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A new interval-valued fuzzy optimization model for truck scheduling in a multi-door cross-docking system by considering transshipment and flexible dock doors extra cost

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Abstract

In a cross-docking system, doors can be used exclusively for receiving or sending operations, or they can be flexible enough to be used for both. Flexible doors have gained attention for improving cross-docking system performance, but their implementation incurs additional costs due to the need for dual-function equipment to handle both receiving and sending operations. Despite the advantages of flexible doors, applying them is usually associated with additional costs due to the need for dual-function equipment to receive and send. Moreover, the distance that goods move within cross-docking facilities, from receiving to sending dock doors, significantly affects how well these facilities work. This paper presents a new bi-objective mixed-integer linear programming model for scheduling the inbound and outbound trucks in a cross-docking facility. The model objectives are to minimize total system costs, including additional costs of applying flexible doors and transshipment (maximize efficiency) and to minimize outbound trucks' tardiness from predetermined due dates (maximize responsiveness). As a result of the uncertainties in cross-dock truck scheduling problems, the parameters are considered triangular interval-valued fuzzy (IVF) numbers. Moreover, a new IVF-uncertain solution approach based on the max-min operator and compromise programming concepts is proposed to solve multi-objective mathematical programming problems with triangular IVF numbers. The proposed model and IVF-solution approach are used for cross-docking operations planning at a well-known food manufacturing group. The results are also analyzed according to their sensitivity to changes in key parameters of the cross-docking problem. Comparison of the proposed approach with other known fuzzy methods for solving multi-objective models demonstrates the superior performance of the proposed approach in the real case study.

Keywords: Cross-docking systems, truck scheduling, flexible dock doors, transshipment, optimization model, interval-valued fuzzy sets.

1 Introduction

Manufacturers often receive orders from customers that are as large as their trucks' capacities, while customers rarely require such large quantities of a product. Intermediary warehouses are a common solution to this problem, but they incur storage costs. An alternative solution is to use cross-docking systems. The goods are transferred to a cross-docking terminal, classified with goods from the same destination as new shipments, and transported together instead of directly to destinations [29, 31, 33]. Consolidation of customer orders from different manufacturers in a cross-docking center leads to maximum use of transportation capacity [37].

Applying cross-docks is as old as the posting service or even older. Ertek [14] stated the Silk Road as an example of cross-docking. However, in the literature, Wal-Mart stores were the first to use cross-docking systems. Analyzing what has made Wal-Mart stores successful in cross-docking systems has been introduced as one of the most important for

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the companys progress [39]. Using this strategy to transfer about 85 percent of products reduced Wal-Mart stores sales costs by 2 to 3 percent compared with competitors. Accordingly, this company became the most profitable retailer in the world in 1992.

The typical cross-docking facility is shaped like an I, with inbound and outbound doors on opposite sides, and trucks are dedicated to receiving and sending docks. However, according to the study by Bartholdi and Gue [3] on cross-dock structure and configuration, only 53.7 percent of the facilities used as cross-docks were designed and built this way. Furthermore, in practical situations, the equipment of many inbound and outbound docks is similar. Despite the implementation and equipment management difficulties, dual functions, namely flexible doors, can improve cross-docking efficiency by allocating the inbound and outbound trucks to the doors [5, 6]. Flexible use of dock doors can reduce wait times for trucks and improve the flow of goods through the facility.

Managing the time to move products from receiving to sending docks is crucial for cross-docking systems. This requires labor and equipment, which is costly. Considering the mentioned costs in determining how to assign inbound and outbound trucks to the doors will improve truck scheduling and reduce system costs. In other words, the trucks' assignment to receive and send docks so that the goods need the slightest movement inside the cross-docking facilities will significantly decrease transshipment costs. Moreover, the time and cost parameters, regarded as uncertain ones, are modeled as triangular interval-valued fuzzy (IVF) numbers. The problem is formulated as a possibilistic bi-objective programming model that aims to minimize outbound trucks' delivery time violations and minimize the sum of internal transportation costs and the costs of utilizing flexible doors. To solve the model, a novel hybrid solution approach was presented that combines the max-min operator and compromise programming concepts, resulting in an efficient method for addressing the uncertainties inherent in cross-docking operations.

The remainder of this paper is organized as follows. The related literature is reviewed in the following section, and the gaps are explained. In section 3, the problem is defined and formulated, and the solution approach is presented. A case study illustrates the implementation of the proposed method in Section 4. Section 5 shows the research results, management insight, and further research.

2 Literature review

Assigning incoming and outgoing trucks to dock doors with high flexibility is an operational issue in a cross-dock scheduling problem. Dock door services have received much attention in the literature; In some cases, researchers have considered a cross-docking facility with one inbound and one outbound door, each with an exclusive service mode. Yu and Egbelu [43] investigated a cross-dock scheduling problem with an inbound dock door and an outbound one. Goods from trucks coming in could be used to meet the needs of trucks leaving. They also proposed a heuristic algorithm based on priority rules to solve the problem. Chen and Lee [9] considered the truck scheduling problem in a cross-dock with an inbound and outbound door as a flow shop scheduling problem. This study developed a heuristic based on branch-and-bound and Johnson's algorithm. Boysen et al. [8] have studied minimizing the maximum processing time of goods in a cross-dock system with one inbound and one outbound door by assuming the goods' interchangeability. The authors developed two different algorithms to solve the assignment of trucks to inbound and outbound doors; They examined the proposed algorithms with extensive numerical experiments. Vahdani and Zandieh [40] and Liao et al. [26] developed several evolutionary algorithms to solve cross-dock truck scheduling problems with single inbound and outbound doors.

Despite the extensive work on cross-docking systems with exclusive service mode dock doors, cross-dock facilities in practice involve mixed-mode dock doors. Real-world applications require a method for assigning incoming and outgoing trucks to dock doors and determining the processing sequence. In a study by Chen and Song [10] that extended Chen and Lee [9], a truck scheduling problem was presented as a two-stage flow shop scheduling problem in cross-docks with mixed-mode dock doors at each workstation; A heuristic was developed to solve the problem. Van Belle et al. [41] addressed a multi-door cross-dock scheduling problem to minimize total tardiness and total travel time. They also developed a tabu search heuristic. Bellanger et al. [4] studied a similar problem and developed a branch-and-bound algorithm and a heuristic for solving small-sized and real-life instances.

Some papers have looked into flexible doors that can be used for receiving and sending operations. Bergman et al. [5] studied the efficiency of flexible doors and exclusive service mode ones on the operational performance of a truck scheduling problem. The authors proposed two integer programming models to minimize products' time on cross-docking. When the proposed models were applied to small-sized instances, the results showed that flexible doors lead to feasible and optimal solutions. They also demonstrated that a limited number of mixed-mode dock doors gave a full benefit cross-docking system. A method was devised to determine how many flexible doors were needed to maximize the system's efficiency.

In a study by Bodnar et al. [6], a cross-docking facility with several doors was considered. Some doors were exclusively used for inbound or outbound operations, while others were flexible and could be used for both. They aimed to find the best allocation of trucks to doors and their processing sequence at a minimum operational cost. The problem was formulated, and a heuristic based on adaptive large neighborhood search was developed to solve it. Computational experiments showed that flexible doors could improve the cross-docking system's efficiency. Gelareh et al. [16] studied the truck-to-door assignment problem in a cross-dock terminal to minimize the total transportation costs. To solve the problem, they developed 11 mixed-integer programming models. Using CPLEX software and a time limit of 7200 seconds, the authors solved 50 sample problems designed by Guignard et al. [18] by the proposed models, and their efficiency was compared.

In a study involving mixed-mode dock doors, Rijal et al. [34] presented an integrated approach to simultaneously consider truck scheduling and dock-door assignment. They aimed to minimize total system costs, including transportation, temporary storage, and total tardiness costs. An adaptive large neighborhood search algorithm was developed to find suitable solutions within a reasonable timescale. Furthermore, the authors assessed the appropriate number of flexible doors required and their impacts on savings. Their research showed that placing the flexible doors in the middle of cross-dock terminals saved more money than placing them at the edges. Correa Issi et al. [12] introduced a mixed-integer programming model for scheduling the inbound and outbound trucks in a cross-dock facility with flexible docks to minimize the time between the arrival and departure of the first and last truck. The proposed model was implemented in a Chilean food manufacturing company, and the results showed the model's efficiency for solving the model in different scenarios.

Shahmardan and Sajadieh [38] investigated a cross-dock truck scheduling problem, in which an incoming truck could eventually be used as an outgoing truck departing to a specific destination. So, unloading the products that belong to that destination was unnecessary, and only those that belong to the other destinations must be unloaded. Next, the truck could be loaded with products with the same destination. The proposed model aimed to assign the inbound trucks to the destinations and schedule them to minimize the makespan. They also developed several local search algorithms for solving the formulated model and improved their performance in various situations by applying some reinforcement learning methods.

Some existing literature on cross-docking systems emphasized transshipment in addition to a truck assignment. Maknoon and Baptise [27] focused on how to model transshipment costs and what factors affected them. Alpan et al. [2] studied a transshipment scheduling problem to minimize the inventory holding and truck replacement costs in a cross-docking system. They also proposed several heuristics based on a dynamic programming model to obtain near-optimal solutions. Sayed et al. [36] presented an integrated model for cross-dock door assignment and truck scheduling problems to minimize the total time required for all trucks to be processed. They developed two hybrid meta-heuristics to solve the formulated mathematical models and evaluated them in some instances.

Considering the uncertainty in the models parameters can also improve the results of implementing the proposed models in a cross-docking system. Researchers developed different methods to deal with uncertainty in cross-dock scheduling problems. Konur and Golias [21] studied the scheduling of inbound trucks in a cross-docking facility, considering the trucks' arrival time uncertainty. They took the arrival time of each truck as a time interval and solved the problem by a bi-level optimization method involving optimistic and pessimistic approaches. A modification of GA was proposed to solve the bi-level optimistic and pessimistic formulations. The new method did better than deterministic, optimistic, and pessimistic methods in test problems.

Mousavi [30] presented a truck scheduling problem in a multi-door cross-docking system, considering a specific delivery time for outbound trucks as a hard constraint that can regroup some of the products in outsourced trucks. Their formulated mixed-integer programming model aimed to minimize operational costs and maximize the quality of products shipped from a cross-dock facility. They considered the uncertainty in the model parameters and developed an IVF-programming method to solve the problem. Essghaier et al. [15] addressed the truck-to-door assignment in a cross-dock scheduling problem. They proposed a new form of collaboration between suppliers to produce identical products by sharing resources for service level improvement and economic cost reduction. The authors modeled the uncertainty in transfer times as a triangular fuzzy number and solved the formulated possibilistic programming problem with the fuzzy chance programming method.

According to the literature review, researchers frequently considered flexible dock doors for truck scheduling problems in cross-docking systems. Moreover, several objective functions were outlined, such as minimizing total operation time [5, 12] and total costs [6, 34]. Despite the ability to improve system performance, using flexible doors lead to higher system costs due to the increased equipment costs and the effort required to provide them. In light of the lack of literature addressing the additional cost of using flexible doors, one should weigh their advantages and costs before deciding whether to use them.

This paper investigates the effect of extra costs that come with using flexible doors. Furthermore, based on the

position of inbound and outbound trucks, the required transshipment time is considered in assigning trucks to the doors. So, as a measure of how well the cross-docking system works, this paper minimizes the costs associated with using flexible doors and transshipment (maximizing efficiency). Another criterion of the cross-docking system is the violation of outgoing trucks' delivery times from predefined due dates. Thus, the second objective function of the proposed model is to minimize the total tardiness of outbound trucks (maximizing responsiveness).

The use of triangular IVF numbers to show the uncertainty in the models parameters is another way this study differs from the previous ones. A new hybrid uncertain solution approach is presented to solve multi-objective programming models with triangular IVF numbers. The proposed method is applied to a well-known food manufacturer in Iran as a case study to schedule trucks at a cross-docking center. Ultimately, the results are analyzed and compared with similar methods.

In summary, the followings are the paper's novelties:

- A new bi-objective mixed-integer linear programming model is introduced for the inbound and outbound truck scheduling in a cross-docking system. This model has two objective functions: minimizing total costs (maximize efficiency) and minimizing the outbound trucks' tardiness (maximize responsiveness).
- Flexible dock doors' extra cost and products' transshipment time and cost within the cross-dock are considered in assigning trucks to inbound, outbound, and flexible dock doors.
- Triangular IVF numbers express the uncertainty in some model parameters.
- A new hybrid uncertain solution approach is proposed to tackle multi-objective mixed-integer linear programming problems with triangular IVF parameters.
- The proposed model and solution approach are used for scheduling trucks at a cross-docking facility in a real-world case study at a reputable food-making group in Iran. The results are compared with similar fuzzy solving methods.

3 Proposed methodology

3.1 Problem definition

This paper studies a multi-door cross-docking system. As described by Boysen and Fliedner [7], a cross-dock truck scheduling problem consists of three essential elements defined by the tuple $[\alpha|\beta|\gamma]$. Door environments are represented by two attributes in the first element, α:

- $\alpha_1 \in \{E, M, EM, G\}$: In a cross-docking terminal, α_1 indicates the dock doors service mode. $\alpha_1 = E$ shows that dock doors can be exclusively dedicated to inbound or outbound operations, while $\alpha_1 = M$ shows an intermixed sequence of inbound and outbound trucks that can be processed on the dock doors. In $\alpha_1 = EM$, both service modes can be applied concurrently; it means that one set of doors can operate exclusively and the other in a mixed mode, and there is a fixed assignment between doors and destinations in $\alpha_1 = G$.
- $\alpha_2 \in \{0, k\}$: The number of dock doors in a multi-cross-docking system indicates by α_2. Using $\alpha_2 = 0$, the number of dock doors varies from terminal to terminal, and using $\alpha_2 = k$, the number of dock doors can be restricted to a given k.

The operational characteristics influencing the structure of a truck scheduling problem in cross-docking systems can be classified by the second element β, which contains nine different attributes (β_1 to β_9):

- $\beta_1 \in \{0, pmtn\}$: The value of β_1 indicates whether interruptions are allowed in unloading and loading operations ($\beta_1 = 0$) or not ($\beta_1 = pmtn$).
- $\beta_2 \in \{0, r_j\}$: When all trucks arrive at time zero (the beginning of the planning period), $\beta_2 = 0$, and when they arrive at different times, $\beta_2 = r_j$.
- $\beta_3 \in \{0, (p_j = p), \underline{p} \leq p \leq \overline{p}\}$: A truck's processing time, or the entire time takes to load or unload its products, is shown by β_3. When each truck has its processing time, $\beta_3 = 0$; when trucks' processing times are equal,$\beta_3 = (p_j = p)$; and when trucks' processing times are regarded as intervals, $\beta_3 = \underline{p} \leq p \leq \overline{p}$.

- $\beta_4 \in \{0, \tilde{d}_\lambda\}$: The deadline for truck arrivals or product deliveries is represented by β_4. $\beta_4 = 0$ when no deadlines are assumed in the system; and $\beta_4 = \tilde{d}_\lambda$ when deadlines are defined for trucks or shipments, with subscript λ indicating which element has due dates.

- $\beta_5 \in \{0, no - wait\}$: Intermediate storage is shown by β_5. If intermediate storage is allowed, $\beta_5 = 0$, and if some products cannot be stored at all, the $no - wait$ property holds.

- $\beta_6 \in \{0, doors\}$: β_6 refers to truck-to-door assignment restrictions. If there are no restrictions, any dock door can be used for truck processing ($\beta_6 = 0$), and there may be some trucks that can be processed only at a particular dock door ($\beta_6 = doors$).

- $\beta_7 \in \{t_{io}, 0, (t_j = 0)\}$: Describing how long it takes for products to be transported between the time they are unloaded from inbound trucks and loaded into outbound trucks. In the case where transshipment times vary according to which dock doors a shipment moves between, $\beta_7 = t_{io}$. Nevertheless, if all the products are assumed to have independent transshipment times, $\beta_7 = 0$, and if no transshipment times are assumed, $\beta_7 = (t_j = 0)$.

- $\beta_8 \in \{0, fix\}$: Outbound truck departure times are determined by β_8, which is the organizational policy. The outbound truck leaves the terminal immediately after it has loaded its predefined set of goods, so $\beta_8 = 0$. However, if the outbound truck leaves at a predetermined time, irrespective of the products loaded, $\beta_8 = fix$.

- $\beta_9 \in \{0, change\}$: A product's interchangeability denotes by β_9, which is determined by whether or not it can be changed with another. In the case that the products arriving at the terminal are assigned to specific outbound trucks, $\beta_9 = 0$, and when the number and type of products loaded per outbound truck might be specified so that any outbound truck can satisfy their demand for this product with product units of a specific type, $\beta_9 = change$.

The third element (γ) consists of the objective functions that guide the evaluation of solutions. Some typical objective functions are the makespan ($\gamma = C_{max}$), the weighted sum of completion times ($\gamma = \sum w_\lambda C_\lambda$), the maximum lateness ($\gamma = L_{max}$), the total weighted tardiness ($\gamma = \sum w_\lambda T_\lambda$).

This study divides the doors into three categories: exclusive service mode dock doors for receiving (in), exclusive service mode dock doors for sending (out), and mixed-mode dock doors for receiving and sending concurrently. So, a subset of doors can be operated exclusively, and the others in a mixed mode ($\alpha_1 = EM$). Flexible dock doors cost more than exclusive service mode doors because they need loading and unloading facilities and human resources to set up and move them. These costs are also factored into the total cost of the system.

Preemption is not allowed in loading and unloading the assigned trucks. After placing them on the docks, the inbound and outbound truck operations continue until all products' unloading and loading processes are completed ($\beta_1 = 0$). Truck arrival times at cross-dock facilities differ from one another ($\beta_2 = r_j$). The time takes to unload and load products depends on how many products are put on each truck. This time also varies from truck to truck ($\beta_3 = 0$). Outgoing trucks have due dates to complete the loading operations; violating them will result in penalties ($\beta_4 = d_\lambda$). Goods can be temporarily stored in cross-docks, and the time and capacity of temporary storage are assumed to be unlimited ($\beta_5 = 0$). There are no limits on how trucks can be assigned to doors. Incoming trucks can be assigned to any inbound or flexible door, and outgoing trucks can be assigned to any outbound or flexible door ($\beta_6 = 0$). How long it takes to move each product within the cross-docking facility depends on the distance between the doors where shipments come in and where they go out ($\beta_7 = t_{io}$). Each outgoing truck leaves the cross-dock when all products are loaded ($\beta_8 = 0$). The number and type of products that need to go on each output truck are known, but determining how incoming goods should be assigned to outgoing trucks is a decision variable ($\beta_9 = change$). The purpose of solving the problem is to assign incoming and outgoing trucks to the doors and figure out their sequence so that the following two objectives can be met:

1. Minimizing outbound truck tardiness equals violating their departure time from their due date ($\gamma_1 = \sum T_j$).

2. Minimizing total costs, including extra costs of using flexible doors and transshipment costs ($\gamma_2 = TC$).

3.2 Problem formulation

Sets and indices:

$R = \{1, ..., \overline{R}\}$	Set of inbound trucks, indexed by $i, j \in R$
$S = \{1, ..., \overline{S}\}$	Set of outbound trucks, indexed by $i, j \in S$
$P = \{1, ..., \overline{P}\}$	Set of different products, indexed by $k \in P$
$In = \{1, ..., \overline{In}\}$	Set of inbound doors
$Out = \{1, ..., \overline{out}\}$	Set of outbound doors
$F = \{1, ..., \overline{F}\}$	Set of flexible doors (allowed to be assigned to inbound and outbound trucks)
$I = In \cup Out \cup F = \{1, ..., \overline{I}\}$	Set of available doors, indexed by $m, n \in I$

Parameters:

r_{ik}	Number of unit loads of product k loaded onto inbound truck i
s_{jk}	Number of unit loads of product k to be loaded onto outbound truck j
$\widetilde{\widetilde{DT}}$	Delay for truck replacement
$\tilde{\tilde{t}}_{mn}$	Transportation time between door m and n
$\widetilde{\widetilde{cost}}_{mn}$	Transportation cost between door m and n
d_j	Due date of outbound truck j
$\widetilde{\widetilde{Al}}_i$	Ready time of inbound truck i
$\widetilde{\widetilde{AO}}_j$	Ready time of outbound truck j
$\widetilde{\widetilde{ULT}}$	Time required for unloading per unit load
$\widetilde{\widetilde{LT}}$	Time required for loading per unit load
$\tilde{\tilde{\delta}}$	Additional costs incurred for every use of every flexible door
M	A big number

Positive variables:

c_i	Start time of truck i processing
C_i	Completion time of truck i processing
T_j	Tardiness of truck j
x_{ijk}	Number of product k moves from inbound truck i into the outbound truck j

Binary variables:

y_{im}	Equals 1, if truck i is assigned to door m; equals 0 otherwise.
q_{ij}	Equals 1, if truck i and truck j are assigned to one door and truck i is processed before truck j processing; equals 0 otherwise.
v_{ij}	Equals 1, if each product of truck i is moved to truck j; equals 0 otherwise.
u_{ijmn}	Equals 1, if truck i and truck j are assigned to door m and door n, respectively, and at least one product goes from truck i to truck j; equals 0 otherwise.

Mathematical model:

$$\text{Min } z_1 = \sum_{j \in S} T_j \tag{1}$$

$$\text{Min } z_2 = \sum_{m \in l} \sum_{n \in l} \sum_{i \in R} \sum_{j \in S} \widetilde{\widetilde{\text{cost}}}_{mn} \cdot u_{ijmn} + \tilde{\tilde{\delta}} \sum_{i \in R \cup S} \sum_{m \in F} y_{im} \tag{2}$$

S.t.

$$\sum_{j \in S} x_{ijk} = r_{ik} \qquad i \in R, k \in P \tag{3}$$

$$\sum_{i \in R} x_{ijk} = s_{jk} \qquad j \in S, k \in P \tag{4}$$

$$\sum_{m \in In \cup F} y_{im} = 1 \qquad i \in R \tag{5}$$

$$\sum_{m \in Out \cup F} y_{im} = 1 \qquad i \in S \tag{6}$$

$$c_i \geq \widetilde{\widetilde{AI}}_i \qquad i \in R \tag{7}$$

$$c_j \geq \widetilde{\widetilde{AO}}_j \qquad j \in S \tag{8}$$

$$y_{im} + y_{jm} \leq q_{ij} + q_{ji} + 1 \qquad i \in R, j \in S (i \neq j), m \in I \tag{9}$$

$$c_j \geq C_i + \widetilde{\widetilde{DT}} - M(3 - q_{ij} - y_{im} - y_{jm}) \qquad i, j \in R \cup S, i \neq j, m \in I \tag{10}$$

$$c_i + \widetilde{\widetilde{ULT}} \sum_{k \in p} r_{ik} \leq C_i \qquad i \in R \tag{11}$$

$$c_j + \widetilde{\widetilde{LT}} \sum_{k \in p} s_{jk} \leq C_j \qquad j \in S \tag{12}$$

$$\sum_{k \in P} x_{ijk} \leq M.v_{ij} \qquad i, j \in R \cup S \tag{13}$$

$$u_{ijmn} \geq v_{ij} + y_{im} + y_{jn} - 2 \qquad i \in R, j \in S, m, n \in I \tag{14}$$

$$C_j \geq c_i + \widetilde{\widetilde{LT}} \sum_{k \in P} x_{ijk} + \widetilde{\widetilde{t}}_{mn} - M(1 - u_{ijmn}) \qquad i \in R, j \in S, m, n \in I \tag{15}$$

$$T_j = max\{0, C_j - d_j\} \qquad j \in S \tag{16}$$

$$c_i, C_i, T_j, x_{ijk} \geq 0 \tag{17}$$

$$y_{im}, v_{ij}, q_{ij}, u_{ijmn} \in \{0, 1\} \tag{18}$$

In the proposed mixed-integer linear programming model, the first objective function (1) represents total tardiness. The second objective function (2) shows the total cost, including the transshipment cost and the additional cost of using flexible doors. Constraint (3) ensures that all shipments are unloaded from inbound trucks and assigned to outbound trucks. Constraint (4) controls that all products unloaded from inbound trucks are loaded onto outbound trucks to the same extent as the definite demand. Constraint (5) indicates that each inbound truck has to be assigned to an outbound door or a flexible one. Similarly, constraint (6) ensures that each outbound truck has to be assigned to an outbound door or a flexible one. Constraints (7) and (8) ensure that the receiving and sending operations will only start when the involved trucks are ready. Constraints (9) and (10) guarantee that if different trucks are assigned to a door, the processing operation of each truck will start after the end of the previous truck operation. Constraint (11) indicates that the process of an inbound truck is completed when all of its products are unloaded. Constraint (12) shows that the process of an outbound truck is completed when all of the products that belong to it are loaded. Constraints (13), (14), and (15) calculate the required time for transferring goods from inbound trucks to outbound ones. Constraint (16) determines the truck's tardiness based on delivery violations of predetermined due dates. Finally, constraints (17) and (18) define the domain of decision variables.

3.3 Proposed IVF solution approach

Due to uncertainty in the time and cost parameters, the normalized triangular interval-valued fuzzy (IVF) numbers are used in this study, and their specifications can be found in the Appendix. Other parameters, e.g., the number of products in inbound trucks and the due dates of outbound trucks, are crisp and predetermined. Uncertain parameters are specified by the double Tilde symbol above them ($\widetilde{\widetilde{\ }}$) in Section 3.2. This research presents a new hybrid IVF solution approach based on the max-min operator and compromise programming concepts using reliable methods in the literature [20, 22, 24, 25, 44]. The most important advantages of the proposed approach over the literature are:

- Grasping the uncertainty and vagueness in model parameters more precisely by using triangular IVF numbers.
- By using the max-min operator, the worst-case satisfaction level of objective functions is maximized, and other objectives cannot compensate for the results with better ones. The proposed method, by using compensatory operators, can obtain a better compromise solution.

Model (19) is a multi-objective linear programming (MOLP) model with normalized triangular IVF parameters.

$$\begin{aligned}
&Min \quad f_r(x) = (c_1^U, c_1^L, c_2, c_3^L, c_3^U)x \qquad r = 1, 2, \cdots, R \\
&Max \quad f_s(x) = (v_1^U, v_1^L, v_2, v_3^L, v_3^U)x \qquad s = 1, 2, \cdots, S \\
&S.t. \\
&(a_1^U, a_1^L, a_2, a_3^L, a_3^U)x \leq (b_1^U, b_1^L, b_2, b_3^L, b_3^U) \\
&x \geq 0
\end{aligned} \tag{19}$$

Where r and s indicate minimization and maximization objective functions. The steps of the proposed procedure are as follows:

Step 1: Using the lower (L) and upper (U) limits of triangular IVF numbers, construct models (20) and (21) as possibilistic programming models with triangular fuzzy numbers (TFNs).

$$\begin{aligned}
&Min \quad f_{rL}(x) = (c_1^L, c_2, c_3^L)x \qquad r = 1, 2, \cdots, R \\
&Max \quad f_{sL}(x) = (v_1^L, v_2, v_3^L)x \qquad s = 1, 2, \cdots, S \\
&S.t. \\
&(a_1^L, a_2, a_3^L)x \leq (b_1^L, b_2, b_3^L) \\
&x \geq 0
\end{aligned} \tag{20}$$

$$\begin{aligned}
&Min \quad f_{rU}(x) = (c_1^U, c_2, c_3^U)x \qquad r = 1, 2, \cdots, R \\
&Max \quad f_{sU}(x) = (v_1^U, v_2, v_3^U)x \qquad s = 1, 2, \cdots, S \\
&S.t. \\
&(a_1^U, a_2, a_3^U)x \leq (b_1^U, b_2, b_3^U) \\
&x \geq 0
\end{aligned} \tag{21}$$

Step 2: Make models (22) and (23) that are crisp equivalents of models (20) and (21) by using the methods proposed by Lai and Hwang [22] for objective functions and Jimenez et al. [20] for constraints. The alpha coefficient (α) is the measure of feasibility introduced by Jimenez et al. [20].

$$
\begin{aligned}
&Min \quad f_{rL1}(x) = (c_3^L - c_2)x \qquad r = 1, 2, \cdots, R\\
&Min \quad f_{rL2}(x) = (c_2)x \qquad r = 1, 2, \cdots, R\\
&Max \quad f_{rL3}(x) = (c_2 - c_1^L)x \qquad r = 1, 2, \cdots, R\\
&Max \quad f_{sL1}(x) = (v_3^L - v_2)x \qquad s = 1, 2, \cdots, S\\
&Max \quad f_{sL2}(x) = (v_2)x \qquad s = 1, 2, \cdots, S\\
&Min \quad f_{sL3}(x) = (v_2 - v_1^L)x \qquad s = 1, 2, \cdots, S\\
&S.t.\\
&[\alpha(a_2 + a_3^L) + (1-\alpha)(a_1^L + a_2)]x \leq [\alpha(b_2 + b_3^L) + (1-\alpha)(b_1^L + b_2)],\\
&x \geq 0
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
&Min \quad f_{rU1} = (c_3^U - c_2)x \qquad r = 1, 2, \cdots, R\\
&Min \quad f_{rU2} = (c_2)x \qquad r = 1, 2, \cdots, R\\
&Max \quad f_{rU3} = (c_2 - c_1^U)x \qquad r = 1, 2, \cdots, R\\
&Max \quad f_{sU1} = (v_3^U - v_2)x \qquad s = 1, 2, \cdots, S\\
&Max \quad f_{sU2} = (v)x \qquad s = 1, 2, \cdots, S\\
&Min \quad f_{sU3} = (v_2 - v_1^U)x \qquad s = 1, 2, \cdots, S\\
&S.t.\\
&[\alpha(a_2 + a_3^U) + (1-\alpha)(a_1^U + a_2)]x \leq [\alpha(b_2 + b_3^U) + (1-\alpha)(b_1^U + b_2)],\\
&x \geq 0
\end{aligned}
\tag{23}
$$

Step 3: Set α as the feasibility degree, solve multi-objective models (22) and (23) separately and create a payoff table for each of them. The payoff table for model (22) is shown in Table 1, for example, where x^*_{rLi} and x^*_{sLj} are the decision variables that make $f_{rLi}(x)$ and $f_{sLi}(x)$ optimal.

Table 1: Payoff table for model (22)

	$f_{rL1}(x)$	$f_{rL2}(x)$	$\cdots$	$f_{sL1}(x)$	$f_{sL2}(x)$	$\cdots$	x
$f_{rL1}(x)$	$f_{rL1}(x^*_{rL1})$	$f_{rL2}(x^*_{rL1})$	$\cdots$	$f_{sL1}(x^*_{rL1})$	$f_{sL2}(x^*_{rL1})$	$\cdots$	x^*_{rL1}
$f_{rL2}(x)$	$f_{rL1}(x^*_{rL2})$	$f_{rL2}(x^*_{rL2})$	$\cdots$	$f_{sL1}(x^*_{rL2})$	$f_{sL2}(x^*_{rL2})$	$\cdots$	x^*_{rL2}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$f_{sL1}(x)$	$f_{rL1}(x^*_{sL1})$	$f_{rL2}(x^*_{sL1})$	$\cdots$	$f_{sL1}(x^*_{sL1})$	$f_{sL2}(x^*_{sL1})$	$\cdots$	x^*_{sL1}
$f_{sL2}(x)$	$f_{rL1}(x^*_{sL2})$	$f_{rL2}(x^*_{sL2})$	$\cdots$	$f_{sL1}(x^*_{sL2})$	$f_{sL2}(x^*_{sL2})$	$\cdots$	x^*_{sL2}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Calculate the positive ideal solution (PIS) and negative ideal solution (NIS) for objective functions using the payoff tables. In the case of minimizing objective functions, the minimum amount under each column shows PIS, and the maximum amount shows NIS. To maximize objective functions, the minimum amount under their columns shows NIS, and the maximum amount shows PIS. Convert multi-objective models (22) and (23) to a bi-objective model using the TOPSIS for MODM method [24]. To create equivalent constraints, define β as an average coefficient for converting triangular IVF numbers to TFNs and build model (24).

$$
\begin{aligned}
&Min \quad DI = \left[\left(\frac{f_{rL1}-f_{rL1}^{PIS}}{f_{rL1}^{NIS}-f_{rL1}^{PIS}}\right)+\left(\frac{f_{rL2}-f_{rL2}^{PIS}}{f_{rL2}^{NIS}-f_{rL2}^{PIS}}\right)+\left(\frac{f_{rL3}^{PIS}-f_{rL3}}{f_{rL3}^{PIS}-f_{rL3}^{NIS}}\right)+\cdots\right],\\
&Max \quad DN = \left[\left(\frac{f_{rL1}^{NIS}-f_{rL1}}{f_{rL1}^{NIS}-f_{rL1}^{PIS}}\right)+\left(\frac{f_{rL2}^{NIS}-f_{rL2}}{f_{rL2}^{NIS}-f_{rL2}^{PIS}}\right)+\left(\frac{f_{rL3}-f_{rL3}^{NIS}}{f_{rL3}^{PIS}-f_{rL3}^{NIS}}\right)+\cdots\right],\\
&S.t. \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (24)\\
&[\alpha(a_2+\beta a_3^L+(1-\beta)a_3^U)+(1-\alpha)(\beta a_1^U+(1-\beta)a_1^L+a_2)]x \leq\\
&[\alpha(b_2+\beta b_3^L+(1-\beta)b_3^U)+(1-\alpha)(\beta b_1^U+(1-\beta)b_1^L+b_2)],\\
&x \geq 0
\end{aligned}
$$

Where DI shows the distance to ideal solutions and DN to non-ideal solutions in TOPSIS for MODM method.

Step 4: Set α and β and solve model (24) and create a payoff table for it. Calculate PIS and NIS for both objectives. Define μ_{DI} and μ_{DN} as membership functions for the objectives based on the concept of fuzzy sets developed by Zimmermann [44]:

$$\mu_{DI} = \frac{DI^{NIS}-DI}{DI^{NIS}-DI^{PIS}}, \tag{25}$$

$$\mu_{DN} = \frac{DN-DN^{NIS}}{DN^{PIS}-DN^{NIS}}. \tag{26}$$

According to the proposed method by Li et al. [25] and using the max-min operator and the degree of satisfaction (λ_1) model (27) can be determined:

$$
\begin{aligned}
&Max \quad \lambda_1\\
&S.t.\\
&\lambda_1 \leq \mu_{DI}\\
&\lambda_1 \leq \mu_{DN} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (27)\\
&[\alpha(a_2+\beta a_3^L+(1-\beta)a_3^U)+(1-\alpha)(\beta a_1^U+(1-\beta)a_1^L+a_2)]x \leq\\
&[\alpha(b_2+\beta b_3^L+(1-\beta)b_3^U)+(1-\alpha)(\beta b_1^U+(1-\beta)b_1^L+b_2)]\\
&x \geq 0
\end{aligned}
$$

Solve model (27) and obtain the minimum satisfaction level of objective functions.

Step 5: Set λ_1^* as the optimal level of λ_1. By adding the optimal satisfaction level as a constraint to model (28), the solution is forced to be better than what the max-min operator came up with.

$$
\begin{aligned}
&Max \quad \overline{\lambda} = \frac{\lambda_2^1+\lambda_2^2}{2}\\
&S.t.\\
&\lambda_1^* \leq \lambda_2^1 \leq \mu_{DI}\\
&\lambda_1^* \leq \lambda_2^2 \leq \mu_{DN} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (28)\\
&[\alpha(a_2+\beta a_3^L+(1-\beta)a_3^U)+(1-\alpha)(\beta a_1^U+(1-\beta)a_1^L+a_2)]x \leq\\
&[\alpha(b_2+\beta b_3^L+(1-\beta)b_3^U)+(1-\alpha)(\beta b_1^U+(1-\beta)b_1^L+b_2)]\\
&x \geq 0
\end{aligned}
$$

Where λ_2^1 and λ_2^2 are new satisfaction levels of DI and DN objectives. The results obtained from solving model (28) are the final preferred solutions. An overview of the proposed approach is shown in Figure 1.

Start

Consider model (19) as a possibilistic multi-objective programming model with triangular IVF parameters.

Step 1: Convert the MOLP model with normalized triangular IVF parameters (model 19) into two separate MOLP models with TFNs (models 20 and 21).

Step 2: Create models (22) and (23) as crisp equivalent forms of models (20) and (21), respectively, by:

2.1. Creating crisp objective functions using Lai and Hwang method,

2.2. Defining α as the feasibility level and developing equivalent constraints using the Jimenez et al. method.

Step 3: Convert two MOLP models (22) and (23) to a crisp bi-objective model (24) by:

3.1. Setting α as the feasibility level,

3.2. Solving models (22) and (23) separately, creating a payoff table for each of them, and calculating the PIS and NIS for objective functions,

3.3. Using TOPSIS for MODM to create objectives,

3.4. Defining β as an average coefficient to convert triangular IVF numbers in constraints in model (19) to TFNs and create equivalent constraints using the Jimenez et al. method.

Step 4: Apply 1st phase for solving model (24) with:

4.1. Setting α and β, solving the model (24), and creating a payoff table,

4.2. Calculating the linear membership function for each objective and creating model (27),

4.3. Solving model (27) and obtaining the minimum satisfaction level of the objective functions.

Step 5: Applying 2nd phase for solving model (24) with:

5.1. Setting the optimal value of λ_1 in step 4 as λ_1^*,

5.2. Defining two variables λ_2^1 and λ_2^2 and then creating model (28),

5.3. Solve model (28) and report the amount of decision variables as the final preferred solution.

End

Figure 1: An overview of the proposed approach

4 Case study

This study used the proposed approach for scheduling trucks in a cross-dock owned by a well-known Iranian food manufacturing group. This group produces various products in several factories in different cities in Iran. It produces approximately 2300 stock-keeping units (SKU) of products, including dairy products, ready-made and semi-finished food products, meat products, beverages, bread, and chocolates. The mentioned products are produced in manufacturing plants, including two dairy factories, two ready-made and semi-finished food factories, two meat products factories, one beverage factory, one chocolate factory, and two bread factories located in different parts of Iran. As a group subsidiary, a distribution company distributes products from all the factories. Various products are sent directly from the factories to various cross-docks located in Iran's north, south, west, east, and center. After the products are unloaded in the cross-docks, they are combined with products manufactured by other factories based on customers' orders, loaded on outbound trucks, and sent to customers. The advantages of cross-docking are that it eliminates the need for sending products directly from factories to customers and improves the efficiency of the group's logistics operations.

4.1 Implementation

A cross-docking center located in Tehran was selected to investigate the proposed approach. This cross-docking terminal covers an area of 7,000 square meters and has seven doors. Two doors are for incoming trucks, three are for outgoing trucks, and two are flexible. The operational plan of this center for one day has been considered in detail in this research. Nine incoming trucks containing around 200 tons of products manufactured by different factories in the group enter the center at the beginning of the shift. Twelve trucks must carry the products to distribution centers. Pallet units are used to process the products. Table 2 lists the service modes of the different doors and the distance between them.

The CPLEX solver and GAMS software were utilized to solve the proposed model, running on a computer with an Intel (R) Core (TM) i7-2670QM CPU @ 2.20GHz, and RAM of 6.00 GB. Table 3 shows how inbound and outbound trucks are assigned to mixed service mode dock doors, along with loading and unloading times and delays experienced

Table 2: Doors service mode and distance between each two of them

Service Mode	Door No.	1	2	3	4	5	6	7
Receive	1	0	10	80	90	10	120	130
Receive	2	10	0	90	100	90	110	120
Flexible	3	80	90	0	10	85	95	95
Flexible	4	90	100	10	0	95	85	85
Send	5	100	90	85	95	0	20	30
Send	6	120	110	95	85	20	0	10
Send	7	130	120	95	85	30	10	0

by outbound trucks. α and β are both assumed to be 0.5. The results were presented according to the three following considerations:

1. Minimizing total tardiness objective function (maximizing responsiveness).
2. Minimizing total cost objective function (maximizing efficiency).
3. Minimizing tardiness and total costs simultaneously by the proposed bi-objective model.

The proposed solution approach was simplified and adapted for solving the first and second cases. Table 4 shows the information regarding the assignment of inbound and outbound trucks to mixed service mode dock doors and the objective function values of the three cases mentioned. In the first case, six of the total inbound and outbound trucks are assigned to flexible doors.

In the second case, five trucks used flexible doors during the receiving and sending processes. The results confirm that considering the additional cost of using flexible doors and the transshipment cost can improve the efficiency of flexible doors. Additionally, examining objective function values reveals the conflict between these two criteria.

In the third case (the proposed bi-objective model), flexible doors were used six times. The best truck-to-door assignment and servicing sequence can be derived when responsiveness and efficiency, which are two conflicting criteria, are considered together. In this situation, the DMs can use the best-preferred solution to solve a problem while keeping both criteria at acceptable levels.

4.2 Discussion of results

One of this paper's contributions is to consider the additional cost of flexible door utilization when determining how many times to employ them. To investigate the effectiveness of this factor, the variation in the number of flexible doors and the increasing cost associated with their use through the three cases presented in Figure 2.

In the first case, where the objective is to maximize responsiveness by decreasing the tardiness of outgoing trucks, increasing the cost of using flexible doors did not affect the number of flexible doors. This occurs because the objective function focuses on minimizing tardiness rather than the costs associated with maximizing responsiveness.

However, in the second and third cases, an increase in the cost of using flexible doors led to a gradual decrease in the number of times they were used. In the second case, an increase of 100% in the cost of using flexible doors made using flexible doors unaffordable. In the third case, the number of flexible doors decreased slower than in the second. Finally, the increased cost of using flexible doors by 120% made their use uneconomical in the third case. As a result, if the only decision criterion is to improve responsiveness (first case), the costs associated with using flexible doors will not affect how frequently they are used. When efficiency is a concern (second and third cases), the extra cost of using it should be considered.

As an evaluation of the proposed IVF solution approach, its efficacy in solving the considered problem was compared with some well-known fuzzy solution methods from the literature, suggested by Lai and Hwang [23], Ozgen and Gulsun [32], and Alavidoost et al. [1]. Table 5 shows the results of comparing solutions derived from different methods with varying degrees of feasibility (α).

Only the upper level of the triangular IVF numbers was considered TFNs, and the beta (β) coefficient and step 1 were skipped to make the comparison conditions more equal. Figures 3 and 4 demonstrate how well the proposed method compares to others.

Table 3: Results obtained from solving the case study problem

Optimization driver	Frequency of using doors with different service modes Using frequency			Objective functions	
	Receiving doors	Flexible doors	Outbound doors	Total Tardiness (hours)	Total Cost (103 Rls.)
(1) Maximizing Responsiveness	5	6	10	7.72	24387
(2) Maximizing Efficiency	6	5	10	35.20	15932
(3) Proposed Bi-objective model	6	6	9	11.61	20730

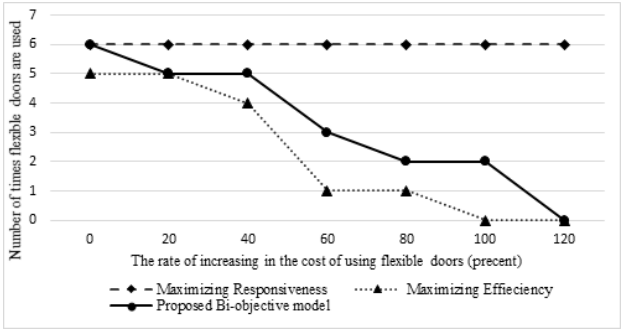


Figure 2: Changes in the number of times flexible doors are used with an increase in the cost of employing them

Table 4: Results obtained from solving the case study problem

Truck No.	(1) Maximizing responsiveness					(2) Maximizing efficiency					(3) Proposed bi-objective model				
	Assigned Door	Service Mode*	Starting Time	Completion Time	Tardiness	Assigned Door	Service Mode*	Starting Time	Completion Time	Tardiness	Assigned Door	Service Mode*	Starting Time	Completion Time	Tardiness
In. 1	4	(F)	7	8.5	-	1	(R)	7	8.5	-	1	(R)	7	8.5	-
In. 2	2	(R)	11	12.5	-	2	(R)	11	12.5	-	2	(R)	11.55	13.05	-
In. 3	1	(R)	8	9.5	-	1	(R)	8.75	10.25	-	4	(F)	8	9.5	-
In. 4	1	(R)	10	11.5	-	2	(R)	14.5	16	-	1	(R)	10	11.5	-
In. 5	1	(R)	12	13.5	-	2	(R)	12.75	14.25	-	1	(R)	12	13.5	-
In. 6	4	(F)	12	13.5	-	4	(F)	12	13.5	-	4	(F)	12	13.5	-
In. 7	2	(R)	9	10.5	-	3	(F)	9	10.5	-	3	(F)	9	10.5	-
In. 8	4	(F)	10	11.3	-	1	(R)	10.5	11.8	-	2	(R)	10	11.3	-
In. 9	3	(F)	8	9.5	-	4	(F)	8	9.5	-	2	(R)	8	9.5	-
Out. 1	5	(S)	8	12.173	1.173	6	(S)	8	15.11	4.11	7	(S)	8	12.235	1.235
Out. 2	3	(F)	9.75	12.17	0.17	5	(S)	6	13.09	1.09	4	(F)	13.75	15.1	3.1
Out. 3	6	(S)	12.403	13.503	1.503	7	(S)	15.37	16.47	4.47	5	(S)	12.45	13.55	1.55
Out. 4	7	(S)	10.67	12.3	0.38	5	(s)	15.54	16.89	4.89	5	(S)	13.8	15.05	3.05
Out. 5	6	(S)	13.753	14.753	0.753	7	(S)	7	15.12	1.12	7	(S)	13.585	14.585	0.585
Out. 6	7	(S)	9.57	10.42	0	6	(S)	16.66	17.51	5.51	6	(S)	8	9.195	0
Out. 7	7	(S)	8	9.32	0	3	(F)	10.75	11.6	0	7	(S)	12.485	13.335	0.335
Out. 8	7	(S)	12.63	13.68	0.68	6	(S)	15.36	16.41	3.41	3	(F)	10.75	12.28	0
Out. 9	3	(F)	12.42	13.57	0.57	7	(S)	16.72	17.87	4.87	6	(S)	9.445	12.378	0
Out. 10	6	(S)	7	12.153	1.153	5	(S)	14.44	15.54	4.54	5	(S)	7	12.2	1.2
Out. 11	5	(S)	12.42	13.273	0.273	5	(S)	13.34	14.19	1.19	3	(F)	12.53	13.38	0.38
Out. 12	5	(S)	13.523	15.073	1.073	3	(F)	11.85	13.4	0	6	(S)	12.627	14.177	0.177

* (R): Receiving door; (F): Flexible door; (S): Sending door.

Table 5: Results obtained by the different methods

α value	Proposed IVF solution approach		Lai and Hwang (L.H.) method		Ozgen and Gulsan (O.G.) method		Alavidoost et al. (Al.) method	
	TT^*	TC^{**}	TT	TC	TT	TC	TT	TC
$\alpha = 0$	11.312	20641	11.312	20651	11.312	20645	11.315	20661
$\alpha = 0.2$	11.512	20649	11.581	20653	11.547	20658	11.581	20653
$\alpha = 0.4$	11.594	20730	11.617	20730	11.59	20730	11.617	20730
$\alpha = 0.6$	11.67	20748	11.815	20771	11.761	20754	11.664	20781
$\alpha = 0.8$	12.041	20782	12.089	20827	12.091	20810	12.056	20785
$\alpha = 1$	12.076	20795	12.174	20827	12.083	20795	12.174	20805

TT^*: Total tardiness (hours)
TC^{**}: Total cost (10^3 Rls.)

According to the results:

- At $\alpha = 0$, the proposed IVF solution approach, the L.H. and O.G methods perform best in the total tardiness objective function. The proposed approach also has the best performance in the total cost function.
- At $\alpha = 0.2$, the proposed IVF solution approach performs best in both objective functions.
- At $\alpha = 0.4$, the O.G. method has the best performance in the total tardiness function, and the performance of all four methods is the same in the total cost function.
- At $\alpha = 0.6$, the Al. method in the total tardiness function and the proposed IVF solution approach in the cost function have the best performance.
- At $\alpha = 0.8$, the proposed IVF solution approach performs best among the methods in both objective functions.
- At $\alpha = 1$, the proposed IVF solution approach has the best performance in the total tardiness function, and this approach, along with the O.G. method, has the best performance in the second objective function.

The results show that the proposed IVF solution approach performs better in solving the case study problem than other fuzzy methods, and for most values of α, it provides the best results. A sensitivity analysis was performed on the average coefficient (β) defined in the proposed solution approach. In this case, α is assumed to be 0.5, and the two objective function values are calculated for different values of β. The results are provided in Table 6 and Figures 5 and 6. According to the results, by increasing the value of β, no particular trend was observed in the efficiency of the proposed solution approach.

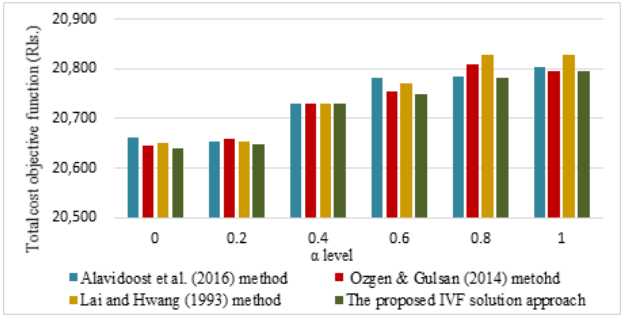


Figure 3: Total cost objective function values obtained by different fuzzy methods

Table 6: Sensitivity analysis of the proposed approach on average coefficient (β)

Objective functions	Different amount of average coefficient (β)										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Total Cost (Rls.)	11.512	11.512	11.761	11.547	11.617	11.67	11.617	11.67	11.581	11.761	11.761
Total Tardiness (Hours)	20683	20649	20754	20658	20730	20730	20730	20748	20683	20754	20748

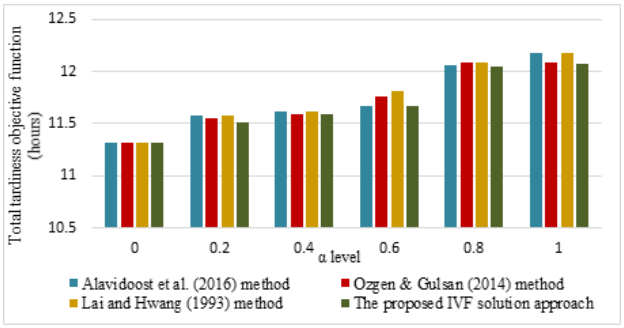


Figure 4: Total tardiness objective function values obtained by different fuzzy methods

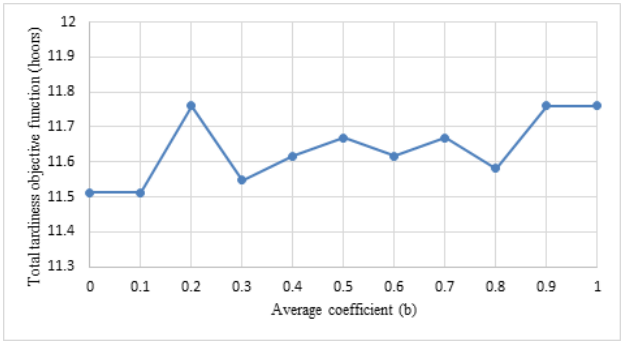


Figure 5: Changes in total tardiness objective function with changes in β coefficient

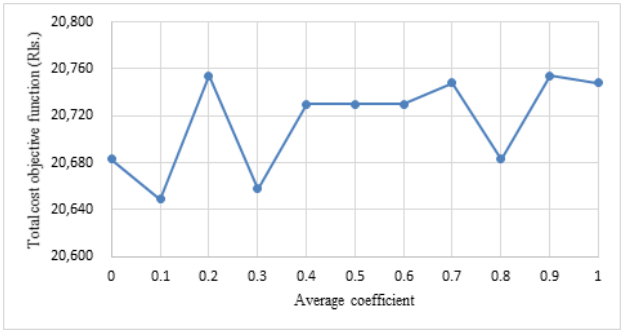


Figure 6: Changes in total cost objective function with changes in β coefficient

5 Conclusions

This paper presented a new bi-objective mixed-integer linear programming model for figuring out how to assign trucks to exclusive and flexible doors and how to find the best order for serving them in a cross-docking center based on two main performance criteria: i) as an efficiency measure, minimizing the costs associated with flexible doors and internal

transshipment; and ii) as a responsiveness measure, minimizing the amount of time that outgoing trucks are late from predefined due times. Also, the uncertainty in some of the input parameters was modeled as triangular IVF numbers, and a new method based on the max-min operator and compromise programming was presented. The proposed model and solution approach were used to schedule the operation of trucks in the cross-docking center using real-world data from a well-known food production group. The proposed method was compared to several well-known fuzzy multi-objective solution approaches to indicate how well it worked. The obtained results represented the optimal performance of the proposed solution approach, as they involved better values than the results obtained by other fuzzy methods. Based on the results, it can be concluded that if the decision-makers want to use flexible doors at a cross-docking facility, they should consider the extra costs as well as the time and cost of transshipments. In addition, addressing the uncertainty in parameters can lead to a stabilized schedule. Future studies can focus on finding other factors that affect whether or not to use flexible doors in cross-docking centers and how those factors affect decision-making. Moreover, given the uncertainties in managing truck scheduling operations through a cross-dock, future research can provide a better understanding of the existing uncertainty through new approaches in uncertainty modeling, such as extended fuzzy sets, and consequently improve the obtained results.

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Appendix

A1. Preliminaries

Fuzzy logic, introduced by Zadeh in 1965, is a multi-valued logic that allows intermediate values to be defined among conventional assessments, such as true/false, high/low, and yes/no, and is regarded with various membership degrees in calculations and modeling. Traditional fuzzy-based methods use classical fuzzy sets, in which decision-makers have trouble giving precise viewpoints in the interval [0, 1] [11, 19, 35]. As a new extension of traditional fuzzy sets, IVF numbers represent the uncertainty as an interval. Based on feelings and experiences rather than pure facts or proof, IVF numbers are suitable for dealing with a lack of information and vagueness [13, 28].

According to Gorzalczany [17], IVF number $\tilde{\tilde{A}}$ is defined as follows:

$$\begin{aligned} &\tilde{\tilde{A}} = \{(x, [\mu_A^L(x), \mu_A^U(x)])\}, \quad x \in (-\infty, +\infty) \\ &\mu_A^L(x), \mu_A^U(x) : (-\infty, +\infty) \to [0, 1] \\ &\mu_A^L(x) \leq \mu_A^U(x), \quad \forall x \in (-\infty, +\infty) \end{aligned} \tag{A.1}$$

Where $\mu_{\tilde{\tilde{A}}}(\mathrm{x})$ is the interval membership function of IVF number $\tilde{\tilde{A}}$, in which $\mu_A^L(x)$,$\mu_A^U(x)$, respectively, denote the lower and upper limits of membership degree:

$$\mu_{\tilde{\tilde{A}}}(x) = [\mu_A^L(x), \mu_A^U(x)]. \tag{A.2}$$

According to Mohagheghi et al. [28], triangular IVF number $\tilde{\tilde{A}}$ can be represented as follows:

$$\tilde{\tilde{A}} = [\tilde{A}^L, \tilde{A}^U]. \tag{A.3}$$

where $\tilde{A}^L = (a_1^L, a_2^L, a_3^L, w^L)$ is the lower limit of $\tilde{\tilde{A}}$ in which w^L denotes the height and $\mu_A^L(x)$ expressed by:

$$\mu_A^L(x) = \begin{cases} w^L \frac{x^L - a_1^L}{a_2^L - a_1^L} & a_1^L \le x \le a_2^L \\ w^L \frac{a_3^L - x^L}{a_3^L - a_2^L} & a_2^L \le x \le a_3^L \\ 0 & Otherwise \end{cases} \tag{A.4}$$

and $\tilde{A}^U = (a_1^U, a_2^U, a_3^U, w^U)$ is the upper limits of $\tilde{\tilde{A}}$ in which w^U denotes the height and $\mu_A^U(x)$ expressed by:

$$\mu_A^U(x) = \begin{cases} w^U \frac{x^U - a_1^U}{a_2^U - a_1^U} & a_1^U \le x \le a_2^U \\ w^U \frac{a_3^U - x^U}{a_3^U - a_2^U} & a_2^U \le x \le a_3^U \\ 0 & Otherwise \end{cases} \tag{A.5}$$

Owing to the proposed definition, the following relations must be met:

$$\begin{aligned} &a_1^U \le a_1^L \\ &a_3^L \le a_3^U \\ &w^L \le w^U \end{aligned} \tag{A.6}$$

Figure A1(a) illustrates triangular IVF number $\tilde{\tilde{A}}$. For ease of calculation, this paper considers a normalized triangular IVF number, as shown in Figure A1(b), which can be represented as $\tilde{\tilde{A}} = [a_1^U, a_1^L, a_2, a_3^L, a_3^U]$ with the following conditions:

$$\begin{aligned} &a_2^L = a_2^U = a_2 \\ &w^L = w^U = 1 \end{aligned} \tag{A.7}$$

The arithmetic operations between two normalized triangular IVF numbers $\tilde{\tilde{A}} = [a_1^U, a_1^L, a_2, a_3^L, a_3^U]$ and $\tilde{\tilde{B}} = [b_1^U, b_1^L, b_2, b_3^L, b_3^U]$ are defined as follows:

1) Addition operation

$$\tilde{\tilde{A}} + \tilde{\tilde{B}} = [a_1^U + b_1^U, a_1^L + b_1^L, a_2 + b_2, a_3^L + b_3^L, a_3^U + b_3^U]. \tag{A.8}$$

2) Multiplication by a scalar k ($k \in R$)

$$k\tilde{\tilde{A}} = \begin{cases} [ka_1^U, ka_1^L, ka_2, ka_3^L, ka_3^U] & k > 0 \\ [ka_3^U, ka_3^L, ka_2, ka_1^L, ka_1^U] & k < 0 \\ 0 & k = 0 \end{cases} \tag{A.9}$$

3) Subtraction

$$\tilde{\tilde{A}} - \tilde{\tilde{B}} = [a_1^U - b_3^U, a_1^L - b_3^L, a_2 - b_2, a_3^L - b_1^L, a_3^U - b_1^U]. \tag{A.10}$$

4) Multiplication

$$\tilde{\tilde{A}} \times \tilde{\tilde{B}} = [a_1^U \times b_1^U, a_1^L \times b_1^L, a_2 \times b_2, a_3^L \times b_3^L, a_3^U \times b_3^U]. \tag{A.11}$$

5) Division

$$\tilde{\tilde{A}} / \tilde{\tilde{B}} = [a_1^U / b_3^U, a_1^L / b_3^L, a_2 / b_2, a_3^L / b_1^L, a_3^U / b_1^U]. \tag{A.12}$$



(a) Triangular IVF number

(b) Normalized triangular IVF number

Figure A1: IVF numbers

A2. Fuzzy solution methods from the literature

The section provides a brief overview of the methods discussed in the paper. Consider the following MOLP model:

$$\begin{aligned} &Max \;\; f_i(x) && i = 1, 2, ..., I, \\ &Min \;\; g_j(x) && j = 1, 2, ..., J, \\ &S.t. \\ &x \in S \end{aligned} \tag{A.13}$$

In the above formulation, S is the set of feasible solutions, and $f_i(x)$ and $g_j(x)$ denote the maximization and minimization objective functions. The PIS and NIS values of each objective function can be determined after solving the MOLP and creating the payoff table. Based on the concept of fuzzy sets proposed by Zimmermann [44], the membership functions for the objectives are defined as:

$$\begin{aligned} &\mu_i = \frac{f_i - f_i^{NIS}}{f_i^{PIS} - f_i^{NIS}} && i = 1, 2, , I, \\ &\mu_j = \frac{g_j^{NIS} - g_j}{g_j^{NIS} - g_j^{PIS}} && j = 1, 2, , J. \end{aligned} \tag{A.14}$$

Lai and Hwangs method [23]:

$$\begin{aligned} &Max \;\; \lambda_0 + \delta \left(\sum_{i=1}^{I} \Theta_i \mu_i(x) + \sum_{j=1}^{J} \Theta_j \mu_j(x) \right), \\ &S.t. \\ &\lambda_0 \leq \mu_i(x) && i = 1, ..., I, \\ &\lambda_0 \leq \mu_j(x) && j = 1, ..., J, \\ &0 \leq \lambda_0 \leq 1 \\ &x \in S \end{aligned} \tag{A.15}$$

Where λ_0 is the level of satisfaction of the objective functions, and Θ_i and Θ_j are their weights, which the DM sets based on his/her preferences. Furthermore, δ refers to a sufficiently small positive number, usually set to 0.01.

Ozgen and Gulsun [32] method:

The first phase:

$$
\begin{aligned}
&Max\ \ \lambda^1 \\
&S.t. \\
&\lambda^1 \le \mu_i(x) \qquad i = 1, ..., I \\
&\lambda^1 \le \mu_j(x) \qquad j = 1, ..., J \\
&0 \le \lambda^1 \le 1 \\
&x \in S
\end{aligned}
\tag{A.16}
$$

In which λ^1 is the degree of satisfaction of objective functions. After solving the first phase and setting λ^{1*} as the optimal value of λ^1, the second phase should be defined as follows:

$$
\begin{aligned}
&Max\ \ \bar{\lambda}^2_{ij} = \frac{1}{I+J}\left(\sum_{i=1}^{I}\lambda_i^2 + \sum_{j=1}^{J}\lambda_j^2\right), \\
&S.t. \\
&\lambda^{1*} \le \lambda_i^2 \le \mu_i(x) \qquad i = 1, ..., I \\
&\lambda^{1*} \le \lambda_j^2 \le \mu_j(x) \qquad j = 1, ..., J \\
&0 \le \lambda_i^2 \le 1 \qquad i = 1, ..., I \\
&0 \le \lambda_j^2 \le 1 \qquad j = 1, ..., J \\
&x \in S
\end{aligned}
\tag{A.17}
$$

Where λ_i^2 and λ_j^2 stand for the more accurate satisfaction levels of objective functions, and $\bar{\lambda}^2_{ij}$ is the average of them.

Alavidoost et al. [1] method:

$$
\begin{aligned}
&Max\ \ \lambda(x) = \lambda_0 + \delta\left(\sum_{i=1}^{I}\Theta_i\mu_i(x) + \sum_{j=1}^{J}\Theta_j\mu_j(x)\right), \\
&S.t. \\
&\Theta_i\lambda_0 + \lambda_i \le \mu_i(x) \qquad i = 1, ..., I \\
&\Theta_j\lambda_0 + \lambda_j \le \mu_j(x) \qquad j = 1, ..., J \\
&0 \le \lambda_i \le 1 \qquad i = 1, ..., I \\
&0 \le \lambda_j \le 1 \qquad j = 1, ..., J \\
&0 \le \lambda_0 \le 1 \\
&x \in S
\end{aligned}
\tag{A.18}
$$

Where λ_0 controls the compromise degree among the objectives and the minimum satisfaction level of those objectives. Θ_i and Θ_j are the relative importance of the objective functions according to DMs preference. δ is the small positive number, typically set to 0.01.