Iranian Journal of Fuzzy Systems

Volume 21, Number 3, (2024), pp. 137-154

Original paper

University of Sistan and Baluchestan

Event-triggered H_∞ filtering for T-S fuzzy discrete-time conic-type nonlinear networked control systems

M. Syed Ali [1], B. Vigneshwar [2], G. Rajchakit [3], B. Priya [4] and G. K. Thakur [5]

[1,2] *Complex Systems and Networked Science Research Laboratory, Department of Mathematics, Thiruvalluvar University, Vellore-632115, Tamil Nadu, India*

[3] *Department of Mathematics, Faculty of Science, Maejo University, Chiang Mai 50290, Thailand*

[4] *Department of Applied Science, G L BAJAJ Institute of Technology and Management, Greater Noida, Uttar Pradesh, India*

[5] *Department of Applied sciences and Humanities, Govenment Engineering College, Vaishali, Bihar, India*

syedjessi@gmail.com, bvigneshwar1998@gmail.com, griengkraj@yahoo.com, bandanapriya@gmail.com, ganesh.thakur@abes.ac.in

Abstract

In this paper event-triggering filtering problem for a class of conic-type nonlinear networked control systems subject to the prescribed disturbance attenuation level. The T-S fuzzy model approach is utilized to approximate the nonlinear dynamics. Conic-type constraints are satisfied by the nonlinear terms within a hyper-sphere centered at an uncertain center. To alleviate the data transmission burden of communication channels, an event-triggering mechanism is adopted, where the data can be transmitted only if certain condition is fulfilled. It is the purpose of this paper to design a filter for the addressed conic-type nonlinear system ensuring that the filtering error system is asymptotically stable whereas the prespecified H_∞ criterion is guaranteed. By using Lyapunov approach in combination with matrix analysis method, the sufficient conditions are established for the existence of desired filtering parameters in terms of the solvability of certain linear matrix inequalities. Finally, a numerical simulation example is forward to demonstrate the validity and effectiveness of the developed theoretical results.

Keywords: T-S fuzzy system, conic-type nonlinearities, H_∞ filter, event-triggered scheme, network control system.

1 Introduction

T-S fuzzy models (Takagi and Sugeno, 1985) have attracted the attention of industry practitioners and academic researchers. These systems are widely used in many fields [13, 16, 28]. There are many examples, such as aircraft flight systems [14], communications networks [26], robotics [24], and mathematical finance [36]. Since its ability to approximate nonlinear dynamics well, T-S fuzzy models have become increasingly popular recently [9]. In order to model nonlinear systems, the T-S fuzzy model utilizes IF-THEN rules in order to effectively express nonlinearities [37]. A non-linear system model is formed through a set of linearized subsystems using fuzzy logic [34]. In recent years, most physical systems and processes, especially networked ones, have become more nonlinear. It has been reported in [7, 25, 40] that some work has been done on filtering, model reduction, and control algorithms in the last few years. As a result, network-based filtering has become increasingly important. However, nonlinear systems cannot be analyzed to determine their stability, making the nonlinear system filter design problem unsolvable [11]. In order to cope with this problem, an increasing number of academics have turned their attention to filtering over the last two decades. The concept of fuzzy logic has been proposed for complex nonlinear systems for many years [29].

Nonlinearities cannot be avoided in real-world applications because of external perturbation, modeling errors, and analysis, among other things [6]. Dynamic systems with this kind of nonlinearity have recently been subjected to significant development in sliding mode control, observer/estimator design, robust H_∞ control and finite-time control

Corresponding Author: M. Syed Ali

Received: August 2023; Revised: July 2024; Accepted: August 2024.

https://doi.org/10.22111/ijfs.2024.46459.8184

and synthesis [2, 5, 30]. Because of this, control theory never stops discussing nonlinearities. In practice, more than one type of engineering nonlinearity may be converted into conic-type nonlinearity, Lipschitz nonlinearity, locally sinusoidal nonlinearity, or dead zone nonlinearity, to name just a few [16, 24]. Nonlinear systems of conic type have received a lot of attention recently as a special class of systems. The nonlinearity of conic type is expressed in a hyper-sphere of n dimensions [23, 38]. There are two linear systems that determine the radius and the center of the hyper-sphere [1]. Thus, the study of nonlinear networked control systems of conic type is of further practical importance. Since network resources are limited, measurements should be transmitted on demand rather than at a set time [32].

Another interesting and fundamental issue in the control area is the filtering problem for practical systems. Due to its powerful application in real world, event-triggered filters or controllers for dynamic systems have become hot topics [27, 39]. As a result of its advantages for improving network bandwidth utilization and saving network resources, event-triggered schemes (ETS) have recently been gaining considerable attention [21]. Several control and filtering issues have been addressed with the event-triggered control, including output feedback, fault detection, H_∞ filtering, tracking and state feedback [10, 42]. Many different types of systems can benefit from event-triggered filters, including sample-data systems, networked control systems, neural networks, T-S fuzzy systems, discrete-time systems, and Markovian jump systems [15, 33]. Event-triggered filtering has been the subject of several studies in recent years [8].

The networked control systems (NCSs) showed great advantages over traditional control systems over the last decade, which prompted both academia and industry to focus on their research [12, 18, 22]. Throughout the past decade, networked systems have seen widespread implementation in a wide range of areas, including smart grids, robotics, artificial intelligence, unmanned vehicles, and smart cities [31]. Researchers have therefore been increasingly interested in stability, estimation, filtering, and control problems for networked control systems over the past few years [3, 35]. Moreover, networked control systems modeling is often accompanied by complex nonlinear factors in actual engineering and practice, including neural network activation functions, complex dynamic network couplings, and multi-agent systems with nonlinear communication topologies [17, 19, 41]. A signal update is scheduled more efficiently by reducing the unnecessary signals using an event-triggered filter (ETF). Designing the event-triggered mechanism (ETM) is a fundamental issue in event-triggered filter development. Once the controlled plant has reached equilibrium, it is not necessary to transmit all sampled signals to the node. Consequently, time-triggered systems consume a lot of network resources [4].

There are many advantages to using networked systems, including their simplicity, low cost, ease of maintenance, and high reliability. In spite of this, networked systems present some challenges. Firstly, network-induced delays are one of the biggest problems in networked systems, and results have been published on this issue. Second, in real-life networked systems, some states are unavailable, and the filter plays a critical role in estimating them. It was studied how optimal estimation is affected by communication delays and packet dropouts in networked systems. There are nonlinear networked control systems as well as event-triggered mechanisms in the model under investigation. Several applications of digital control execute most control tasks periodically.

A conic-type nonlinear network based on T-S fuzzy systems is discussed in this article as an event-triggered H_∞ filtering technique implemented to regulate the data transmission and reduce communication cost. Some integral inequalities are used for analyzing the stability of discrete-time systems with time-varying delays. By choosing an appropriate Lyapunov-Krasovskii functions and in terms of certain linear matrix inequalities, sufficient condition of the existence for the desired filtering parameters are formulated with the guarantee of both asymptotic stability and the prespecified H_∞ specification. Finally, the proposed theoretical results are verified via a numerical simulation example. The main contributions of this work are summarized as follows:

1. Considering the event-triggered mechanism, the H_∞ filtering of nonlinear networked control systems via discrete T-S fuzzy models is investigated. The model of the system under investigation is comprehensive which covers the event triggered mechanism and nonlinear networked control systems.

2. By the Lyapunov-Krasovskii method, some sufficient conditions are derived to ensure asymptotic stability and the H_∞ performance of he filtering error systems.

The remaining of this paper is arranged as follows. In Section 2, the event-triggered condition and time delay transformation model are given, and the filtering error system is constructed by the means of a polynomial fuzzy model.

Section 3 presents the main results of designing the desired H_∞ filter. In the Section 4, a numerical simulation example is provided to demonstrate the effectiveness of the proposed method. Section 5 draws the conclusions.

Notation: The n-dimensional Euclidean space and its set of all $m \times n$ real matrices are denoted by the symbols $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$, respectively. $\mathbf{M}^T$ and $\mathbf{M}^{-1}$ represent the transpose and inverse matrices, respectively. The matrix $\mathbf{P} > 0$ is positive-definite. Unit and zero matrices are denoted with $\mathcal{I}$ and 0, respectively. $\mathbf{Z}^+$ represents the set of all nonnegative integers. $*$ stands for the symmetric term and $sym\{\mathbf{P}\} = \mathbf{P} + \mathbf{P}^T$.

2 Problem formulation

The architecture of the filtering problem addressed in this paper is illustrated in Fig.1, which, in comparison to the traditional system structure, includes an event detector and a zero order hold (ZOH) that are connected via network. The problem formulation is expressed as follows.

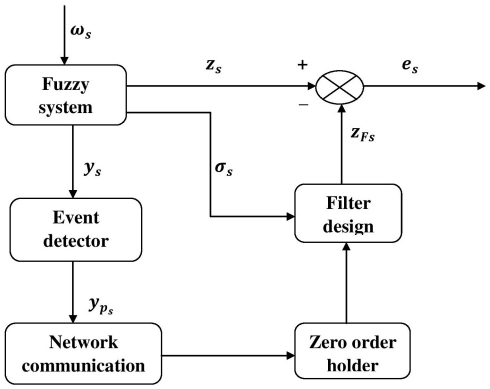


Figure 1: Nonlinear NCSs with event-triggered mechanism

We consider nonlinear discrete-time conic T-S fuzzy systems in terms of formulas:

$$\begin{cases} x_{\mathbf{s}+1} = \varphi(x_{\mathbf{s}}, \omega_{\mathbf{s}}), \\ y_{\mathbf{s}} = Cx_{\mathbf{s}} + Dw_{\mathbf{s}}, \\ z_{\mathbf{s}} = Lx_{\mathbf{s}}, \end{cases} \tag{1}$$

where $x_{\mathbf{s}} \in \mathfrak{R}^n$ is the state vector; $\omega_{\mathbf{s}} \in \mathfrak{R}^q$ is the disturbance input, which belongs to $L_2[0,\infty)$; $y_{\mathbf{s}} \in \mathfrak{R}^m$ is the measurement output; and $z_{\mathbf{s}} \in \mathfrak{R}^p$ is the estimated output. For the nonlinear function $\varphi(x_{\mathbf{s}}, \omega_{\mathbf{s}})$ satisfies the following conic sector description:

$$2\|\varphi(x_{\mathbf{s}}, \omega_{\mathbf{s}}) - (Ax_{\mathbf{s}} + B\omega_{\mathbf{s}})\| \leq \|Mx_{\mathbf{s}} + N\omega_{\mathbf{s}}\|. \tag{2}$$

Remark 2.1. [40] *Conic-type nonlinear function $\varphi(x_{\mathbf{s}}, \omega_{\mathbf{s}})$ lies in an n-dimensional hyper-sphere whose center describes a linear system $Ax_{\mathbf{s}} + B\omega_{\mathbf{s}}$ and whose radius describes another linear system $Mx_{\mathbf{s}} + N\omega_{\mathbf{s}}$. For, our convenience we take $\phi_{\mathbf{s}} = \|\varphi(x_{\mathbf{s}}, \omega_{\mathbf{s}}) - (Ax_{\mathbf{s}} + B\omega_{\mathbf{s}})\|$. Then, we obtain*

$$\phi_{\mathbf{s}}^T \phi_{\mathbf{s}} \leq (Mx_{\mathbf{s}} + N\omega_{\mathbf{s}})^T (Mx_{\mathbf{s}} + N\omega_{\mathbf{s}}). \tag{3}$$

Plant rule i: IF $\alpha_{1\mathbf{s}}$ is $z_{p1}, ..., \alpha_{q\mathbf{s}}$ is z_{pq}. and $\alpha_{r\mathbf{s}}$ is z_{pr}, THEN

$$\begin{cases} x_{\mathbf{s}+1} = A_p x_{\mathbf{s}} + B_p \omega_{\mathbf{s}} + \phi_{\mathbf{s}}, \\ y_{\mathbf{s}} = C_p x_{\mathbf{s}} + D_p \omega_{\mathbf{s}}, \\ z_{\mathbf{s}} = L_p x_{\mathbf{s}}, \end{cases} \tag{4}$$

In this case, A_p, B_p, C_p, D_p, $L_p(p = 1, 2, ..., i)$ refers to constant known matrices with appropriate dimensions, i is the set of IF-THEN rules, and $\alpha_{r\mathbf{s}}$ represents the fuzzy set and $z_{pq}(q = 1, 2, ..., j)$ represents the premise variable.

Basic fuzzy functions are defined as follows:

$$\mathfrak{h}_p(\alpha_{\mathbf{s}}) = \frac{N_p(\alpha_{\mathbf{s}})}{\sum_p^i N_p(\alpha_{\mathbf{s}})}, \tag{5}$$

where $N_p(\alpha_{\mathbf{s}}) = \Pi_{q=1}^{j} z_{pq}(\alpha_{q\mathbf{s}})$, and $z_{pq}(\alpha_{q\mathbf{s}})$ stands for the grade of the membership of $\alpha_{q\mathbf{s}}$ in the fuzzy set z_{pq}. Therefore, for all t, we have $\mathfrak{h}_p(\alpha_{\mathbf{s}}) \geq 0$, $\sum_{p=1} \mathfrak{h}_p(\alpha_{\mathbf{s}}) = 1$.

The fuzzy system can be expressed as follows:

$$\begin{cases} x_{\mathbf{s}+1} = A(\mathfrak{h}_{\mathbf{t}})x_{\mathbf{s}} + B(\mathfrak{h}_{\mathbf{t}})\omega_{\mathbf{s}} + \phi_{\mathbf{s}}, \\ y_{\mathbf{s}} = C(\mathfrak{h}_{\mathbf{t}})x_{\mathbf{s}} + D(\mathfrak{h}_{\mathbf{t}})\omega_{\mathbf{s}}, \\ z_{\mathbf{s}} = L(\mathfrak{h}_{\mathbf{t}})x_{\mathbf{s}}, \end{cases} \tag{6}$$

where

$$A(\mathfrak{h}_{\mathbf{t}}) = \sum_p^i \mathfrak{h}_p(\alpha_{\mathbf{s}})A_p, \qquad B(\mathfrak{h}_{\mathbf{t}}) = \sum_p^i \mathfrak{h}_p(\alpha_{\mathbf{s}})B_p, \qquad C(\mathfrak{h}_{\mathbf{t}}) = \sum_p^i \mathfrak{h}_p(\alpha_{\mathbf{s}})C_p,$$

$$D(\mathfrak{h}_{\mathbf{t}}) = \sum_p^i \mathfrak{h}_p(\alpha_{\mathbf{s}})D_p, \qquad L(\mathfrak{h}_{\mathbf{t}}) = \sum_p^i \mathfrak{h}_p(\alpha_{\mathbf{s}})L_p.$$

3 Event-triggered mechanism

A time-triggered mechanism is presented for determining when sampling data can be transmitted via a communication network to the estimator. A sampling package should be transferred if the event detector detects a change in the current sampling package. Firstly, we define:

$$\sigma_{\mathbf{s}} = y_{\mathbf{s}} - y(p_{\mathbf{s}}), \tag{7}$$

A propagating time instant is $p_{\mathbf{s}}$, and a transmission data is $y(p_{\mathbf{s}})$ with $\mathbf{t} \in [p_{\mathbf{s}}, p_{\mathbf{t}+1})$. In order to trigger the event, we need the following conditions:

$$\sigma_{\mathbf{s}}^T \Sigma \sigma_{\mathbf{s}} \leq \delta y^T(p_{\mathbf{s}})\Sigma y(p_{\mathbf{s}}), \tag{8}$$

With $\Sigma > 0$ denoting a matrix of suitable dimensions, and $\delta > 0$ denoting a scalar of appropriate dimensions. Only measurements signals satisfying (8) will be transmitted over the communication channel.

Remark 3.1. [5] *An event detector determines whether the remote sensor should transmit data to the estimator based on the triggering condition (8). Time-triggered sensor data is received by the estimator, while event-triggered data is received by the sensor. As a result of differences between the latest propagated information $y(p_{\mathbf{s}})$ and the measured output $y_{\mathbf{s}}$, it determines the transmitted instant $p_{\mathbf{s}}$. The measurements will only be transmitted if they differ significantly from $y(p_{\mathbf{s}})$. This mechanism could reduce network communication costs by reducing the frequency of data transmission. As a result, all measurements will be transmitted if $\Sigma = 0$.*

Zero order holder: Event detectors cause signals selected to be transmitted by the event-trigger mechanism to be nonuniform. It is used here to keep the signal available for estimation at all times through implementation of a zero order holder (ZOH). As data transmission causes time delays, ι_t is introduced with $0 \leq \iota_t \leq \iota_{\mathfrak{X}}$ implementing the constant positive integer $\iota_{\mathfrak{X}}$ as a transmission delay for the network. Passing through the zero order holder results in an output signal that satisfies the following conditions.

$$\tilde{y}_{\mathbf{s}} = y(p_{\mathbf{s}}), \mathbf{s} \in [p_{\mathbf{s}} + \iota_{p_{\mathbf{s}}}, p_{\mathbf{s}+1} + \iota_{p_{\mathbf{s}+1}}). \tag{9}$$

Define

$$\begin{cases} I_0 = & [p_{\mathbf{s}} + \iota(p_{\mathbf{s}}), p_{\mathbf{s}} + \iota(p_{\mathbf{s}}+1)+1) \\ I_1 = & [p_{\mathbf{s}} + n + \iota(p_{\mathbf{s}}+n), p_{\mathbf{s}} + n + \iota(p_{\mathbf{s}}+n+1)+1) \\ I_d = & [p_{\mathbf{s}} + d + \iota(p_{\mathbf{s}}+d), p_{\mathbf{s}} + d + \iota(p_{\mathbf{s}}+d+1)+1) \end{cases} \tag{10}$$

where $n = 1, 2, ..., d-1$.
Moreover, define $d_{\mathbf{s}}$ and $\sigma_{\mathbf{s}}$ as

$$d_{\mathbf{s}} = \begin{cases} \mathbf{s} - \iota_{\mathbf{s}}, & \mathbf{s} \in I_0 \\ \mathbf{s} - \iota_{\mathbf{s}} - n, & \mathbf{s} \in I_1, n = 1, 2, ..., d-1 \\ \mathbf{s} - \iota_{\mathbf{s}}, & \mathbf{s} \in I_d. \end{cases} \tag{11}$$

$$\sigma_{\mathbf{s}} = \begin{cases} 0, & \mathbf{s} \in I_0 \\ y(p_{\mathbf{s}}) - y(p_{\mathbf{s}} + n), & \mathbf{s} \in I_1, n = 1, 2, ..., d-1 \\ y(p_{\mathbf{s}}) - y(p_{\mathbf{s}} + d), & \mathbf{s} \in I_d. \end{cases} \tag{12}$$

Then it follows that

$$\begin{cases} \iota_{p_{\mathbf{s}}} \le d_{\mathbf{s}} \le 1 + \iota_{\mathfrak{X}} d^{\mathfrak{X}}, & \mathbf{s} \in I_0 \\ \iota_{p_{\mathbf{s}}} \le d_{\mathbf{s}} \le d^{\mathfrak{X}}, & \mathbf{s} \in I_1, n = 1, 2, ..., d-1 \\ \iota_{p_{\mathbf{s}}} \le d_{\mathbf{s}} \le d^{\mathfrak{X}}, & \mathbf{s} \in I_d. \end{cases} \tag{13}$$

from which, we can easily see that

$$0 \le \iota_{p_{\mathbf{s}}} \le d_{\mathbf{s}} \le d^{\mathfrak{X}},$$

Furthermore, it is acquired that

$$\tilde{y}_{\mathbf{s}} = [y(p_{\mathbf{s}}) = [y(\mathbf{s} - d_{\mathbf{s}}) - \sigma_{\mathbf{s}}], \mathbf{s} \in [p_{\mathbf{s}} + \iota_{p_{\mathbf{s}}}, p_{\mathbf{s}+1} + \iota_{p_{\mathbf{s}+1}}). \tag{14}$$

Remark 3.2. [6] *When the ZOH receives a data packet from the network $(p_{\mathbf{s}}, y(p_{\mathbf{s}}))$, the filter can work with the signal $y(p_{\mathbf{s}})$. ZOHs use event-driven data transmission in the process of transmitting data. Once the ZOH receives a new data packet, the filter will update the signal. Transmission can be prevented thus by avoiding out-of-order packets.*

4 Filter structure

The following fuzzy filter will be constructed in this paper.
Plant rule p: IF $\alpha_{1\mathbf{s}} isz_{p1}, ..., \alpha_{q\mathbf{s}} isz_{pq}$. and $\alpha_{r\mathbf{s}} isz_{pr}$, THEN

$$\begin{cases} x_{F\mathbf{s}+1} = A_{Fp} x_{F\mathbf{s}} + B_{Fp} \tilde{y}_{\mathbf{s}}, \\ z_{F\mathbf{s}} = C_{Fp} x_{F\mathbf{s}} + D_{Fp} \tilde{y}_{\mathbf{s}}, \end{cases} \tag{15}$$

where $x_F \in \mathfrak{R}^n$ is the state variable of the filter; $z_{F\mathbf{s}} \in \mathfrak{R}^p$ is the estimation of $z_{\mathbf{s}}$, and A_{Fp}, B_{Fp}, C_{Fp} and D_{Fp} are constant matrices with appropriate dimensions.
According to (4) and (14), we have

$$\begin{cases} x_{F\mathbf{s}+1} = A_F(\mathfrak{h}_{\mathbf{s}}) x_{F\mathbf{s}} + B_F(\mathfrak{h}_{\mathbf{s}})[y(\mathbf{s} - d_{\mathbf{s}}) - \sigma_{\mathbf{s}}], \\ z_{F\mathbf{s}} = C_F(\mathfrak{h}_{\mathbf{s}}) x_{F\mathbf{s}} + D_F(\mathfrak{h}_{\mathbf{s}})[y(\mathbf{s} - d_{\mathbf{s}}) - \sigma_{\mathbf{s}}], \end{cases} \tag{16}$$

where

$$A_F(\mathfrak{h}_{\mathbf{s}}) = \sum_p^i \mathfrak{h}_p(\alpha_{\mathbf{s}}) A_{Fp}, \qquad B_F(\mathfrak{h}_{\mathbf{s}}) = \sum_p^i \mathfrak{h}_p(\alpha_{\mathbf{s}}) B_{Fp},$$

$$C_F(\mathfrak{h}_{\mathbf{s}}) = \sum_p^i \mathfrak{h}_p(\alpha_{\mathbf{s}}) C_{Fp}, \qquad D_F(\mathfrak{h}_{\mathbf{s}}) = \sum_p^i \mathfrak{h}_p(\alpha_{\mathbf{s}}) D_{Fp}.$$

Filtering error system

Thus, the conic-type error filter is as follows:

$$\begin{cases} \zeta_{\mathbf{s}+1} = \sum_{p=1}^{s}\sum_{q=1}^{s}\{\tilde{A}_{pq}\zeta_{\mathbf{s}} + \tilde{B}_{pq}\zeta_{\mathbf{s}-d(\mathbf{s})} + \tilde{C}_{pq}v_{\mathbf{s}} + \tilde{D}_{pq}\sigma_{\mathbf{s}} + \tilde{\phi}_{\mathbf{s}}\}, \\ e_{\mathbf{s}} = \sum_{p=1}^{s}\sum_{q=1}^{s}\{\tilde{E}_{pq}\zeta_{\mathbf{s}} + \tilde{F}_{pq}\zeta_{\mathbf{s}-d(\mathbf{s})} + \tilde{G}_{pq}v_{\mathbf{s}} + \tilde{H}_{pq}\sigma_{\mathbf{s}}\}, \end{cases} \tag{17}$$

Denote

$$\zeta_{\mathbf{s}}^T = \left[\begin{array}{cc} x_{\mathbf{s}}^T & x_{F\mathbf{s}}^T \end{array}\right], e_{\mathbf{s}} = z_{\mathbf{s}} - z_{F\mathbf{s}},$$

where

$$\tilde{A}_{pq} = \left[\begin{array}{cc} A_p & 0 \\ 0 & A_{Fq} \end{array}\right], \tilde{B}_{pq} = \left[\begin{array}{cc} 0 & 0 \\ B_{Fq}C_p & 0 \end{array}\right], \tilde{C}_{pq} = \left[\begin{array}{cc} B_p & 0 \\ 0 & B_{Fq}D_p \end{array}\right], \tilde{D}_{pq} = \left[\begin{array}{c} 0 \\ -B_{Fq} \end{array}\right],$$

$$\tilde{E}_{pq} = \left[\begin{array}{cc} L_p & -C_{Fq} \end{array}\right], \tilde{F}_{pq} = \left[\begin{array}{cc} -D_{Fq}C_p & 0 \end{array}\right], \tilde{G}_{pq} = \left[\begin{array}{cc} 0 & -D_{Fq}D_p \end{array}\right], \tilde{H}_{pq} = \left[\begin{array}{cc} D_{Fp} & 0 \end{array}\right],$$

$$v_{\mathbf{s}} = \left[\begin{array}{c} \omega_{\mathbf{s}} \\ \omega_{\mathbf{s}-d(\mathbf{s})} \end{array}\right], \tilde{\phi}_{\mathbf{s}} = \left[\begin{array}{c} \phi_{\mathbf{s}} \\ 0 \end{array}\right].$$

Definition 4.1. *The estimation error $e_{\mathbf{s}}$ is asymptotically stable under the given H_∞ performance index given a prespecified disturbance attenuation level γ:*

$$\|e_{\mathbf{s}}\|_2 \leq \gamma\|\omega_{\mathbf{s}}\|_2, \tag{18}$$

under zero initial condition holds for all non-zero $\omega_{\mathbf{s}} \in l_2[0,\infty)$.

Lemma 4.2. (Schur Complement)[6] *Given constant matrices $\mathbf{G}_1$, $\mathbf{G}_2$ and $\mathbf{G}_3$ with appropriate dimensions, where $\mathbf{G}_1^T = \mathbf{G}_1$ and $\mathbf{G}_2^T = \mathbf{G}_2 > 0$, then $\mathbf{G}_1 - \mathbf{G}_3^T\mathbf{G}_2^{-1}\mathbf{G}_3 > 0$ if and only if*

$$\left[\begin{array}{cc} \mathbf{G}_1 & \mathbf{G}_3 \\ \mathbf{G}_3^T & \mathbf{G}_2 \end{array}\right] > 0. \tag{19}$$

Lemma 4.3. [7] *Consider two real matrices of appropriate dimensions, $\mathbf{X}$ and $\mathbf{Y}$. Using the scalar $\alpha > 0$, and vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^{\mathbf{n}}$, we have:*

$$2\mathbf{x}^T\mathbf{X}^T\mathbf{Y}\mathbf{y} \leq \alpha^{-1}\mathbf{x}^T\mathbf{X}^T\mathbf{X}\mathbf{x} + \alpha\mathbf{y}^T\mathbf{Y}^T\mathbf{Y}\mathbf{y}. \tag{20}$$

Lemma 4.4. [34] *Let $U_{\mathbf{k}}$, $\mathcal{P}$, and $\mathcal{Q}$ be real matrices of the proper dimensions, where $U_{\mathbf{k}}^T U_{\mathbf{k}} \leq I$. Then there exists a constant $\epsilon > 0$ such that the inequality:*

$$\mathcal{P}U_{\mathbf{k}}\mathcal{Q} + \mathcal{Q}^T U_{\mathbf{k}}^T \mathcal{Q}^T \leq \epsilon\mathcal{P}\mathcal{P}^T + \epsilon^{-1}\mathcal{Q}^T\mathcal{Q}. \tag{21}$$

5 Main results

Theorem 5.1. *For given positive scalars γ, δ, c and d with $c - d < 0$, the filtering error system (17) is asymptotically stable, if there exist positive definite matrices $X > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$ and Σ satisfying the following inequality hold:*

$$\left[\begin{array}{cc} \Xi_a & \Xi_b \\ * & \Xi_c \end{array}\right] < 0, \tag{22}$$

$$\left[\begin{array}{cc} -X & X \\ * & -(\frac{c}{3})I \end{array}\right] < 0, \tag{23}$$

where

$$\Xi_a = \begin{bmatrix} \Gamma_{11} & 0 & 0 & 0 & 0 & 0 & \Gamma_{17} \\ * & -Q_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & -Q_3 & 0 & 0 & 0 & \Gamma_{37} \\ * & * & * & -Q_2 & 0 & 0 & 0 \\ * & * & * & * & \gamma^2 I & 0 & \Gamma_{57} \\ * & * & * & * & * & -\Sigma & \Gamma_{67} \\ * & * & * & * & * & * & -X \end{bmatrix},$$

$$\Xi_b = \begin{bmatrix} \Gamma_{18} & \Gamma_{19} & \Gamma_{110} & \Gamma_{111} & \Gamma_{112} & 0 & \Gamma_{114} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Gamma_{311} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \Gamma_{58} & \Gamma_{59} & \Gamma_{510} & \Gamma_{511} & 0 & \Gamma_{513} & \Gamma_{514} \\ 0 & 0 & 0 & \Gamma_{611} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Xi_c = \begin{bmatrix} -\mu_1 I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & -\mu_2 I & 0 & 0 & 0 & 0 & 0 \\ * & * & -\delta\Sigma & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & -\mu_1^{-1} I & 0 & 0 \\ * & * & * & * & * & -\mu_2^{-1} I & 0 \\ * & * & * & * & * & * & -\frac{d}{2} I \end{bmatrix},$$

$$\begin{aligned}
\Gamma_{11} =& -X + Q_1 + Q_2 + Q_3, & \Gamma_{17} =& 2\tilde{A}_{pq}^T X, & \Gamma_{18} =& \mu_1 \tilde{M}_{pq}^T, \\
\Gamma_{19} =& \mu_2 \tilde{M}_{pq}^T, & \Gamma_{110} =& \delta \tilde{L}_{pq}^T \Sigma, & \Gamma_{111} =& \tilde{E}_{pq}^T, \\
\Gamma_{112} =& \mu_1^{-1} \tilde{A}_{pq}^T X, & \Gamma_{37} =& \sqrt{5} \tilde{B}_{pq}^T X, & \Gamma_{311} =& \tilde{F}_{pq}^T, \\
\Gamma_{67} =& \sqrt{5} \tilde{D}_{pq}^T X, & \Gamma_{57} =& 2\tilde{C}_{pq}^T X, & \Gamma_{58} =& \mu_1 \tilde{N}_{pq}^T, \\
\Gamma_{59} =& \mu_2 \tilde{N}_{pq}^T, & \Gamma_{510} =& \delta \tilde{R}_{pq}^T \Sigma, & \Gamma_{511} =& \tilde{G}_{pq}^T, \\
\Gamma_{513} =& \mu_2^{-1} \tilde{C}_{pq}^T X, & \Gamma_{611} =& \tilde{H}_{pq}^T, & \Gamma_{114} =& d\tilde{M}_{pq}^T, \\
\Gamma_{514} =& d\tilde{N}_{pq}^T.
\end{aligned}$$

Proof. Lyapunov's functional is constructed as follows:

$$V_{\mathbf{s}} = V_{1\mathbf{s}} + V_{2\mathbf{s}}. \tag{24}$$

with

$$\begin{aligned}
V_{1\mathbf{s}} =& \zeta_{\mathbf{s}}^T X \zeta_{\mathbf{s}}, \\
V_{2\mathbf{s}} =& \sum_{\mathbf{s}-d_1}^{\mathbf{s}-1} \zeta^T(i) Q_1 \zeta(i) + \sum_{\mathbf{s}-d_2}^{\mathbf{s}-1} \zeta^T(i) Q_2 \zeta(i) + \sum_{\mathbf{s}-d_{\mathbf{s}}}^{\mathbf{s}-1} \zeta^T(i) Q_3 \zeta(i),
\end{aligned}$$

Thus, we compute the difference of $V_{\mathbf{s}}$ along the filtering error system by $\Delta V_{t\mathbf{s}} = V_{t\mathbf{s}+1} - V_{t\mathbf{s}} (t = 1, 2, 3)$:

$$\begin{aligned}\Delta V_{1\mathbf{s}} =&V_{1\mathbf{s}+1} - V_{1\mathbf{s}},\\ =&\zeta_{\mathbf{s}+1}^T X\zeta_{\mathbf{s}+1} - \zeta_{\mathbf{s}}^T X\zeta_{\mathbf{s}},\\ =&(\tilde{A}_{pq}\zeta_{\mathbf{s}} + \tilde{B}_{pq}\zeta_{\mathbf{s}-d(\mathbf{s})} + \tilde{C}_{pq}v_{\mathbf{s}} + \tilde{D}_{pq}\sigma_{\mathbf{s}} + \tilde{\phi}_{\mathbf{s}})^T X\\ &(\tilde{A}_{pq}\zeta_{\mathbf{s}} + \tilde{B}_{pq}\zeta_{\mathbf{s}-d(\mathbf{s})} + \tilde{C}_{pq}v_{\mathbf{s}} + \tilde{D}_{pq}\sigma_{\mathbf{s}} + \tilde{\phi}_{\mathbf{s}}) - \zeta_{\mathbf{s}}^T X\zeta_{\mathbf{s}}.\\ =&\zeta_{\mathbf{s}}^T\tilde{A}_{pq}^T X\tilde{A}_{pq}\zeta_{\mathbf{s}} + 2\zeta_{\mathbf{s}}^T\tilde{A}_{pq}^T X\tilde{B}_{pq}\zeta_{\mathbf{s}-d(\mathbf{s})} + 2\zeta_{\mathbf{s}}^T\tilde{A}_{pq}^T X\tilde{C}_{pq}v_{\mathbf{s}} + 2\zeta_{\mathbf{s}}^T\tilde{A}_{pq}^T X\tilde{\phi}_{\mathbf{s}}\\ &+ 2\zeta_{\mathbf{s}}^T\tilde{A}_{pq}^T X\tilde{D}_{pq}\sigma_{\mathbf{s}} + \zeta_{\mathbf{s}-d(\mathbf{s})}^T\tilde{B}_{pq}^T X\tilde{B}_{pq}\zeta_{\mathbf{s}-d(\mathbf{s})} + 2\zeta_{\mathbf{s}-d(\mathbf{s})}^T\tilde{B}_{pq}^T X\tilde{C}_{pq}v_{\mathbf{s}} - \zeta_{\mathbf{s}}^T X\zeta_{\mathbf{s}}\\ &+ 2\zeta_{\mathbf{s}-d(\mathbf{s})}^T)\tilde{B}_{pq}^T X\tilde{D}_{pq}\sigma_{\mathbf{s}} + 2\zeta_{\mathbf{s}-d(\mathbf{s})}^T)\tilde{B}_{pq}^T X\tilde{\phi}_{\mathbf{s}} + v_{\mathbf{s}}^T\tilde{C}_{pq}^T X\tilde{C}_{pq}v_{\mathbf{s}} + 2v_{\mathbf{s}}^T\tilde{C}_{pq}^T X\tilde{D}_{pq}\sigma_{\mathbf{s}}\\ &+ 2v_{\mathbf{s}}^T\tilde{C}_{pq}^T X\tilde{\phi}_{\mathbf{s}} + \tilde{\phi}_{\mathbf{s}}^T X\tilde{\phi}_{\mathbf{s}} + \sigma_{\mathbf{s}}^T\tilde{D}_{pq}^T X\tilde{D}_{pq}\sigma_{\mathbf{s}} + 2\sigma_{\mathbf{s}}^T\tilde{D}_{pq}^T X\tilde{\phi}_{\mathbf{s}}.\end{aligned}$$

$$\begin{aligned}\Delta V_{2\mathbf{s}} =&\zeta_{\mathbf{s}}^T(Q_1+Q_2+Q_3)\zeta_{\mathbf{s}} - \zeta^T(\mathbf{s}-d_1)Q_1\zeta(\mathbf{s}-d_1) - \zeta^T(\mathbf{s}-d_2)Q_2\zeta(\mathbf{s}-d_2)\\ &- \zeta_{\mathbf{s}-d(\mathbf{s})}^T Q_3\zeta_{\mathbf{s}-d(\mathbf{s})} + \sum_{s=t-d_2+1}^{t-d_1}\zeta^T(i)Q_3\zeta(i).\end{aligned}$$

Considering the inequality (3) and Lemma 2, the following inequality can be obtained:

$$2\zeta_{\mathbf{s}}^T\tilde{A}_{pq}^T X\tilde{\phi}_{\mathbf{s}} \le \mu_1^{-1}\zeta_{\mathbf{s}}^T\tilde{A}_{pq}^T\tilde{A}_{pq}\zeta_{\mathbf{s}} + \mu_1\tilde{\phi}_{\mathbf{s}}^T\tilde{\phi}_{\mathbf{s}}, \tag{25}$$

$$2v_{\mathbf{s}}^T\tilde{C}_{pq}^T X\tilde{\phi}_{\mathbf{s}} \le \mu_2^{-1}v_{\mathbf{s}}^T\tilde{C}_{pq}^T\tilde{C}_{pq}v_{\mathbf{s}} + \mu_2\tilde{\phi}_{\mathbf{s}}^T\tilde{\phi}_{\mathbf{s}}, \tag{26}$$

$$2\zeta_{\mathbf{s}-d(\mathbf{s})}^T)\tilde{B}_{pq}^T X\tilde{\phi}_{\mathbf{s}} \le \zeta_{\mathbf{s}-d(\mathbf{s})}^T\tilde{B}_{pq}^T X\tilde{B}_{pq}\zeta_{\mathbf{s}-d(\mathbf{s})} + \tilde{\phi}_{\mathbf{s}}^T X\tilde{\phi}_{\mathbf{s}}, \tag{27}$$

$$2\sigma_{\mathbf{s}}^T\tilde{D}_{pq}^T X\tilde{\phi}_{\mathbf{s}} \le \sigma_{\mathbf{s}}^T\tilde{D}_{pq}^T X\tilde{D}_{pq}\sigma_{\mathbf{s}} + \tilde{\phi}_{\mathbf{s}}^T X\tilde{\phi}_{\mathbf{s}}, \tag{28}$$

From (24)-(27), we obtain

$$\begin{aligned}\Delta V_{\mathbf{s}} =& \zeta_{\mathbf{s}}^T(4\tilde{A}_{pq}^T X\tilde{A}_{pq} + Q_1 + Q_2 + Q_3 + \mu_1^{-1}\tilde{A}_{pq}^T\tilde{A}_{pq} + \mu_1\tilde{M}_{pq}^T\tilde{M}_{pq} + \mu_2\tilde{M}_{pq}^T\tilde{M}_{pq})\zeta_{\mathbf{s}}\\ &- \zeta_{\mathbf{s}}^T X\zeta_{\mathbf{s}} + \zeta_{\mathbf{s}-d(\mathbf{s})}^T(5\tilde{B}_{pq}^T X\tilde{B}_{pq} - Q_3)\zeta_{\mathbf{s}-d(\mathbf{s})} + \sigma_{\mathbf{s}}^T(5\tilde{D}_{pq}^T X\tilde{D}_{pq})\sigma_{\mathbf{s}}\\ &+ v_{\mathbf{s}}^T(4\tilde{C}_{pq}^T X\tilde{C}_{pq} + \mu_2^{-1}\tilde{C}_{pq}^T\tilde{C}_{pq} + \mu_1\tilde{N}_{pq}^T\tilde{N}_{pq} + \mu_2\tilde{N}_{pq}^T\tilde{N}_{pq})v_{\mathbf{s}} + 3\tilde{\phi}_{\mathbf{s}}^T X\tilde{\phi}_{\mathbf{s}}\\ &- \zeta^T(\mathbf{s}-d_1)Q_1\zeta(\mathbf{s}-d_1) - \zeta^T(\mathbf{s}-d_2)Q_2\zeta(\mathbf{s}-d_2),\end{aligned}$$

According to the event triggered condition (8),

$$\sigma_{\mathbf{s}}^T\Sigma\sigma_{\mathbf{s}} - \delta y^T(p_{\mathbf{s}})\Sigma y(p_{\mathbf{s}}) \le 0. \tag{29}$$

If the initial condition is zero, then the closed system (17) will satisfy the H_∞ performance index:

$$\mathrm{J} = \sum_{t=0}^{\infty}\{\Delta V_{\mathbf{s}} + e_{\mathbf{s}}^T e_{\mathbf{s}} - \gamma^2\omega_{\mathbf{s}}^T\omega_{\mathbf{s}}\} \tag{30}$$

$$\le \sum_{t=0}^{\infty}\{\Delta V_{\mathbf{s}} + e_{\mathbf{s}}^T e_{\mathbf{s}} - \gamma^2\omega_{\mathbf{s}}^T\omega_{\mathbf{s}} + \sigma_{\mathbf{s}}^T\Sigma\sigma_{\mathbf{s}} - \delta y^T(p_{\mathbf{s}})\Sigma y(p_{\mathbf{s}})\} \tag{31}$$

Then,

$$
\begin{aligned}
\mathrm{J} = {} & \zeta_{\mathbf{s}}^T(4\tilde{A}_{pq}^T X\tilde{A}_{pq} + Q_1 + Q_2 + Q_3 + \mu_1^{-1}\tilde{A}_{pq}^T\tilde{A}_{pq} + \mu_1\tilde{M}_{pq}^T\tilde{M}_{pq} + \mu_2\tilde{M}_{pq}^T\tilde{M}_{pq})\zeta_{\mathbf{s}} \\
& - \zeta_{\mathbf{s}}^T X\zeta_{\mathbf{s}} + \zeta_{\mathbf{s}-d(\mathbf{s})}^T(5\tilde{B}_{pq}^T X\tilde{B}_{pq} - Q_3)\zeta_{\mathbf{s}-d(\mathbf{s})} + \sigma_{\mathbf{s}}^T(5\tilde{D}_{pq}^T X\tilde{D}_{pq})\sigma_{\mathbf{s}} \\
& + v_{\mathbf{s}}^T(4\tilde{C}_{pq}^T X\tilde{C}_{pq} + \mu_2^{-1}\tilde{C}_{pq}^T\tilde{C}_{pq} + \mu_1\tilde{N}_{pq}^T\tilde{N}_{pq} + \mu_2\tilde{N}_{pq}^T\tilde{N}_{pq})v_{\mathbf{s}} + 3\tilde{\phi}_{\mathbf{s}}^T X\tilde{\phi}_{\mathbf{s}} \\
& - \zeta^T(\mathbf{s}-d_1)Q_1\zeta(\mathbf{s}-d_1) - \zeta^T(\mathbf{s}-d_2)Q_2\zeta(\mathbf{s}-d_2) - \gamma^2\omega_{\mathbf{s}}^T\omega_{\mathbf{s}} + \zeta_{\mathbf{s}}^T\tilde{E}_{pq}^T\tilde{E}_{pq}\zeta_{\mathbf{s}} \\
& + 2\zeta_{\mathbf{s}}^T\tilde{E}_{pq}^T\tilde{F}_{pq}\zeta_{\mathbf{s}-d(\mathbf{s})} + 2\zeta_{\mathbf{s}}^T\tilde{E}_{pq}^T\tilde{G}_{pq}v_{\mathbf{s}} + 2\zeta_{\mathbf{s}}^T\tilde{E}_{pq}^T\tilde{H}_{pq}\sigma_{\mathbf{s}} - \sigma_{\mathbf{s}}^T\Sigma\sigma_{\mathbf{s}} \\
& + \zeta_{\mathbf{s}-d(\mathbf{s})}^T\tilde{F}_{pq}^T\tilde{F}_{pq}\zeta_{\mathbf{s}-d(\mathbf{s})} + 2\zeta_{\mathbf{s}-d(\mathbf{s})}^T\tilde{F}_{pq}^T\tilde{G}_{pq}v_{\mathbf{s}} + 2\zeta_{\mathbf{s}-d(\mathbf{s})}^T\tilde{F}_{pq}^T\tilde{H}_{pq}\sigma_{\mathbf{s}} \\
& + \delta[C_p x_{\mathbf{s}} + D_p\omega_{\mathbf{s}}]^T\Sigma[C_p x_{\mathbf{s}} + D_p\omega_{\mathbf{s}}] + v_{\mathbf{s}}^T\tilde{G}_{pq}^T\tilde{G}_{pq}v_{\mathbf{s}} + 2v_{\mathbf{s}}^T\tilde{G}_{pq}^T\tilde{H}_{pq}\sigma_{\mathbf{s}} \\
& + \sigma_{\mathbf{s}}^T\tilde{H}_{pq}^T\tilde{H}_{pq}\sigma_{\mathbf{s}}.
\end{aligned}
$$

By using the Schur complement, we can rewrite inequality (30) as:

$$
\mathrm{J} \leq \bar{\mathrm{J}} = \begin{bmatrix} \Gamma & -3\tilde{\phi}_{\mathbf{s}}^T X \\ * & -3X \end{bmatrix}, \tag{32}
$$

where

$$
\begin{aligned}
\Gamma = {} & \zeta_{\mathbf{s}}^T(4\tilde{A}_{pq}^T X\tilde{A}_{pq} + Q_1 + Q_2 + Q_3 + \mu_1^{-1}\tilde{A}_{pq}^T\tilde{A}_{pq} + \mu_1\tilde{M}_{pq}^T\tilde{M}_{pq} + \mu_2\tilde{M}_{pq}^T\tilde{M}_{pq})\zeta_{\mathbf{s}} \\
& - \zeta_{\mathbf{s}}^T X\zeta_{\mathbf{s}} + \zeta_{\mathbf{s}-d(\mathbf{s})}^T(5\tilde{B}_{pq}^T X\tilde{B}_{pq} - Q_3)\zeta_{\mathbf{s}-d(\mathbf{s})} + \sigma_{\mathbf{s}}^T(5\tilde{D}_{pq}^T X\tilde{D}_{pq})\sigma_{\mathbf{s}} \\
& + v_{\mathbf{s}}^T(4\tilde{C}_{pq}^T X\tilde{C}_{pq} + \mu_2^{-1}\tilde{C}_{pq}^T\tilde{C}_{pq} + \mu_1\tilde{N}_{pq}^T\tilde{N}_{pq} + \mu_2\tilde{N}_{pq}^T\tilde{N}_{pq})v_{\mathbf{s}} + \sigma_{\mathbf{s}}^T\tilde{H}_{pq}^T\tilde{H}_{pq}\sigma_{\mathbf{s}} \\
& - \zeta^T(\mathbf{s}-d_1)Q_1\zeta(\mathbf{s}-d_1) - \zeta^T(\mathbf{s}-d_2)Q_2\zeta(\mathbf{s}-d_2) - \gamma^2\omega_{\mathbf{s}}^T\omega_{\mathbf{s}} + \zeta_{\mathbf{s}}^T\tilde{E}_{pq}^T\tilde{E}_{pq}\zeta_{\mathbf{s}} \\
& + 2\zeta_{\mathbf{s}}^T\tilde{E}_{pq}^T\tilde{F}_{pq}\zeta_{\mathbf{s}-d(\mathbf{s})} + 2\zeta_{\mathbf{s}}^T\tilde{E}_{pq}^T\tilde{G}_{pq}v_{\mathbf{s}} + 2\zeta_{\mathbf{s}}^T\tilde{E}_{pq}^T\tilde{H}_{pq}\sigma_{\mathbf{s}} - \sigma_{\mathbf{s}}^T\Sigma\sigma_{\mathbf{s}} \\
& + \zeta_{\mathbf{s}-d(\mathbf{s})}^T\tilde{F}_{pq}^T\tilde{F}_{pq}\zeta_{\mathbf{s}-d(\mathbf{s})} + 2\zeta_{\mathbf{s}-d(\mathbf{s})}^T\tilde{F}_{pq}^T\tilde{G}_{pq}v_{\mathbf{s}} + 2\zeta_{\mathbf{s}-d(\mathbf{s})}^T\tilde{F}_{pq}^T\tilde{H}_{pq}\sigma_{\mathbf{s}} \\
& + \delta[C_p x_{\mathbf{s}} + D_p\omega_{\mathbf{s}}]^T\Sigma[C_p x_{\mathbf{s}} + D_p\omega_{\mathbf{s}}] + v_{\mathbf{s}}^T\tilde{G}_{pq}^T\tilde{G}_{pq}v_{\mathbf{s}} + 2v_{\mathbf{s}}^T\tilde{G}_{pq}^T\tilde{H}_{pq}\sigma_{\mathbf{s}}.
\end{aligned}
$$

On the other hand, by considering two positive scalars c and d with $c - d < 0$, the following formula is established:

$$
-2d\tilde{\phi}_{\mathbf{s}}^T\tilde{\phi}_{\mathbf{s}} + c\tilde{\phi}_{\mathbf{s}}^T\tilde{\phi}_{\mathbf{s}} < 0. \tag{33}
$$

Using Schur complement lemma in (32), we have

$$
\begin{bmatrix} -2d\tilde{\phi}_{\mathbf{s}}^T\tilde{\phi}_{\mathbf{s}} & 0 \\ * & -9c^{-1}X^2 \end{bmatrix} < \begin{bmatrix} 0 & 3\tilde{\phi}_{\mathbf{s}}^T X \\ * & 0 \end{bmatrix}. \tag{34}
$$

From the inequality (30) we have $\tilde{\mathrm{J}} < 0$, that is

$$
\begin{bmatrix} \Gamma & 0 \\ * & -3X \end{bmatrix} < \begin{bmatrix} 0 & 3\tilde{\phi}_{\mathbf{s}}^T X \\ * & 0 \end{bmatrix}, \tag{35}
$$

which is assured by

$$
\begin{bmatrix} \Gamma & 0 \\ * & -3X \end{bmatrix} < \begin{bmatrix} -2d\tilde{\phi}_{\mathbf{s}}^T\tilde{\phi}_{\mathbf{s}} & 0 \\ * & -9c^{-1}X^2 \end{bmatrix}. \tag{36}
$$

Then, inequality (35) can be modified as

$$
\begin{bmatrix} \Gamma_1 & 0 \\ 0 & \Gamma_2 \end{bmatrix} < 0, \tag{37}
$$

where

$$\Gamma_1 = \Gamma + 2d\tilde{\phi}_{\mathbf{s}}^T\tilde{\phi}_{\mathbf{s}} < 0, \tag{38}$$

$$\Gamma_2 = -3X + 9c^{-1}X^2 < 0. \tag{39}$$

Thus, LMI (10) holds, by implementing Schur complement to inequality (38). Moreover, from inequality (37) along with Schur complement lemma, we get

$$\mathrm{J} \leq \chi_{\mathbf{s}}^T \Xi \chi_{\mathbf{s}}. \tag{40}$$

where

$$\chi_{\mathbf{s}} = \begin{bmatrix} \zeta_{\mathbf{s}}^T & \zeta^T(\mathbf{s}-d_1) & \zeta_{\mathbf{s}-d(\mathbf{s})}^T & \zeta^T(\mathbf{s}-d_2) & v_{\mathbf{s}}^T & \sigma_{\mathbf{s}}^T \end{bmatrix},$$

$$\Xi = \begin{bmatrix} \Gamma_{11}^{\upsilon} & 0 & \Gamma_{13}^{\upsilon} & 0 & \Gamma_{15}^{\upsilon} & \Gamma_{16}^{\upsilon} \\ * & -Q_1 & 0 & 0 & 0 & 0 \\ * & * & \Gamma_{33}^{\upsilon} & 0 & \Gamma_{35}^{\upsilon} & \Gamma_{36}^{\upsilon} \\ * & * & * & -Q_2 & 0 & 0 \\ * & * & * & * & \Gamma_{55}^{\upsilon} & \Gamma_{56}^{\upsilon} \\ * & * & * & * & * & \Gamma_{66}^{\upsilon} \end{bmatrix} + \delta[C_p x_{\mathbf{s}} + D_p\omega_{\mathbf{s}}]^T\Sigma[C_p x_{\mathbf{s}} + D_p\omega_{\mathbf{s}}],$$

$$\begin{aligned} \Gamma_{11}^{\upsilon} =& -X + 4\tilde{A}_{pq}^T X\tilde{A}_{pq} + Q_1 + Q_2 + Q_3 + \mu_1^{-1}\tilde{A}_{pq}^T\tilde{A}_{pq} + \mu_1\tilde{M}_{pq}^T\tilde{M}_{pq} + \mu_2\tilde{M}_{pq}^T\tilde{M}_{pq} + \tilde{E}_{pq}^T\tilde{E}_{pq}, \\ \Gamma_{55}^{\upsilon} =& 4\tilde{C}_{pq}^T X\tilde{C}_{pq} - \gamma^2 I + \mu_2^{-1}\tilde{C}_{pq}^T\tilde{C}_{pq} + \mu_1\tilde{N}_{pq}^T\tilde{N}_{pq} + \mu_2\tilde{N}_{pq}^T\tilde{N}_{pq} + \tilde{G}_{pq}^T\tilde{G}_{pq}, \end{aligned}$$

$$\begin{aligned} \Gamma_{13}^{\upsilon} =& \tilde{E}_{pq}^T\tilde{F}_{pq}, & \Gamma_{15}^{\upsilon} =& \tilde{E}_{pq}^T\tilde{G}_{pq}, & \Gamma_{16}^{\upsilon} =& \tilde{E}_{pq}^T\tilde{H}_{pq}, \\ \Gamma_{35}^{\upsilon} =& \tilde{F}_{pq}^T\tilde{G}_{pq}, & \Gamma_{36}^{\upsilon} =& \tilde{F}_{pq}^T\tilde{H}_{pq}, & \Gamma_{56}^{\upsilon} =& \tilde{G}_{pq}^T\tilde{H}_{pq}. \end{aligned}$$

$$\Gamma_{33}^{\upsilon} = 5\tilde{B}_{pq}^T X\tilde{B}_{pq} - Q_3 + \tilde{F}_{pq}^T\tilde{F}_{pq}, \qquad \Gamma_{66}^{\upsilon} = 5\tilde{D}_{pq}^T X\tilde{D}_{pq} + \tilde{H}_{pq}^T\tilde{H}_{pq} - \Sigma.$$

By utilizing the Schur complement, this is equivalent to condition (21). Therefore the system is asymptotically stable. This completes the proof.

□

Remark 5.2. *In inequality (30), the conic-type linearization problem is solved using the Schur complement and relevant matrix inequality transformations. According to inequality (32),* $\mathrm{J} < 0$ *can be guaranteed by inequality (40). The conic-type nonlinear term* $\phi_{\mathbf{s}}$ *can be expressed linearly by combining inequalities (3) and (37). Additionally, the LMI method is used to determine the asymptotic stability of error system (17) and the attenuation performance from* $e_{\mathbf{s}}$ *to* $\omega_{\mathbf{s}}$ *for* H_∞.

Theorem 5.3. *Given the upper and lower limits of time delays* d_2 *and* d_1, *and the noise attenuation level* γ, *positive scalars* c, d, μ_1, μ_2, δ *and matrices* A_{Fq}, B_{Fq}, C_{Fq}, D_{Fq} $(q = 1, 2, ..)$, *the filtering conic type error system (17) is asymptotically stable with an* H_∞ *performance* γ^2 *under zero initial condition, if there exist matrices* $X_p > 0$, $Q_{pq} > 0$, A_f, B_f, C_{Fq}, D_{Fq} $(p = 1, 2, ..)$, $(q = 1, 2, ...)$ *such that*

$$\tilde{\Xi}_{22\times 22} < 0, \tag{41}$$

$$\begin{bmatrix} -X_1 & -X_2 & X_1 & X_2 \\ * & -X_2 & X_2^T & X_2 \\ * & * & -\frac{c}{3}I & 0 \\ * & * & * & -\frac{c}{3}I \end{bmatrix}. \tag{42}$$

$$\tilde{\Gamma}_{11} = -X_1 + Q_{11} + Q_{21} + Q_{31}, \qquad \tilde{\Gamma}_{12} = -X_2 + Q_{12} + Q_{22} + Q_{32}, \qquad \tilde{\Gamma}_{22} = -X_2 + Q_{13} + Q_{23} + Q_{33},$$

$$\begin{array}{lllll}
\tilde{\Gamma}_{112}=L_p^T, & \tilde{\Gamma}_{113}=2A_p^TX_1, & \tilde{\Gamma}_{114}=2A_p^TX_2, & \tilde{\Gamma}_{115}=\delta C_p^T\Sigma, & \tilde{\Gamma}_{116}=A_p^TX_1,\\
\tilde{\Gamma}_{117}=A_p^TX_2, & \tilde{\Gamma}_{120}=dM^T, & \tilde{\Gamma}_{121}=\mu_1M^T, & \tilde{\Gamma}_{122}=\mu_2M^T, & \tilde{\Gamma}_{212}=-\tilde{C}_{Fq}^T,\\
\tilde{\Gamma}_{213}=2\tilde{A}_{Fq}^T, & \tilde{\Gamma}_{214}=2\tilde{A}_{Fq}^T, & \tilde{\Gamma}_{216}=\tilde{A}_{Fq}^T, & \tilde{\Gamma}_{217}=\tilde{A}_{Fq}^T, & \tilde{\Gamma}_{33}=-Q_{11},\\
\tilde{\Gamma}_{34}=-Q_{12}, & \tilde{\Gamma}_{44}=-Q_{13}, & \tilde{\Gamma}_{55}=-Q_{31}, & \tilde{\Gamma}_{56}=-Q_{32}, & \tilde{\Gamma}_{512}=-\tilde{D}_{Fq}^TC_p^T,\\
\tilde{\Gamma}_{513}=\sqrt{5}\tilde{B}_{Fq}^TC_p^T, & \tilde{\Gamma}_{514}=\sqrt{5}\tilde{B}_{Fq}^TC_p^T, & \tilde{\Gamma}_{66}=-Q_{33}, & \tilde{\Gamma}_{77}=-Q_{21}, & \tilde{\Gamma}_{78}=-Q_{22},\\
\tilde{\Gamma}_{88}=-Q_{23}, & \tilde{\Gamma}_{99}=-\gamma^2I, & \tilde{\Gamma}_{913}=2B_p^TX_1, & \tilde{\Gamma}_{914}=2B_p^TX_2, & \tilde{\Gamma}_{915}=\delta D_p^T\Sigma,\\
\tilde{\Gamma}_{918}=B_p^TX_1, & \tilde{\Gamma}_{919}=B_p^TX_2, & \tilde{\Gamma}_{920}=dN^T, & \tilde{\Gamma}_{921}=\mu_1N^T, & \tilde{\Gamma}_{922}=\mu_2N^T,\\
\tilde{\Gamma}_{1010}=-\gamma^2I, & \tilde{\Gamma}_{1012}=-\tilde{D}_{Fq}^TD_p^T, & \tilde{\Gamma}_{1013}=2\tilde{B}_{Fq}^TD_p^T, & \tilde{\Gamma}_{1014}=2\tilde{B}_{Fq}^TD_p^T, & \tilde{\Gamma}_{1018}=\tilde{B}_{Fq}^TD_p,\\
\tilde{\Gamma}_{1019}=\tilde{B}_{Fq}^TD_p, & \tilde{\Gamma}_{1111}=-\Sigma, & \tilde{\Gamma}_{1112}=-\tilde{D}_{Fq}^T, & \tilde{\Gamma}_{1113}=-\sqrt{5}\tilde{B}_{Fq}^T, & \tilde{\Gamma}_{1114}=-\sqrt{5}\tilde{B}_{Fq}^T,\\
\tilde{\Gamma}_{1212}=-I, & \tilde{\Gamma}_{1313}=-X_1, & \tilde{\Gamma}_{1414}=-X_2, & \tilde{\Gamma}_{1515}=-\delta\Sigma, & \tilde{\Gamma}_{1616}=-\mu_1I,\\
\tilde{\Gamma}_{1717}=-\mu_1I, & \tilde{\Gamma}_{1818}=-\mu_2I, & \tilde{\Gamma}_{1919}=-\mu_2I, & \tilde{\Gamma}_{2020}=-dI, & \tilde{\Gamma}_{2121}=-\mu_1I,\\
\tilde{\Gamma}_{2222}=-\mu_2I, & & & &
\end{array}$$

Moreover, the gain matrices of the filter as $A_{Fp}=\tilde{A}_{Fp}X_2^{-1}$, $B_{Fp}=\tilde{B}_{Fp}X_2^{-1}$, $C_{Fp}=\tilde{C}_{Fp}$ and $D_{Fp}=\tilde{D}_{Fp}$.

Proof. On the basic of Theorem 1, to ensure the required results. We consider the matrices in the form as

$$X=\begin{bmatrix} X_1 & X_2\\ * & X_2\end{bmatrix}, Q_1=\begin{bmatrix} Q_{11} & Q_{12}\\ * & Q_{13}\end{bmatrix}, Q_2=\begin{bmatrix} Q_{21} & Q_{22}\\ * & Q_{23}\end{bmatrix}, Q_3=\begin{bmatrix} Q_{31} & Q_{32}\\ * & Q_{33}\end{bmatrix}.$$

Substituting X, Q_1, Q_2, Q_3 into (21) and defining $\tilde{A}_{Fp}=A_{Fp}X_2$, $\tilde{B}_{Fp}=B_{Fp}X_2$, $\tilde{C}_{Fp}=C_{Fp}$ and $\tilde{D}_{Fp}=D_{Fp}$. we can get LMI (40). Hence, proof of this theorem is completed. □

6 Numerical example

Example 6.1. *In order to illustrate the effectiveness of the proposed method in this work, we consider the conic-type filtering error system is of the form:*

$$\begin{cases} \zeta_{\mathbf{s}+1}=\sum_{p=1}^{s}\sum_{q=1}^{s}\{\tilde{A}_{pq}\zeta_{\mathbf{s}}+\tilde{B}_{pq}\zeta_{\mathbf{s}-d(\mathbf{s})}+\tilde{C}_{pq}v_{\mathbf{s}}+\tilde{D}_{pq}\sigma_{\mathbf{s}}+\tilde{\phi}_{\mathbf{s}}\}\\ e_{\mathbf{s}}=\sum_{p=1}^{s}\sum_{q=1}^{s}\{\tilde{E}_{pq}\zeta_{\mathbf{s}}+\tilde{F}_{pq}\zeta_{\mathbf{s}-d(\mathbf{s})}+\tilde{G}_{pq}v_{\mathbf{s}}+\tilde{H}_{pq}\sigma_{\mathbf{s}}\},\end{cases} \tag{43}$$

discrete fuzzy model is obtained as shown in equation (1) when the sampling time is $0.02s$*, and its parameters are as follows:*

$$\begin{aligned}
&A_1=\begin{bmatrix}2&1\\3&5\end{bmatrix}, A_2=\begin{bmatrix}2.3&1\\2&5\end{bmatrix}, B_1=\begin{bmatrix}1.3&4\\0&2.06\end{bmatrix}, B_2=\begin{bmatrix}1.3&2\\0&2.5\end{bmatrix},\\
&C_1=\begin{bmatrix}3&1\\1.6&6\end{bmatrix}, C_2=\begin{bmatrix}3&1.6\\2&4\end{bmatrix}, D_1=\begin{bmatrix}1&4\\2&6\end{bmatrix}, D_2=\begin{bmatrix}1.5&4\\2&7\end{bmatrix},\\
&L_1=\begin{bmatrix}2.3&0.7\\0.3&1.5\end{bmatrix}, L_2=\begin{bmatrix}2.3&1\\0.3&1.7\end{bmatrix}, M_1=M_2=\begin{bmatrix}0.02&0\\0&0\end{bmatrix}, N_1=N_2=\begin{bmatrix}0.02&0\\0&0\end{bmatrix}.
\end{aligned}$$

The membership functions are taken as

$$\begin{aligned}\mathfrak{h}_1(\alpha_{\mathbf{s}})&=sin^2(x_{1\mathbf{s}}),\\ \mathfrak{h}_2(\alpha_{\mathbf{s}})&=1-\mathfrak{h}_1(\alpha_{\mathbf{s}}).\end{aligned}$$

$d_1=1$ and $d_2=4$, respectively, are the lower and upper limits of time delay. A parameter that triggers an event is $\delta=1$. Assume that $\mu_1=0.5$, $\mu_2=0.4$, $m=0.7$, $n=0.06$ and $\gamma=0.6$. The filtering parameters can be calculated by

utilizing LMI toolbox in Matlab. They are shown as follows:

$$A_{f1} = 1.0e-004 * \begin{bmatrix} -0.1121 & -0.0659 \\ -0.0659 & -0.0037 \end{bmatrix}, A_{f2} = 1.0e-003 * \begin{bmatrix} 0.0671 & 0.0161 \\ 0.0161 & -0.1672 \end{bmatrix},$$
$$B_{f1} = \begin{bmatrix} -0.00106 & 0.0017 \\ 0.0017 & 0.0001 \end{bmatrix}, B_{f2} = \begin{bmatrix} -0.0102 & 0.0051 \\ 0.0051 & -0.0010 \end{bmatrix}, C_{f1} = \begin{bmatrix} -0.0064 & 0.0015 \\ 0.0015 & -0.0018 \end{bmatrix},$$
$$C_{f2} = \begin{bmatrix} -0.0024 & 0.0011 \\ 0.0011 & -0.0020 \end{bmatrix}, D_{f1} = \begin{bmatrix} 0.0094 & 0.0077 \\ 0.0077 & -0.0189 \end{bmatrix}, D_{f2} = \begin{bmatrix} 0.0054 & 0.0055 \\ 0.0055 & -0.0263 \end{bmatrix},$$
$$\Sigma_1 = \begin{bmatrix} 1.3441 & -0.8529 \\ -0.8529 & 0.5943 \end{bmatrix}, \Sigma_2 = \begin{bmatrix} 1.7772 & -0.9922 \\ -0.9922 & 0.5936 \end{bmatrix}.$$

Assuming that the disturbance is $\omega_s = 2/(1+0.8k)$, the responses of z_s and z_{Fs} can be obtained by iterative computations from initial value $x_0 = [0.50.3]^T$, and the error $e_s = z_s - z_{Fs}$ can therefore be easily obtained. The evolutions of z_s, z_{Fs} and e_s are shown in Fig.3 and Fig.4. We can see from the figures that the filtering error system is asymptotically stable by implementing the designed parameters.

In addition, on account of event-triggered mechanism, the release instants and release intervals are shown in Fig.5. We can see that the number of sampled-data transmission is 67, which is nearly one tenth of the time-triggered mechanism (50 times). Thus, the event-triggered mechanism can greatly reduce the data communication burden.

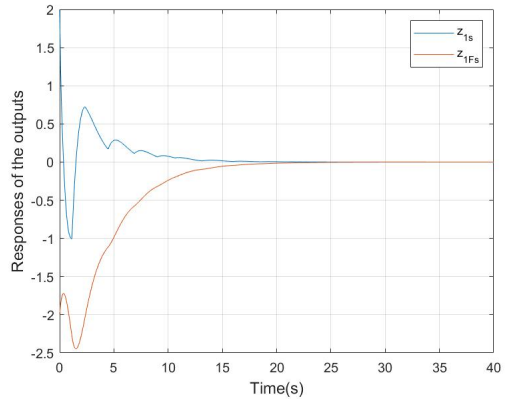


Figure 2: control output z and its estimation z_F

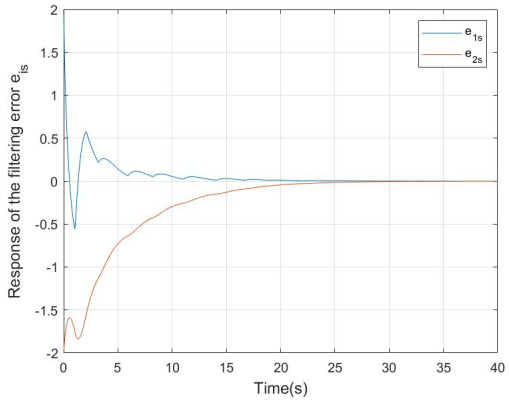


Figure 3: Filtering error e_s

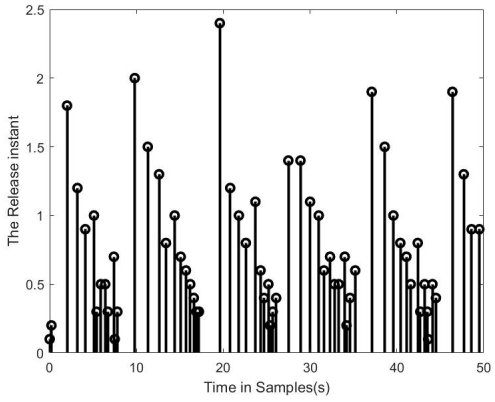


Figure 4: Release instants and time interval

Example 6.2. *In order to illustrate the effectiveness of the proposed method in this work, we consider the conic-type filtering error system of the form:*

$$\begin{cases} \zeta_{\mathbf{s}+1} = \sum_{p=1}^{s}\sum_{q=1}^{s}\{\tilde{A}_{pq}\zeta_{\mathbf{s}} + \tilde{B}_{pq}\zeta_{\mathbf{s}-d(\mathbf{s})} + \tilde{C}_{pq}v_{\mathbf{s}} + \tilde{D}_{pq}\sigma_{\mathbf{s}} + \tilde{\phi}_{\mathbf{s}}\} \\ e_{\mathbf{s}} = \sum_{p=1}^{s}\sum_{q=1}^{s}\{\tilde{E}_{pq}\zeta_{\mathbf{s}} + \tilde{F}_{pq}\zeta_{\mathbf{s}-d(\mathbf{s})} + \tilde{G}_{pq}v_{\mathbf{s}} + \tilde{H}_{pq}\sigma_{\mathbf{s}}\}, \end{cases} \tag{44}$$

discrete fuzzy model is obtained as shown in equation (1) when the sampling time is $0.02s$*, and its parameters are as follows:*

$$A_1 = \begin{bmatrix} -0.3 & 2 & 1 \\ 0 & 2 & 5 \\ -3 & 6 & 0.7 \end{bmatrix}, A_2 = \begin{bmatrix} -0.3 & 2.5 & 1 \\ 0 & 2 & 5 \\ -2 & 4 & 0.7 \end{bmatrix}, B_1 = \begin{bmatrix} 1 & 3 & 2 \\ 0 & -2 & 5 \\ 4 & 0.2 & 1 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} 2 & 3.2 & 2 \\ 0 & -1 & 5 \\ 4 & 0.2 & 1.8 \end{bmatrix}, C_1 = \begin{bmatrix} 3 & 1 & -5 \\ 1 & 8 & 2 \\ 0 & 6 & 1.3 \end{bmatrix}, C_2 = \begin{bmatrix} 3 & 1.3 & -2 \\ 1 & 7 & 2 \\ 0 & 4 & 1.3 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} 1 & 0 & 4 \\ 2.2 & 3 & -7 \\ 0.4 & 2 & 0 \end{bmatrix}, D_2 = \begin{bmatrix} 1.3 & 1 & 4 \\ 2.7 & 3 & -5 \\ 1 & 2 & 0.5 \end{bmatrix}, M_1 = M_2 = \begin{bmatrix} 0.02 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
$$L_1 = \begin{bmatrix} 2 & 3 & 1 \\ 0.3 & 1.6 & 0 \\ 0 & 0 & -2 \end{bmatrix}, L_2 = \begin{bmatrix} 0 & 3.5 & 2 \\ 3 & 1.6 & 0 \\ 1 & 0 & -2 \end{bmatrix}, N_1 = N_2 = \begin{bmatrix} 0.02 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The membership functions are given as

$$\mathfrak{h}_1(\alpha_{\mathbf{s}}) = sin^2(x_{1\mathbf{s}}),$$
$$\mathfrak{h}_2(\alpha_{\mathbf{s}}) = 1 - \mathfrak{h}_1(\alpha_{\mathbf{s}}).$$

$d_1 = 1$ *and* $d_2 = 4$*, respectively, are the lower and upper limits of time delay. A parameter that triggers an event is* $\delta = 1$*. Assume that* $\mu_1 = 0.5$*,* $\mu_2 = 0.4$*,* $m = 0.7$*,* $n = 0.06$ *and* $\gamma = 0.6$*. The filtering parameters can be calculated by*

utilizing LMI toolbox in Matlab. They are shown as follows:

$$A_{f1} = 1.0e-004 * \begin{bmatrix} -0.1870 & 0.0665 & 0.2976 \\ 0.0665 & 0.0316 & -0.2822 \\ 0.2976 & -0.2822 & -0.1098 \end{bmatrix}, B_{f1} = \begin{bmatrix} 12.4906 & -0.0011 & -0.0130 \\ -0.1583 & 12.2701 & -0.1052 \\ -0.0130 & -0.1052 & 12.4236 \end{bmatrix},$$

$$C_{f1} = \begin{bmatrix} -0.0012 & -0.0003 & -0.0002 \\ -0.0003 & -0.0004 & 0.0000 \\ -0.0002 & 0.0000 & -0.0009 \end{bmatrix}, D_{f1} = \begin{bmatrix} 0.0061 & -0.0005 & -0.0002 \\ -0.0005 & -0.0196 & -0.0088 \\ -0.0002 & -0.0088 & 0.0036 \end{bmatrix},$$

$$\Sigma_1 = \begin{bmatrix} -0.2564 & 0.1190 & -0.1652 \\ 0.1190 & 0.0316 & -0.1190 \\ -0.1652 & -0.1190 & 0.4243 \end{bmatrix}, A_{f2} = 1.0e-04 * \begin{bmatrix} -0.1999 & 0.1230 & 0.2504 \\ 0.1230 & 0.0407 & -0.2794 \\ 0.2504 & -0.2794 & -0.0539 \end{bmatrix},$$

$$B_{f2} = \begin{bmatrix} 11.8763 & -0.1204 & 0.0539 \\ -0.1204 & 11.7476 & -0.01870 \\ 0.0539 & -0.01870 & 11.9229 \end{bmatrix}, C_{f2} = \begin{bmatrix} 0.0013 & 0.0018 & 0.0015 \\ 0.0018 & -0.0003 & 0.0006 \\ 0.0015 & 0.0006 & -0.0019 \end{bmatrix},$$

$$D_{f2} = \begin{bmatrix} 0.0072 & 0.0028 & -0.0014 \\ 0.0028 & -0.0202 & -0.0094 \\ -0.0014 & -0.0094 & 0.0041 \end{bmatrix}. \Sigma_2 = \begin{bmatrix} 0.2874 & 0.1451 & -0.3679 \\ 0.1451 & 0.1281 & -0.2428 \\ -0.3679 & -0.2428 & 0.8007 \end{bmatrix}.$$

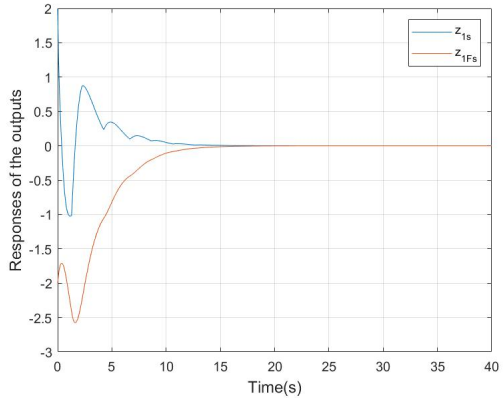


Figure 5: control output z and its estimation z_F

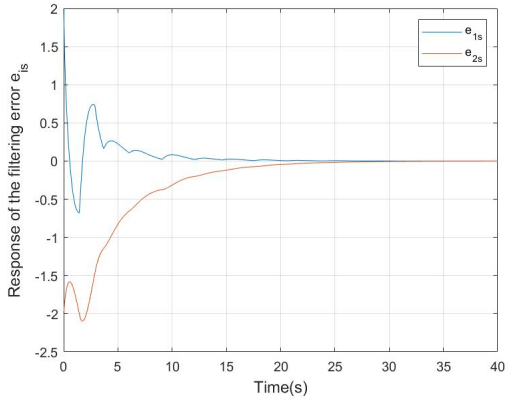


Figure 6: Filtering error e_s

Assuming that the disturbance is $\omega_s = 2/(1+0.8k)$, the responses of z_s and z_{Fs} can be obtained by iterative computations from initial value $x_0 = [0.50.3]^T$, and the error $e_s = z_s - z_{Fs}$ can therefore be easily obtained. The evolutions of

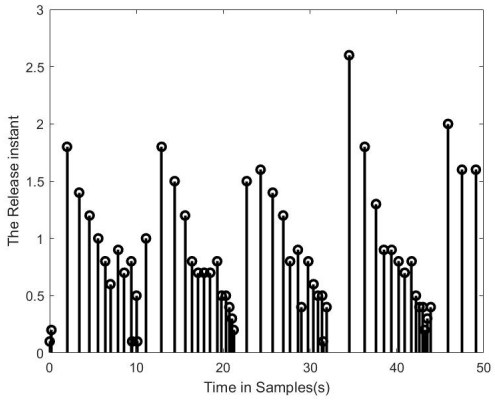


Figure 7: Release instants and time interval

z_s, z_{Fs} and e_s are shown in Fig.6 and Fig.7. We can see from the figures that the filtering error system is asymptotically stable by implementing the designed parameters.

In addition, on account of event-triggered mechanism, the release instants and release intervals are shown in Fig.8. We can see that the number of sampled-data transmission is 57, which is nearly one tenth of the time-triggered mechanism (50 times). Thus, the event-triggered mechanism can greatly reduce the data communication burden.

7 Conclusions

This paper investigates discrete-time nonlinear networked fuzzy systems and their H_∞ filtering. Communication costs can be reduced by using an event-triggered mechanism. Filtering error systems subject to transmission delay and event triggers are shown to have asymptotically stable conditions using integral inequalities. We have derived a sufficient condition that holds for a fuzzy system addressed in terms of linear matrix inequality. The proposed theoretical results are finally illustrated by numerical examples to illustrate their correctness and effectiveness. It may be possible to extend the proposed approach to the research of event-based filters that accommodate cyber-attacks and robotics in T-S fuzzy networked control systems.

Data Availability Statement

The authors does not have any conflict of interest.

Acknowledgement

This research has been funded by Scientific Research Deanship at University of Ha'il-Saudi Arabia through Project Number RG-23 005.

References

[1] Z. Chen, B. Zhang, V. Stojanovic, Y. Zhang, Z. Zhang, *Event-based fuzzy control for T-S fuzzy networked systems with various data missing*, Neurocomputing, **417** (2020), 322-332. https://doi.org/10.1016/j.neucom.2020.08.063

[2] P. Cheng, J. Wang, S. He, X. Luan, F. Liu, *Observer-based asynchronous fault detection for conic-type nonlinear jumping systems and its application to separately excited DC motor*, IEEE Transactions on Circuits and Systems I: Regular Papers, **67**(3) (2019), 951-962. https://doi.org/10.1109/TCSI.2019.2949368

[3] A. M. El-Nagar, T. R. Khalifa, M. A. El-Brawany, M. El-Bardini, E. A. El-Araby, *Adaptive interval type-2 fuzzy controller for nonlinear networked Wiener systems subject to packet dropout and time-varying delay*, ISA Transactions, **128** (2022), 565-580. https://doi.org/10.1016/j.isatra.2021.10.018

[4] Z. Gu, P. Shi, D. Yue, Z. T. Ding, *Decentralized adaptive event-triggered H_1 filtering for a class of networked nonlinear interconnected systems*, IEEE Transactions on Cybernetics, **99** (2018), 1-10. https://doi.org/10.1109/TCYB.2018.2802044

[5] S. He, W. Lyu, F. Liu, *Robust H_∞ sliding mode controller design of a class of time-delayed discrete conic-type nonlinear systems*, IEEE Transactions on Systems, Man, and Cybernetics Systems, **51**(2) (2018), 885-892. https://doi.org/10.1109/TSMC.2018.2884491

[6] S. He, J. Song, *Finite-time sliding mode control design for a class of uncertain conic nonlinear systems*, IEEE/CAA Journal of Automatica Sinica, **4**(4) (2017), 809-816. https://doi.org/10.1109/JAS.2017.7510643

[7] X. Jia, S. Xu, X. Shi, Z. Zhang, *Global practical tracking for nonlinear systems with uncertain dead-zone input via output feedback*, Journal of the Franklin Institute, **358**(6) (2021), 2987-3009. https://doi.org/10.1016/j.jfranklin.2021.02.013

[8] T. R. Khalifa, A. M. El-Nagar, M. A. El-Brawany, E. A. El-Araby, M. El-Bardini, *A novel Hammerstein model for nonlinear networked systems based on an interval type-2 fuzzy Takagi Sugeno Kang system*, IEEE Transactions on Fuzzy Systems, **29**(2) (2020), 275-285. https://doi.org/10.1109/TFUZZ.2020.3007460

[9] C. C. Ku, W. J. Chang, M. H. Tsai, Y. C. Lee, *Observer-based proportional derivative fuzzy control for singular Takagi-Sugeno fuzzy systems*, Information Sciences, **570** (2021), 815-830. https://doi.org/10.1016/j.ins.2021.01.023

[10] Z. M. Li, J. H. Park, *Dissipative fuzzy tracking control for nonlinear networked systems with quantization*, IEEE Transactions on Systems, Man, and Cybernetics: Systems, **50**(12) (2018), 5130-5141. https://doi.org/10.1109/TSMC.2018.2866996

[11] J. Li, Z. Wang, C. K. Ahn, Y. Shen, *Fault detection for Lipschitz nonlinear systems with restricted frequency-domain specifications*, IEEE Transactions on Systems, Man, and Cybernetics: Systems, **51**(12) (2020), 7486-7496. https://doi.org/10.1007/s40815-022-01336-6

[12] G. Li, R. Yang, *Observer-based hybrid-triggered control for nonlinear networked control systems with disturbances*, International Journal of Fuzzy Systems, (2022), 1-10. https://doi.org/10.1007/s40815-022-01336-6

[13] Y. Liu, Y. Ma, *Finite-time non-fragile extended dissipative control for T-S fuzzy system via augmented Lyapunov-Krasovskii functional*, ISA Transactions, **117** (2021), 1-15. https://doi.org/10.1016/j.isatra.2021.01.038

[14] Y. Liu, F. Wu, X. Ban, *Dynamic output feedback control for continuous-time T-S fuzzy systems using fuzzy Lyapunov functions*, IEEE Transactions on Fuzzy Systems, **25**(5) (2017), 1155-1167. https://doi.org/10.1109/TFUZZ.2016.2598852

[15] Q. Lu, P. Shi, L. Wu, H. Zhang, *Event-triggered interval type-2 T-S fuzzy control for nonlinear networked systems*, Journal of the Franklin Institute, **357**(14) (2020), 9834-9852. https://doi.org/10.1016/j.jfranklin.2020.08.001

[16] J. Mrazgua, R. Chaibi, E. H. Tissir, M. Ouahi, *Static output feedback stabilization of T-S fuzzy active suspension systems*, Journal of Terramechanics, **97** (2021), 19-27. https://doi.org/10.1016/j.jterra.2021.05.001

[17] G. Nagamani, Y. H. Joo, G. Soundararajan, R. Mohajerpoor, *Robust event-triggered reliable control for TS fuzzy uncertain systems via weighted based inequality*, Information Sciences, **512** (2020), 31-49. https://doi.org/10.1016/j.ins.2019.09.034

[18] T. G. Oliveira, R. M. Palhares, V. C. Campos, P. S. Queiroz, E. N. Gonçalves, *Improved Takagi-Sugeno fuzzy output tracking control for nonlinear networked control systems*, Journal of the Franklin Institute, **354**(16) (2017), 7280-7305. https://doi.org/10.1016/j.jfranklin.2017.08.042

[19] Y. Pan, G. H. Yang, *Event-based output tracking control for fuzzy networked control systems with network-induced delays*, Applied Mathematics and Computation, **346** (2019), 513-530. https://doi.org/10.1016/j.amc.2018.10.038

[20] J. Priyadharsini, P. Balasubramaniam, *Controllability of fractional non-instantaneous impulsive integro-differential stochastic delay system*, IMA Journal of Mathematical Control and Information, **38**(2) (2021), 654-683. https://doi.org/10.1093/imamci/dnab004

[21] D. Shah, A. Mehta, K. Patel, A. Bartoszewicz, *Event-triggered discrete higher-order SMC for networked control system having network irregularities*, IEEE Transactions on Industrial Informatics, **16**(11) (2019), 6837-6847. https://doi.org/10.1109/TII.2020.2973739

[22] H. Shen, F. Li, H. Yan, H. R. Karimi, H. K. Lam, *Finite-time event-triggered H_∞ control for T-S fuzzy Markov jump systems*, IEEE Transactions on Fuzzy Systems, **26**(5) (2018), 3122-3135. https://doi.org/10.1109/TFUZZ.2017.2788891

[23] K. Shi, J. Wang, S. Zhong, Y. Tang, J. Cheng, *Non-fragile memory filtering of T-S fuzzy delayed neural networks based on switched fuzzy sampled-data control*, Fuzzy Sets and Systems, **394** (2020), 40-64. https://doi.org/10.1016/j.fss.2019.09.001

[24] X. Sun, Q. Zhang, *Admissibility analysis for interval type-2 fuzzy descriptor systems based on sliding mode control*, IEEE Transactions on Cybernetics, **49**(8) (2019), 3032-3040. https://doi.org/10.1109/TCYB.2018.2837890

[25] M. Syed Ali, S. Saravanan, A. Sabri, *Finite-time H_∞ state estimation for switched neural networks with time-varying delays*, Neurocomputing, **207** (2016), 580-589. https://doi.org/10.1016/j.neucom.2016.05.037

[26] M. Syed Ali, M. Usha, Q. Zhu, S. Saravanan, *Synchronization analysis for stochastic T-S fuzzy complex networks with Markovian jumping parameters and mixed time-varying delays via impulsive control*, Mathematical Problems in Engineering, (2020), 1-27. https://doi.org/10.1155/2020/9739876

[27] M. Syed Ali, R. Vadivel, R. Saravanakumar, *Event-triggered state estimation for Markovian jumping impulsive neural networks with interval time-varying delays*, International Journal of Control, **92**(2) (2019), 270-290. https://doi.org/10.1080/00207179.2017.1350884

[28] T. Takagi, M. Sugeno, *Fuzzy identification of systems and its applications to modeling and control*, IEEE Transactions on Systems, Man, and Cybernetics: Systems, **15**(1) (1985), 116-132. https://doi.org/10.1016/B978-1-4832-1450-4.50045-6

[29] J. Wang, S. He, *Fuzzy filtering based fault detection for a class of discrete-time conic-type nonlinear systems*, IET Signal Processing, **15**(3) (2021), 153-161. https://doi.org/10.1049/sil2.12016

[30] L. Wang, X. Liu, H. Zhang, *Further studies on H_∞ observer design for continuous-time Takagi-Sugeno fuzzy model*, Information Sciences, **422** (2018), 396-407. https://doi.org/10.1109/TCYB.2019.2933696

[31] W. Wang, R. Postoyan, D. Neic, W. P. M. H. Heemels, *Periodic event-triggered control for nonlinear networked control systems*, IEEE Transactions on Automatic Control, **65**(2) (2019), 620-635. https://doi.org/10.1109/TAC.2019.2914255

[32] Y. Wang, Y. Xia, H. Shen, P. Zhou, *SMC design for robust stabilization of nonlinear Markovian jump singular systems*, IEEE Transactions Automatic Control, **63**(1) (2018), 219-224. https://doi.org/10.1109/TAC.2017.2720970

[33] J. Wang, C. Yang, J. Xia, Z. G. Wu, H. Shen, *Observer-based sliding mode control for networked fuzzy singularly perturbed systems under weighted try-once-discard protocol*, IEEE Transactions on Fuzzy Systems, **30**(6) (2021), 1889-1899. https://doi.org/10.1109/TFUZZ.2021.3070125

[34] X. Xiao, Z. Lei, *Mode-dependent IOSS conditions for continuous-time switched nonlinear systems*, International Journal of Control, Automatic and Systems, **19**(11) (2021), 3580-3587. https://doi.org/10.1007/s12555-020-0786-x

[35] X. M. Zhang, Q. L. Han, *Event-triggered dynamic output feedback control for networked control systems*, IET Control Theory and Applications, **8**(4) (2014), 226-234. https://doi.org/10.1049/iet-cta.2013.0253

[36] D. Zhang, S. K. Nguang, D. Srinivasan, L. Yu, *Distributed filtering for discrete-time T-S fuzzy systems with incomplete measurements*, IEEE Transactions on Fuzzy Systems, **26**(3) (2018), 1459-1471. https://doi.org/10.1109/TFUZZ.2017.2725228

[37] M. Zhang, P. Shi, C. Shen, Z. G. Wu, *Static output feedback control of switched nonlinear systems with actuator faults*, IEEE Transactions on Fuzzy Systems, **28**(8) (2019), 1600-1609. https://doi.org/10.1109/TFUZZ.2019.2917177

[38] X. Zhang, Y. Yin, H. Wang, S. He, *Finite-time dissipative control for time-delay Markov jump systems with conic-type nonlinearities under guaranteed cost controller and quantiser*, IET Control Theory and Applications, **15**(4) (2021), 489-498. https://doi.org/10.1049/cth2.12031

[39] J. Zhang, F. Zhu, H. R. Karimi, F. Wang, *Observer-based sliding mode control for T-S fuzzy descriptor systems with time delay*, IEEE Transactions on Fuzzy Systems, **27**(10) (2019), 2009-2023. https://doi.org/10.1109/TFUZZ.2019.2893220

[40] Z. Zheng, J. Gao, J. Mo, L. Zhang, Q. Zhang, *A fast self-correction method for nonlinear sinusoidal fringe images in 3-D measurement*, IEEE Transactions on Instrumentation and Measurement, **70** (2021), 1-9. https://doi.org/10.1109/TIM.2021.3066535

[41] Q. Zheng, S. Xu, B. Du, *Quantized guaranteed cost output feedback control for nonlinear networked control systems and its applications*, IEEE Transactions on Fuzzy Systems, **30**(7) (2021), 2402-2411. https://doi.org/10.1109/TFUZZ.2021.3082691

[42] G. Zong, H. Ren, H. R. Karimi, *Event-triggered communication and annular finite-time H_∞ filtering for networked switched systems*, IEEE Transactions on Cybernetics, **51**(1) (2020), 309-317. https://doi.org/10.1109/TCYB.2020.3010917