Iranian Journal of Fuzzy Systems

Volume 21, Number 1, (2024), pp. 83-101

Original paper

University of Sistan and Baluchestan

H_∞ control for discrete-time IT2 fuzzy system with infinite distributed-delay via adaptive event-triggered scheme and its application

M. Syed Ali [1], M. Mubeen Tajudeen [2], G. Rajchakit [3] and A. Jirawattanapanit [4]

[1,2] *Complex Systems and Networked Science Research Laboratory, Department of Mathematics, Thiruvalluvar University, Vellore-632115, Tamil Nadu, India.*

[3] *Department of Mathematics, Faculty of Science, Maejo University, Chiang Mai 50290, Thailand.*

[4] *Department of Mathematics, Faculty of Science, Phuket Rajabhat University (PKRU), 6 Thekasattree Road, Raddasa, Phuket 83000, Thailand.*

syedgru@gmail.com, mubeentajudeen19@gmail.com, kreangkri@mju.ac.th, anuwat.j@pkru.ac.th

Abstract

This article investigates the H_∞ control problem for discrete-time interval type-2 (IT2) fuzzy systems with infinite distributed delay via an adaptive event-triggered scheme. The IT2 T-S fuzzy system, which is a development over the (type-1) T-S fuzzy system, has greater effectiveness for the expression of system uncertainty, which will improve the difficulty of analysis. Our main goal is to make more efficient use of network resources by developing an adaptive event-triggered controller for interval type-2 fuzzy systems. In contrast to the traditional triggering method, an adaptive event-triggered technique is proposed to improve bandwidth consumption and network control performance. The triggering function's parameters are based on an adaptive law. Moreover, employing the Lyapunov functional method, the resultant criterion gives sufficient conditions to guarantee that discrete time IT2 fuzzy systems are mean-square exponentially stable with a H_∞ performance. Finally, a single-link robot arm model and a DC motor are employed to show the usefulness and efficiency of the obtained theoretical results.

Keywords: Adaptive event-triggered scheme, infinite-distributed delay, discrete-time IT2 fuzzy system.

1 Introduction

The Takagi-Sugeno(T-S) fuzzy model has been demonstrated in engineering applications and theoretical research fields to better deal with the nonlinearity of the system [2]. The T-S fuzzy model has received extensive attention from researchers in sliding mode control design, output feedback control, event-triggered control, and so on [12, 23, 24]. Due to its uniform approximation property, this model can be transformed into a linear system with a few local subsystems. The type-1 T-S model has been proven to be an effective tool for analyzing nonlinear systems by converting them into some sub-local linear systems with weighting summation. The parameter uncertainties in the membership functions, which are present in every practical application, cannot be handled by type-1 fuzzy systems [7]. Also, with the IT2 fuzzy system, the parameter uncertainties can be effectively extracted by the following bounds, that is, by the bounds of the upper member function and the lower member function. The T-S fuzzy system was proposed to describe such systems in [34] where there are unclear parameters in discrete-time systems. In fact, IT2 fuzzy systems are usually preferred due to their easy design and beneficial applicability to larger networks. It has been demonstrated that IT2 T-S fuzzy systems are superior in various fields compared to type-1 T-S fuzzy models [21]. Recently, dwell-time-dependent H_∞ bumpless transfer control for discrete-time switched interval type-2 fuzzy systems described in [33]. The IT2 fuzzy model has attracted a lot of interest from researchers in recent years [10, 35]. The sliding mode control problem of discrete-time IT2 Markov jump systems subject to external disturbances and time-varying delays is described in [17].

Corresponding Author: M. Syed Ali

Received: October 2022; Revised: October 2023; Accepted: November 2023.

https://doi.org/10.22111/IJFS.2023.43679.7686

Discrete-time systems have a strong tradition in engineering applications. The study of time-delay systems stability analysis and controller design has become a popular research topic in recent years, with a number of famous results being reported [5, 19, 26, 36]. Recent developments in discrete-time delay state systems involve tighter sum over bounds Wirtinger's inequality, resulting in less conservative analysis requirements. Synchronization and state estimation for discrete-time complex networks with distributed delays is studied in [13]. Quantized static output feedback control for discrete-time systems. The $l_2 - l_\infty$ control of discrete-time with randomly occurring delays via IT2 fuzzy model [31]. Time delay fuzzy system have been effectively employed in a wide range of domains, including system identification, modeling chemical plants and other industrial processes [11, 20, 29, 30]. The delay-dependent reliable control problem for discrete-time systems with H_∞ are discussed by many researchers [14, 18].

Event-triggered control (ETC) techniques are thought to be a good way to save communication resources. When compared to a traditional sampled-data control, an event-triggered control can help reduce the number of control tasks [9, 22, 28]. A number of control strategies are based on an event-triggered scheme as discussed in [1, 6]. However, the event-triggered scheme greatly aids in more efficient utilisation of network resources and decreases the transmission load of the broadband network, and many successful results have been attained. It has a wide range of applications since it is inspired by the event-triggered scheme [4]. Traditional ETC for nonlinear systems was developed into adaptive event-triggered control. In recent years, the literature has described a variety of event-triggered methods as an alternative methodology for reducing communication resources, one of which is event-triggered control (ETC) in [27]. Further, a new ETS with an adaptive law was proposed to reduce the use of communication resources, and it is utilised to address the challenging issue of fuzzy network systems in [25]. Some recent results about AETS is described in [11, 15, 16, 32]. H_∞ control problem for networked contorl system via AETS discussed in [25]. Resilient adaptive event-triggered fuzzy tracking control and filtering discussed in [3]. It is essential to develop a more effective AETS for discrete-time IT2 T-S fuzzy systems in light of the aforementioned concerns.

The goal of this research work is to develop a H_∞ control structure for discrete-time IT2 fuzzy systems via adaptive event-triggered scheme in order to increase effective communication while maintaining desirable control performance. The below are the major contributions:
(1) We focus on the H_∞ control problem for the discrete-time IT2 fuzzy system with infinite distributed delay via adaptive event-triggered scheme.
(2) In contrast to traditional triggering scheme, the adaptive event-triggered scheme is proposed to reduce energy consumption. Furthermore, we verify that this adaptive event-triggered scheme has no Zeno behaviour associated with it.
(3) This study is distinct from previous work [7, 34], since it is the first to describe an IT2 fuzzy system with infinite distributed delay.
(4) The fundamental aim is to create an adaptive event-triggered control and it ensure exponentially mean-square stable with H_∞ performance index.
(5) The numerical results include a single-link robot arm model and a DC motor are employed to show the usefulness and efficiency of the obtained theoretical results.

Notations: In this study, $\mathbb{R}^n$ denotes n-dimensional real vector space. The transpose of vector A is A^T. The space of summable sequences over $[0, \infty)$ is denoted as $l_2[0, \infty)$. Set of positive reals are denoted as $\mathbb{R}_{\geq 0}$. To establish symmetric structure, the symbol $*$ will be employed in matrix expressions. $n \times n$ identity matrix is denoted by I_n. $\mathbb{E}(x)$ represents the mathematical expectation of x. $\|\cdot\|$ denotes the Euclidean norm for vectors.

2 Preliminaries

2.1 System description:

The IT2 T-S fuzzy model can be used to estimate a class of nonlinear plants that are uncertain. Consider the discrete-time IT2 T-S fuzzy system is described as follows.

Plant Rule i: IF $g_1(\varpi(\wp))$ is G_1^i and ... and $g_r(\varpi(\wp))$ is G_r^i, THEN

$$\begin{cases} \varpi(\wp+1) & = A_i\varpi(\wp) + A_{\iota i}\sum_{d=1}^{\infty}\lambda_d\varpi(\wp-d) + B_{1i}w(\wp) + C_{1i}u(\wp), \\ z(\wp) & = D_i\varpi(\wp) + B_{2i}w(\wp) + C_{2i}u(\wp), \\ \varpi(\wp) & = \theta(\wp), \quad \wp \in \mathcal{Z}^-. \end{cases} \tag{1}$$

where $G_{\bar{r}}^i(\bar{r} = 1, 2, ..., r)$ and $g(\varpi(\wp)) = [g_1(\varpi(\wp)), g_2(\varpi(\wp)), ..., g_r(\varpi(\wp))]$ denotes fuzzy sets and premise variables, respectively. The scalar q is the number of IF-THEN rules of the system. $u(\wp) \in \mathcal{R}^m$ is the controlled input vector, $\phi(\wp)$

is the initial state vector, $w(\wp) \in \mathcal{R}^p$ is the disturbance input vector, $z(\wp) \in \mathcal{R}^s$ is the output vector, which is unknown but that belongs to $l_2[0,\infty)$ and $\varpi(\wp) \in \mathcal{R}^n$ is the state vector. Constant matrices are $A_i, A_{\iota i}, B_{1i}, B_{2i}, C_{1i}, C_{2i}, D_{1i}$. The constants $\lambda_d \geq 0, (d = 1, 2, ...)$ satisfies the converging condition given below [26]:

$$\bar{\lambda} = \sum_{d=1}^{\infty} \lambda_d < \infty, \quad \sum_{d=1}^{\infty} d\lambda_d < \infty. \tag{2}$$

Remark 2.1. *In system (1), the term $\sum_{d=1}^{\infty} \lambda_d \varpi(\wp - d)$ represents the infinite distributed delay in discrete-time model* [22]*, which is equivalent to the infinite-distributed delay $\int_{-\infty}^{t} \wp(t-g)dg$ in continuous time system. The convergent condition (2) guarantees the fuzzy Lyapunov functional (to be developed later) convergent and $\int_{d=1}^{\infty} \lambda_d \varpi(\wp - d)$ is ensured, which is necessary for the system's stability.*

The interval firing strength corresponding to ith plant rule is represented as follows:

$$\Omega_i(\varpi(\wp)) = [\underline{\alpha}_i(\varpi(\wp)), \overline{\alpha}_i(\varpi(\wp))], \tag{3}$$

where $\overline{\alpha}_i(\varpi(\wp))$ and $\underline{\alpha}_i(\varpi(\wp))$ the denote the upper and lower firing strengths,

$$\overline{\alpha}_i(\varpi(\wp)) = \Pi_{m=1}^{r} \overline{\varphi}_{G_{im}}(g_m(\varpi(\wp))) \geq 0, \tag{4}$$

$$\underline{\alpha}_i(\varpi(\wp)) = \Pi_{m=1}^{r} \underline{\varphi}_{G_{im}}(g_m(\varpi(\wp))) \geq 0. \tag{5}$$

where $\overline{\varphi}_{G_{im}}(g_m(\varpi(\wp))) \in [0,1]$ and $\underline{\varphi}_{G_{im}}(g_m(\varpi(\wp))) \in [0,1]$ are represent the upper and lower grade memberships of $(g_m(\varpi(\wp)))$, respectively. $\overline{\varphi}_{G_{im}}(g_m(\varpi(\wp))) \geq \underline{\varphi}_{G_{im}}(g_m(\varpi(\wp))) \geq 0$ and $\overline{\alpha}_i(\varpi(\wp)) \geq \underline{\alpha}_i(\varpi(\wp)) \geq 0$.

The global model of system (1) is described as,

$$\begin{cases} \varpi(\wp+1) & = \Sigma_{i=1}^{q} \alpha_i(\varpi(\wp))[A_i\varpi(\wp) + A_{\iota i}\sum_{d=1}^{\infty} \lambda_d \varpi(\wp - d) + B_{1i}w(\wp) + C_{1i}u(\wp)], \\ z(\wp) & = \Sigma_{i=1}^{q} \alpha_i(\varpi(\wp))[D_i\varpi(\wp) + B_{2i}w(\wp) + C_{2i}u(\wp)]. \end{cases} \tag{6}$$

The variable $\alpha_i(\varpi(\wp))$ is the membership of the plant and satisfy $\Sigma_{i=1}^{q} \alpha_i(\varpi(\wp)) = 1$, and $\alpha_i(\varpi(\wp)) = \frac{\tilde{\alpha}_i(\varpi(\wp))}{\Sigma_{\varepsilon=1}^{q} \tilde{\alpha}_\varepsilon(\varpi(\wp))} \geq 0$ with $\tilde{\alpha}_i(\varpi(\wp)) = \underline{\pi}_i(\varpi(\wp))\underline{\alpha}_i(\varpi(\wp)) + \overline{\pi}_i(\varpi(\wp))\overline{\alpha}_i(\varpi(\wp)) \geq 0$. $\underline{\pi}_i(\varpi(\wp))$ and $\overline{\pi}_i(\varpi(\wp))$ denote non-linear weighting functions and satisfy $\overline{\pi}_i(\varpi(\wp)) + \underline{\pi}_i(\varpi(\wp)) = 1$, $0 \leq \underline{\pi}_i(\varpi(\wp)) \leq 1$ and $0 \leq \overline{\pi}_i(\varpi(\wp)) \leq 1$.

Remark 2.2. *In recent years, research on type-2 fuzzy systems has attracted a lot of attention* [10, 35]*. The nonlinearities but not the uncertainties can be efficiently captured by type-1 fuzzy sets. The Type-2 fuzzy model handles uncertainty well, in contrast to the conventional T-S fuzzy model. The generalized type-2 fuzzy set can be computed more quickly using the IT2 fuzzy set while still having the additional benefit of being able to handle uncertainty with simplicity.*

2.2 Adaptive event-triggered scheme:

An adaptive event-triggered scheme (AETS) introduced between the sensor and the controller to reduce effectively the communication burden of the shared network. $\wp_l^i h (l = 0, 1, 2, ...)$, where h is the sampling period, $\wp_l^i$ is non-negative integer. The initial instant is $\wp_0^i h = 0$.

The mathematical model of the adaptive event-triggered law is written as follows

$$e_i^T(\wp)\Theta e_i(\wp) \leq \gamma_i(t)\varpi_i^T((\wp_l^i + j)h)\Theta\varpi_i((\wp_l^i + j)h), \tag{7}$$

where $e_i(\wp) = \varpi_i(\wp_l^i h) - \varpi_i((\wp_l^i + j)h), \quad \Theta > 0$ is the parameter of adaptive event-triggered scheme to be determined, and $\gamma_i(\wp)$ satisfies the following condition.

$$\gamma_i(\wp) = \frac{\theta_i}{\gamma_i(\wp)}\big[\frac{1}{\gamma_i(\wp)} - \vartheta_i(\wp)\big]e_i^T(\wp)\Theta e_i(\wp), \tag{8}$$

where $0 \leq \gamma_i(\wp) < 1, \vartheta_i > 0, \theta_i > 0$.

Thus, based on the triggering law (8), the next released instant can be determined by the following expression.

$$\wp_{l+1}^i h = \wp_l^i h + min\{jh | e_i^T(t)\Theta_i e_i(t) > \gamma_i(\wp)\varpi_i^T((\wp_l^i + j)h)\Theta_i\varpi_i((\wp_l^i + j)h\}. \tag{9}$$

It is unavoidable that the occurrence of communication delay when signals are transmitted in the network channels. One can assume that the delay is expressed by ι_l^i, where $\iota_l^i \in [0, \iota_M^i]$, then the time $\wp$ of the transmitted data arriving at controller satisfies $\wp \in [\wp_l^i + \iota_l^i, \wp_{l+1}^i h + \iota_{k+1}^i)$.

Defining $\iota_i(\wp) = \wp - \wp_{l+1}^i, \wp \in [\wp_l^i + \iota_l^i, \wp_{l+1}^i h + \iota_{l+1}^i)$, the actual sensor measurement error $\bar{\varpi}_i(\wp)$ can be written as

$$\bar{\varpi}(\wp) = \varpi(\wp_l^i h) = \varpi(\wp - \iota(\wp)) + e(\wp). \tag{10}$$

Remark 2.3. *Different from traditional event-triggered scheme* [6, 9]*, the adaptive event-triggered scheme is introduced. According to equation (8) using the dynamically adjusted threshold $\gamma_i(\wp)$ and reducing the number of unnecessary data packet transmissions is possible using the event generator with adaptive law. In (8), when the parameter $\vartheta_i(\wp) = \frac{1}{\gamma(0)}$, one can get the adaptive event-triggered law $\dot{\gamma}(\wp) = 0$. That indicates that the proposed AETS will become a standard time-triggered strategy. The adaptation law $\gamma(\wp) \to 0$ indicates that the thresold of AETS does not need adjustment if the system gradually obtains stability.*

2.3 Controller design:

The design interval type-2 fuzzy controller with mismatched premises can improve design flexibility against deception attacks.
The fuzzy control law is considered as follows:
Controller Rule j: IF $f_1(\varpi(\wp))$ is F_1^j and $f_2(\varpi(\wp))$ is F_2^j and ,...,$f_s(\varpi(\wp))$ is F_s^j, THEN

$$u(\wp) = K_j \bar{\varpi}(\wp), \quad (j = 1, ..., q) \tag{11}$$

where $F_{\bar{s}}^j (\bar{s} = 1, 2, ..., s)$ and $f(\varpi(\wp)) = [f_1(\varpi(\wp)), f_2(\varpi(\wp)), ..., f_s(\varpi(\wp))]$ denotes the related fuzzy set and premise variables, respectively. The scalar q is the number of IF-THEN rules of the system. K_j are controller gains to be designed, $\bar{\varpi}(\nu)$ is the signals transmitted through the network to the controller.
The interval firing strength corresponding to jth plant rule is represented as,

$$\Gamma_i(\varpi(\wp)) = [\underline{\beta}_i(\varpi(\wp)), \overline{\beta}_i(\varpi(\wp))], \tag{12}$$

where $\overline{\beta}_j(\varpi(\wp))$ and $\underline{\beta}_j(\varpi(\wp))$ are denotes the upper and lower firing strengths,

$$\overline{\beta}_j(\varpi(\wp)) = \Pi_{n=1}^s \overline{\varphi}_{F_{jn}}(f_n(\varpi(\wp))) \geq 0, \tag{13}$$

$$\underline{\beta}_j(\varpi(\wp)) = \Pi_{n=1}^s \underline{\varphi}_{F_{jn}}(f_n(\varpi(\wp))) \geq 0. \tag{14}$$

where $\overline{\varphi}_{F_{jn}}(f_n(\varpi(\wp))) \in [0, 1]$ and $\underline{\varphi}_{F_{jn}}(f_n(\varpi(\wp))) \in [0, 1]$ are represent the upper and lower grade memberships of $(f_n(\varpi(\wp)))$, respectively. $\overline{\varphi}_{F_{jn}}(f_n(\varpi(\wp))) \geq \underline{\varphi}_{F_{jn}}(f_n(\varpi(\wp))) \geq 0$ and $\overline{\beta}_j(\varpi(\wp)) \geq \underline{\beta}_j(\varpi(\wp)) \geq 0$.

Based on fuzzy reasoning process, the over all controller is derived as:

$$u(\wp) = \Sigma_{j=1}^q \beta_j(\varpi(\wp)) K_j \bar{\varpi}(\wp), \tag{15}$$

where $\beta_j(\varpi(\wp))$ represent the grade membership and satisfy $\Sigma_{j=1}^q \beta_j(\varpi(\wp)) = 1$ and $\beta_j(\varpi(\wp)) = \frac{\tilde{\beta}_j(\varpi(\wp))}{\Sigma_{\epsilon=1}^q \tilde{\beta}_\epsilon(\varpi(\wp))}$ with $\tilde{\beta}_j(\varpi(\wp)) = \underline{\nu}_j(\varpi(\wp))\underline{\beta}_j(\varpi(\wp)) + \overline{\nu}_j(\varpi(\wp))\overline{\beta}_j(\varpi(\wp)) \geq 0$. $\underline{\nu}_j(\varpi(\wp))$ and $\overline{\nu}_j(\varpi(\wp))$ denote non-linear weighting functions and satisfy $\overline{\nu}_j(\varpi(\wp)) + \underline{\nu}_j(\varpi(\wp)) = 1$, $0 \leq \underline{\nu}_j(\varpi(\wp)) \leq 1$ and $0 \leq \overline{\nu}_j(\varpi(\wp)) \leq 1$.

Substituting (15) into (10), the control input is described as follows:

$$u(\wp) = \Sigma_{j=1}^q \beta_j(\varpi(\wp)) K_j [\varpi(\wp - \iota(\wp)) + e(\wp)]. \tag{16}$$

Based on the above discussions, by using (6) and (16), the closed-loop discrete-time IT2 fuzzy systems is formulated as follows:

$$\begin{cases} \varpi(\wp + 1) & = \Sigma_{i=1}^q \Sigma_{j=1}^q \alpha_i \beta_j \{A_i \varpi(\wp) + A_{\iota i} \sum_{d=1}^{\infty} \lambda_d \varpi(\wp - d) + B_{1i} w(\wp) + C_{1i} K_j [e(\wp) + \varpi(\wp - \iota(\wp))]\}, \\ z(\wp) & = \Sigma_{i=1}^q \Sigma_{j=1}^q \alpha_i \beta_j \{D_i \varpi(\wp) + B_{2i} w(\wp) + C_{2i} K_j [\varpi(\wp - \iota(\wp)) + e(\wp)]\}. \end{cases} \tag{17}$$

Let $A = \Sigma_{i=1}^{q}\alpha_i A_i$, $A_\iota = \Sigma_{i=1}^{q}\alpha_i A_\iota$, $B_1 = \Sigma_{i=1}^{q}\alpha_i B_{1i}$, $B_2 = \Sigma_{i=1}^{q}\alpha_i B_{2i}$, $C_1 = \Sigma_{i=1}^{q}\Sigma_{j=1}^{q}\alpha_i\beta_j C_{1i}$, and

$$C_2 = \Sigma_{i=1}^{q}\Sigma_{j=1}^{q}\alpha_i\beta_j C_{2i}.$$

The equation (17) can be written in compact form as

$$\begin{cases} \varpi(\wp+1) & = A\varpi(\wp) + A_\iota \sum_{d=1}^{\infty}\lambda_d\varpi(\wp-d) + B_1 w(\wp) + C_1 K_j[e(\wp) + \varpi(\wp-\iota(\wp))], \\ z(\wp) & = D\varpi(\wp) + B_2 w(\wp) + C_2 K_j[\varpi(\wp-\iota(\wp)) + e(\wp)]. \end{cases} \tag{18}$$

Definition 2.4. [26] *In the mean square sense, the system (10) is considered to be exponentially stable, with initial condition $\psi(\wp) = 0$, if there exist constants α and β with, $\alpha > 0$ and $0 < \beta < 1$, such that*

$$\| \varpi(\wp) \|^2 < \alpha\beta^{\wp} \sup_{s\in\mathcal{Z}^-} \| \psi(s) \|^2 . \tag{19}$$

Definition 2.5. [26] *The system (10) with $w(\wp) = 0$ is said to be exponentially mean-square stable. If for a given disturbance attention level $\delta > 0$, then the system (10) is said to be exponentially mean-square stable with H_∞ performance δ and under zero initial condition,*

$$\sum_{\wp=0}^{\infty} \| z(\wp) \|^2 < \delta^2 \sum_{\wp=0}^{\infty} \| w(\wp) \|^2, \tag{20}$$

is satisfied and $\omega(\wp) \in l_2[0, +\infty)$.

Lemma 2.6. [13] *For any matrix $M > 0$, vectors $\varpi_i \in \mathcal{R}^n$ and scalar constants $a_i (i = 1, 2, ...)$ if the series concerned are convergent, then the following inequality holds:*

$$\Big(\sum_{i=1}^{\infty} a_i\varpi_i\Big)^T M \Big(\sum_{i=1}^{\infty} a_i\varpi_i\Big) \le \Big(\sum_{i=1}^{\infty} a_i\Big)\sum_{i=1}^{\infty} a_i\varpi_i^T M\varpi_i.$$

Lemma 2.7. [19] *(Schur Complement) Given constant matrices S_1, S_2, S_3, where $S_1 = S_1^T$ and $S_3 < 0$, then $S_1 - S_2 S_3^{-1} S_2^T < 0$ if and only if*

$$\begin{bmatrix} S_1 & S_2 \\ S_2^T & S_3 \end{bmatrix} < 0.$$

Lemma 2.8. [14] *For any positive symmetric constant matrix $\mathcal{R} \in \mathbb{R}^{n\times n}$. ι_m, ι_M are any scalar which satisfies the condition $\iota_m \le \iota_M$. The vector valued function $\xi(\wp) = \eta(\wp+1) - \eta(\wp)$, we have*

$$-\sum_{i=\wp-\iota_M}^{\wp-\iota_m-1} \xi^T(i)\mathcal{R}\xi(i) \le -\frac{1}{\iota_M - \iota_m} \sum_{i=\wp-\iota_M}^{\wp-\iota_m-1} \xi^T(i)\mathcal{R} \sum_{i=\wp-\iota_M}^{\wp-\iota_m-1} \xi(i),$$

$$-\sum_{j=-\iota_M}^{\iota_m-1}\sum_{i=\wp+j}^{\wp-1} \xi^T(i)\mathcal{R}\xi(i) \le -\frac{2}{(\iota_M - \iota_m)(\iota_M + \iota_m + 1)} \sum_{j=-\iota_M}^{\iota_m-1}\sum_{i=\wp+j}^{\wp-1} \xi^T(i)\mathcal{R}\xi(i) \sum_{j=-\iota_M}^{\iota_m-1}\sum_{i=\wp+j}^{\wp-1} \xi(i).$$

3 Main results

Theorem 3.1. *For a given constant $\delta > 0, \vartheta > 0$, discrete-time IT2 T-S fuzzy system (18) is mean-square exponentially stable with H_∞ performance δ, if there exist a positive definite matrices $R_1 > 0, Q_1 > 0, R_2 > 0, Q_2 > 0, R_3 > 0, W > 0, R > 0, P_i > 0$ and event triggered parameter matrix Θ, such that the following inequalities hold.*

$$\begin{bmatrix} \Phi & \delta I \\ * & -P_i \end{bmatrix} < 0, \tag{21}$$

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & 0 & \Phi_{15} & 0 & \Phi_{17} & \Phi_{18} & 0 & 0 & 0 \\ * & \Phi_{22} & 0 & 0 & 0 & 0 & \Phi_{27} & 0 & 0 & 0 & 0 \\ * & * & \Phi_{33} & 0 & 0 & 0 & \Phi_{37} & 0 & 0 & 0 & 0 \\ * & * & * & \Phi_{44} & \Phi_{45} & 0 & 0 & 0 & \Phi_{49} & 0 & 0 \\ * & * & * & * & \Phi_{55} & \Phi_{56} & \Phi_{57} & 0 & \Phi_{59} & \Phi_{5,10} & 0 \\ * & * & * & * & * & \Phi_{66} & 0 & 0 & 0 & 0 & \Phi_{6,11} \\ * & * & * & * & * & * & \Phi_{77} & \Phi_{78} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Phi_{88} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \Phi_{99} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \Phi_{10,10} & 0 \\ * & * & * & * & * & * & * & * & * & * & \Phi_{11,11} \end{bmatrix}, \tag{22}$$

where,
$\Phi_{11} = -P_i + W + (\iota_*)R + R_{11} + R_{21} + R_{31} + \iota_2 R_1 - \frac{Q_2}{\iota_M} + 2N_1[A(\wp) - I]$, $\Phi_{12} = 2N_1 A_\iota(\wp)$,
$\Phi_{13} = 2N_1 B_1(\wp)$, $\Phi_{15} = 2N_1 K C_1(\wp) + \gamma\Theta + \frac{Q_2}{\iota_M}$, $\Phi_{17} = 2N_1 A[\wp - I_1] - 2N_1 + R_{12} + R_{22} + R_{32}$,
$\Phi_{18} = -2N_1 K C_1(\wp)$, $\Phi_{22} = -\lambda^{-1}R$, $\Phi_{27} = 2N_1 A_\iota(\wp)$, $\Phi_{33} = -\delta^2 I$, $\Phi_{37} = 2N_1 B_1(\wp)$,
$\Phi_{44} = -R_{21} - \frac{Q_1}{\iota_*}$, $\Phi_{45} = \frac{Q_1}{\iota_*}$, $\Phi_{49} = -2R_{22}$, $\Phi_{55} = -W - R_{11} - \frac{2Q_1}{\iota_*} + \Theta - \frac{2Q_2}{\iota_*} - \frac{Q_2}{\iota_M}$,
$\Phi_{57} = 2N_1 C_1(\wp)$, $\Phi_{5,10} = -2R_{12}$, $\Phi_{66} = -\frac{Q_1}{\iota_*} - \frac{Q_2}{\iota_*} - R_{31}$, $\Phi_{6,11} = -R_{32}$, $\Phi_{77} = R_{13} + R_{23} + R_{33} + \iota_* R_1 + \iota_* Q_1 + \iota_M Q_2 - 2N_1$, $\Phi_{78} = -2N_1 K C_1(\wp)$ $\Phi_{88} = -\vartheta\Theta$, $\Phi_{99} = -R_{23}$,
$\Phi_{10,10} = -R_{13}$, $\Phi_{11,11} = -R_{33}$, $\Phi_{56} = \frac{Q_1}{\iota_*} + \frac{Q_2}{\iota_*}$, *and* $\iota_* = \iota_M - \iota_m$

Proof. The following Lyapunov functional candidate is considered:

$$\begin{aligned}
V_1(\wp) =& \varpi^T(\wp)P_i(\wp)\varpi(\wp) + \sum_{d=1}^{\infty}\lambda_d \sum_{\bar{\iota}=\wp-d}^{\wp-1} \varpi(\bar{\iota})^T R\varpi(\bar{\iota}),\\
V_2(\wp) =& \sum_{i=\wp-\iota(\wp)}^{\wp-1} \varpi^T(i)W\varpi(i),\\
V_3(\wp) =& \sum_{i=\wp-\iota_M}^{\wp-\iota_m} \varpi^T(i)W\varpi(i),\\
V_4(\wp) =& \sum_{i=\wp-\iota(\wp)}^{\wp-1} \mu^T(i)R_1\mu(i) + \sum_{i=\wp-\iota_m}^{\wp-1} \mu^T(i)R_2\mu(i) + \sum_{i=\wp-\iota_m}^{\wp-1} \mu^T(i)R_3\mu(i),\\
V_5(\wp) =& \sum_{j=-\iota_M+1}^{-\iota_m}\sum_{i=\wp+j}^{\wp-1} \mu^T(i)R_1\mu(i) + \sum_{j=-\iota_M}^{-\iota_m-1}\sum_{i=\wp+j}^{\wp-1} x^T(i)Q_1x(i) + \sum_{j=\iota_M}^{-1}\sum_{i=\wp+j}^{\wp-1} x^T(i)Q_2x(i),\\
V_6(\wp) =& \frac{1}{2}\gamma^T(\wp)\gamma(\wp).
\end{aligned}$$

Here $\mu(\wp) = [\varpi^T(\wp) \quad x^T(\wp)]^T$ and $x^T(\wp) = \varpi(\wp+1) - \varpi(\wp)$.

$$J = \sum_{\wp=0}^{N-1} \big(z^T(\wp)z(\wp) - \delta^2 w^T(\wp)w(\wp)\big). \tag{23}$$

Then, we get

$$\dot{V}_1(\wp) = \{\varpi(\wp+1)^T P_i(\wp+1)\varpi(\wp+1) - \varpi(\wp)^T P_i(\wp)\varpi(\wp) + \bar{\lambda}\varpi(\wp)^T R\varpi(\wp) - \sum_{d=1}^{\infty}\lambda_d \varpi(\wp-d)^T R\varpi(\wp-d)\}. \tag{24}$$

By applying (2) and Lemma 2.6, we get

$$-\sum_{d=1}^{\infty}\lambda_d\varpi(\wp-d)^T R\varpi(\wp-d) \le -\bar{\lambda}^{-1}\Big[\sum_{d=1}^{\infty}\lambda_d\varpi(\wp-d)\Big]^T R\Big[\sum_{d=1}^{\infty}\lambda_d\varpi(\wp-d)\Big].$$

$$\begin{aligned}
\dot{V}_2(\wp) =&\{\sum_{i=\wp+1-\iota(\wp+1)}^{\wp}\varpi^T(i)W\varpi(i)-\sum_{i=\wp-\iota(\wp)}^{\wp-1}\varpi^T(i)W\varpi(i)\},\\
=&\{\varpi^T(\wp)W\varpi(\wp)-\varpi^T(\wp-\iota(\wp))W\varpi(\wp-\iota(\wp))+\sum_{i=\wp-\iota(\wp+1)+1}^{\wp}\varpi^T(i)W\varpi(i)-\sum_{i=\wp-\iota(\wp)+1}^{\wp-1}\varpi^T(i)W\varpi(i)\},\\
=&\{\varpi^T(\wp)W\varpi(\wp)-\varpi^T(\wp-\iota(\wp))W\varpi(\wp-\iota(\wp))+\sum_{i=\wp-\iota_M+1}^{\wp-\iota_m}\varpi^T(i)W\varpi(i)\}. \qquad (25)\\
\dot{V}_3(\wp) =&\{\sum_{j=\wp-\iota_M+2}^{\wp-\iota_m+1}\sum_{i=j}^{\wp}\varpi^T(i)W\varpi(i)-\sum_{j=\wp-\iota_M+1}^{\wp-\iota_m}\sum_{i=j}^{\wp-1}\varpi^T(i)W\varpi(i)\},\\
=&\{\sum_{j=\wp-\iota_M+1}^{\wp-\iota_m}\sum_{i=j+1}^{\wp}\varpi^T(i)W\varpi(i)-\sum_{j=\wp-\iota_M+1}^{\wp-\iota_m}\sum_{i=j}^{\wp-1}\varpi^T(i)W\varpi(i)\},\\
=&\{(\iota_M-\iota_m)\varpi^T(i)W\varpi(i)-\sum_{j=\wp-\iota_M+1}^{\wp-\iota_m}\varpi^T(i)W\varpi(i)\}. \qquad (26)\\
\dot{V}_4(\wp) =&\{\mu^T(\wp)R_1\mu(\wp)-\mu^T(\wp-\iota(\wp))R_1\mu(\wp-\iota(\wp))+\sum_{i=\wp+1-\iota(\wp+1)}^{\wp-\iota_m}\mu^T(i)R_1\mu(i)\\
&+\sum_{i=\wp+1-\iota_m}^{\wp-1}\mu^T(i)R_1\mu(i)-\sum_{i=\wp+1-\iota(\wp)}^{\wp-1}\mu^T(i)\quad R_1\quad\mu(i)+\mu^T(\wp)R_2\mu(\wp)\\
&-\mu^T(\wp-\iota_m)\quad R_2\quad\mu(\wp-\iota_m)+\mu^T(\wp)R_3\mu(\wp)-\mu^T(\wp-\iota_M)R_3\mu(\wp-\iota_M)\},\\
\le&\{\mu^T(\wp)(R_1+R_2+R_3)\mu(\wp)-\mu^T(\wp-\iota(\wp))R_1\mu(\wp-\iota(\wp))+\sum_{i=\wp+1-\iota_M}^{\wp-\iota_m}\mu^T(i)R_1\mu(i)\\
&-\mu^T(\wp-\iota_m)R_2\mu(\wp-\iota_m)-\mu^T(\wp-\iota_M)R_3\mu(\wp-\iota_M)\},\\
\le&\{\varpi^T(\wp)(R_{11}+R_{21}+R_{31})\varpi(\wp)+2\varpi^T(\wp)(R_{12}+R_{22}+R_{32})x(\wp)+x^T(\wp)(R_{13}+R_{23}\\
&+R_{33})x(\wp)-\varpi^T(\wp-\iota(\wp))R_{11}\varpi(\wp-\iota(\wp))-2\varpi^T(\wp-\iota(\wp))R_{12}x(\wp-\iota(\wp))\\
&-x^T(\wp-\iota(\wp))R_{13}x(\wp-\iota(\wp))-\varpi^T(\wp-\iota_m)R_{21}\varpi(\wp-\iota_m)-2\varpi^T(\wp-\iota_m)R_{22}x(\wp-\iota_m)\\
&-x^T(\wp-\iota_m)R_{23}x(\wp-\iota_m)-\varpi^T(\wp-\iota_M)R_{31}\varpi(\wp-\iota_M)-2\varpi^T(\wp-\iota_M)R_{32}x(\wp-\iota_M)\\
&-x^T(\wp-\iota_M)R_{33}x(\wp-\iota_M)+\sum_{i=\wp+1-\iota_M}^{\wp-\iota_m}\mu^T(i)R_1\mu(i)\}. \qquad (27)\\
\dot{V}_5(\wp) =&\Big(\sum_{j=-\iota_M+1}^{-\iota_m}\Big[\sum_{i=\wp+1+j}^{i}\mu^T(i)R_1\mu(i)-\sum_{i=\wp+j}^{\wp-1}\mu^T(i)R_1\mu(i)\Big]+\sum_{j=-\iota_M}^{-\iota_m-1}\Big[\sum_{i=\wp+1+j}^{\wp}x^T(i)Q_1x(i)-\sum_{i=\wp+j}^{\wp-1}x^T(i)Q_1x(i)\Big]\\
&+\sum_{j=-\iota_M}^{-1}\Big[\sum_{i=\wp+1+j}^{\wp}x^T(i)Q_2x(i)-\sum_{i=\wp+j}^{\wp-1}x^T(i)Q_2x(i)\Big]\Big),\\
=&\Big[\iota_2\mu^T(\wp)R_1\mu(\wp)-\sum_{i=\wp+1-\iota_M}^{\wp-\iota_m}\mu^T(i)R_1\mu(i)+\iota_2x^T(\wp)R_1x(\wp)-\sum_{i=\wp-\iota_M}^{\wp-\iota_m-1}x^T(i)Q_1x(i)+\iota_Mx^T(\wp)Q_2x(\wp)\\
&-\sum_{i=\wp-\iota_M}^{\wp-1}x^T(i)Q_2x(i)\Big]. \qquad (28)
\end{aligned}$$

By applying Lemma 2.8, we get

$$
\begin{aligned}
-\sum_{i=\wp-\iota_m}^{\wp-\iota_m-1} x^T(i)Q_1x(i) &= -\sum_{i=\wp-\iota_M}^{\wp-\iota(\wp)-1} x^T(i)Q_1x(i) - \sum_{i=\wp-\iota(\wp)}^{\wp-\iota_m-1} x^T(i)Q_1x(i),\\
&\leq -\frac{1}{\iota_2}\sum_{i=\wp-\iota_M}^{\wp-\iota(\wp)-1} x^T(i)Q_1\sum_{i=\wp-\iota_M}^{\wp-\iota(\wp)-1} x(i) - \frac{1}{\iota_2}\sum_{i=\wp-\iota(\wp)}^{\wp-\iota_m-1} x^T(i)Q_1\sum_{i=\wp-\iota(\wp)}^{\wp-\iota_m-1} x(i),\\
&\leq -\frac{1}{\iota_2}\Big(\{\varpi[\wp-\iota(\wp)]-\varpi[\wp-\iota_M]\}^TQ_1\{\varpi[\wp-\iota(\wp)]-\varpi[\wp-\iota_M]\}+\\
&\quad \{\varpi[\wp-\iota_m]-\varpi[\wp-\iota(\wp)]\}^T\times Q_1\{\varpi[\wp-\iota_m]-\varpi[\wp-\iota(\wp)]\}\Big),\\
-\sum_{i=\wp-\iota_M}^{\wp-1} x^T(i)Q_2x(i) &= -\sum_{i=\wp-\iota_M}^{\wp-\iota(\wp)-1} x^T(i)Q_2x(i) - \sum_{i=\wp-\iota(\wp)}^{\wp-1} x^T(i)Q_2x(i),\\
&\leq -\frac{1}{\iota_2}\sum_{i=\wp-\iota_M}^{\wp-\iota(\wp)-1} x^T(i)Q_2\sum_{i=\wp-\iota_M}^{\wp-\iota(\wp)-1} x(i) - \frac{1}{\iota_M}\sum_{i=\wp-\iota(\wp)}^{\wp-1} x^T(i)Q_2\sum_{i=\wp-\iota(\wp)}^{\wp-1} x(i),\\
&\leq -\frac{1}{\iota_2}\Big(\{\varpi[\wp-\iota(\wp)]-\varpi[\wp-\iota_M]\}^TQ_2\{\varpi[\wp-\iota(\wp)]-\varpi[\wp-\iota_M]\}\\
&\quad -\frac{1}{\iota_M}\{\varpi[\wp]-\varpi[\wp-\iota(\wp)]\}^T\times Q_2\{\varpi[\wp]-\varpi[\wp-\iota(\wp)]\}\Big).
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_6(\wp) =&\gamma^T(\wp)\dot{\gamma}(\wp),\\
=&\theta_1(\frac{1}{\gamma_1(\wp)}-\vartheta_1)e_1^T(\wp)\Theta_1e_1(\wp)+\theta_2(\frac{1}{\gamma_2(\wp)}-\vartheta_2)e_2^T(\wp)\Theta_2e_2(\wp)+...+\theta_N(\frac{1}{\gamma_N(\wp)}-\vartheta_N)\\
&\times e_N^T(\wp)\Theta_Ne_N(\wp),\\
=&\frac{\theta_1}{\gamma_1}^T(\wp)\Theta_1\gamma_1(\wp)+\frac{\theta_2}{\gamma_2}^T(\wp)\Theta_2\gamma_2(\wp)+...+\frac{\theta_N}{\gamma_N}^T(\wp)\Theta_N\gamma_N(\wp)-\theta_1\vartheta_1e_1^T(\wp)\Theta_1e_1(\wp)\\
&-\theta_2\vartheta_2e_2^T(\wp)\Theta_2e_2(\wp)-...-\theta_N\vartheta_Ne_N^T(\wp)\Theta_Ne_N(\wp).
\end{aligned}
\tag{29}
$$

Combining (8) and (29), one can get the following inequality

$$
\begin{aligned}
\dot{V}_6(\wp) \leq&\theta_1\varpi_1^T(\wp-\iota_1(\wp))\Theta_1\varpi_1(\wp-\iota_1(\wp))+\theta_2\varpi_2^T(\wp-\iota_2(\wp))\Theta_2\varpi_2(\wp-\iota_2(\wp))+...\\
&+\theta_N\varpi_N^T(\wp-\iota_N(\wp))\Theta_N\varpi_N(\wp-\iota_N(\wp))-\theta_1\vartheta_1e_1^T(\wp)\Theta_1e_1(\wp)-\theta_2\vartheta_2e_2^T(\wp)\Theta_2e_2(\wp)-...\\
&-\theta_N\vartheta_Ne_N^T(\wp)\Theta_Ne_N(\wp),\\
=&\varpi^T(\iota-\iota(\wp))\Theta\varpi(\iota-\iota(\wp))-e^T(\wp)\Theta\vartheta e(\wp).
\end{aligned}
\tag{30}
$$

In addition, we have

$$
\begin{aligned}
[x(\wp)] =&[\varpi(\wp+1)-\varpi(\wp)],\\
0 =&\Big[2\Big([x(\wp)N_1+\varpi(\wp)N_1][\varpi(\wp+1)-\varpi(\wp)-x(\wp)]\Big)\Big],
\end{aligned}
\tag{31}
$$

$$
\begin{aligned}
0 =&\Big[2\Big([x(\wp)N_1+\varpi(\wp)N_1][A(\wp)\varpi(\wp)+A_\iota(\wp)\sum_{d=1}^{\infty}\lambda_d\varpi(\wp-d)+B_1(\wp)w(\wp)\\
&+C_1(\wp)K\varpi(\wp-\iota(\wp))-C_1K(k)e(i_\wp h)-\varpi(\wp)-x(\wp)]\Big)\Big].
\end{aligned}
\tag{32}
$$

From the equations (23)-(32), we get

$$\begin{aligned}
\dot{V}(\wp) =&\{\varpi(\wp+1)^T P_i(\wp+1)\varpi(\wp+1) - \varpi(\wp)^T P_i(\wp)\varpi(\wp) + \bar{\lambda}\varpi(\wp)^T R\varpi(\wp) - \bar{\lambda}^{-1}\Big[\sum_{d=1}^{\infty}\lambda_d\varpi(\wp-d)\Big]^T \\
&R\Big[\sum_{d=1}^{\infty}\lambda_d\varpi(\wp-d)\Big] + \varpi^T(\wp)W\varpi(\wp) - \varpi^T(\wp-\iota(\wp))W\varpi(\wp-\iota(\wp)) + \sum_{i=\wp-\iota_M+1}^{\wp-\iota_m}\varpi^T(i)W\varpi(i) \\
&+ (\iota_M-\iota_m)\varpi^T(i)W\varpi(i) - \sum_{j=\wp-\iota_M+1}^{\wp-\iota_m}\varpi^T(i)W\varpi(i) + \sum_{i=\wp+1-\iota_M}^{\wp-\iota_m}\mu^T(i)R_1\mu(i) - e^T(\wp)\Theta\vartheta e(\wp) \\
&- \varpi^T(\wp-\iota(\wp))R_{11}\varpi(\wp-\iota(\wp)) - 2\varpi^T(\wp-\iota(\wp))R_{12}x(\wp-\iota(\wp)) - x^T(\wp-\iota(\wp))R_{13}x(\wp-\iota(\wp)) \\
&+ \varpi^T(\wp)(R_{11}+R_{21}+R_{31})\varpi(\wp) + 2\varpi^T(\wp)(R_{12}+R_{22}+R_{32})x(\wp) + x^T(\wp)(R_{13}+R_{23}+R_{33}) \\
&x(\wp) - \varpi^T(\wp-\iota_m)R_{21}\varpi(\wp-\iota_m) - 2\varpi^T(\wp-\iota_m)R_{22}x(\wp-\iota_m) - x^T(\wp-\iota_m)R_{23}x(\wp-\iota_m) \\
&- \varpi^T(\wp-\iota_M)R_{31}\varpi(\wp-\iota_M) - 2\varpi^T(\wp-\iota_M)R_{32}x(\wp-\iota_M) - x^T(\wp-\iota_M)R_{33}x(\wp-\iota_M) \\
&+ \Big[\iota_2\mu^T(\wp)R_1\mu(\wp) - \sum_{i=\wp+1-\iota_M}^{\wp-\iota_m}\mu^T(i)R_1\mu(i) + \iota_2 x^T(\wp)R_1x(\wp) + \iota_M x^T(\wp)R_1x(\wp)\Big] - \frac{1}{\iota_2} \\
&\Big(\{\varpi[\wp-\iota(\wp)] - \varpi[\wp-\iota_M]\}^T Q_1\{\varpi[\wp-\iota(\wp)] - \varpi[\wp-\iota_M]\} + \{\varpi[\wp-\iota_m] - \varpi[\wp-\iota(\wp)]\}^T Q_1 \\
&\{\varpi[\wp-\iota_m] - \varpi[\wp-\iota(\wp)]\}\Big) - \frac{1}{\iota_2}\Big(\{\varpi[\wp-\iota(\wp)] - \varpi[\wp-\iota_M]\}^T Q_2\{\varpi[\wp-\iota(\wp)] - \varpi[\wp-\iota_M]\} \\
&- \frac{1}{\iota_M}\{\varpi[\wp] - \varpi[\wp-\iota(\wp)]\}^T Q_2\{\varpi[\wp] - \varpi[\wp-\iota(\wp)]\}\Big) + \Big[2N_1\Big([x(\wp)+\varpi(\wp)][A(\wp)\varpi(\wp)A_\iota(\wp) \\
&+ \sum_{d=1}^{\infty}\lambda_d\varpi(\wp-d) + B_1(\wp)w(\wp) + C_1(\wp)K\varpi(\wp-\iota(\wp)) - C_1K(\wp)e(i_\wp h) - \varpi(\wp) - x(\wp)]\Big)\Big]\} \\
&+ \varpi^T(\iota-\iota(\wp))\Theta\varpi(\iota-\iota(\wp)).
\end{aligned} \tag{33}$$

Therefore, under zero initial condition, it follows from (23) and (33), that

$$\begin{aligned}
J \le& \sum_{\wp=0}^{N-1}\big(z^T(\wp)z(\wp) - \delta^2 w^T(\wp)w(\wp)\big) + V(\wp+1) - V(\wp) + V(0) - V(\infty), \\
J \le& \xi^T(\wp)\big[\Phi + \Upsilon_1^T(P_i)\Upsilon_1 + \Upsilon_2^T I\Upsilon_2\big]\xi(\wp).
\end{aligned} \tag{34}$$

where,
$\xi(\wp) = \Big[\varpi^T(\wp) \quad \sum_{d=1}^{\infty}\lambda_d\varpi^T(\wp-d) \quad w^T(\wp) \quad \varpi^T(\wp-\iota_m(\wp)) \quad \varpi^T(\wp-\iota(\wp)) \quad \varpi^T(\wp-\iota_M(\wp))$
$x^T(\wp) \quad e^T(i_\wp h) \quad x^T(\wp-\iota_m(\wp)) \quad x^T(\wp-\iota(\wp)) \quad x^T(\wp-\iota_M(\wp))\Big]^T$, $\Upsilon_1(\wp) = \Big[A^T(\wp) \quad A_\iota^T(\wp) \quad B_{1i}^T(\wp)$
$0 \quad KC_{1i}^T(\wp) \quad 0 \quad 0 \quad -KC_{1i}^T(\wp) \quad 0 \quad 0 \quad 0\Big]^T$ and $\Upsilon_2(\wp) = \Big[D_i^T(\wp) \quad 0 \quad B_{2i}^T(\wp) \quad 0 \quad KC_{2i}^T(\wp) \quad 0 \quad 0$
$-KC_{2i}^T(\wp) \quad 0 \quad 0 \quad 0\Big]^T$.

By using the Schur Complement (Lemma 2.8), we get (21). Therefore, $J < 0$ for any nonzero $w(\wp) \in l_2[0,\infty)$. Letting $N \to \infty$, then the inequality (20) holds. Therefore the system (18) is exponentially mean square stable with H_∞ performance. □

Corollary 3.2. *The discrete-time IT2 T-S fuzzy system (18) with $w(\wp) = 0$ is mean-square exponentially stable, if there exist a positive definite matrices $R_1 > 0, Q_1 > 0, R_2 > 0, Q_2 > 0, R_3 > 0, W > 0, R > 0, P_i > 0$, and event triggered parameter matrix Θ, such that the following inequalities hold.*

$$\begin{bmatrix} \bar{\Phi} & \delta I \\ * & -P_i \end{bmatrix} < 0, \tag{35}$$

$$\bar{\Phi}=\begin{bmatrix} \bar{\Phi}_{11} & \bar{\Phi}_{12} & 0 & \bar{\Phi}_{14} & 0 & \bar{\Phi}_{16} & \bar{\Phi}_{17} & 0 & 0 & 0 \\ * & \bar{\Phi}_{22} & 0 & 0 & 0 & \bar{\Phi}_{26} & 0 & 0 & 0 & 0 \\ * & * & \bar{\Phi}_{33} & \bar{\Phi}_{34} & 0 & 0 & 0 & \bar{\Phi}_{38} & 0 & 0 \\ * & * & * & \bar{\Phi}_{44} & \bar{\Phi}_{45} & \bar{\Phi}_{46} & 0 & \bar{\Phi}_{4,8} & \bar{\Phi}_{4,9} & 0 \\ * & * & * & * & \bar{\Phi}_{55} & 0 & 0 & 0 & 0 & \bar{\Phi}_{5,10} \\ * & * & * & * & * & \bar{\Phi}_{66} & \bar{\Phi}_{67} & 0 & 0 & 0 \\ * & * & * & * & * & * & \bar{\Phi}_{77} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \bar{\Phi}_{88} & 0 & 0 \\ * & * & * & * & * & * & * & * & \bar{\Phi}_{99} & 0 \\ * & * & * & * & * & * & * & * & * & \bar{\Phi}_{10,10} \end{bmatrix}, \tag{36}$$

where,
$\bar{\Phi}_{11}=-P_i+W+(\iota_*)R+R_{11}+R_{21}+R_{31}+\iota_2R_1-\frac{Q_2}{\iota_M}+2N_1[A(\wp)-I]$, $\bar{\Phi}_{12}=2N_1A_\iota(\wp)$,
$\bar{\Phi}_{14}=2N_1KC_1(\wp)+\gamma_1\Theta+\frac{Q_2}{\iota_M}$, $\bar{\Phi}_{16}=2N_1A[\wp-I_1]-2N_1+R_{12}+R_{22}+R_{32}$, $\bar{\Phi}_{17}=-2N_1KC_1(\wp)$,
$\bar{\Phi}_{22}=-\lambda^{-1}R$, $\Phi_{27}=2N_1A_\iota(\wp),\bar{\Phi}_{33}=-R_{21}-\frac{Q_1}{\iota_*}$, $\bar{\Phi}_{34}=\frac{Q_1}{\iota_*}$, $\bar{\Phi}_{38}=-2R_{22}$, $\bar{\Phi}_{4,9}=-2R_{12}$,
$\bar{\Phi}_{44}=-W-R_{11}-\frac{2Q_1}{\iota_*}+\Theta-\frac{2Q_2}{\iota_*}-\frac{Q_2}{\iota_M}$, $\bar{\Phi}_{45}=\frac{Q_1}{\iota_*}+\frac{Q_2}{\iota_*}$, $\bar{\Phi}_{46}=2N_1C_1(\wp),\bar{\Phi}_{55}=-\frac{Q_1}{\iota_*}-\frac{Q_2}{\iota_*}-R_{31}$,
$\bar{\Phi}_{5,10}=-R_{32},\bar{\Phi}_{66}=R_{13}+R_{23}+R_{33}+\iota_*R_1+\iota_*Q_1+\iota_MQ_2-2N_1$, $\bar{\Phi}_{67}=-2N_1KC_1(\wp)$
$\bar{\Phi}_{77}=-\vartheta\Theta$, $\bar{\Phi}_{88}=-R_{23}$, $\bar{\Phi}_{9,9}=-R_{13}$, $\bar{\Phi}_{10,10}=-R_{33}$, $\bar{\Phi}_8=\gamma_1\Theta$ *and* $\iota_*=\iota_M-\iota_m$.

Proof. Following the same procedure in Theorem 3.1, we get

$$\dot{V}(\wp)\}=\tilde{\xi}^T(\wp)\Big[\bar{\Phi}+\tilde{\Upsilon}_1^T(P_i)\tilde{\Upsilon}_1+\tilde{\Upsilon}_2^TI\tilde{\Upsilon}_2\Big]\tilde{\xi}(\wp)<-\kappa\|\varpi(\wp)\|^2, \tag{37}$$

where,
$\tilde{\xi}(\wp)=\Big[\varpi^T(\wp) \quad \sum_{d=1}^{\infty}\lambda_d\varpi^T(\wp-d) \quad \varpi^T(\wp-\iota_m(\wp)) \quad \varpi^T(\wp-\iota(\wp)) \quad \varpi^T(\wp-\iota_M(\wp)) \quad x^T(\wp) \quad e^T(i_\wp h)$
$x^T(\wp-\iota_m(\wp)) \quad x^T(\wp-\iota(\wp)) \quad x^T(\wp-\iota_M(\wp))\Big]^T$, $\tilde{\Upsilon}_1(\wp)=\Big[A^T(\wp) \quad A_{\iota i}^T(\wp) \quad 0 \quad KC_{1i}^T(\wp) \quad 0 \quad 0$
$-KC_{1i}^T(\wp) \quad 0 \quad 0 \quad 0\Big]^T$ and $\tilde{\Upsilon}_2(\wp)=\Big[D_{1i}^T(\wp) \quad 0 \quad 0 \quad KC_{2i}^T(\wp) \quad 0 \quad 0 \quad -KC_2^T(\wp) \quad 0 \quad 0 \quad 0\Big]^T$.
By using a method of analysis similar to that used in [9] and from Definition 2.4, system (18) with $w(\wp)=0$ is exponentially mean-square stable. This concludes the proof. □

Theorem 3.3. *The discrete-time IT2 T-S fuzzy system (18) is mean-square exponentially stable with H_∞ performance δ, if there exist matrices $\widetilde{R}_1>0,\widetilde{Q}_1>0,\widetilde{R}_2>0,\widetilde{Q}_2>0,\widetilde{R}_3>0,\widetilde{R}>0,\widetilde{V}>0,\widetilde{P}_i>0$, event triggered parameter matrix Θ and positive diagonal matrix $\widetilde{N}_1$ with proper dimensions such that,*

$$\widetilde{\Phi}=\begin{bmatrix} \widetilde{\Phi}_{11} & \widetilde{\Phi}_{12} & \widetilde{\Phi}_{13} & 0 & \widetilde{\Phi}_{15} & 0 & \widetilde{\Phi}_{17} & \widetilde{\Phi}_{18} & 0 & 0 & 0 & P_iA_i & D_i \\ * & \widetilde{\Phi}_{22} & 0 & 0 & 0 & 0 & \widetilde{\Phi}_{27} & 0 & 0 & 0 & 0 & P_iA_{\iota i} & 0 \\ * & * & \widetilde{\Phi}_{33} & 0 & 0 & 0 & \widetilde{\Phi}_{37} & 0 & 0 & 0 & 0 & P_iB_{1i} & B_{2i} \\ * & * & * & \widetilde{\Phi}_{44} & \widetilde{\Phi}_{45} & 0 & 0 & 0 & \widetilde{\Phi}_{49} & 0 & 0 & 0 & 0 \\ * & * & * & * & \widetilde{\Phi}_{55} & \widetilde{\Phi}_{56} & \widetilde{\Phi}_{57} & 0 & \widetilde{\Phi}_{59} & \widetilde{\Phi}_{5,10} & 0 & P_iC_{1i}K & C_{2i}K \\ * & * & * & * & * & \widetilde{\Phi}_{66} & 0 & 0 & 0 & 0 & \widetilde{\Phi}_{6,11} & 0 & 0 \\ * & * & * & * & * & * & \widetilde{\Phi}_{77} & \widetilde{\Phi}_{78} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \widetilde{\Phi}_{88} & 0 & 0 & 0 & -P_iC_{1i}K & -C_{2i}K \\ * & * & * & * & * & * & * & * & \widetilde{\Phi}_{99} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \widetilde{\Phi}_{10,10} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & \widetilde{\Phi}_{11,11} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -P_i & 0 \\ * & * & * & * & * & * & * & * & * & * & * & * & -I \end{bmatrix}<0 \tag{38}$$

where,
$\widetilde{\Phi}_{11}=-\widetilde{P}_i+\widetilde{W}+(\iota_*)\widetilde{R}+\widetilde{R}_{11}+\widetilde{R}_{21}+\widetilde{R}_{31}+\iota_2\widetilde{R}_1-\frac{\widetilde{Q}_2}{\iota_M}+2\epsilon\widetilde{P}[A(\wp)-I]$, $\widetilde{\Phi}_{12}=2\epsilon\widetilde{P}A_\iota(\wp)$,
$\widetilde{\Phi}_{13}=2\epsilon\widetilde{P}B_1(\wp)$, $\widetilde{\Phi}_{15}=2\epsilon XC_1(\wp)+\frac{Q_2}{\iota_M}+\gamma_1\widetilde{\Theta}$, $\widetilde{\Phi}_{17}=2\epsilon\widetilde{P}A[\wp-I_1]-2\epsilon\widetilde{P}+\widetilde{R}_{12}+\widetilde{R}_{22}+\widetilde{R}_{32}$,
$\widetilde{\Phi}_{18}=-2\epsilon XC_1(\wp)$, $\widetilde{\Phi}_{1,12}=\widetilde{P}A_i(k)$ $\widetilde{\Phi}_{1,13}=\widetilde{P}D_i$, $\widetilde{\Phi}_{22}=-\lambda^{-1}\widetilde{R}$, $\widetilde{\Phi}_{27}=2\epsilon\widetilde{P}A_\iota(\wp)$,
$\widetilde{\Phi}_{2,12}=\widetilde{P}A_{\iota i}(k)$, $\widetilde{\Phi}_{33}=-\widehat{\delta}^2I$, $\widetilde{\Phi}_{37}=2\epsilon\widetilde{P}B_1(\wp)$, $\widetilde{\Phi}_{3,12}=\widetilde{P}B_{1i}$, $\widetilde{\Phi}_{3,13}=B_{2i}\widetilde{\Phi}_{44}=-\widetilde{R}_{21}-\frac{\widetilde{Q}_1}{\iota_*}$,

$\widetilde{\Phi}_{45} = \frac{\widetilde{Q}_1}{\iota_*}$, $\widetilde{\Phi}_{49} = -2\widetilde{R}_{22}$, $\widetilde{\Phi}_{55} = -\widetilde{W} - \widetilde{R}_{11} + \widetilde{\Theta} - \frac{2\widetilde{Q}_1}{\iota_*} - \frac{2\widetilde{Q}_2}{\iota_*} - \frac{\widetilde{Q}_2}{\iota_M}$, $\widetilde{\Phi}_{56} = \frac{\widetilde{Q}_1}{\iota_*} + \frac{\widetilde{Q}_2}{\iota_*}$,
$\widetilde{\Phi}_{5,10} = -2\widetilde{R}_{12}$, $\widetilde{\Phi}_{5,12} = XC_1$, $\widetilde{\Phi}_{5,13} = XC_2$, $\widetilde{\Phi}_{66} = -\frac{\widetilde{Q}_1}{\iota_*} - \frac{\widetilde{Q}_2}{\iota_*} - \widetilde{R}_{31}$, $\widetilde{\Phi}_{6,11} = -\widetilde{R}_{32}$,
$\widetilde{\Phi}_{77} = \widetilde{R}_{13} + \widetilde{R}_{23} + \widetilde{R}_{33} + \iota_*\widetilde{R}_1 + \iota_*\widetilde{Q}_1 + \iota_M\widetilde{Q}_2 - 2\epsilon\widetilde{P}$, $\widetilde{\Phi}_{78} = -2\epsilon XC_1(\wp)$, $\widetilde{\Phi}_{88} = -\vartheta\widetilde{\Theta}$,
$\widetilde{\Phi}_{8,12} = -XC_1$, $\widetilde{\Phi}_{8,13} = -XC_2$, $\widetilde{\Phi}_{99} = -\widetilde{R}_{23}$, $\widetilde{\Phi}_{10,10} = -\widetilde{R}_{13}$, $\widetilde{\Phi}_{11,11} = -\widetilde{R}_{33}$,
$\widetilde{\Phi}_{12,12} = -\widetilde{P}$, $\widetilde{\Phi}_{13,13} = -I$, $\widetilde{\Phi}_{57} = 2\epsilon\widetilde{P}C_1(\wp)$ *and* $\iota_* = \iota_M - \iota_m$. *The control gain in (15) are given by*

$$K = \widetilde{P}^{-1}X. \tag{39}$$

Proof. We calculate $\widetilde{\Phi}_{ij} = \Lambda\Phi_{ij}\Lambda^T$ with

$$\Lambda = diag\{\widetilde{P}, \widetilde{P}, I, \widetilde{P}, \widetilde{P}, \widetilde{P}, \widetilde{P}, \widetilde{P}, \widetilde{P}, \widetilde{P}, \widetilde{P}, \widetilde{P}, I\},$$

where $\widetilde{P} = P^{-1}$. Defining $\widetilde{W} = \widetilde{P}W\widetilde{P}$, $\widetilde{Q}_i = \widetilde{P}Q_i\widetilde{P}, (i = 1, 2, 3)$, $\widetilde{R}_{ij} = \widetilde{P}R_{ij}\widetilde{P}, (i, j = 1, 2, 3)$ and letting $N_1 = \epsilon\widetilde{P}$, we obtain $\widetilde{\Phi}_{ij}$ in (38). If the conditions (38) hold, event-triggered control gain matrices are K are given by (39). □

4 Numerical examples

To illustrate the theoretical results, numerical examples are provided.

Example 4.1. *Consider the following single link robot arm system in* [24, 33].

$$\ddot{\varpi}(t) = \frac{LgM_k}{J_k}sin(\varpi(t)) - \frac{Q_k}{J_k}\dot{\varpi}(t) + \frac{1}{J_k}u(t) + Bw(t). \tag{40}$$

In this example $w(t)$ and $u(t)$ represents the disturbance input and control input, respectively. $\dot{\varpi}$ and ϖ indicates the angular velocity of the robot arm and the angel, respectively. The system parameters include, L is the arm's length, g is the acceleration of gravity, the mass of the payload is M_k, J_k is the moment of inertia, ϱ is retarded coefficient and the coefficient of viscous friction is Q_k.
The values of the parameters $g = 9.81, Q_1 = Q_2 = 2, L = 0.5, M_1 = J_1 = 1, M_2 = J_2 = 5, M_3 = J_3 = 10, \varrho = 0.85$ and $T = 0.1$.
Then, the single-link robot arm system can be approximated by the following IT2 T-S fuzzy model.
Pant rule 1: *IF $\varpi_1(\wp)$ is $1 - \frac{\varpi_1^2(\wp)}{\Im}$, THEN*

$$\begin{cases} \varpi(\wp+1) & = \Sigma_{i=1}^q\Sigma_{j=1}^q\alpha_i\beta_j\{A_1\varpi(\wp) + A_{\iota 1}\sum_{d=1}^\infty\lambda_d\varpi(\wp-d) + B_{11}w(\wp) + C_{11}u(\wp)\}, \\ z(\wp) & = \Sigma_{i=1}^q\Sigma_{j=1}^q\alpha_i\beta_j\{D_1\varpi(\wp) + B_{21}w(\wp) + C_{21}u(\wp)\}. \end{cases} \tag{41}$$

Pant rule 2: *IF $\varpi_1(\wp)$ is $\frac{\varpi_1^2(\wp)}{\Im}$, THEN*

$$\begin{cases} \varpi(\wp+1) & = \Sigma_{i=1}^q\Sigma_{j=1}^q\alpha_i\beta_j\{A_2\varpi(\wp) + A_{\iota 2}\sum_{d=1}^\infty\lambda_d\varpi(\wp-d) + B_{12}w(\wp) + C_{12}u(\wp))\}, \\ z(\wp) & = \Sigma_{i=1}^q\Sigma_{j=1}^q\alpha_i\beta_j\{D_2\varpi(\wp) + B_{22}w(\wp) + C_{22}u(\wp)\}. \end{cases} \tag{42}$$

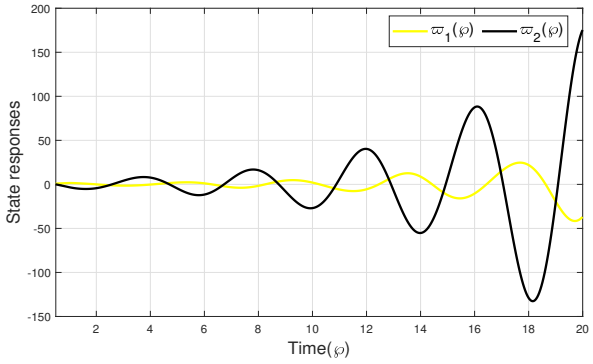


Figure 1: State response for discrete-time IT2 fuzzy system without control input.

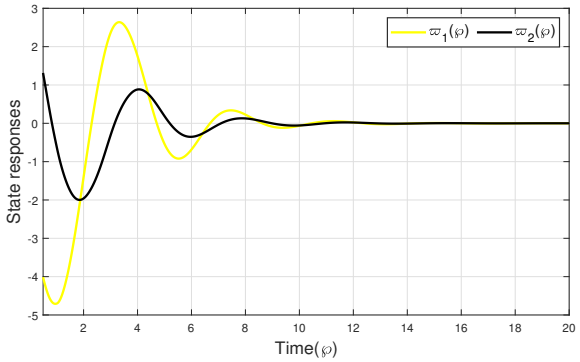


Figure 2: State response for discrete-time IT2 fuzzy system with control input.

where,

$$A_1 = \begin{bmatrix} 1 & T\varrho \\ -\frac{TgLM_b}{J_b} & 1-\frac{T\varrho Q_a}{J_b} \end{bmatrix}, A_2 = \begin{bmatrix} 1 & T\varrho \\ -\varsigma\frac{TgLM_b}{J_b} & 1-\frac{T\varrho Q_a}{J_b} \end{bmatrix}, A_{\iota 1} = A_{\iota 2} = \begin{bmatrix} 0 & T(1-\varrho) \\ 0 & -\frac{T(1-\varrho)Q_a}{J_b} \end{bmatrix},$$

$$B_{11} = B_{12} = \begin{bmatrix} 0 \\ T \end{bmatrix}, C_{11} = C_{12} \begin{bmatrix} 0 \\ \frac{Q}{J_b} \end{bmatrix}, D_1 = D_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, B_{21} = B_{22} = 0.1, C_{21} = C_{22} = 0.$$

Choose $\lambda_d = 2^{-d-3}$*, then we find that*

$$\bar{\lambda}_d = \sum_{d=1}^{\infty} \lambda_d = \frac{1}{8} < \sum_{d=1}^{\infty} d\lambda_d = 2 < +\infty,$$

which satisfies the convergence condition (2).

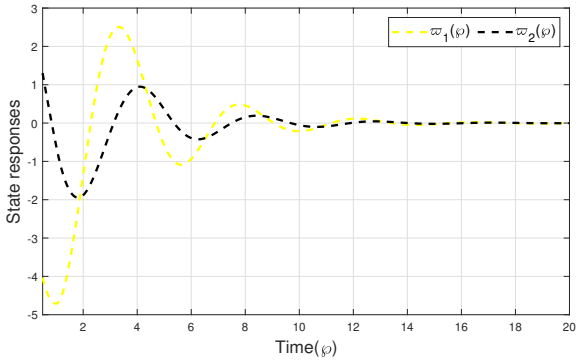


Figure 3: State response for discrete-time type-1 fuzzy system.

In addition, $\Im \in [9, 27]$ *is the uncertain parameter. The IT2 fuzzy model can be used to effectively describe the uncertain parameters in this scenario. Table I and Table II, respectively, list the lower and upper membership functions of the original system and controller. Assume that the external disturbance* $w(\wp) = e^{1.5\wp}$*.*

Table 1: The upper and lower membership functions of the original system.

The upper membership functions	*The lower membership functions*
$\overline{\alpha}_1(\varpi(\wp)) = 1 - \frac{e^{-\varpi_1^2(\wp)}}{3}$	$\underline{\alpha}_1(\varpi(\wp)) = 1 - \frac{e^{-\varpi_1^2(\wp)}}{6}$
$\overline{\alpha}_2(\varpi(\wp)) = 1 - \overline{\alpha}_1(\varpi(\wp))$	$\underline{\alpha}_2(\varpi(\wp)) = 1 - \underline{\alpha}_1(\varpi(\wp))$

Let $\underline{\pi}(\varpi(\wp)) = 0.6sin^2(\varpi(\wp)), \overline{\pi}(\varpi(\wp)) = 1 - \underline{\pi}(\varpi(\wp)), \underline{\nu}(\varpi(\wp)) = \overline{\nu}(\varphi(\wp)) = 0.5$. Assume the generalized ETC performance be specified by $\iota_m = 1.2, \iota_M = 3.30, \lambda = 0.1$ and H_∞ performance level is chosen $\delta = 2.75$. By applying

Theorem 3.3, the controller gain and event-triggered parameter matrices are obtained as follows:

$$K_1 = \begin{bmatrix} 0.5831 & 0.2233 \end{bmatrix}, K_2 = \begin{bmatrix} -0.4992 & -0.3959 \end{bmatrix}, \Theta = \begin{bmatrix} 0.1856 & 0.0498 \\ 0.0493 & 0.1332 \end{bmatrix}. \tag{43}$$

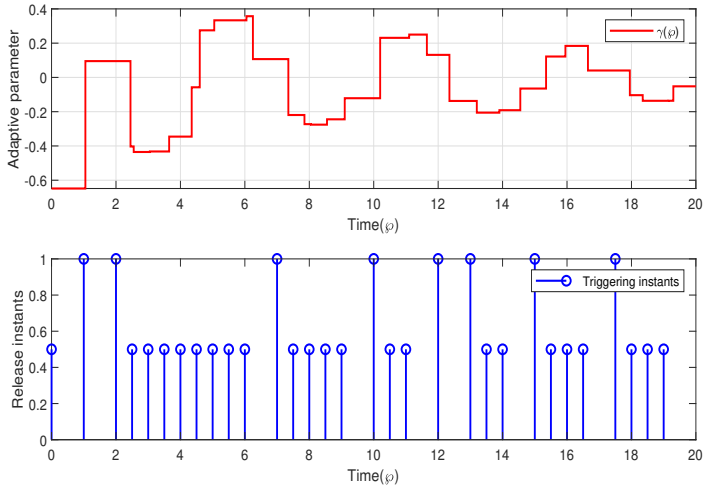


Figure 4: Curve of adaptive triggering parameter $\gamma(\wp)$ and Triggering instants and intervals.

Table 2: The upper and lower membership functions for controller.	
The upper membership functions	The lower membership functions
$\overline{\beta}_1(\varpi(\wp)) = \underline{\beta}_1(\varpi(\wp))$ $\overline{\beta}_2(\varpi(\wp)) = 1 - \underline{\beta}_1(\varpi(\wp))$	$\underline{\beta}_1(\varpi(\wp)) = \frac{e^{-\varpi_1^2(\wp)}}{6}$ $\underline{\beta}_2(\varpi(\wp)) = 1 - \underline{\beta}_1(\varpi(\wp))$

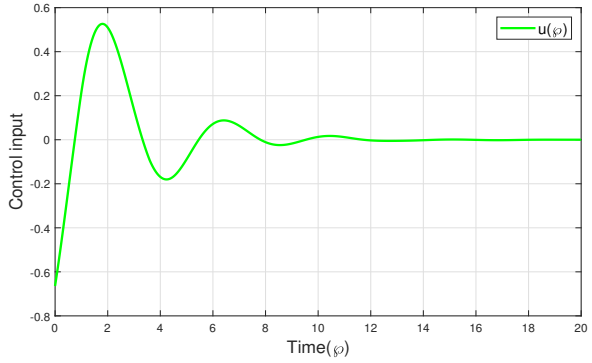


Figure 5: Trajectory of control input.

By choosing the sampling interval as $h = 0.05$ and assume the initial condition $\varpi(0) = [3.5, -1.5]^T$. State responses discrete-time IT2 fuzzy system without control input depicted in Fig. 1. When there is no control input, the system is unstable. State response for discrete-time IT2 fuzzy system with control input shown in Fig. 2. According to Fig. 2, it is clear that the state trajectories under the proposed control scheme are converging to zero within $10s$. In Fig. 3, propose the state responses for discrete-time type-1 fuzzy system and its converging to zero within $14s$. From Figure 2 and Figure 3, it is clear that the proposed IT2 fuzzy model is effective. The adaptive triggering parameter $\gamma(\wp)$ and the triggered instants with intervals are described in Fig. 4. In Fig. 5, present trajectories of control input. We can conclude from the above simulation findings that the technique for discrete-time IT2 fuzzy system with designing AETS controllers in this work is effective.

Example 4.2. *Consider an inverted pendulum controlled by a DC motor via a gear train* [7] *with discrete-time IT2 fuzzy systems under AETS.*

We add some disturbance terms and a controllable output for simulation as follows.

Pant rule 1: *IF* $\varpi_1(\wp)$ *is* $1-\frac{\varpi_1^2(\wp)}{\Im}$, *THEN*

$$\begin{cases} \varpi(\wp+1) & = \Sigma_{i=1}^{q}\Sigma_{j=1}^{q}\alpha_i\beta_j\{A_1\varpi(\wp)+A_{\iota 1}\sum_{d=1}^{\infty}\lambda_d\varpi(\wp-d)+B_{11}w(\wp)+C_{11}u(\wp)\}, \\ z(\wp) & = \Sigma_{i=1}^{q}\Sigma_{j=1}^{q}\alpha_i\beta_j\{D_1\varpi(\wp)+B_{21}w(\wp)+C_{21}u(\wp)\}. \end{cases} \tag{44}$$

Pant rule 2: *IF* $\varpi_1(\wp)$ *is* $\frac{\varpi_1^2(\wp)}{\Im}$, *THEN*

$$\begin{cases} \varpi(\wp+1) & = \Sigma_{i=1}^{q}\Sigma_{j=1}^{q}\alpha_i\beta_j\{A_2\varpi(\wp)+A_{\iota 2}\sum_{d=1}^{\infty}\lambda_d\varpi(\wp-d)+B_{12}w(\wp)+C_{12}u(\wp))\}, \\ z(\wp) & = \Sigma_{i=1}^{q}\Sigma_{j=1}^{q}\alpha_i\beta_j\{D_2\varpi(\wp)+B_{22}w(\wp)+C_{22}u(\wp)\}. \end{cases} \tag{45}$$

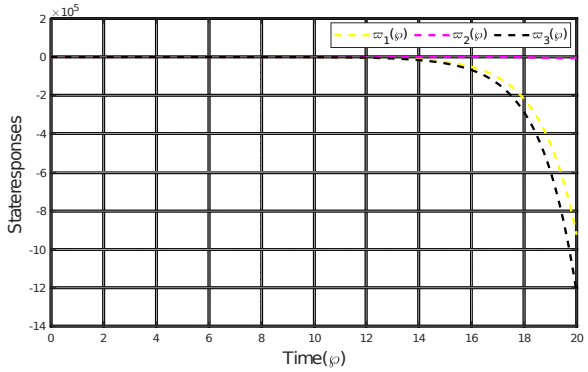


Figure 6: State response for discrete-time IT2 fuzzy system without control input.

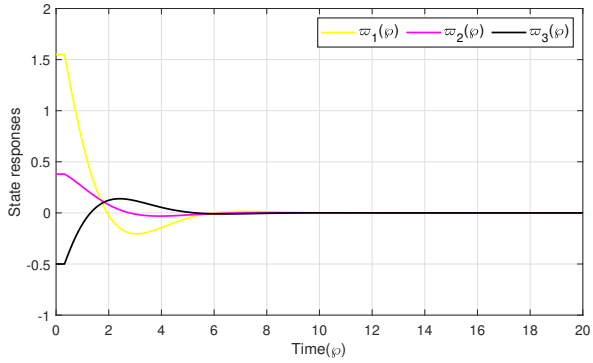


Figure 7: State response for discrete-time IT2 fuzzy system with control input.

where,

$$A_1 = \begin{bmatrix} 1.002 & 0.02 & 0.02 \\ 0.196 & 1.001 & 0.0181 \\ -0.0184 & -0.1813 & 0.8170 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 0.02 & 0.0002 \\ 0 & 0.9981 & 0.0181 \\ 0 & -0.1811 & 0.8170 \end{bmatrix}, \quad B_{11} = \begin{bmatrix} 0.054 \\ 0.094 \\ 0 \end{bmatrix},$$
$$B_{12} = \begin{bmatrix} -0.054 \\ -0.094 \\ 0 \end{bmatrix}, C_{11} = \begin{bmatrix} 0 \\ 0.0019 \\ 0.1811 \end{bmatrix}, C_{12} = \begin{bmatrix} 0 \\ 0.0019 \\ 0.1811 \end{bmatrix}, D_1 = \begin{bmatrix} 0.054 \\ 0.005 \\ 0.1 \end{bmatrix}, D_2 = \begin{bmatrix} 0.054 \\ 0.005 \\ 0.1 \end{bmatrix},$$
$$B_{21} = 0.1, B_{22} = 0.1, C_{21} = 0.1, C_{22} = 0.1.$$

Choose $\lambda_d = 2^{-d-3}$*, the we find that*

$$\bar{\lambda}_d = \sum_{d=1}^{\infty} \lambda_d = \frac{1}{8} < \sum_{d=1}^{\infty} d\lambda_d = 2 < +\infty,$$

which satisfies the convergence condition (2).

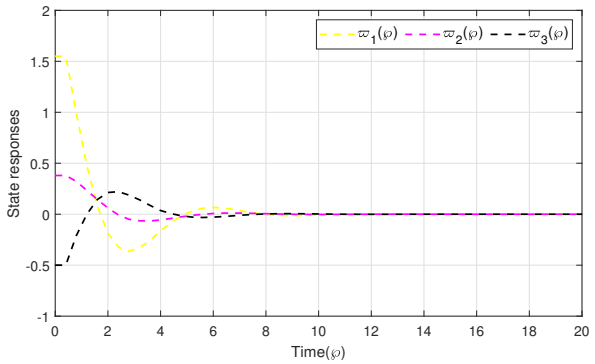


Figure 8: State response for discrete-time type-1 fuzzy system.

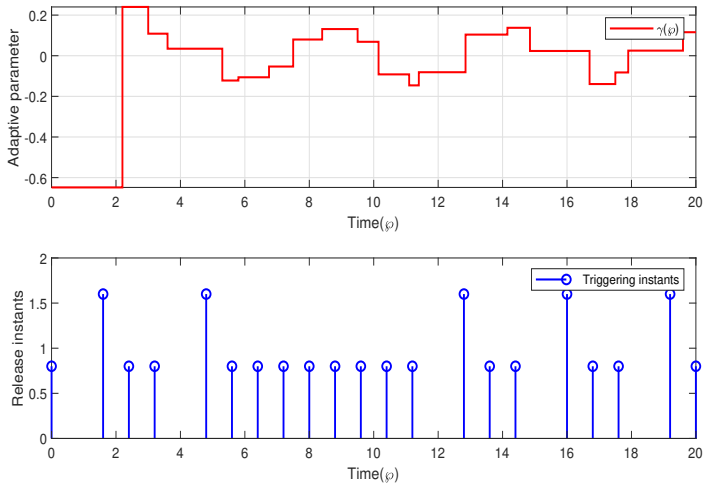


Figure 9: Curve of adaptive triggering parameter $\gamma(\wp)$ and Triggering instants and intervals.

The IT2 fuzzy model can be used to effectively describe the uncertain parameters in this scenario. Table III and Table IV, respectively, list the lower and upper membership functions of the original system and controller. The external

disturbance $w(\wp)$ is is assumed to be

$$w(\wp) = \begin{cases} 0.2, & 0 \leq \wp \leq 1 \\ -0.2, & 1 \leq \wp \leq 2 \\ 0, & others \end{cases}$$

Table 3: The upper and lower membership functions of the original system.	
The upper membership functions	The lower membership functions
$\overline{\alpha}_1(\varpi(\wp)) = 1 - \frac{1}{1+e^{-(2-\varpi_1(\wp)+3)}}$ $\overline{\alpha}_2(\varpi(\wp)) = 1 - \frac{1}{1+e^{-(2-\varpi_1(\wp)-3)}}$	$\underline{\alpha}_1(\varpi(\wp)) = 1 - \frac{1}{1+e^{-(2-\varpi_1(\wp)+5)}}$ $\underline{\alpha}_2(\varpi(\wp)) = 1 - \frac{1}{1+e^{-(2-\varpi_1(\wp)-5)}}$

Let $\underline{\pi}(\varpi(\wp)) = 0.4sin^2(\varpi(\wp)), \overline{\pi}(\varpi(\wp)) = 1 - \underline{\pi}(\varpi(\wp)), \underline{\nu}(\varpi(\wp)) = \overline{\nu}(\varphi(\wp)) = 0.6$. Assume the generalized ETC performance be specified by $\iota_m = 1.4, \iota_M = 3.10, \lambda = 0.2$, and H_∞ performance level is chosen as $\delta = 1.5$. By applying Theorem 3.3,for $\gamma_1 = 0.45$, the feedback controller gain can be obtained as

$$K_1 = \begin{bmatrix} 0.6844 & 0.0952 & 0.0479 \end{bmatrix}, K_2 = \begin{bmatrix} 0.6435 & -0.1995 & 0.0775 \end{bmatrix}.$$

Table 4: The upper and lower membership functions for controller.	
The upper membership functions	The lower membership functions
$\overline{\beta}_1(\varpi(\wp)) = \underline{\beta}_1(\varpi(\wp))$ $\overline{\beta}_2(\varpi(\wp)) = 1 - \underline{\beta}_1(\varpi(\wp))$	$\underline{\beta}_1(\varpi(\wp)) = \frac{e^{-\varpi_1^2(\wp)}}{6}$ $\underline{\beta}_2(\varpi(\wp)) = 1 - \underline{\beta}_1(\varpi(\wp))$

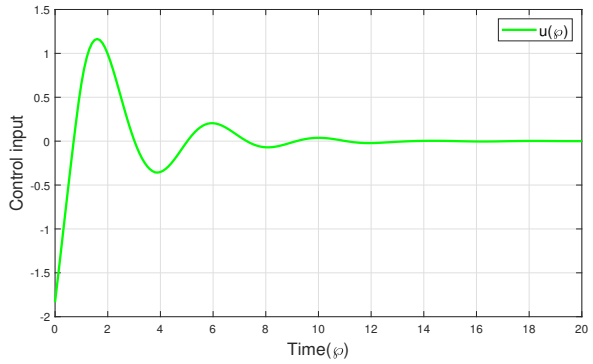


Figure 10: Trajectory of control input.

By choosing the sampling interval as $h = 0.07$ and assume the initial condition $\varpi(0) = [0.3, -0.8]^T$. State responses discrete-time IT2 fuzzy system without control input depicted in Fig. 6. State response for discrete-time IT2 fuzzy system with control input shown in Fig. 7. According to Fig. 7, it is clear that the state trajectories under the proposed control scheme are converging to zero within $5s$. In Fig. 8. propose the state response for discrete-time type-1 fuzzy system are converging to zero within $7s$. From Fig. 7 and Fig. 8, it is clear that the proposed IT2 fuzzy model is effective. The adaptive triggering parameter $\gamma(\wp)$ and the triggered instants with intervals are is described in Fig. 9. In Fig. 10. present trajectories of control input. The simulation results clearly demonstrate the utility of the established theoretical results in the study of H_∞ control for discrete-time IT2 fuzzy systems via AETS.

Remark 4.3. *It must be observed that the type-1 fuzzy-model-based results cannot handle uncertain parameters. The lower and upper member functions in the IT2 fuzzy model of this paper are used to describe the uncertainties. The proposed IT2 fuzzy model approach is more general and effective in contrast to existing results* [7, 23, 24]. *In compared to existing work* [17]*, our results are quickly converge, where the trajectories are converges to origin within* 14*s as shown in Fig.3.*

5 Conclusion

In this work, we have investigated the problem of H_∞ control for discrete-time IT2 fuzzy systems with infinite distributed delay via adaptive event-triggered scheme. Furthermore, the adaptive event-triggered technique was developed to re-

duce communication's power consumption. The importance of the proposed technique in terms of state convergence, stability, and avoiding unwanted triggering events are illustrated by numerical simulations achieved with the examined discrete-time IT2 fuzzy system via an adaptive event-triggered scheme. In addition, IT2 is used to approximate the external disturbances of the system due to its better estimation ability to describe the fuzzy system uncertainties. Also, by using the Lyapunov functional method, the resulting criterion guarantees that discrete-time IT2 fuzzy systems with efficiency H_∞ are mean-square exponentially stable. The effectiveness of the IT2 fuzzy system by using AETS approach is demonstrated by two examples of numerical simulation and real-world application. The authors will try to extend this in further studies by making resilient tracking control susceptible to a variety of attacks, including DoS, reply, and deception attacks. The theory's capacity to be applied to actual engineering practice is significantly improved.

Conflict of Interest The authors does not have any conflict of interest.

Acknowledgement:

The financial support from the National Research Council of Thailand (Talented Mid-Career Researchers) Grant Number N42A650250.

References

[1] M. S. Ali, *Robust stability of stochastic uncertain recurrent neural networks with Markovian jumping parameters and time-varying delays*, International Journal of Machine Learning and Cybernetics, **5**(1) (2014), 13-22. https://doi.org/10.1007/s13042-012-0124-6

[2] M. S. Aslam, X. Dai, T. Zhao, *Stability and stabilization of network T-S fuzzy systems with random packet-loss for synchronous machine*, Iranian Journal of Fuzzy Systems, **18**(1) (2021), 35-51. https://doi.org/10.22111/IJFS.2021.5870

[3] P. Balasubramaniam, L. J. Banu, *Robust stability criterion for discrete-time nonlinear switched systems with randomly occurring delays via T-S fuzzy approach*, Information Sciences, **20**(6) (2015), 49-61. https://doi.org/10.1002/cplx.21530

[4] B. Ding, B. Wang, R. Zhang, *Event-triggered control for hybrid power supply of fuel-cell heavy-duty truck*, Journal of Energy Storage, **41** (2021), 102985. https://doi.org/10.1016/j.est.2021.102985

[5] L. Fan, Q. Zhu, *Mean square exponential stability of discrete-time Markov switched stochastic neural networks with partially unstable subsystems and mixed delays*, Information Sciences, **580** (2021), 243-259. https://doi.org/10.1016/j.ins.2021.08.068

[6] M. Gao, L. Zhang, W. Qi, J. Cao, *Security control for T-S fuzzy systems with multi-sensor saturations and distributed event-triggered mechanism*, Journal of the Franklin Institute, **357**(5) (2020), 2851-2867. https://doi.org/10.1016/j.jfranklin.2020.02.013

[7] H. Gao, Y. Zhao, T. Chen, H_∞ *fuzzy control of nonlinear systems under unreliable communication links*, ISA Transactions, **17**(2) (2009), 96-104. https://doi.org/10.1109/TFUZZ.2008.924315

[8] A. Kazemy, J. Lam, Z. Chang, *Adaptive event-triggered mechanism for networked control systems under deception attacks with uncertain occurring probability*, International Journal of Systems Science, **52**(7) (2021), 1426-1439. https://doi.org/10.1080/00207721.2020.1858205

[9] B. A. Khashooei, D. J. Antunes, W. P. M. H. Heemels, *Output-based event-triggered control with performance guarantees*, IEEE Transactions on Automatic Control, **62**(7) (2017), 3646-3652. https://doi.org/10.1109/TAC.2017.2672201

[10] Q. Li, Y. Pan, Z. Zhang, H. K. Lam, *Reliable dissipative interval type-2 fuzzy control for nonlinear systems with stochastic incomplete communication route and actuator failure*, International Journal of Fuzzy Systems, **22** (2020), 368-379. https://doi.org/10.1007/s40815-020-00807-y

[11] Z. Lian, P. Shi, C. C. Lim, *Hybrid-triggered interval type-2 fuzzy control for networked systems under attacks*, Information Science, **567** (2021), 332-347. https://doi.org/10.1016/j.ins.2021.03.050

[12] J. Liu, Q. Liu, J. Cao, Y. Zhang, *Adaptive event-triggered H_∞ filtering for T-S fuzzy system with time delay*, Neurocomputing, **189** (2015), 86-94. https://doi.org/10.1016/j.neucom.2015.12.049

[13] Y. Liu, Z. Wang, J. Liang, X. Liu, *Synchronization and state estimation for discrete-time complex networks with distributed delays*, IEEE Transactions on Systems, Man, and Cybernetics: Systems, **38**(5) (2008), 1314-1325. https://doi.org/10.1109/TSMCB.2008.925745

[14] K. Mathiyalagan, R. Sakthivel, *Robust stabilization and H_∞ control for discrete-time stochastic genetic regulatory networks with time delays*, Canadian Journal of Physics, **90**(10) (2012), 939-953. https://doi.org/10.1139/p2012-088

[15] M. Mubeen Tajudeen, M. S. Ali, S. A. Kauser, K. Subkrajang, A. Jirawattanapanit, G. Rajchakit, *Adaptive event-triggered control for complex dynamical network with random coupling delay under stochastic deception attacks*, Hindawi Complexity, (2022). https://doi.org/10.1155/2022/8761612

[16] R. Pan, Y. Tana, D. Du, S. Fei, *Adaptive event-triggered synchronization control for complex networks with quantization and cyber-attacks*, Neurocomputing, **382** (2020), 249-258. https://doi.org/10.1016/j.neucom.2019.11.096

[17] B. Priya, M. Mubeen Tajudeen, M. Syed Ali, G. Rajchakit, G. Kumar Thakur, *Event-triggered H_∞ control of complex dynamical networks subject to stochastic cyber-attacks by new two-sided delay-dependent LKF functional*, Journal of Control and Decision, (2023). https://doi.org/10.1080/23307706.2023.2240338

[18] Z. Qu, Z. Du, G. Zhang, *Sampled-data H_∞ control for nonlinear dynamical intelligent fuzzy systems with time-varying delay*, 2022 34th Chinese Control and Decision Conference (CCDC), (2022), 4077-4084.

[19] N. Rong, Z. Wang, *State-dependent asynchronous intermittent control for IT2 T-S fuzzy interconnected systems under deception attacks*, Nonlinear Dynamics, **100** (2020), 3433-3448. https://doi.org/10.1007/s11071-020-05669-5

[20] A. W. A. Saif, M. Mudasar, M. Mysorewala, M. Elshafei, *Observer-based interval type-2 fuzzy logic control for nonlinear networked control systems with delays*, International Journal of Fuzzy Systems, **22** (2020), 380-399. https://doi.org/10.1007/s40815-020-00799-9

[21] J. Sun, H. Zhang, Y. Wang, S. Sun, *Fault-tolerant control for stochastic switched IT2 fuzzy uncertain time-delayed nonlinear systems*, IEEE Transactions on Cybernetics, **52**(2) (2022), 1335-1346. https://doi.org/10.1109/TCYB.2020.2997348

[22] J. Suo, Z. Wang, B. Shen, *Pinning synchronization control for a class of discrete-time switched stochastic complex networks under event-triggered mechanism*, Nonlinear Analysis: Hybrid Systems, **37** (2020), 100886. https://doi.org/10.1016/j.nahs.2020.100886

[23] M. Syed Ali, M. Mubeen Tajudeen, G. Rajchakit, B. Priya, G. Kumar Thakur, *Adaptive event-triggered pinning synchronization control for complex networks with random saturation subject to hybrid cyber-attacks*, International Journal of Adaptive Control and Signal Processing, **37**(8) (2023), 2041-2062. https://doi.org/10.1002/acs.3625

[24] J. Tao, R. Lu, P. Shi, H. Su, Z. G. Wu, *Dissipativity-based reliable control for fuzzy Markov jump systems with actuator faults*, IEEE Transactions on Cybernetics, **47**(9) (2017), 2377-2388. https://doi.org/10.1109/TCYB.2016.2584087

[25] X. Wang, J. H. Park, H. Yang, Z. Yu, *Sampled-data-based $\mathcal{H}_\infty$ fuzzy pinning synchronization of complex networked systems with adaptive event-triggered communications*, IEEE Transactions on Fuzzy Systems, **30**(7) (2022), 2254-2265. https://doi.org/10.1109/TFUZZ.2021.3078643

[26] G. Wei, G. Feng, Z. Wang, *Robust $\mathcal{H}_\infty$ control for discrete-time fuzzy systems with infinite-distributed delays*, IEEE Transactions on Fuzzy Systems, **17**(1) (2009). https://doi.org/10.1109/TFUZZ.2008.2006621

[27] Y. Yan, C. Yang, X. Ma, L. Zhou, *Observer-based event-triggered control for singularly perturbed systems with saturating actuator*, International Journal of Robust and Nonlinear Control, **29**(12) (2019), 3954-3970. https://doi.org/10.1002/rnc.4589

[28] L. Yang, C. Guan, Z. Fei, *Finite-time asynchronous filtering for switched linear systems with an event-triggered mechanism*, Journal of the Franklin Institute, **356**(10) (2015), 5503-5520. https://doi.org/10.1016/j.jfranklin.2019.03.019

[29] X. Yang, H. Wang, Q. Zhu, *Event-triggered predictive control of nonlinear stochastic systems with output delay*, Automatica, **140** (2022), 110230. https://doi.org/10.1016/j.automatica.2022.110230

[30] X. Yang, Q. Zhu, *Stabilization of stochastic retarded systems based on sampled-data feedback control*, IEEE Transactions on Systems, Man, and Cybernetics: Systems, **51**(9) (2021), 5895-5904. https://doi.org/10.1109/TSMC.2019.2956757

[31] Y. Zeng, H. K. Lam, B. Xiao, L. Wu, $l_2 - l_\infty$ *control of discrete-time state-delay interval type-2 fuzzy systems via dynamic output feedback*, IEEE Transactions on Cybernetics, **52**(6) (2022), 4198-4208. https://doi.org/10.1109/TCYB.2020.3024754

[32] H. Zhang, Q. Ma, J. Lu, Y. Chu, Y. Li, *Synchronization control of neutral-type neural networks with sampled-data via adaptive event-triggered communication scheme*, Journal of the Franklin Institute, **358**(3) (2021), 1999-2014. https://doi.org/10.1016/j.jfranklin.2021.01.005

[33] S. Zhang, J. Zhao, *Dwell-time-dependent H_∞ bumpless transfer control for discrete-time switched interval type-2 fuzzy systems*, IEEE Transactions on Fuzzy Systems, **30**(7) (2022), 2426-2437. https://doi.org/10.1109/TFUZZ.2021.3083322

[34] D. Zhang, Z. Zhou, X. Ji, *Networked fuzzy output feedback control for discrete-time Takagi-Sugeno fuzzy systems with sensor saturation and measurement noise*, Information Sciences, **457**(8) (2018), 182-194. https://doi.org/10.1016/j.ins.2018.02.026

[35] N. Zhao, P. Shi, W. Xing, C. P. Lim, *Resilient adaptive event-triggered fuzzy tracking control and filtering for nonlinear networked systems under denial-of-service attacks*, IEEE Transactions on Fuzzy Systems, **30**(8) (2022), 3191-3201. https://doi.org/10.1109/TFUZZ.2021.3106674

[36] Q. Zhu, *Stabilization of stochastic nonlinear delay systems with exogenous disturbances and the event-triggered feedback control*, IEEE Transactions on Automatic Control, **64**(9) (2019), 3764-3771. https://doi.org/10.1109/TAC.2018.2882067