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Most dangerous and safest countries for women: Analysis by fuzzy similarity measures

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Abstract

In this paper, we evaluate the fuzzy similarity measures of different country rankings with respect to the most dangerous and safest for women. We find that the fuzzy similarity measures are high. We develop some theoretical results concerning fuzzy similarity measures. The goal of the paper is to examine the similarity of the rankings of countries with respect to the most dangerous countries for women to live and the peace and security of the countries for women. We determine a formula for the difference of rankings of the fuzzy similarity measure of two rankings A and B and the opposite of A and B.

Keywords: Female well-being, organized violence, disempowerment, country rankings, fuzzy similarity measures.

1 Introduction

The Peace, and Security Index, [5], systematically measures and ranks women's well-being worldwide. Trends in women's empowerment are heading in the right direction globally. Some 59 countries recorded significant progress since the first edition of [5]. According to [5], the regions Middle East and North Africa perform poorly overall which is traceable largely to high levels of organized violence and discriminatory laws that disempower women, especially in paid employment.

In [5], it is determined which countries are the most dangerous for women to live in. The Women's Danger Index ranks each country based on eight factors: street safety for women, intentional homicide of women, non-partner sexual violence, intimate partner sexual violence, legal discrimination, global gender gap, gender inequality index, and violence against women attitudes. We present this ranking in Table 3.1, the column headed MDCW (Most Dangerous Countries for Women). A low number represents a high risk. The column headed WPSI is from [2].

In [2], women's inclusion, justice, and security are examined. They are used to rank countries with respect to women's well-being. The ranking is presented in Table 3.2 in the column headed WSPI. It is actually the reverse ranking of that of [5] so that a low number represents well-being is low. In this way, we can get better obtain a comparison of both of these two issues.

We determine the fuzzy similarity measure of these two rankings MDCW and WPSI. We find that the fuzzy similarity measures are high.

2 Preliminary results

We apply similarity measures to rankings of members of a finite set.

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We let $\mathcal{FP}(X)$ the fuzzy power set of a set X, i.e., the set of all fuzzy subsets of X. We let $\wedge$ denote the minimum or infimum of a set of real numbers and we let $\vee$ denote the maximum or supremum. For two fuzzy subsets of X, we $\mu \subseteq \nu$ if $\mu(x) \leq \nu(x)$ for all $x \in X$.

Definition 2.1. *Let S be a function of $\mathcal{FP}(X) \times \mathcal{FP}(X)$ into $[0,1]$. Then S is called a* ***fuzzy similarity measure*** *on $\mathcal{FP}(X)$ if the following properties hold: $\forall \mu, \nu, \rho \in \mathcal{FP}(X)$*

(1) $S(\mu, \nu) = S(\nu, \mu)$;

(2) $S(\mu, \nu) = 1$ *if and only if* $\mu = \nu$;

(3) *If* $\mu \subseteq \nu \subseteq \rho$, *then* $S(\mu, \rho) \leq S(\mu, \nu) \wedge S(\nu, \rho)$;

(4) *If* $S(\mu, \nu) = 0$, *then* $\forall x \in X, \mu(x) \wedge \nu(x) = 0$.

Let A be a one-to-one function of X onto $\{1, 2, ..., n\}$. Then A is called a **ranking** of X. Define the fuzzy subset μ_A of X into $[0,1]$ by for all $x \in X, \mu_A(x) = A(x)/n$. Then μ_A is called the **fuzzy subset associated** with A. For μ_A a fuzzy ranking of X, we have that $\sum_{x \in X} A(x) = \frac{n(n+1)}{2}$ and $\sum_{x \in X} \mu_A(x) = \frac{(n+1)}{2}$ since $\sum_{x \in X} A(x) = 1 + 2 + ... + n$.

Define $A^* : X \to [0,1]$ by $\forall x \in X, A^*(x) = n + 1 - A(x)$. Then A^* is a ranking of X. Note also that $\mu_{A^*}(x) = 1 + \frac{1}{n} - \mu_A(x)$. A^* is called the reverse ranking of A. It should be noted that A^* yields the smallest fuzzy similarity measure that A can have with any ranking of X, [4]. Note also that $\mu_{A^*}(x) = \mu_{A^c}(x) + \frac{1}{n}$, where μ_{A^c} is the compliment of μ_A.

Example 2.2. *Let μ_A and μ_B be fuzzy subsets associated with two rankings A and B of X, respectively. Then M and S are fuzzy similarity measures, where*

$$\begin{aligned} M(\mu_A, \mu_B) &= \frac{\sum_{x \in X} \mu_A(x) \wedge \mu_B(x)}{\sum_{x \in} \mu_A(x) \vee \mu_B(x)}, \\ S(\mu_A, \mu_B) &= 1 - \frac{\sum_{x \in X} |\mu_A(x) - \mu_B(x)|}{\sum_{x \in X} (\mu_A(x) + \mu_B(x))}. \end{aligned}$$

Theorem 2.3. [1] *Let μ_A and μ_B be fuzzy subsets associated with two rankings A and B of X, respectively. Let M and S be defined as in Example 1.2. Then $S = \frac{2M}{1+M}$ and $M = \frac{S}{2-S}$.*

Theorem 2.4. [6] *Let X be a finite set with n elements. Let μ_A and μ_B be fuzzy subsets of X associated with two rankings A and B of X, respectively.*

(1) *If n is even, then the smallest $M(\mu_A, \mu_B)$ can be is $\frac{n+2}{3n+2}$ and the smallest $S(\mu_A, \mu_B)$ is $\frac{n/2+1}{n+1}$.*

(2) *If n is odd, then the smallest $M(\mu_A, \mu_B)$ can be is $\frac{n+1}{3n-1}$ and the smallest $S(\mu_A, \mu_B)$ is $\frac{1}{2} + \frac{1}{2n}$.*

Consider Theorem 2.4. Suppose that s denotes the smallest value for S. Define $\widehat{S}(\mu_A, \mu_B) = \frac{S(\mu_A, \mu_B) - s}{1 - s}$. Then $\widehat{S}(\mu_A, \mu_B)$ varies between 0 and 1. For values of $\widehat{S}(\mu_A, \mu_B)$ between 0 and 0.2, we say the the fuzzy similarity is very low, between 0.2 and 0.4 low, between 0.4 and 0.6 medium, between 0.6 and 0.8 high, and between 0.8 and 1 very high. A similar approach can be taken for M.

3 Application

We allow for ties in the following rankings. This has very little effect on the results.

Table 3.1 Peace and Security

Country	MDCW Rank / WPSI Rank	Country	MDCW Rank / WPSI Rank
Argentina	14 / 25	Japan	31 / 31
Australia	29 / 36	Malaysia	11 / 19
Austria	47 / 47	Mexico	4 / 9
Bahrain	17 / 16	Morocco	8 / 2.5
Belgium	28 / 33	Netherlands	39 / 44.5
Brazil	2 / 11	Norway	45 / 50
Bulgaria	26 / 27	Philippines	25 / 15
Cambodia	16 / 10	Poland	41 / 34
Canada	42 / 43	Portugal	44 / 39.5
Chile	15 / 21	Russia	3 / 22.5
China	23 / 17	Saudi Arabia	12 / 5
Croatia	43 / 29	Singapore	49 / 35
Czech Rep.	35 / 30	South Africa	1 / 20
Denmark	37 / 48	South Korea	30 / 28
Dominican Rep.	6 / 18	Spain	50 / 42
Egypt	7 / 1	Sweden	34 / 44.5
France	38 / 37	Switzerland	46 / 49
Germany	33 / 39.5	Thailand	10 / 13.5
Greece	32 / 22.5	Tunisia	18 / 4
Hungary	27 / 24	Turkey	13 / 7
India	9 / 2.5	Ukraine	20 / 8
Indonesia	24 / 12	UAE	21 / 26
Iran	5 / 6	U. Kingdom	40 / 46
Ireland	48 / 41	U. States	19 / 38
Italy	36 / 32	Vietnam	22 / 13.5

Here $n = 50$.

$$S = 1 - \frac{359}{2550} = 1 - 0.1408 = 0.8592.$$

The smallest S can be is $\frac{n/2+1}{n+1} = \frac{26}{51} = 0.5098$. Thus

$$\widehat{S} = \frac{0.8592 - 0.5098}{1 - 0.5098} = \frac{0.3494}{0.4902} = 0.7128.$$

Now

$$M = \frac{S}{2 - S} = \frac{0.8592}{2 - 0.8592} = 0.7532.$$

The smallest M can be is $\frac{0.5098}{2-0.5098} = 0.3421$. Thus

$$\widehat{M} = \frac{0.7532 - 0.3421}{1 - 0.3421} = \frac{0.4111}{0.6579} = 0.6249.$$

In Table 3.2, we determine the reverse ranking WPSI of the WPSI ranking appearing in the WPSI rank column of Table 3.1. MDCW is a second ranking of countries provided in [5].

Table 3.2 Dangerous Countries

Country	MDCW Rank [5] / WPSI* Rank	Country	MDCW Rank [5] / WPSI* Rank
Argentina	27 / 26	Japan	16 / 20
Australia	7 / 15	Malaysia	32 / 32
Austria	5 / 4	Mexico	50 / 42
Bahrain	31 / 35	Morocco	44 / 48.5
Belgium	8 / 18	Netherlands	6 / 6.5
Brazil	45 / 40	Norway	4 / 1
Bulgaria	25 / 24	Philippines	47 / 36
Cambodia	43 / 41	Poland	21 / 17
Canada	11 / 8	Portugal	13 / 11.5
Chile	34 / 30	Russia	30 / 28.5
China	36 / 34	Saudi Arabia	33 / 46
Croatia	18 / 22	Singapore	10 / 16
Czech Rep.	12 / 21	South Africa	38 / 31
Denmark	1 / 3	South Korea	22 / 23
Dominican Rep.	40 / 33	Spain	20 / 9
Egypt	42 / 50	Sweden	3 / 6.5
France	17 / 14	Switzerland	2 / 2
Germany	14 / 11.5	Thailand	29 / 37.5
Greece	28 / 28.5	Tunisia	39 / 47
Hungary	23 / 27	Turkey	41 / 44
India	48 / 48.5	Ukraine	46 / 43
Indonesia	37 / 39	UAE	15 / 25
Iran	49 / 45	U. Kingdom	19 / 5
Ireland	9 / 10	U. States	26 / 13
Italy	24 / 19	Vietnam	35 / 37.5

$$S = 1 - \frac{235}{2550} = 1 - 0.0922 = 0.9078.$$

The smallest S can be is $\frac{26}{51} = 0.5098$. Thus

$$\widehat{S} = \frac{0.9078 - 0.5098}{1 - 0.5098} = \frac{0.3980}{0.4902} = 0.8119.$$

:Now

$$M = \frac{S}{2 - S} = \frac{0.9078}{2 - 0.9078} = 0.8312.$$

The smallest M can be is $\frac{n+2}{3n+2} = \frac{52}{152} = 0.3421$. Hence

$$\widehat{M} = \frac{0.8312 - 0.3421}{1 - 0.3421} = \frac{0.4891}{0.6579} = 0.7434.$$

We note that

$$\begin{aligned} S(\mu_{A^*}, \mu_{B^*}) &= \frac{1}{n+1} \sum_{x \in X} |\mu_{A^*}(x) - \mu_{B^*}(x)| \\ &= \frac{1}{n+1} \sum_{x \in X} |1 - \mu_A(x) + \frac{1}{n} - (1 - \mu_B(x) + \frac{1}{n})| \\ &= \frac{1}{n+1} \sum_{x \in X} | - \mu_A(x) + \mu_B(x)| \\ &= \frac{1}{n+1} \sum_{x \in X} |\mu_A(x) - \mu_B(x)| = S(\mu_A, \mu_B). \end{aligned}$$

Hence if one of $S(\mu_{A^*}, \mu_{B^*})$ and $S(\mu_A, \mu_B)$ is known, the value of other follows immediately.

4 Theoretical results

Let A and B be rankings of X. Then B satisfies, for all $x \in X$, either (1) or (2) or (3) or (4) holds, where

(1) $\mu_A(x) \leq \mu_B(x) \leq \mu_{A^*}(x)$,

(2) $\mu_A(x) \geq \mu_B(x) \geq \mu_{A^*}(x)$,

(3) $\mu_B(x) < \mu_A(x) \wedge \mu_{Ah*}(x)$,

(4) $\mu_B(x) > \mu_A(x) \vee \mu_{Ah*}(x)$.

Let $E = \{x \in X \mid A(x) = B(x) = A^*(x)\}$ and $X_i = \{x \in X \mid x \notin E$ and (i) holds$\}, i = 1, 2$. Let $X_i = \{x \in X \mid (i)$ holds$\}, i = 3, 4$. Let n_i denote the cardinality of $X_i, i = 1, 2, 3, 4$.

We see that $\{\{X_i | i = 1, 2, 3, 4\}\} \cup \{E\}$is a partition of X.

Recall

$$S(\mu_A, \mu_B) = 1 - \frac{\sum_{x\in X} |\mu_A(x) - \mu_B(x)|}{\sum_{x\in X}(\mu_A(x) + \mu_B(x))} \text{ and } S(\mu_{A^*}, \mu_B) = 1 - \frac{\sum_{x\in X} |\mu_{*A}(x) - \mu_B(x)|}{\sum_{x\in X}(\mu_{A^*}(x) + \mu_B(x))}.$$

In the following result, we provide a measure of closeness of μ_B with respect to μ_A and μ_{A^*}.

Theorem 4.1. *Let A, A^*, B be as above. Then $S(\mu_A, \mu_B) - S(\mu_{A^*}, \mu_B) =$*

$$\frac{n_1 - n_2 + n_3 - n_4}{n} + \frac{1}{n+1}(-\sum_{x\in X_1} 2\mu_B(x) + \sum_{x\in X_2}(2\mu_B(x)) - \sum_{x\in X_3}(2(\mu_A(x)) + \sum_{x\in X_4}(2\mu_A(x)).$$

Proof. We have that

$$\begin{aligned}\sum_{x\in X} |\mu_A - \mu_B(x)| &= \sum_{x\in X_1}(\mu_B(x) - \mu_A(x)) + \sum_{x\in X_2}(\mu_A(x) - \mu_B(x) \\ &\quad + \sum_{x\in X_3}(\mu_A(x) - \mu_B(x) + \sum_{x\in X_4}(\mu_B(x) - \mu_A(x)),\end{aligned}$$

and

$$\begin{aligned}\sum_{x\in X} |\mu_{A^*} - \mu_B(x)| &= \sum_{x\in X_1}(\mu_{A^*}(x) - \mu_B(x)) + \sum_{x\in X_2}(\mu_B(x) - \mu_{A^*}(x) \\ &\quad + \sum_{x\in X_3}(\mu_{A^*}(x) - \mu_B(x) + \sum_{x\in X_4}(\mu_B(x) - \mu_{A^*}(x)).\end{aligned}$$

Thus,

$$\begin{aligned}\sum_{x\in X} |\mu_A - \mu_B(x)| - \sum_{x\in X} |\mu_{A^*} - \mu_B(x)| &= \sum_{x\in X_1}(2\mu_B(x) - \mu_A(x) - \mu_{A^*}(x)) \\ &\quad + \sum_{x\in X_2}(-2\mu_B(x) + \mu_A(x) + \mu_{A^*}(x)) \\ &\quad + \sum_{x\in X_3}((\mu_A(x) - \mu_{A^*}(x)) + \sum_{x\in X_4}(-\mu_A(x)) + \mu_{A^*}(x)).\end{aligned}$$

Since $\mu_{A^*}(x) = 1 - \mu_A(x) + \frac{1}{n}$ for all $x \in X$, we have that

$$\begin{aligned}\sum_{x\in X} |\mu_A - \mu_B(x)| - \sum_{x\in X} |\mu_{A^*} - \mu_B(x)| &= \sum_{x\in X_1}(2\mu_B(x) - 1 - \frac{1}{n}) + \sum_{x\in X_2}(-2\mu_B(x) + 1 + \frac{1}{n}) \\ &\quad + \sum_{x\in X_3}((\mu_A(x) - 1 + \mu_A(x) - \frac{1}{n}) + \\ &\quad \sum_{x\in X_4}(-\mu_A(x)) + 1 - \mu_A(x) + \frac{1}{n}).\end{aligned}$$

Hence

$$\begin{aligned}\sum_{x\in X}|\mu_A-\mu_B(x)|-\sum_{x\in X}|\mu_{A^*}-\mu_B(x)| &= -n_1-\frac{n_1}{n}+\sum_{x\in X_1}2\mu_B(x)+n_2+\frac{n_2}{n}\\ &\quad+\sum_{x\in X_2}(-2\mu_B(x))-n_3-\frac{n_3}{n}+\sum_{x\in X_3}(2(\mu_A(x))\\ &\quad+n_4+\frac{n_4}{n}+\sum_{x\in X_4}(-2\mu_A(x)).\end{aligned}$$

Therefore,

$$\begin{aligned}&S(\mu_A,\mu_B)-S(\mu_{A^*},\mu_B)\\ =\;& 1-\frac{\sum_{x\in X}|\mu_A-\mu_B(x)|}{n+1}-1+\frac{\sum_{x\in X}|\mu_{A^*}-\mu_B(x)|}{n+1}\\ =\;& \frac{-1}{n+1}(-n_1-\frac{n_1}{n}+\sum_{x\in X_1}2\mu_B(x)+n_2+\frac{n_2}{n}+\sum_{x\in X_2}(-2\mu_B(x))\\ &-n_3-\frac{n_3}{n}+\sum_{x\in X_3}(2(\mu_A(x))+n_4+\frac{n_4}{n}+\sum_{x\in X_4}(-2\mu_A(x))\\ =\;& \frac{n_1-n_2+n_3-n_4}{n}+\frac{1}{n+1}(-\sum_{x\in X_1}2\mu_B(x)+\sum_{x\in X_2}(2\mu_B(x))-\sum_{x\in X_3}(2(\mu_A(x))+\sum_{x\in X_4}(2\mu_A(x)).\end{aligned}$$

The desired result now holds. □

Example 4.2. *Let* $X=\{x_i\mid i=1,2,...,6\}$. *Let* $A(x_i)=i$, $i=1,2,...,6$. *Then* $\mu_A(x_i)=\frac{i}{6}, i=1,2,...,6$. *Let*

$$B(x_1)=2, B(x_2)=1, B(x_3)=5, B(x_4)=3, B(x_5)=4, B(x_6)=6.$$

It follows that

$$\mu_{A^*}(x_i)=1-\frac{i}{6}+\frac{1}{6}, i=1,2,...,6.$$

Thus $X_1=\{x_1\}, X_2=\{x_4,x_5,x_6\}$, $X_3=\{x_2\}$ *and* $X_4=\{x_3\}$. *Now*

$$S(\mu_A,\mu_B)=1-\frac{\frac{1}{6}(1+1+2+1+1+0)}{7}=1-\frac{1}{7}=\frac{6}{7},$$

and

$$S(\mu_{A^*},\mu_B)=1-\frac{\frac{1}{6}(4+4+1+0+2+5)}{7}=1-\frac{16}{42}=1-\frac{8}{21}=\frac{13}{21}.$$

Thus

$$S(\mu_A,\mu_B)-S(\mu_{A^*},\mu_B)=\frac{6}{7}-\frac{13}{21}=\frac{5}{21}.$$

We now use Theorem 4.1 to find $S(\mu_A,\mu_B)-S(\mu_{A^*},\mu_B)$. *We have*

$$\begin{aligned}S(\mu_A,\mu_B)-S(\mu_{A^*},\mu_B) &= \frac{n_1-n_2+n_3-n_4}{n}+\frac{1}{n+1}(\sum_{x\in X_1}-2\mu_B(x)+\sum_{x\in X_2}(2\mu_B(x))\\ &\quad-\sum_{x\in X_3}(2(\mu_A(x))+\sum_{x\in X_4}(2\mu_A(x))\\ &= \frac{1-3+1-1}{6}+\frac{1}{7}\frac{1}{6}(-2(2)+2(3+4+5))-2(2)+2(3))\\ &= -\frac{1}{3}+\frac{4}{7}=\frac{5}{21}.\end{aligned}$$

Example 4.3. *In the following table, A denotes WPSI and B denotes MDCW of Table 3.1. The column headed C provides the category* (1), (2), (3), *or* (4) *the country is in. Our purpose is to illustrate Theorem 4.1 with a real situation.*

Table 4.1 Peace and Danger

Country	A	B	A^*	C	*Country*	A	B	A^*	C
Argentina	24	14	27	3	*Japan*	30	31	21	4
Australia	35	29	16	2	*Malaysia*	18	11	33	3
Austria	46	47	5	4	*Mexico*	8	4	43	3
Bahrain	15	17	36	1	*Morocco*	2	8	49	1
Belgium	32	28	19	2	*Netherlands*	43.5	39	7.5	2
Brazil	10	2	41	3	*Norway*	50	45	1	2
Bulgaria	26	26	25	2	*Philippines*	14	25	37	1
Cambodia	9	16	42	1	*Poland*	33	41	18	4
Canada	42	42	9	2	*Portugal*	38.5	44	12.5	4
Chile	20	15	31	3	*Russia*	21.5	3	29.5	3
China	16	23	35	1	*Saudi Arabia*	4	12	47	1
Croatia	28	43	23	4	*Singapore*	34	49	17	4
Czech Rep.	29	35	22	4	*South Africa*	19	1	32	3
Denmark	47.5	37	3.5	2	*South Korea*	27	30	24	4
Dominican Rep.	17	6	34	3	*Spain*	41	50	10	4
Egypt	1	7	50	1	*Sweden*	43.5	34	7.5	2
France	36	38	15	4	*Switzerland*	49	46	2	2
Germany	38.5	33	12.5	2	*Thailand*	12.5	10	38.5	3
Greece	21.5	32	29.5	4	*Tunisia*	3	18	48	1
Hungary	23	27	28	1	*Turkey*	6	13	45	1
India	47.5	9	3.5	2	*Ukraine*	7	20	44	1
Indonesia	11	24	40	1	*UAE*	25	21	26	3
Iran	5	5	46	1	*U. Kingdom*	45	40	6	2
Ireland	40	48	11	4	*U. States*	37	19	14	3
Italy	31	36	20	4	*Vietnam*	12.5	22	38.5	1

Now

$$S(\mu_A, \mu_B) = 1 - \frac{395}{2550} = 0.8451,$$

and

$$S(\mu_{A^*}, \mu_B) = 1 - \frac{1193}{2550} = 0.5322.$$

Thus

$$S(\mu_A, \mu_B) - S(\mu_{A^*}, \mu_B) = 0.8451 - 0.5322 = 0.3129.$$

The smallest S can be is 0.5098. *Thus*

$$\widehat{S}(\mu_A, \mu_B) = \frac{S(\mu_A, \mu_B) - 0.5098}{1 - 0.5098} = \frac{0.8451 - 0.5098}{1 - 0.5098} = 0.6840.$$

Consequently,

$$\begin{aligned} \widehat{S}(\mu_A, \mu_B) - \widehat{S}(\mu_{A^*}, \mu_B) &= \frac{0.8451 - 0.5322}{1 - 0.5098} = \frac{0.3129}{1 - 0.5098} \\ &= \frac{0.3129}{0.4902} = 0.6383. \end{aligned}$$

Hence the difference between $\widehat{S}(\mu_A, \mu_B)$ and $\widehat{S}(\mu_{A^*}, \mu_B)$ is high.

We next find $S(\mu_A, \mu_B) - S(\mu_{A^*}, \mu_B)$ by using Theorem 4.1. Here $n_1 = 14, n_2 = 12, n_3 = 11,$ and $n_4 = 13$. Thus

$$n_1 - n_2 + n_3 - n_4 = 14 - 12 + 11 - 13 = 0.$$

We also have $\sum_{x\in X_1} B(x) = 237$, $\sum_{x\in X_2} B(x) = 408$, $\sum_{x\in X_3} A(x) = 212$, *and* $\sum_{x\in X_4} A(x) = 435$. *Hence*

$$\begin{aligned} & -\sum_{x\in X_1} 2\mu_B(x) + \sum_{x\in X_2} 2\mu_B(x) - \sum_{x\in X_3} 2\mu_A(x) + \sum_{x\in X_4} 2\mu_A(x) \\ = \; & \frac{1}{50}(-2(237) + 2(408) - 2(212) + 2(435)) \\ = \; & \frac{788}{50} = 15.76. \end{aligned}$$

Thus by Theorem 4.1,

$$\begin{aligned} & S(\mu_A, \mu_B) - S(\mu_{A^*}, \mu_B) \\ = \; & \frac{n_1 - n_2 + n_3 - n_4}{n} + \frac{1}{n+1}(-\sum_{x\in X_1} 2\mu_B(x) + \sum_{x\in X_2} (2\mu_B(x)) - \sum_{x\in X_3} (2(\mu_A(x)) + \sum_{x\in X_4} (2\mu_A(x)) \\ = \; & 0 + \frac{1}{51}(15.76) = 0.3090. \end{aligned}$$

5 Conclusions

We determined the fuzzy similarity measures of rankings of countries with respect to the well-being of women. We found that these measures were high. We then determined the fuzzy similarity measures of the most dangerous place for women to live and the above issues concerning well-being. We found the fuzzy similarity measures to be high. One could consider [1] and [6] for future research projects. A companion paper to the one here can be found in [3].

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