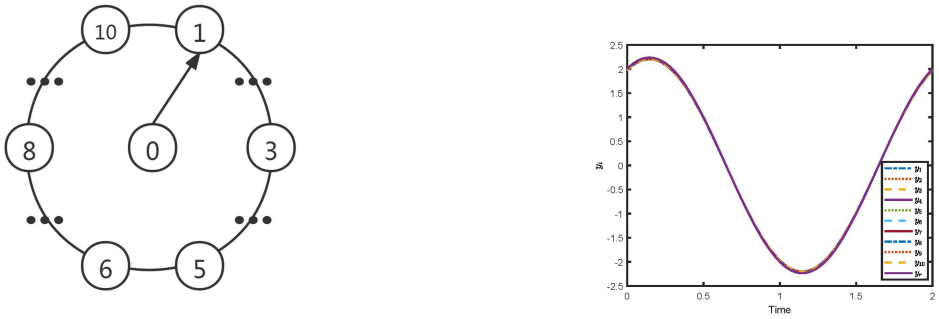
Example 4.1. *Consensus control for MASs.*

The leader is described as $y_r(t) = \sin(\pi t) + 2\cos(\pi t), t \in [0, 2]$. *Consider the dynamics of MAS (1) as* $f_{i,1}^k = 0.1\cos\left(x_{i,1}^k\right)$, $f_{i,2}^k = \cos(i\pi t)\left(-0.5 - x_{i,1}^k \sin\left(x_{i,2}^k\right)^2\right)$, $\sigma_{i,1}^k = -0.1\sin(2\pi t), \sigma_{i,2}^k(t) = -0.01\coth(\pi t)$, $\mathrm{i} = 1, 2, \cdots, 10$, *and* $p_1(t) = \cdots = p_5(t) = 0.1 + \tanh(\pi t) > 0$, $p_6(t) = \cdots = p_{10}(t) = -0.1 - \tanh(\pi t) < 0$, $q_3 = q_6 = q_9 = 2, q_l = -1, l = 1, 2, 4, 5, 7, 8, 10$. *Choose* $c_1 = c_2 = [3, 5, \cdots, 5]^{\mathrm{T}}$;

$$\Gamma_1 = [0.2, 1, 1.5, 1.6, 1.7, 1.8, 1.9, 1.8, 1.9, 1.7]^{\mathrm{T}},$$
$$\Gamma_2 = [-0.1, -0.8, 1, 1.2, 1.4, 1.6, 1.8, 1.9, 1.7, -0.8]^{\mathrm{T}},$$

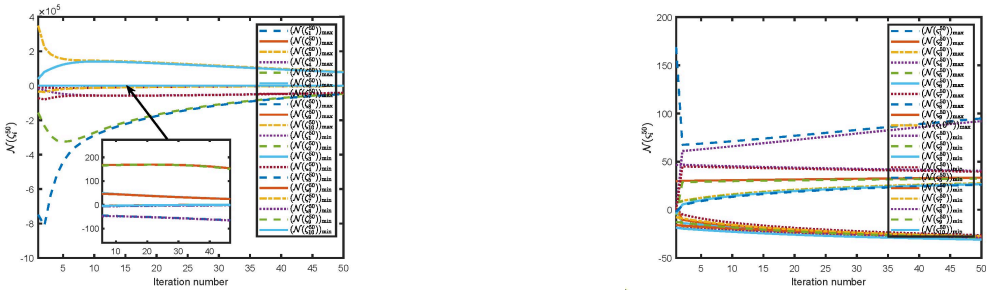
$r_1 = [2, 5, 2, 2.1, 2.3, 2.5, 2.7, 2.8, 2.9, 2.1]^{\mathrm{T}}$; $r_2 = [3, \cdots, 3]^{\mathrm{T}}$; $x_1(0) = [0.5, 0.1, \cdots, 0.1]^{\mathrm{T}}$; $\varsigma(0) = [-1, \cdots, -1]^{\mathrm{T}}$; $x_2(0) = [2.5, 2.2, 1.6, \cdots, 1.6]^{\mathrm{T}}$; $\zeta(0) = [-1.5, 2, -4, 4, \cdots, 4]^{\mathrm{T}}$.

Therefore, by using adaptive laws (23-24), we get the simulation results as Fig.1-4. Fig.1(b) depicts the output trajectories of all agents in the 50th iteration, it is easily seen that the output trajectories perfectly track the desired trajectories in $[0, 2]$, *which achieves the control objective of this paper. Fig.2 shows the change of upper and lower bounds of the Nussbaum-type function* $\mathcal{N}\left(\zeta_i^k\right), \mathcal{N}\left(\varsigma_i^k\right)$ *for ith agent as the number of iterations increases. The responses of consensus are illustrated in Fig.3, (a) represents the control error and it converges to* 0 *as the number of iterations approaches 50; (b) shows the boundedness of* $u_i^k(t)$ *for each follower agent; (c) and (d) imply the boundedness of adaptive parameters* ζ_i^k *and* ς_i^k *respectively. The boundedness of the other adaptive parameters could be illustrated in Fig.4. Finally, simulation results accord with Theorem 3.4, which verifies the feasibility of our proposed algorithm.*



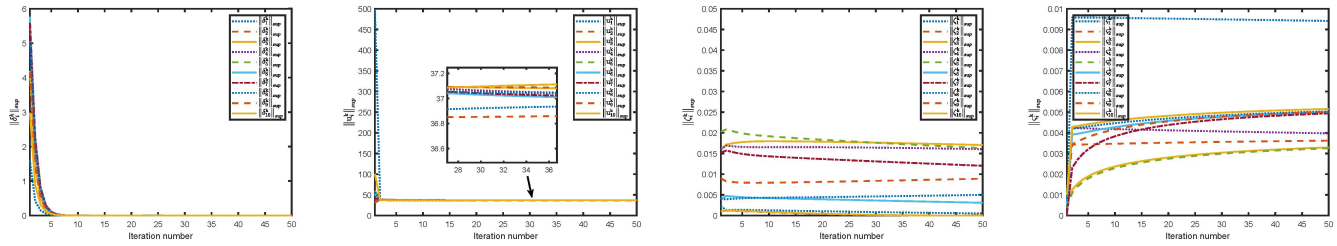
(a) (b)

Figure 1: The topology and the output trajectories of Example 4.1.



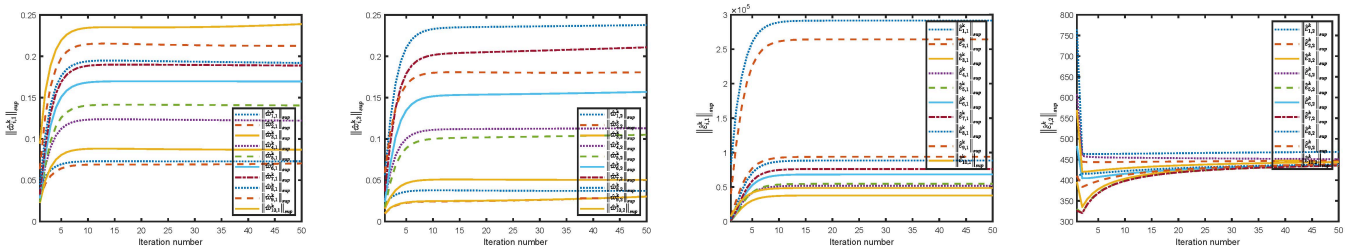
(a) (b)

Figure 2: Evolution of the Nussbaum gain of Example 4.1.



(a) (b) (c) (d)

Figure 3: Response results of consensus of Example 4.1.



(a) (b) (c) (d)

Figure 4: The signals of Example 4.1.

Example 4.2. *Formation control for MASs.*

Next we consider the strict-feedback network single-link robot system in repeated operation, and describe its dynamics as

$$\begin{aligned}\dot{x}_{i,1}^{k} &= G_i x_{i,2}^{k} + 0.1\cos\left(x_{i,1}^{k}\right) + \sigma_{i,1}^{k},\\ \dot{x}_{i,2}^{k} &= J_i^{-1}\left(K_i u_i^{k} - B_i x_{i,2}^{k} - Q_i \sin\left(x_{i,1}^{k}\right)\right) + \sigma_{i,2}^{k},\end{aligned} \tag{66}$$

where $Q_i = x_{i,1}^{k}, \sigma_{i,1}^{k} = -0.1\sin(2\pi t), \sigma_{i,2}^{k} = -0.1\tanh(2\pi t), i = 1,2,3,4$. *Meanwhile, the leader is described by* $y_r(t) = \sin(\pi t) + 2\cos(\pi t), t \in [0,2]$. *Choose* $\bar{\Delta}_1 = -2, \bar{\Delta}_2 = -1, \bar{\Delta}_3 = 1, \bar{\Delta}_4 = 2; J_1 = J_2 = J_3 = J_3 = 2.5; K_1 = K_2 = 2, K_3 = K_4 = -1; G_2 = 3, G_1 = G_3 = G_4 = -2; B_1 = B_2 = B_3 = B_4 = 0.5; c_1 = [0.3, 0.5, 0.5, 0.5]^{\mathrm{T}}; c_2 = [0.3, 0.5, 0.5, 0.5]^{\mathrm{T}};$ $\Gamma_1 = [0.8, 1, 1.5, 1]^{\mathrm{T}}, \Gamma_2 = [0.1, 0.2, 0.3, 0.3]^{\mathrm{T}}; r_1 = [1, 1.5, 0.8, 0.8]^{\mathrm{T}}, r_2 = [0.1, 0.2, 0.3, 0.3]^{\mathrm{T}}; x_1(0) = [1.1, 1.2, 0.1, 0.4]^{\mathrm{T}}, x_2(0) = [-0.4, -0.3, 0.2, 0.3]^{\mathrm{T}};$ $\zeta(0) = [3, -8, -3, -3]^{\mathrm{T}}; \varsigma(0) = [-0.1, -0.2, 0.2, -0.5]^{\mathrm{T}}$.

The the formation control results are shown in Fig.5-8. Apparently, Fig.5(b) depicts the output trajectories of all agents in the 50th iteration, all agents including the leader form the desired formation with the leader, which achieves formation control objective of this paper. Fig.6 shows the change of upper and lower bounds of the Nussbaum-type function $\mathcal{N}(\zeta_i^k), \mathcal{N}(\varsigma_i^k)$ *for ith agent as the number of iterations increases. The responses of formation are showed in Fig.7, (a) represents the formation control error and it converges to* 0 *as the number of iterations approaches 50; (b) shows the boundedness of* $u_i^k(t)$ *for each follower agent; (c) and (d) imply the boundedness of adaptive parameters* ζ_i^k *and* ς_i^k *respectively. The boundedness of the adaptive parameters could be illustrated in Fig.8. The formation control results well fit Theorem 3.6, which further verifies the practicability of the control algorithm.*



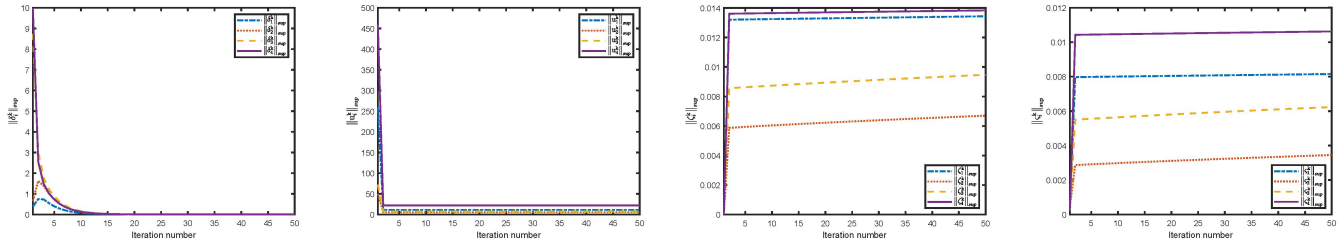
(a) (b)

Figure 5: The topology and the output trajectories of Example 4.2.



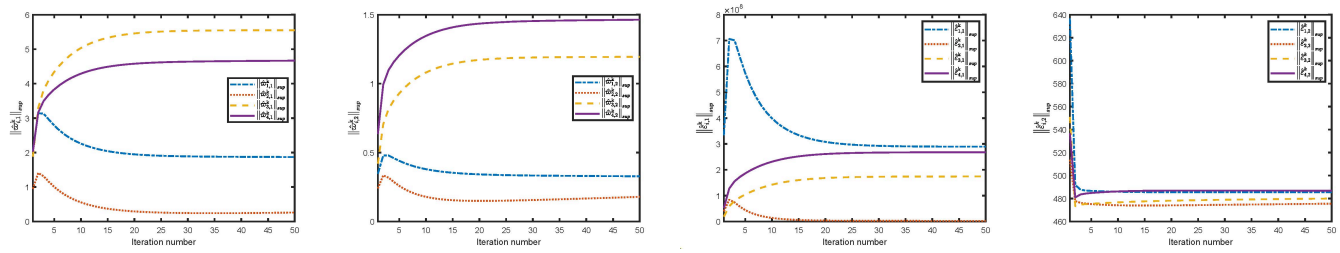
(a) (b)

Figure 6: Evolution of the Nussbaum gain of Example 4.2.



(a) (b) (c) (d)

Figure 7: Response results of formation of Example 4.2.



(a) (b) (c) (d)

Figure 8: The signals of Example 4.2.

5 Conclusions

This paper has solved the ILC consensus for second-order nonlinear strict-feedback MASs with partially unknown control directions on finite time interval. By applying backstepping technique and Nussbaum function, we designed a distributed fuzzy AILC scheme using neighborhood information among all multiple agents. Ultimately, under our new protocols, all follower agents would exactly track the leader's output curve, and the control directions can be adaptively sought by Nussbaum-type items. In addition, the formation results are also confirmed by our protocols. Simulation results further verify the effectiveness of our control algorithm. In the later work, the consensus of high-order pure-feedback or non-strict-feedback MASs will be the focus of our works.

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