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An enhanced interval rough numbers-based outranking technique for evaluation of technology for sustainable mining

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Abstract

This study pioneers an innovative decision-making framework, integrating interval rough numbers with the elimination and choice translating reality method, enhanced by the level-based weight assessment approach, specifically designed to assess sustainable mining technologies. The proposed model overcomes the limitations of traditional multi-criteria decision-making methods by effectively addressing uncertainty and imprecision through interval rough numbers. In contrast to the conventional versions of outranking methods, which proceed upon the principles of concordance and discordance sets, the proposed approach has the additional potential to meticulously handle the pseudo criteria to facilitate the proper examination of the outranking relations. By closely evaluating each criterion's explicit behavior in reference to the veto, preference, and indifference threshold values, the suggested technique has the advantage of being able to infer preferences in terms of strong and weak preference relations. The level-based weight assessment technique ensures a systematic and balanced determination of criteria weights, reflecting their relative importance. A comprehensive case study on sustainable mining technologies highlights the practicality and robustness of the proposed approach. Through the evaluation of fully autonomous, semi-autonomous, and labor-intensive mining systems, the findings establish fully autonomous systems as the most dependable choice. A comparison study with existing techniques is carried out to elucidate the validity, rationality, superiority, and effectiveness of the results.

Keywords: Interval rough number, outranking relation, concordance index, discordance index, threshold values.

1 Introduction

Effective decision-making in today's fast-paced technological landscape requires careful consideration of intricate details and influencing factors. The evolution of multi-criteria decision-making (MCDM) has been significantly shaped by mathematicians who have enhanced its framework. This development includes multi-criteria group decision-making (MCGDM), which involves expert committees to provide a more authentic approach to practical decisions. The existing literature covers a wide range of MCDM methodologies grounded in various theoretical frameworks. Outranking techniques are notably based on pairwise comparisons of alternatives against competing criteria. Among these, the elimination and choice translating reality (ELECTRE) methods are valuable tools for comparative analysis and effective decision-making [7, 22]. While other methods, such as MARCOS, MABAC, VIKOR, TOPSIS, and AROMAN, are also widely used, ELECTRE is often preferred for its ability to handle both qualitative and quantitative data. In contrast to methods like VIKOR and TOPSIS, which rank alternatives based on distance measures, ELECTRE adopts an outranking approach, which more accurately captures the relative strengths and weaknesses of alternatives. This makes it particularly useful when expert preferences are uncertain or imprecise. A prominent feature of this family lies in the exploration of the optimal solution by assessing the superiorities and inferiorities among options. This assessment is carried out using the concepts of concordance and discordance, ensuring a balanced evaluation. The ELECTRE-III strategy belonging to the ELECTRE family is the primary subject of this investigation. By presenting the ELECTRE-I

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technique, Benayoun et al., [4] were stationed the pioneers to present the foundations of ELECTRE approaches in the literature. The main motivation behind the improvements proposed in [9] to the ELECTRE-I model was to resolve its limitation of not providing a ranking list, with the goal of achieving more effective outcomes. This improved version named as the ELECTRE-II approach. Strong and weak outranking relations constituted the cornerstone for decision-making in these versions. They were unable to deal with pseudo criteria since they take into account small variations in the opponent's superiority or inferiority. The indifference spectrum for the pseudo criterion in the ELECTRE-III variation is recognized as having been considered by Roy [21]. Takeda [28], Roy et al., [23], and Lopez et al., [12] investigated the ELECTRE-III technique's parameters. The effectiveness of the ELECTRE method encouraged other researchers to incorporate it to real-world decisions.

Conventional MCDM methods typically assume that human judgments and preferences are precise and exact, which may not always reflect the complexities of real-world decision-making. These conventional approaches often rely on crisp values and deterministic assessments, neglecting the inherent uncertainty and imprecision in human opinions. However, the evolution of MCDM has led to the integration of models specifically designed to address this uncertainty. Notably, fuzzy set theory, introduced by Zadeh [30], captures membership grades and linguistic variables, while rough set theory, proposed by Pawlak [19], quantifies uncertainty through approximation spaces. These advancements have enabled MCDM to tackle complex decision-making problems, accommodating both qualitative and quantitative criteria, and providing decision-makers with a more comprehensive and informed decision-making framework. To address the ambiguity and hesitation inherent in human beliefs, most researchers employ fuzzy set theory along with its modified variants [20]. Fuzzy sets are a popular and highly effective technique for handling uncertain and vague human opinions. Nevertheless, fuzzy sets demand a special attention while choosing a suitable membership function which may involve subjectivity. Rough set theory (RST), introduced by Pawlak [19], revolutionized the handling of uncertainty and vagueness in data analysis. This approach enables the approximation of complex concepts through lower and upper approximations, providing a robust foundation for decision-making [1, 2, 27].

Interval sets [14] can be used in uncertain reasoning by providing a liberty to the decision-maker (DM) who can present his judgments in the form of intervals ranging between two different linguistic terms instead of an exact value. Pamucar et al., [17] introduced the interval rough number (IRN) being an insightful instrument reflecting the degree of ambiguity inherent with individual judgments by integrating the interval-based information with the concept of RST. IRNs offer a powerful tool for managing numerical uncertainty. By encapsulating uncertainty within interval boundaries, these numbers facilitate robust decision-making in various fields, including image processing, natural language processing, and multi-criteria decision analysis. Researchers have been paying close consideration to IRNs recently, and a significant body of literature has been focused about integration of MCDM with IRNs. Deveci et al., [5] integrated IRNs into the MARCOS and best-worst approaches to address the issue of choosing offshore wind farm sites. In order to tackle difficult MCDM situations, Pamucar et al., [17] examined various important aspects of IRNs and created a DEMATEL-ANP-MAIRCA combo based on them. The IRN-SWARA-MABAC technique was developed by Ghosh et al., [8], who also showed how well it worked for assessing tourism sites that included IR information. Pamucar et al., [18] studied the evaluation of university websites considering interval rough data by implementing the AHP-MABAC methodology. The interval rough SAW technique was created by Stević et al., [26] to aid in the trustworthy supplier selection process. To prioritize the switch of natural gas grid stations to hydrogen, Iordache et al., [10] introduced an innovative hybrid approach composed of the interval rough Dombi MARCOS technique. In order to assess concepts, Sarwar [24] came up with a MCGDM model inside an interval rough cloud VIKOR structure. Sarwar et al., [25] came up with an innovative interval rough integrated cloud COPRAS technique and successfully applied it to remanufacturing system selection. A fuzzy rough Hamacher operator with an MCDM technique was proposed by Pamucar et al., [16] to handle the unmanned aerial vehicle (UAV) prioritization in transportation networks. In their study, Ecer et al., [6] skillfully assessed renewable energy resources by applying IRNs to both the level-based weight evaluation and the combinative distance-based assessment. Mardanya et al., [13] emphasized the interval rough technique's relevance in multi-modal transportation. The interval rough incorporated SWARA ELECTRE-I technique was developed by Akram et al., [3] to evaluate machine tool choices for remanufacturing.

1.1 Main motivation of the research

Motives for the research conducted are outlined in the following list:

- RST and its extension through IRNs have emerged as pivotal tools in handling uncertainty and imprecision within decision-making frameworks. Originally conceived by Pawlak, RST provides a mathematical foundation for dealing with vague and incomplete information, allowing for the extraction of valuable insights from ambiguous data. IRNs further extend this capability by incorporating intervals to represent imprecision more accurately, thus enhancing the robustness of decision models. Together, these methodologies empower researchers and practitioners

across various disciplines, offering innovative approaches to tackle complex problems where traditional methods fall short. Embracing RST and IRNs not only enriches theoretical frameworks but also propels practical applications in fields such as data mining, artificial intelligence, and MCDM, promising novel solutions to real-world challenges.

- The ELECTRE-III approach, in contrast to earlier ELECTRE method extensions, considers pseudo-criteria that capture every minor detail regarding comparison between pair of alternatives through three threshold values corresponding to each criterion (indifference, preference, veto), indicating the relation of strong preference or weak preference or indifference, and finally produce a complete preference ordering/ranking of available options. The outstanding and exceptional capability of IRNs to handle uncertainties appeared in human assessments in the process of MCDM made us to develop a modified structure of ELECTRE-III technique which is capable to incorporate interval information rather than exact values and can provide more authentic and reliable outcomes.
- The relatively simple framework of Level Based Weight Assessment (LBWA) approach for evaluation of criteria weight vector inspired us to utilize it to systematically determine the weights of criteria, ensuring a balanced and accurate representation of their relative importance.
- Aiming to advance the field of MCDM by integrating IRNs and LBWA into the ELECTRE-III method, demonstrating its effectiveness and applicability through a comprehensive case study on sustainable mining technology.

1.2 Innovative contributions

The presented decision support system (DSS) introduces significant advancements over traditional decision-making frameworks, particularly within the context of sustainable mining. It combines the robust and reliable ELECTRE approach with the adaptable uncertainty-handling capabilities of IRNs, resulting in a hybrid MCDM technique that enhances decision-making processes. This integration enables the DSS to effectively manage both qualitative and quantitative data, accounting for uncertainties and imprecisions in expert judgments. By doing so, the system improves the precision, effectiveness, and resilience of decision-making, ensuring that sustainable mining alternatives are evaluated more comprehensively and reliably. The following points highlight the key contributions of the presented work:

- This research integrates the ELECTRE methodology with the IRN framework, delivering a more structured outranking MCDM approach for tackling complex real-world problems. The proposed method establishes outranking relations among alternatives through a comprehensive evaluation of pairwise comparisons. It introduces interval rough concordance and discordance indices by defining a systematic way to establish indifference, preference, and veto thresholds for interval rough assessment data. The method then generates a complete preference ordering of alternatives by evaluating their credibility indices.
- The successful adaptation and application of the IRN-based outranking approach to the assessment of sustainable mining technologies represents a significant accomplishment of this research, demonstrating its practical utility and impact.
- The outranking-based methods greatly relies upon significance of conflicting criteria in the decision-making process, the simplified algorithm of LBWA approach is employed to determine the criteria weights, leveraging the capabilities of IRNs to capture the nuances of uncertain data and ensure a more accurate representation of criteria importance.
- Furthermore, the proposed methodology's novelty and effectiveness are validated through comparative analysis with IRN-based HPA, MAIRCA, and rough CoCoSo methods, demonstrating its ability to improve decision-making accuracy and robustness. Additionally, a sensitivity analysis is performed to confirm its reliability and effectiveness.

1.3 Layout of paper

The article is organized as follows: Section 2 introduces the foundational concepts of IRNs, including key definitions and preliminaries. Section 3 outlines the methodology, combining the IRN-based LBWA technique for criteria weight calculation with the interval rough ELECTRE-III approach for ranking alternatives. Flowcharts for both phases are provided for clarity. Section 4 details the application of the developed approach to evaluating sustainable mining systems, explaining the methodology and implementation. Section 5 offers a discussion and comparative analysis with existing studies, highlighting the model's advantages. Section 6 incorporates a comprehensive sensitivity analysis of decision results to demonstrate proposed method's compatibility and authenticity. Limitations and future directions

are highlighted in Section 7. Finally, Section 8 concludes with key findings, implications, and suggestions for future research.

2 Interval rough numbers

As human beings, there is always a possibility of vagueness or impreciseness in the assessments provided by each DM, who has their own unique set of skills. When it comes to interpreting the hazy and unclear parts of decision-making processes, IRN is an effective instrument. To approximate interval information and bring assessments closer to reality, IRNs take the upper and lower bounds of both end components. Let $\mathcal{T} = \{\tau^{\hbar} = \left[\underline{\tau}^{\hbar}, \overline{\tau}^{\hbar}\right],\ \hbar = 1, 2, \ldots, \ell'\}$ be the collection of performance ratings provided by group of ℓ' decision makers, where $\underline{\tau}^{\hbar}$ and $\overline{\tau}^{\hbar}$ are the lower and upper limits of the assessment values, respectively. The IRN of each element of the set $\mathcal{T}$ is denoted by $\mathrm{IRN}(\tau^{\hbar}) = \left(\left[\underline{\tau}^{\hbar L}, \underline{\tau}^{\hbar U}\right], \left[\overline{\tau}^{\hbar L}, \overline{\tau}^{\hbar U}\right]\right)$, where lower and upper approximation set of each element can be computed by employing following Equations [17]:

$$\underline{Apr}(\underline{\tau}^{\hbar}) = \cup\{\underline{\tau} \mid \underline{\tau} \leq \underline{\tau}^{\hbar}\ ,\ \tau \in \mathcal{T}\}, \qquad \underline{Apr}(\overline{\tau}^{\hbar}) = \cup\{\overline{\tau} \mid \overline{\tau} \leq \overline{\tau}^{\hbar}\ ,\ \tau \in \mathcal{T}\}, \tag{2.1}$$

$$\overline{Apr}(\underline{\tau}^{\hbar}) = \cup\{\underline{\tau} \mid \underline{\tau} \geq \underline{\tau}^{\hbar}\ ,\ \tau \in \mathcal{T}\}, \qquad \overline{Apr}(\overline{\tau}^{\hbar}) = \cup\{\overline{\tau} \mid \overline{\tau} \geq \overline{\tau}^{\hbar}\ ,\ \tau \in \mathcal{T}\}, \tag{2.2}$$

and lower and upper limits are calculated with the help of following Equations:

$$\underline{\tau}^{\hbar L} = \underline{Lim}(\underline{\tau}^{\hbar}) = \frac{1}{\underline{N}^{L}} \sum_{\underline{\tau} \in \underline{Apr}(\underline{\tau}^{\hbar})} \underline{\tau}, \qquad \overline{\tau}^{\hbar L} = \underline{Lim}(\overline{\tau}^{\hbar}) = \frac{1}{\overline{N}^{L}} \sum_{\overline{\tau} \in \underline{Apr}(\overline{\tau}^{\hbar})} \overline{\tau}, \tag{2.3}$$

$$\underline{\tau}^{\hbar U} = \overline{Lim}(\underline{\tau}^{\hbar}) = \frac{1}{\underline{N}^{U}} \sum_{\underline{\tau} \in \overline{Apr}(\underline{\tau}^{\hbar})} \underline{\tau}, \qquad \overline{\tau}^{\hbar U} = \overline{Lim}(\overline{\tau}^{\hbar}) = \frac{1}{\overline{N}^{U}} \sum_{\overline{\tau} \in \overline{Apr}(\overline{\tau}^{\hbar})} \overline{\tau}, \tag{2.4}$$

where $\underline{N}^{L}$, $\overline{N}^{L}$, $\underline{N}^{U}$ and $\overline{N}^{U}$ represent the cardinalities of sets $\underline{Apr}(\underline{\tau}^{\hbar})$, $\underline{Apr}(\overline{\tau}^{\hbar})$, $\overline{Apr}(\underline{\tau}^{\hbar})$, and $\overline{Apr}(\overline{\tau}^{\hbar})$, respectively.

2.1 Arithmetic operations on interval rough numbers

Given two IRNs $IRN(\tau^1) = \left(\left[\underline{\tau}^{1L}, \underline{\tau}^{1U}\right], \left[\overline{\tau}^{1L}, \overline{\tau}^{1U}\right]\right)$, and $IRN(\tau^2) = \left(\left[\underline{\tau}^{2L}, \underline{\tau}^{2U}\right], \left[\overline{\tau}^{2L}, \overline{\tau}^{2U}\right]\right)$, the arithmetic operations are defined as follows [17]:

(1) Addition & subtraction: $IRN(\tau^1) \pm IRN(\tau^2) = \left(\left[\underline{\tau}^{1L} \pm \underline{\tau}^{2L}, \underline{\tau}^{1U} \pm \underline{\tau}^{2U}\right], \left[\overline{\tau}^{1L} \pm \overline{\tau}^{2L}, \overline{\tau}^{1U} \pm \overline{\tau}^{2U}\right]\right)$.

(2) Multiplication: $IRN(\tau^1) \otimes IRN(\tau^2) = \left(\left[\underline{\tau}^{1L} \times \underline{\tau}^{2L}, \underline{\tau}^{1U} \times \underline{\tau}^{2U}\right], \left[\overline{\tau}^{1L} \times \overline{\tau}^{2L}, \overline{\tau}^{1U} \times \overline{\tau}^{2U}\right]\right)$.

(3) Division: $IRN(\tau^1)/IRN(\tau^2) = \left(\left[\underline{\tau}^{1L}/\overline{\tau}^{2U}, \underline{\tau}^{1U}/\overline{\tau}^{2L}\right], \left[\overline{\tau}^{1L}/\underline{\tau}^{2U}, \overline{\tau}^{1U}/\underline{\tau}^{2L}\right]\right)$.

(4) Scalar multiplication: $k \times IRN(\tau^1) = \left(\left[k \times \underline{\tau}^{1L}, k \times \underline{\tau}^{1U}\right], \left[k \times \overline{\tau}^{1L}, k \times \overline{\tau}^{1U}\right]\right)$.

2.2 Comparison between interval rough numbers

The process for the comparison between two IRNs is motivated by the comparison among two rough numbers. It employs the terminology of intersection point being an approximate middle point of an IRN. The comparison of two IRNs $IRN(\tau^1) = \left(\left[\underline{\tau}^{1L}, \underline{\tau}^{1U}\right], \left[\overline{\tau}^{1L}, \overline{\tau}^{1U}\right]\right)$ and $IRN(\tau^2) = \left(\left[\underline{\tau}^{2L}, \underline{\tau}^{2U}\right], \left[\overline{\tau}^{2L}, \overline{\tau}^{2U}\right]\right)$ is performed by following these two points [17]:

1. If one of the IRN does not strictly bound the other one, then:
   (a) if $\left\{\overline{\tau}^{1U} > \overline{\tau}^{2U} \text{ and } \underline{\tau}^{1L} \geq \underline{\tau}^{2L}\right\}$ or $\left\{\overline{\tau}^{1U} \geq \overline{\tau}^{2U} \text{ and } \underline{\tau}^{1L} < \underline{\tau}^{2L}\right\}$ is satisfied then $IRN(\tau^1) \succ IRN(\tau^2)$.
   (b) if $\left\{\overline{\tau}^{1U} = \overline{\tau}^{2U} \text{ and } \underline{\tau}^{1L} = \underline{\tau}^{2L}\right\}$, then $IRN(\tau^1) \approx IRN(\tau^2)$.
2. If one of the IRN strictly bound the other one, then intersection points $I(\tau^1)$ and $I(\tau^2)$ of each IRN is calculated to compare them. If $\overline{\tau}^{1U} > \overline{\tau}^{2U}$ and $\underline{\tau}^{1L} < \underline{\tau}^{2L}$, then:
   (a) if $I(\tau^1) \leq I(\tau^2)$ then $IRN(\tau^1) \prec IRN(\tau^2)$,
   (b) if $I(\tau^1) > I(\tau^2)$ then $IRN(\tau^1) \succ IRN(\tau^2)$,

   where

$$I(\tau^j) = \left(\frac{\overline{\tau}^{jU} - \overline{\tau}^{jL}}{\overline{\tau}^{jU} - \overline{\tau}^{jL} + \underline{\tau}^{jU} - \underline{\tau}^{jL}}\right) \cdot \underline{\tau}^{jL} + \left(j - \frac{\overline{\tau}^{jU} - \overline{\tau}^{jL}}{\overline{\tau}^{jU} - \overline{\tau}^{jL} + \underline{\tau}^{jU} - \underline{\tau}^{jL}}\right) \cdot \overline{\tau}^{jU}, \tag{2.5}$$

   where $j = 1, 2$. Similar concept can be applied if $\overline{\tau}^{1U} < \overline{\tau}^{2U}$ and $\underline{\tau}^{1L} > \underline{\tau}^{2L}$.

3 Proposed MCGDM framework

The fundamental framework of interval rough (IR) information-based multi-criteria group decision-making (MCGDM) is presented in this section. The interval rough LBWA technique is employed of determine the criteria weight vector. The interval rough ELECTRE-III approach is developed to tackle the assessment and ranking problem of alternatives in the presence of interval rough evaluation information.

3.1 LBWA method

The level based weight assessment (LBWA) approach was first put forward by Žižović and Pamucar, [31]. This technique is a reliable tool to deduce subjective criteria weight vector both in group and individual decision-making, and requires a rather simple mathematical equipment. The collection of criteria $\zeta = \{\zeta_1, \zeta_2, \ldots, \zeta_m\}$, where m is the number of factors (or criteria) impacting the selection, is defined at the initial stage of the LBWA technique's implementation. The working principal of LBWA technique initiates by dividing the criteria set ζ into different levels of significance classes. The following points outline the process of the LBWA approach, which are summarized in Figure 1.

Step 5(a). The initial step of the LBWA process is identifying the criterion that has the most influence on the decision-making process, or the most crucial criterion, among the collection of specified criteria ζ. Let that criterion be ζ_1.

Step 5(b). Put the criteria into groups based on their significance degree. The following defines the significance levels:

Level $\mathcal{L}_1$: Level $\mathcal{L}_1$ comprises criteria from the collection ζ with significance that is up to twice as low as or equivalent to the significance of the most important criterion;

Level $\mathcal{L}_2$: Level $\mathcal{L}_2$ comprises criteria from the collection ζ with significance that is exactly twice or up to thrice as low from the significance of the most important criterion;

$\vdots$

Level $\mathcal{L}_q$: Criteria belonging to the set ζ whose significance is precisely q times lower than the highest significant criterion's importance, that is, up to $q+1$ times less than the highest significant criterion's importance, are clustered at the level $\mathcal{L}_q$.

A decision maker develops a preliminary categorization of the given criteria by using the above stated rules. Let $s(\zeta_j)$ denote the significance of a specific criterion ζ_j, $j = 1, 2, \ldots, m$, then $\zeta = \mathcal{L}_1 \cup \mathcal{L}_2 \cup \ldots \cup \mathcal{L}_q$, where

$$\mathcal{L}_\imath = \{\zeta_{1_\imath}, \zeta_{2_\imath}, \ldots, \zeta_{p_\imath}\} = \{\zeta_j \in \zeta \mid \imath \leq s(\zeta_j) < \imath + 1\}, \quad \imath = 1, 2, \ldots, q. \tag{3.6}$$

Also $\mathcal{L}_\hbar \cap \mathcal{L}_g = \emptyset$, for any $\hbar, g = 1, 2, \ldots, q$, $\hbar \neq g$, which shows that collection of significance level sets defines partition of the criteria set ζ.

Step 5(c). Compare the importance of the criteria within the previously formed significance levels. Allot an integer $\mathcal{I}_{j_\hbar} \in \{0, 1, \ldots, \mathfrak{z}\}$ to the criterion $\zeta_{j_\hbar}$ belonging to the level $\mathcal{L}_\hbar = \{\zeta_{1_\hbar}, \zeta_{2_\hbar}, \ldots, \zeta_{p_\hbar}\}$, in such a way that most influential criterion ζ_j is assigned $\mathcal{I}_j = 0$ and $\mathcal{I}_{j_\hbar} < \mathcal{I}_{j'_\hbar}$ if criterion $\zeta_{j_\hbar}$ is more important than $\zeta_{j'_\hbar}$. $\mathcal{I}_{j_\hbar} = \mathcal{I}_{j'_\hbar}$ if both criteria $\zeta_{j_\hbar}$ and $\zeta_{j'_\hbar}$ are equally important. Determine the highest value of the rating scale for the criteria comparison by employing the following equation:

$$\mathfrak{z} = \max\{|\mathcal{L}_1|, |\mathcal{L}_2|, \ldots, |\mathcal{L}_q|\}. \tag{3.7}$$

Step 5(d). Specify the elasticity coefficient denoted by $\mathfrak{z}_0 \in \mathbb{R}$, with condition that it should always be less than the maximum criteria comparison rating scale $\mathfrak{z}$ (that is, $\mathfrak{z}_0 > \mathfrak{z}$). It is suggested to take $\mathfrak{z}_0 = \mathfrak{z} + 1$.

Step 5(e). For each $\zeta_{j_\hbar} \in \mathcal{L}_\hbar$, define the influence function $\phi : \zeta \to \mathbb{R}$ as follows:

$$\phi(\zeta_{j_\hbar}) = \frac{\mathfrak{z}_0}{\hbar \cdot \mathfrak{z}_0 + \mathcal{I}_{j_\hbar}}, \tag{3.8}$$

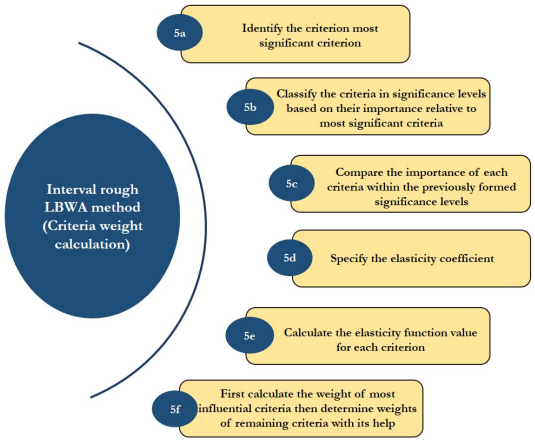

Figure 1: LBWA approach

where $\hbar$ is the number of the level in which corresponding criteria is categorized, $\mathfrak{z}_0$ is the elasticity coefficient and $\mathcal{I}_{j_\hbar}$ represents the influence level allotted to the criterion $\zeta_{j_\hbar}$.

Step 5(f). Compute the weight coefficient of the most influential criterion ζ_1 by utilizing the following equation:

$$\omega_1 = \frac{1}{1 + \phi(\zeta_2) + \phi(\zeta_3) + \ldots + \phi(\zeta_m)}. \tag{3.9}$$

Calculate the weights of the remaining criteria with the help of following expression:

$$\omega_j = \phi(\zeta_j) \cdot \omega_1, \quad j = 2, 3, \ldots, m. \tag{3.10}$$

3.2 Interval rough ELECTRE-III approach

The IRNs-based ELECTRE-III approach, a proficient extension of the ELECTRE strategy, is demonstrated in this section along with its methodology. By applying three types of preferences, the suggested technique produces consistent results by eliminating (outranking) undesirable options. This decision-making process is compatible with interval rough information and accommodates multiple criteria for evaluating alternatives, aligning naturally with the MCGDM process. The mathematical framework for the MCGDM problem comprises three main components. The first is the set of alternatives $\mathcal{A} = \{\alpha_i; i = 1, 2, \ldots, n\}$, where the abilities of the alternatives are evaluated. The second is the set of criteria $\zeta = \{\zeta_j; j = 1, 2, \ldots, m\}$, which defines the m parameters used to analyze the alternatives. The third component is the decision-making committee $\mathcal{D} = \{\mathcal{D}_\hbar; \hbar = 1, 2, \ldots, \ell'\}$, consisting of ℓ' experts, with a weight vector $\epsilon = (\epsilon_1, \epsilon_2, \ldots, \epsilon_{\ell'})$ representing the importance of their opinions. The steps for the IRN-ELECTRE-III method are as follows:

Step 1. Assessment of alternatives by group of DMs

The first step in solving any MCDM problem is gathering evaluations from each decision committee member. A key challenge is the presence of vague opinions and the difficulty in quantifying certain phenomena. To address this, an appropriate linguistic scale (defined in Table 1) is used to represent the committee's evaluations of the alternatives across all criteria. In some cases, a DM may believe that an alternative's performance for a specific criterion lies between two linguistic terms, rather than a single value. This requires a model that allows DMs to assign a range (i.e., an interval) as assessment information. Such ambiguous interval data is handled through RST, forming IRNs. The interval decision matrix $\mathcal{T}^\hbar$, given in Equation 3.11, contains evaluations in the form of intervals $[\underline{\tau}_{ij}^\hbar, \overline{\tau}_{ij}^\hbar]$ provided by the $\hbar$th DM for n alternatives across m decision criteria.

$$\mathcal{T}^\hbar = \begin{array}{c} \\ \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{array} \begin{array}{c} \begin{array}{cccc} \zeta_1 & \zeta_2 & \cdots & \zeta_m \end{array} \\ \begin{pmatrix} [\underline{\tau}_{11}^\hbar, \overline{\tau}_{11}^\hbar] & [\underline{\tau}_{12}^\hbar, \overline{\tau}_{12}^\hbar] & \cdots & [\underline{\tau}_{1m}^\hbar, \overline{\tau}_{1m}^\hbar] \\ [\underline{\tau}_{21}^\hbar, \overline{\tau}_{21}^\hbar] & [\underline{\tau}_{22}^\hbar, \overline{\tau}_{22}^\hbar] & \cdots & [\underline{\tau}_{2m}^\hbar, \overline{\tau}_{2m}^\hbar] \\ \vdots & \vdots & \ddots & \vdots \\ [\underline{\tau}_{n1}^\hbar, \overline{\tau}_{n1}^\hbar] & [\underline{\tau}_{n2}^\hbar, \overline{\tau}_{n2}^\hbar] & \cdots & [\underline{\tau}_{nm}^\hbar, \overline{\tau}_{nm}^\hbar] \end{pmatrix} \end{array}. \tag{3.11}$$

Table 1: Linguistic scale for the performance ratings of alternatives

Performance	Rating
`Extremely Poor` ($\mathbb{EP}$)	1
`Very Poor` ($\mathbb{VP}$)	2
`Poor` ($\mathbb{P}$)	3
`Slightly Poor` ($\mathbb{SP}$)	4
`Fair` ($\mathbb{F}$)	5
`Slightly Good` ($\mathbb{SG}$)	6
`Good` ($\mathbb{G}$)	7
`Very Good` ($\mathbb{VG}$)	8
`Extremely Good` ($\mathbb{EG}$)	9

Step 2. Conversion of interval-based assessments to interval rough numbers
The next step captures the undecidedness and incompleteness of interval based evaluation information provided by each DM through the concept of interval rough theory and converts the interval decision data into the interval rough evaluation information by employing the definition of IRNs given in Equations 2.1-2.4. Arrange the interval rough assessments into the individual interval rough decision matrix $\mathcal{T}^{[\hbar]}$ according to Equation 3.12.

$$\mathcal{T}^{[\hbar]} = \begin{array}{c} \\ \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{array} \begin{array}{c} \begin{array}{cccc} \zeta_1 & \zeta_2 & \cdots & \zeta_m \end{array} \\ \begin{pmatrix} \left(\left[\underline{\tau}_{11}^{\hbar L}, \underline{\tau}_{11}^{\hbar U}\right], \left[\overline{\tau}_{11}^{\hbar L}, \overline{\tau}_{11}^{\hbar U}\right]\right) & \left(\left[\underline{\tau}_{12}^{\hbar L}, \underline{\tau}_{12}^{\hbar U}\right], \left[\overline{\tau}_{12}^{\hbar L}, \overline{\tau}_{12}^{\hbar U}\right]\right) & \cdots & \left(\left[\underline{\tau}_{1m}^{\hbar L}, \underline{\tau}_{1m}^{\hbar U}\right], \left[\overline{\tau}_{1m}^{\hbar L}, \overline{\tau}_{1m}^{\hbar U}\right]\right) \\ \left(\left[\underline{\tau}_{21}^{\hbar L}, \underline{\tau}_{21}^{\hbar U}\right], \left[\overline{\tau}_{21}^{\hbar L}, \overline{\tau}_{21}^{\hbar U}\right]\right) & \left(\left[\underline{\tau}_{22}^{\hbar L}, \underline{\tau}_{22}^{\hbar U}\right], \left[\overline{\tau}_{22}^{\hbar L}, \overline{\tau}_{22}^{\hbar U}\right]\right) & \cdots & \left(\left[\underline{\tau}_{2m}^{\hbar L}, \underline{\tau}_{2m}^{\hbar U}\right], \left[\overline{\tau}_{2m}^{\hbar L}, \overline{\tau}_{2m}^{\hbar U}\right]\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\left[\underline{\tau}_{n1}^{\hbar L}, \underline{\tau}_{n1}^{\hbar U}\right], \left[\overline{\tau}_{n1}^{\hbar L}, \overline{\tau}_{n1}^{\hbar U}\right]\right) & \left(\left[\underline{\tau}_{n2}^{\hbar L}, \underline{\tau}_{n2}^{\hbar U}\right], \left[\overline{\tau}_{n2}^{\hbar L}, \overline{\tau}_{n2}^{\hbar U}\right]\right) & \cdots & \left(\left[\underline{\tau}_{nm}^{\hbar L}, \underline{\tau}_{nm}^{\hbar U}\right], \left[\overline{\tau}_{nm}^{\hbar L}, \overline{\tau}_{nm}^{\hbar U}\right]\right) \end{pmatrix} \end{array}. \tag{3.12}$$

Step 3. Aggregation of individual interval rough decision matrices
Specify the decision power of each DM $\mathcal{D}_{\hbar}; \hbar = 1, 2, \ldots, \ell$ through assigning their weight vector $\epsilon = (\epsilon_1, \epsilon_2, \ldots, \epsilon_\ell)$. Combine the individual assessment values regarding each alternative's performance corresponding to each criterion by employing interval rough aggregating operator defined in Equation 3.13. Arrange the aggregated data into the interval rough aggregated decision matrix $\widetilde{\mathcal{T}} = (\widetilde{\tau}_{ij})_{n\times m}$.

$$\left[\widetilde{\underline{\tau}}_{ij}^{L}, \widetilde{\underline{\tau}}_{ij}^{U}\right] = \left[\sum_{\hbar=1}^{\ell}\left(\epsilon_{\hbar}\cdot\underline{\tau}_{ij}^{\hbar L}\right), \sum_{\hbar=1}^{\ell}\left(\epsilon_{\hbar}\cdot\underline{\tau}_{ij}^{\hbar U}\right)\right], \quad \left[\widetilde{\overline{\tau}}_{ij}^{L}, \widetilde{\overline{\tau}}_{ij}^{U}\right] = \left[\sum_{\hbar=1}^{\ell}\left(\epsilon_{\hbar}\cdot\overline{\tau}_{ij}^{\hbar L}\right), \sum_{\hbar=1}^{\ell}\left(\epsilon_{\hbar}\cdot\overline{\tau}_{ij}^{\hbar U}\right)\right], \tag{3.13}$$

Step 4. Normalization of aggregated interval rough decision matrix
Normalize the averaged assessment data in correspondence to the set of cost criteria ζ^C and the set of benefit type criteria ζ^B, according to Equation 3.14. Gather the normalized interval rough information into the normalized aggregated interval rough decision matrix $\widehat{\mathcal{T}} = (\widehat{\tau}_{ij})_{n\times m}$.

$$\left(\left[\widehat{\underline{\tau}}_{ij}^{L}, \widehat{\underline{\tau}}_{ij}^{U}\right], \left[\widehat{\overline{\tau}}_{ij}^{L}, \widehat{\overline{\tau}}_{ij}^{U}\right]\right) = \begin{cases} \left(\left[\frac{\widetilde{\underline{\tau}}_{ij}^{L}}{\max_{i=1}^{n}\widetilde{\overline{\tau}}_{ij}^{U}}, \frac{\widetilde{\underline{\tau}}_{ij}^{U}}{\max_{i=1}^{n}\widetilde{\overline{\tau}}_{ij}^{L}}\right], \left[\frac{\widetilde{\overline{\tau}}_{ij}^{L}}{\max_{i=1}^{n}\widetilde{\underline{\tau}}_{ij}^{U}}, \frac{\widetilde{\overline{\tau}}_{ij}^{U}}{\max_{i=1}^{n}\widetilde{\underline{\tau}}_{ij}^{L}}\right]\right) & \text{if } \zeta_j \in \zeta^B, \\ \left(\left[\frac{\min_{i=1}^{n}\widetilde{\underline{\tau}}_{ij}^{L}}{\widetilde{\overline{\tau}}_{ij}^{U}}, \frac{\min_{i=1}^{n}\widetilde{\underline{\tau}}_{ij}^{U}}{\widetilde{\overline{\tau}}_{ij}^{L}}\right], \left[\frac{\min_{i=1}^{n}\widetilde{\overline{\tau}}_{ij}^{L}}{\widetilde{\underline{\tau}}_{ij}^{U}}, \frac{\min_{i=1}^{n}\widetilde{\overline{\tau}}_{ij}^{U}}{\widetilde{\underline{\tau}}_{ij}^{L}}\right]\right) & \text{if } \zeta_j \in \zeta^C. \end{cases} \tag{3.14}$$

Step 5. Computation of criteria weight vector
Construct the weight vector $\omega = (\omega_1, \omega_2, \ldots \omega_m)$ comprising of weights of each criteria present in the set $\zeta = \{\zeta_j; j = 1, 2, \ldots, m\}$ by following the interval rough LBWA technique narrated in Section 3.1 with steps 5(a)−5(f).
Step 6. Construction of weighted normalized aggregated interval rough decision matrix
In order to determine the fused assessment information, combine the criteria weight vector ω and normalized aggregated interval rough decision matrix $\widehat{\mathcal{T}}$ through interval rough aggregating operator given in Equation 3.15, and build weighted normalized aggregated interval rough decision matrix $\mathcal{T}$.

$$\left(\left[\underline{\tau}_{ij}^{L}, \underline{\tau}_{ij}^{U}\right], \left[\overline{\tau}_{ij}^{L}, \overline{\tau}_{ij}^{U}\right]\right) = \left(\left[\omega_{\zeta_j}\times\widehat{\underline{\tau}}_{ij}^{L}, \omega_{\zeta_j}\times\widehat{\underline{\tau}}_{ij}^{U}\right], \left[\omega_{\zeta_j}\times\widehat{\overline{\tau}}_{ij}^{L}, \omega_{\zeta_j}\times\widehat{\overline{\tau}}_{ij}^{U}\right]\right), \tag{3.15}$$

Step 7. Specification of suitable threshold values
The ELECTRE-III approach is specifically designed to capture the pseudo-criterion, where small variations do not lead

to significant changes in the overall evaluation. Within the pseudo criteria, three threshold values are utilized to examine the pairwise relations among alternatives. The three levels that make up the thresholds are organized in ascending order to distinguish between the various degrees of dominance between the alternative options. The indifference threshold (q) refers to the lowest threshold value. Next are the threshold values for preference (ρ) and veto (υ), respectively. It often occurs that the performance differences between two alternatives are too small to clearly label one as better or worse than the other. These situations are addressed by the indifference threshold, which defines the maximum degree of dominance for one option that is still insufficient to outrank the other. The following inequality defines the necessary mathematical condition to recognize the indifference relation:

$$\zeta(\alpha_y) < \zeta(\alpha_x) \leq \zeta(\alpha_y) + q \quad \Leftrightarrow \quad \alpha_x \mathbb{I} \alpha_y, \tag{3.16}$$

where $\zeta(\alpha_x) = (\zeta_1(\alpha_x), \zeta_2(\alpha_x), \ldots, \zeta_m(\alpha_x))$ presents the collection of performance ratings of alternative α_x over the considered set of criteria. Similarly, $\zeta(\alpha_y) = (\zeta_1(\alpha_y), \zeta_2(\alpha_y), \ldots, \zeta_m(\alpha_y))$. $\mathbb{I}$ indicates the relation of indifference. Between the veto and indifference threshold levels comes the next threshold level, referred to as the preference threshold. The distinction between the weak and strong preference levels is made by the preference threshold. When difference between the assessments of pair of alternatives exceeds indifference level (threshold), there is an indication of strong or weak preference. In order to exceed a comparable alternative with a strong preference, one must surpass the preference threshold value, which displays the highest value for analyzing weak preferences. The following inequalities outline the necessary mathematical criteria to detect weak preference relation $\mathbb{Q}$ and strong preference relation $\mathbb{P}$, respectively.

$$\zeta(\alpha_y) + q < \zeta(\alpha_x) \leq \zeta(\alpha_y) + \rho \quad \Leftrightarrow \quad \alpha_x \mathbb{Q} \alpha_y, \tag{3.17}$$

$$\zeta(\alpha_x) > \zeta(\alpha_y) + \rho \quad \Leftrightarrow \quad \alpha_x \mathbb{P} \alpha_y. \tag{3.18}$$

Step 8. Computation of interval rough concordance indices

Aiming to examine the degree of superiority of alternative α_x over the alternative α_y, interval rough concordance indices are computed. In order to evaluate the alternatives' performances in relation to a certain criterion, the difference between the IRN intersection points that appear in the weighted normalized aggregated interval rough decision matrix $\mathcal{T}$ is examined with respect to the strong and weak threshold values. The comparison among pairs is categorized into two sets $\Delta_{xy}^{\mathbb{P}}$ and $\Delta_{xy}^{\mathbb{Q}}$, depicting different levels of dominance of the alternative α_x over α_y corresponding to respective collection of criteria ζ. The set $\Delta_{xy}^{\mathbb{P}}$ is composed of all those criteria according to which alternative α_x is strongly preferable over α_y, defined in Equation 3.19. Similarly, $\Delta_{xy}^{\mathbb{Q}}$ is the collection of criteria for which α_x is weakly preferable over α_y, defined in Equation 3.20.

$$\Delta_{xy}^{\mathbb{P}} = \{\zeta_j \in \zeta \mid \mathcal{J}(\tau_{yj}) - \mathcal{J}(\tau_{xj}) \leq q_j\}, \tag{3.19}$$

$$\Delta_{xy}^{\mathbb{Q}} = \{\zeta_j \in \zeta \mid q_j \leq \mathcal{J}(\tau_{yj}) - \mathcal{J}(\tau_{xj}) \leq \rho_j\}. \tag{3.20}$$

The numerical measure of dominance of alternative α_x over the alternative α_y with respect to criterion ζ_j is termed as interval rough partial concordance index, denoted by $\mu_j(\alpha_x, \alpha_y)$ and is calculated with the help of sets $\Delta_{xy}^{\mathbb{P}}$ and $\Delta_{xy}^{\mathbb{Q}}$, as defined in Equation 3.21.

$$\mu_j(\alpha_x, \alpha_y) = \begin{cases} 1 & , \quad \text{if} \quad \zeta_j \in \Delta_{xy}^{\mathbb{P}} \\ \frac{\rho_j - (\mathcal{J}(\tau_{yj}) - \mathcal{J}(\tau_{xj}))}{\rho_j - q_j} & , \quad \text{if} \quad \zeta_j \in \Delta_{xy}^{\mathbb{Q}} \\ 0 & , \quad \text{otherwise.} \end{cases} \tag{3.21}$$

The comprehensive concordance index indicates the overall dominance of one alternative over the other one in reference to the complete set of criteria and is computed by weighted sum of partial concordance indices and corresponding criteria weight, as defined in Equation 3.22. Organize the indices μ in concordance matrix $\mathcal{C}$, as constructed in Equation 3.23.

$$\mu(\alpha_x, \alpha_y) = \sum_{j=1}^{m} \omega_j \cdot \mu_j(\alpha_x, \alpha_y), \tag{3.22}$$

$$\mathcal{C} = \begin{array}{c} \\ \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{array} \begin{array}{c} \begin{array}{cccc} \alpha_1 & \alpha_2 & \cdots & \alpha_n \end{array} \\ \begin{pmatrix} - & \mu(\alpha_1, \alpha_2) & \cdots & \mu(\alpha_1, \alpha_n) \\ \mu(\alpha_2, \alpha_1) & - & \cdots & \mu(\alpha_2, \alpha_n) \\ \vdots & \vdots & \ddots & \vdots \\ \mu(\alpha_n, \alpha_1) & \mu(\alpha_n, \alpha_2) & \cdots & - \end{pmatrix} \end{array}. \tag{3.23}$$

Step 9. Calculation of interval rough discordance indices

The discordance index presents the opposite argument against the fact that alternative α_x is outranking other alternative α_y. That is, it measures the degree of inferiority of one alternative over the other. The discordance index λ_j of the pair (α_x, α_y) with respect to the criterion ζ_j is computed through preference and veto threshold values as defines in Equation 3.24.

$$\lambda_j(\alpha_x, \alpha_y) = \begin{cases} 1 & , \quad \text{if} \quad \mathcal{g}(\tau_{yj}) - \mathcal{g}(\tau_{xj}) \geq \upsilon_j \\ 0 & , \quad \text{if} \quad \mathcal{g}(\tau_{yj}) - \mathcal{g}(\tau_{xj}) \leq \rho_j \\ \frac{\mathcal{g}(\tau_{yj}) - \mathcal{g}(\tau_{xj}) - \rho_j}{\upsilon_j - \rho_j} & , \quad \text{otherwise.} \end{cases} \tag{3.24}$$

In order to counteract the discordance effect, the veto threshold value υ_j shows the lowest performance difference. Hence, the greatest discordance value, that is 1, will be allocated to the values that are higher than the veto threshold. When discordant action is negligible or the performance variation is below the preference threshold, the discordance index is going to be at its lowest. Employing a mathematical means, the discordance indices for intermediate values are calculated, which fall into the range $[0, 1]$. Construct the discordance matrix $\mathcal{D}_j$ comprising of discordances indices relative to the criterion ζ_j, as in Equation 3.25.

$$\mathcal{D}_j = \begin{array}{c} \\ \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{array} \begin{array}{c} \begin{array}{cccc} \alpha_1 & \alpha_2 & \cdots & \alpha_n \end{array} \\ \begin{pmatrix} - & \lambda_j(\alpha_1, \alpha_2) & \cdots & \lambda_j(\alpha_1, \alpha_n) \\ \lambda_j(\alpha_2, \alpha_1) & - & \cdots & \lambda_j(\alpha_2, \alpha_n) \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_j(\alpha_n, \alpha_1) & \lambda_j(\alpha_n, \alpha_2) & \cdots & - \end{pmatrix} \end{array}. \tag{3.25}$$

Step 10. Determination of credibility indices

The inspection of available set of alternatives with respect to both aspects of concordance and discordance is followed by determination of collective behavior by identifying that which behavior is more dominant. The credibility index depicts the strength of outranking relation between a pair of alternatives by assuring that amount of concordance effect of one action over the other is enough to neglect the corresponding discordance effect. Compute the credibility degree π for a pair (α_x, α_y) by employing the following Equation 3.26.

$$\pi(\alpha_x, \alpha_y) = \begin{cases} \mu(\alpha_x, \alpha_y) & , \quad \text{if} \quad \lambda_j(\alpha_x, \alpha_y) \leq \mu(\alpha_x, \alpha_y) \ \ \forall j \in \zeta \\ \mu(\alpha_x, \alpha_y) \times \prod\limits_{j \in \zeta'(\alpha_x, \alpha_y)} \frac{1 - \lambda_j(\alpha_x, \alpha_y)}{1 - \mu(\alpha_x, \alpha_y)} & , \quad \text{otherwise} \end{cases}, \tag{3.26}$$

where

$$\zeta'(\alpha_x, \alpha_y) = \{ j \in \zeta : \ \lambda_j(\alpha_x, \alpha_y) > \mu(\alpha_x, \alpha_y) \}. \tag{3.27}$$

Step 11. Evaluation of preference ordering(ranking) of alternatives

The traditional process for evaluation of preference ordering in ELECTRE-III method through credibility indices involves an iterative distillation process whose implementation becomes time taking and complex when number of available options is large in number. Li et al., [11] came up with a relatively simple process to extract ranking of alternatives by finding their net credibility index. Calculate the concordance credibility index $\gamma_{\mathcal{C}}$ of each alternative α_x $(x = 1, 2, \ldots, n)$ by employing Equation 3.28.

$$\gamma_{\mathcal{C}}(\alpha_x) = \sum_{\alpha_y \in \mathcal{A}, y \neq x} \pi(\alpha_x, \alpha_y). \tag{3.28}$$

The index $\gamma_{\mathcal{C}}(\alpha_x)$ highlights the overall outranking capability of alternative α_x relative to other available alternatives. On the other hand, discordance credibility index $\gamma_{\mathcal{D}}(\alpha_x)$ reveals the degree to which alternative α_x is inferior than other ones. Compute the discordance credibility index $\gamma_{\mathcal{D}}$ of each alternative α_x $(x = 1, 2, \ldots, n)$ by employing Equation 3.29.

$$\gamma_{\mathcal{D}}(\alpha_x) = \sum_{\alpha_y \in \mathcal{A}, y \neq x} \pi(\alpha_y, \alpha_x). \tag{3.29}$$

Finally, deduce the net credibility index γ of each alternative (α_x) with the help of Equation 3.30.

$$\gamma(\alpha_x) = \gamma_{\mathcal{C}}(\alpha_x) - \gamma_{\mathcal{D}}(\alpha_x). \tag{3.30}$$

Rank the alternatives in descending order of net credibility index. The alternative with maximum net credibility index will be considered as optimal solution to the underlying problem. Figure 2 presents a comprehensive layout of the proposed methodology in interval rough environment.

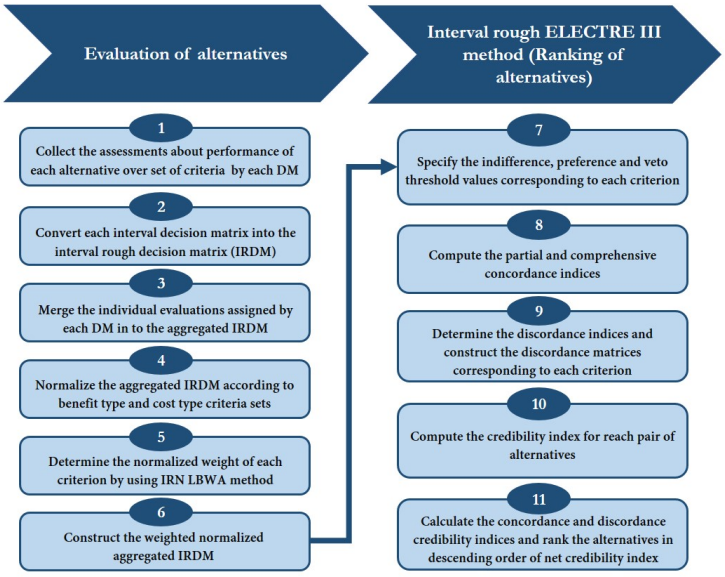

Figure 2: Framework of proposed decision support system

4 Case study (Evaluation of sustainable mining technologies)

This section demonstrates the applicability of the proposed interval rough ELECTRE-III approach by addressing a real-life MCDM problem related to the evaluation of sustainable mining systems (adapted from [15]). The model's authenticity and reliability are established through a comparative analysis with existing MCDM techniques in the literature.

4.1 An overview of the problem

Mining involves extracting valuable minerals and geological materials from the Earth's surface, providing essential resources for technology and economic growth. It remains a key source of income for many nations, contributing to community development and individual well-being. However, mining generates significant waste and can harm the environment if not properly managed. Autonomous mining systems now improve safety, efficiency, and performance. When selecting technology for sustainable mining, decision-makers must evaluate multiple alternatives. The set of fourteen decision criteria $\{\zeta_1, \zeta_2, \ldots, \zeta_{14}\}$, is chosen to evaluate available sustainable mining systems. The criteria selected for evaluating sustainable mining technologies, shown in Figure 3, were carefully chosen to cover the essential aspects influencing mining system performance and sustainability. These include cost, efficiency, environmental impact, and social aspects, each further divided into relevant sub-criteria. This ensures a well-rounded evaluation that captures operational, economic, environmental, and social considerations, addressing challenges such as cost efficiency, environmental responsibility, and public acceptance. Figure 3 illustrates the decision hierarchy for this problem. The data used in this study was sourced from [15], incorporating government reports, industry surveys, expert opinions, and publicly available mining datasets. Key criteria, including operational efficiency, environmental sustainability, and economic viability, were collected for each technology alternative. Ambiguous interval-based subjective data was transformed into IRNs using linguistic terms for integration into the ELECTRE III framework. A sensitivity analysis was also conducted to assess the robustness of the results, ensuring the decision-making process was reliable and applicable to sustainable mining scenarios.

4.1.1 Description of alternatives

$\alpha_{\mathcal{A}}$: ***Fully autonomous systems***: The mining sector is increasingly using automation to operate with minimal human involvement. Originally applied in agriculture, automation in mining relies on advanced sensor technologies to model mines and detect geological features. Automated systems, such as gamma-detecting sensors, enable safer, more efficient analysis with improved resolution, although the field still lacks sufficient attention.

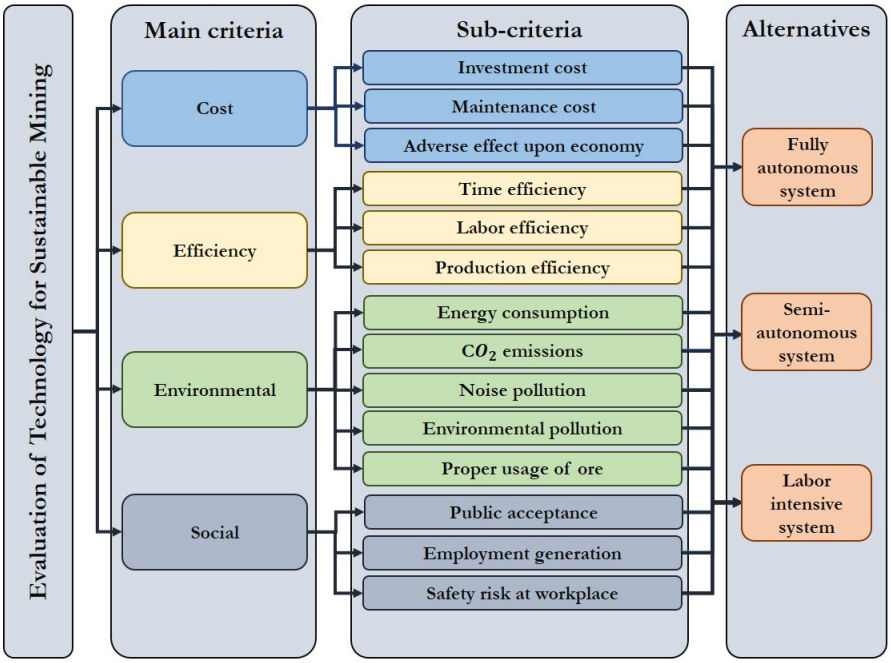

Figure 3: Hierarchal structure of evaluation of technology for sustainable mining

$\alpha_{\mathcal{B}}$: ***Semi-autonomous systems***: Semi-autonomous systems in mining combine automation with human oversight to improve efficiency and safety. Machines like trucks and drills can operate mostly on their own but require human intervention for complex tasks and safety management. For example, trucks can navigate mining sites autonomously, but operators handle challenging situations. Collision avoidance systems use sensors but still need human attention for unexpected issues. This integration aims to boost efficiency while reducing risks in mining operations.

$\alpha_{\mathcal{C}}$: ***System continues with its current technology (labor-intensive)***: Labor-intensive mining relies on manual labor for tasks like drilling, blasting, and material transport, requiring significant physical effort. Despite technological advancements, many operations, especially in developing regions or artisanal mining, still depend on human labor due to economic limitations. This reliance makes mining more time-consuming, increases accident risks, and impacts worker health, emphasizing the need for better technology and safer working conditions.

4.2 Implementation of proposed methodology

Step 1. Group of four DMs $\mathcal{D}_{\hbar}$ $(\hbar = 1, 2, 3, 4)$ conduct the decision-making process aiming to choose a sustainable mining option by evaluating three mining automation alternatives $\alpha_{\mathcal{A}}$, $\alpha_{\mathcal{B}}$, and $\alpha_{\mathcal{C}}$ in the presence of fourteen factors ζ_j $(j = 1, 2, \ldots, 14)$ and present their opinions in the form of linguistic terms defined in Table 1. The complete assessment information in the form of interval data about mining process technologies provided by four DMs is compiled in Table 2.

Step 2. Aiming to capture the roughness and undecidedness appeared in the human opinions, each entry in the form of interval present in individual interval decision matrix is transformed into IRNs by employing Equations 2.1-2.4. Correspondingly interval rough decision matrices $\mathcal{T}^{[\hbar]}$ associated with each DM are constructed in Table 3.

Step 3. All of the DMs are assigned with equal opinion significance weights $(\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4) = (0.25, 0.25, 0.25, 0.25)$. The combined interval rough decision information is calculated with the help of Equation 3.13 and is arranged in aggregated interval rough decision matrix $\widetilde{\mathcal{T}}$, constructed in Table 4.

Step 4. The benefit criteria set is $\zeta^B = \{\zeta_4, \zeta_5, \zeta_6, \zeta_{11}, \zeta_{12}, \zeta_{13}\}$ and cost criteria set is $\zeta^C = \{\zeta_1, \zeta_2, \zeta_3, \zeta_7, \zeta_8, \zeta_9, , \zeta_{10}, \zeta_{14}\}$. According to this division, the decision information in the matrix $\widetilde{\mathcal{T}}$ is normalized by following the Equation 3.14 and

Table 2: Interval decision matrix

Alternatives	DMs	Criteria													
		ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6	ζ_7	ζ_8	ζ_9	ζ_{10}	ζ_{11}	ζ_{12}	ζ_{13}	ζ_{14}
$\alpha_{\mathcal{A}}$	$\mathcal{D}_1$	[1, 1]	[1, 1]	[1, 1]	[9, 9]	[9, 9]	[9, 9]	[1, 1]	[1, 1]	[7, 7]	[9, 9]	[8, 8]	[1, 1]	[8, 8]	[1, 2]
	$\mathcal{D}_2$	[1, 1]	[1, 2]	[3, 3]	[9, 9]	[2, 2]	[2, 2]	[3, 3]	[2, 2]	[4, 4]	[1, 1]	[8, 9]	[8, 9]	[2, 3]	[1, 1]
	$\mathcal{D}_3$	[1, 1]	[2, 2]	[2, 2]	[8, 8]	[9, 9]	[7, 7]	[2, 2]	[1, 1]	[3, 4]	[2, 3]	[8, 8]	[3, 4]	[1, 2]	[2, 2]
	$\mathcal{D}_4$	[1, 2]	[1, 1]	[4, 5]	[9, 9]	[1, 1]	[8, 9]	[1, 1]	[1, 1]	[1, 1]	[3, 3]	[1, 1]	[4, 5]	[1, 2]	[4, 5]
Alternatives	DMs	Criteria													
		ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6	ζ_7	ζ_8	ζ_9	ζ_{10}	ζ_{11}	ζ_{12}	ζ_{13}	ζ_{14}
$\alpha_{\mathcal{B}}$	$\mathcal{D}_1$	[3, 3]	[3, 3]	[2, 2]	[8, 8]	[8, 8]	[7, 7]	[2, 2]	[2, 2]	[2, 2]	[9, 9]	[8, 9]	[1, 1]	[8, 8]	[1, 1]
	$\mathcal{D}_2$	[2, 3]	[4, 5]	[5, 6]	[7, 8]	[4, 5]	[4, 5]	[5, 6]	[4, 4]	[5, 5]	[3, 3]	[3, 4]	[7, 8]	[4, 4]	[2, 3]
	$\mathcal{D}_3$	[4, 4]	[4, 4]	[4, 5]	[7, 7]	[6, 7]	[6, 7]	[3, 4]	[2, 2]	[4, 4]	[3, 4]	[7, 7]	[5, 6]	[3, 4]	[3, 4]
	$\mathcal{D}_4$	[2, 2]	[2, 3]	[5, 5]	[8, 9]	[7, 8]	[8, 8]	[1, 2]	[1, 2]	[2, 3]	[2, 2]	[5, 6]	[6, 7]	[1, 2]	
Alternatives	DMs	Criteria													
		ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6	ζ_7	ζ_8	ζ_9	ζ_{10}	ζ_{11}	ζ_{12}	ζ_{13}	ζ_{14}
$\alpha_{\mathcal{C}}$	$\mathcal{D}_1$	[8, 8]	[8, 8]	[3, 3]	[8, 8]	[7, 7]	[8, 8]	[3, 3]	[3, 3]	[1, 1]	[9, 9]	[9, 9]	[2, 2]	[9, 9]	[1, 1]
	$\mathcal{D}_2$	[7, 8]	[7, 8]	[8, 9]	[2, 3]	[7, 8]	[8, 8]	[7, 8]	[8, 8]	[3, 3]	[7, 8]	[7, 7]	[4, 5]	[7, 7]	[5, 5]
	$\mathcal{D}_3$	[9, 9]	[9, 9]	[7, 8]	[3, 4]	[2, 3]	[3, 3]	[7, 8]	[8, 9]	[7, 7]	[8, 8]	[3, 3]	[8, 8]	[9, 9]	[6, 7]
	$\mathcal{D}_4$	[2, 3]	[3, 3]	[5, 6]	[7, 8]	[9, 9]	[7, 8]	[2, 3]	[4, 4]	[3, 3]	[1, 2]	[2, 3]	[8, 9]	[9, 9]	[1, 1]

Table 3: Interval rough performance ratings of sustainable mining options

Alternative	DMs	Criteria				
		ζ_1	ζ_2	ζ_3	$\cdots$	ζ_{14}
$\alpha_{\mathcal{A}}$	$\mathcal{D}_1$	([1, 1], [1, 1.25])	([1, 1.25], [1, 1.5])	([1, 2.5], [1, 2.75])	$\cdots$	([1, 2], [1.6667, 3])
	$\mathcal{D}_2$	([1, 1], [1, 1.25])	([1, 1.25], [1.5, 2])	([2, 3.5], [2, 4])	$\cdots$	([1, 2], [1, 2.5])
	$\mathcal{D}_3$	([1, 1], [1, 1.25])	([1.25, 2], [1.5, 2])	([1.5, 3], [1.5, 3.3333])	$\cdots$	([1.3333, 3], [1.6667, 3])
	$\mathcal{D}_4$	([1, 1], [1.25, 2])	([1, 1.25], [1, 1.5])	([2.5, 4], [2.75, 5])	$\cdots$	([2, 4], [2.5, 5])
Alternative	DMs	Criteria				
		ζ_1	ζ_2	ζ_3	$\cdots$	ζ_{14}
$\alpha_{\mathcal{B}}$	$\mathcal{D}_1$	([2.3333, 3.5], [2.6667, 3.3333])	([2.5, 3.6667], [3, 3.75])	([2, 4], [2, 4.5])	$\cdots$	([1, 1.75], [1, 2.5])
	$\mathcal{D}_2$	([2, 2.75], [2.6667, 3.3333])	([3.25, 4], [3.75, 5])	([4, 5], [4.5, 6])	$\cdots$	([1.3333, 2.5], [2, 3.5])
	$\mathcal{D}_3$	([2.75, 4], [3, 4])	([3.25, 4], [3.3333, 4.5])	([3, 4.6667], [4, 5.3333])	$\cdots$	([1.75, 3], [2.5, 4])
	$\mathcal{D}_4$	([2, 2.75], [2, 3])	([2, 3.25], [3, 3.75])	([4, 5], [4, 5.3333])	$\cdots$	([1, 1.75], [1.5, 3])
Alternative	DMs	Criteria				
		ζ_1	ζ_2	ζ_3	$\cdots$	ζ_{14}
$\alpha_{\mathcal{C}}$	$\mathcal{D}_1$	([5.6667], [8.5], [6.3333], [8.3333])	([6, 8.5], [6.3333, 8.3333])	([3, 5.75], [3, 6.5])	$\cdots$	([1, 3.25], [1, 3.5])
	$\mathcal{D}_2$	([4.5, 8], [6.3333, 8.3333])	([5, 8], [6.3333, 8.3333])	([5.75, 8], [6.5, 9])	$\cdots$	([2.3333, 5.5], [2.3333, 6])
	$\mathcal{D}_3$	([6.5, 9], [7, 9])	([6.75, 9], [7, 9])	([5, 7.5], [5.6667, 8.5])	$\cdots$	([3.25, 6], [3.5, 7])
	$\mathcal{D}_4$	([2, 6.5], [3, 7])	([3, 6.75], [3, 7])	([4, 6.6667], [4.5, 7.6667])	$\cdots$	([1, 3.25], [1, 3.5])

normalized assessments are arranged in normalized aggregated interval rough decision matrix $\widehat{\mathcal{T}}$, constructed in Table 5.

Step 5. The importance/weight of each criterion involved in the assessment of sustainable mining options are evaluated through interval rough LBWA technique whose implementation of computational process is explained as follows:

Step 5a. The initial step of LBWA method to identify a criterion from the available set of criteria $\{\zeta_1, \zeta_2, \ldots, \zeta_{14}\}$ that has the most influence on the decision-making process. The group of DMs considered safety risk at workplace (ζ_{14}) as most crucial criterion.

Step 5b. Criteria are placed into the groups based on their significance degree as follows:

$$\mathcal{L}_1 = \{\zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5, \zeta_6, \zeta_{12}, \zeta_{14}\}, \quad \mathcal{L}_2 = \{\zeta_{13}\}, \quad \mathcal{L}_3 = \{\zeta_7, \zeta_8, \zeta_9, \zeta_{10}, \zeta_{11}\}.$$

Table 4: Aggregated interval rough evaluation matrix

$\widetilde{\mathcal{T}}$	ζ_1	ζ_2	ζ_3	$\cdots$	ζ_{14}
$\alpha_{\mathcal{A}}$	([1.0000, 1.0000], [1.0625, 1.4375])	([1.0625, 1.4375], [1.2500, 1.7500])	([1.7500, 3.2500], [1.8125, 3.7708])	$\cdots$	([1.3333, 2.7500], [1.7084, 3.3750])
$\alpha_{\mathcal{B}}$	([2.2708, 3.25], [2.5834, 3.4167])	([2.7500, 3.7292], [3.2708, 4.2500])	([3.2500, 4.6667], [3.6250, 5.2917])	$\cdots$	([1.2708, 2.2500], [1.7500, 3.2500])
$\alpha_{\mathcal{C}}$	([4.6667, 8.0000], [5.6667, 8.1667])	([5.1875, 8.0625], [5.6667, 8.1667])	([4.4375, 6.9792], [4.9167, 7.9167])	$\cdots$	([1.8958, 4.5000], [1.9583, 5.0000])

Table 5: Normalized aggregated interval rough decision matrix

$\widehat{\mathcal{T}}$	ζ_1	ζ_2	ζ_3	$\cdots$	ζ_{14}
$\alpha_{\mathcal{A}}$	$([0.6957, 0.9412], [1.0625, 1.4375])$	$([0.6071, 1.1500], [0.8696, 1.6471])$	$([0.4641, 1.7931], [0.5577, 2.1547])$	$\cdots$	$([0.3765, 1.3170], [0.6212, 2.4376])$
$\alpha_{\mathcal{B}}$	$([0.2927, 0.3871], [0.3269, 0.6330])$	$([0.2500, 0.4395], [0.3352, 0.6364])$	$([0.3307, 0.8966], [0.3884, 1.1602])$	$\cdots$	$([0.391, 1.2857], [0.7593, 2.5574])$
$\alpha_{\mathcal{C}}$	$([0.1224, 0.1765], [0.1328, 0.3080])$	$([0.1301, 0.2537], [0.1550, 0.3373])$	$([0.2211, 0.6610], [0.2597, 0.8498])$	$\cdots$	$([0.2542, 1.1490], [0.3796, 1.7143])$

Step 5c. The importance of criteria within the above formed significance levels is compared in this step. The highest value of the rating scale for the criteria comparison is determined by employing Equation 3.7 as follows:

$$\mathfrak{z} = \max\{|\mathcal{L}_1|, |\mathcal{L}_2|, |\mathcal{L}_3|\} = \max\{8, 1, 5\} = 8.$$

Allotting the integer $\mathcal{I}_{j_\hbar} \in \{0, 1, \ldots, \mathfrak{z}\} = \{0, 1, \ldots, 8\}$ to the criterion $\zeta_{j_\hbar}$ belonging to the level $\mathcal{L}_\hbar = \{\zeta_{1_\hbar}, \zeta_{2_\hbar}, \ldots, \zeta_{p_\hbar}\}$ $(\hbar = 1, 2, 3)$, in such a way that most influential criterion ζ_j is assigned $\mathcal{I}_j = 0$ and $\mathcal{I}_{j_\hbar} < \mathcal{I}_{j'_\hbar}$ if criterion $\zeta_{j_\hbar}$ is more important than $\zeta_{j'_\hbar}$. $\mathcal{I}_{j_\hbar} = \mathcal{I}_{j'_\hbar}$ if both criteria $\zeta_{j_\hbar}$ and $\zeta_{j'_\hbar}$ are equally important. In level $\mathcal{L}_1$: $\mathcal{I}_{14} = 0$, $\mathcal{I}_1 = 1 = \mathcal{I}_6$, $\mathcal{I}_4 = 2$, $\mathcal{I}_5 = 3$, $\mathcal{I}_3 = 4$, $\mathcal{I}_2 = 5 = \mathcal{I}_{12}$. In level $\mathcal{L}_2$: $\mathcal{I}_{13} = 1$. In level $\mathcal{L}_3$: $\mathcal{I}_7 = 1$, $\mathcal{I}_8 = 2 = \mathcal{I}_{10}$, $\mathcal{I}_{11} = 3$, $\mathcal{I}_9 = 4$.

Step 5d. The elasticity coefficient $\mathfrak{z}_0$ is determined by $\mathfrak{z}_0 = \mathfrak{z} + 1 = 8 + 1 = 9$, satisfying the condition that $\mathfrak{z}_0 > \mathfrak{z}$.

Step 5e. The influence function value of each criterion is calculated by employing Equation 3.8. As an example, the calculation for the influence function value of the criterion ζ_7 belonging to the significance level $\mathcal{L}_3$ is presented in Equation 4.31. By following the same pattern influence function values for all of the criteria are determined and are organized in Table 6.

$$\phi(\zeta_7) = \frac{\mathfrak{z}_0}{\hbar \cdot \mathfrak{z}_0 + \mathcal{I}_{7_\hbar}} = \frac{9}{(3)(9) + 1} = 0.3214. \tag{4.31}$$

Table 6: Influence function value

$\phi(\zeta_1)$	$\phi(\zeta_2)$	$\phi(\zeta_3)$	$\phi(\zeta_4)$	$\phi(\zeta_5)$	$\phi(\zeta_6)$	$\phi(\zeta_7)$
0.9000	0.6429	0.6923	0.8182	0.7500	0.9000	0.3214
$\phi(\zeta_8)$	$\phi(\zeta_9)$	$\phi(\zeta_{10})$	$\phi(\zeta_{11})$	$\phi(\zeta_{12})$	$\phi(\zeta_{13})$	$\phi(\zeta_{14})$
0.3103	0.2903	0.3103	0.3000	0.6429	0.4737	1

Step 5f. The weight coefficient of the most influential criterion ζ_{14} is obtained by employing Equation 3.9 as follows:

$$\omega_{14} = \frac{1}{1 + \phi(\zeta_1) + \phi(\zeta_2) + \ldots + \phi(\zeta_{13})} = \frac{1}{0.9 + 0.6429 + \ldots + 0.4737} = 0.1539.$$

The weights of remaining criteria are calculated with the help of Equation 3.10 and are presented in Table 7.

Table 7: Criteria weight

ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
0.1385	0.0989	0.1065	0.1259	0.1154	0.1385	0.0495
ω_8	ω_9	ω_{10}	ω_{11}	ω_{12}	ω_{13}	ω_{14}
0.0478	0.0447	0.0478	0.0462	0.0989	0.0729	0.1539

Step 6. In order to examine the effect of different significance level of each criteria upon the performance assessment process of available alternatives, the criteria weights ω_j $(j = 1, 2, \ldots, 14)$ and assessment information of each alternative appeared in the normalized aggregated interval rough decision matrix $\widehat{\mathcal{T}}$ are combined with the help of Equation 3.15 and weighted normalized aggregated interval rough decision matrix $\mathcal{T}$ is obtained, as constructed in Table 8.

Table 8: Weighted normalized aggregated interval rough decision matrix

$\mathcal{T}$	ζ_1	ζ_2	ζ_3	$\cdots$	ζ_{14}
$\alpha_{\mathcal{A}}$	$([0.0964, 0.1304], [0.1472, 0.1991])$	$([0.06, 0.1137], [0.086, 0.1629])$	$([0.0494, 0.191], [0.0594, 0.2295])$	$\cdots$	$([0.0579, 0.2027], [0.0956, 0.3751])$
$\alpha_{\mathcal{B}}$	$([0.0405, 0.0536], [0.0453, 0.0877])$	$([0.0247, 0.0435], [0.0332, 0.0629])$	$([0.0352, 0.0955], [0.0414, 0.1236])$	$\cdots$	$([0.0602, 0.1979], [0.1169, 0.3936])$
$\alpha_{\mathcal{C}}$	$([0.017, 0.0244], [0.0184, 0.0427])$	$([0.0129, 0.0251], [0.0153, 0.0334])$	$([0.0235, 0.0704], [0.0277, 0.0905])$	$\cdots$	$([0.0391, 0.1768], [0.0584, 0.2638])$

Step 7. The main parameter for eliminating the degree of superiority between alternatives in relation to pseudo criterion is the threshold values. Table 9 contains the values for thresholds corresponding to each criterion that

Table 9: Criteria thresholds

Threshold values	Criteria													
	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6	ζ_7	ζ_8	ζ_9	ζ_{10}	ζ_{11}	ζ_{12}	ζ_{13}	ζ_{14}
$q(\zeta_j)$	0.04	0.02	0.02	0.01	0.003	0.001	0.007	0.008	0.004	0.01	0.003	0.007	0.02	0.01
$\rho(\zeta_j)$	0.07	0.04	0.03	0.03	0.005	0.002	0.01	0.01	0.005	0.02	0.005	0.01	0.03	0.02
$v(\zeta_j)$	0.1	0.06	0.05	0.04	0.01	0.003	0.03	0.02	0.006	0.03	0.007	0.02	0.04	0.04

the decision-making panel determined to be appropriate for evaluating the weak preference, strong preference and indifference relation.

Step 8. The intersection point of each entry present in the matrix $\mathcal{T}$ are calculated through Equation 2.5 and are arranged in Table 10. Table 11 comprises of performance comparison between each pair of alternatives in terms of difference between their corresponding performance rating presented in the form of intersection points. The partial concordance indices μ_j, highlighting the partial amount of dominance of one alternative over the other with respect to each criterion ζ_j $(j = 1, 2, \ldots, 14)$ for every pair of alternatives (α_x, α_y) are obtained by using Equation 3.21, and are tabulated in Table 11. The comprehensive concordance index μ indicating the overall dominance of one alternative over the other one in reference to the complete set of criteria is computed by employing Equation 3.22 and these indices are organized in comprehensive concordance matrix $\mathcal{C}$, given in Table 12.

Table 10: Intersection points of decision matrix $\mathcal{T}$

$\mathcal{T}$	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6	ζ_7	ζ_8	ζ_9	ζ_{10}	ζ_{11}	ζ_{12}	ζ_{13}	ζ_{14}
$\alpha_{\mathcal{A}}$	0.137	0.1023	0.1312	0.126	0.0988	0.1397	0.0578	0.05	0.0442	0.0709	0.0516	0.093	0.031	0.1662
$\alpha_{\mathcal{B}}$	0.0516	0.0395	0.0726	0.111	0.116	0.1366	0.033	0.0267	0.0522	0.0519	0.0438	0.0979	0.0479	0.171
$\alpha_{\mathcal{C}}$	0.023	0.0212	0.0521	0.0788	0.1145	0.1395	0.0192	0.0112	0.0491	0.0363	0.0449	0.1167	0.0732	0.1293

Table 11: Partial concordance indices

Partial concordance indices	Criteria													
	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6	ζ_7	ζ_8	ζ_9	ζ_{10}	ζ_{11}	ζ_{12}	ζ_{13}	ζ_{14}
$g_j(\alpha_{\mathcal{B}}) - g_j(\alpha_{\mathcal{A}})$	−0.0854	−0.0628	−0.0586	−0.015	0.0172	−0.0031	−0.0248	−0.0233	0.008	−0.019	−0.0078	0.0049	0.0169	0.0048
$\mu_j(\alpha_{\mathcal{A}}, \alpha_{\mathcal{B}})$	1	1	1	1	0	1	1	1	0	1	1	1	1	1
$g_j(\alpha_{\mathcal{C}}) - g_j(\alpha_{\mathcal{A}})$	−0.114	−0.0811	−0.0791	−0.0472	0.0157	−0.0002	−0.0386	−0.0388	0.0049	−0.0346	−0.0067	0.0237	0.0422	−0.0369
$\mu_j(\alpha_{\mathcal{A}}, \alpha_{\mathcal{C}})$	1	1	1	1	0	1	1	1	0.1	1	1	0	0	1
$g_j(\alpha_{\mathcal{A}}) - g_j(\alpha_{\mathcal{B}})$	0.0854	0.0628	0.0586	0.015	−0.0172	0.0031	0.0248	0.0233	−0.008	0.019	0.0078	−0.0049	−0.0169	−0.0048
$\mu_j(\alpha_{\mathcal{B}}, \alpha_{\mathcal{A}})$	0	0	0	0.75	1	0	0	0	1	0.1	0	1	1	1
$g_j(\alpha_{\mathcal{C}}) - g_j(\alpha_{\mathcal{B}})$	−0.0286	−0.0183	−0.0205	−0.0322	−0.0015	0.0029	−0.0138	−0.0155	−0.0031	−0.0156	0.0011	0.0188	0.0253	−0.0417
$\mu_j(\alpha_{\mathcal{B}}, \alpha_{\mathcal{C}})$	1	1	1	1	1	0	1	1	1	1	1	0	0.47	1
$g_j(\alpha_{\mathcal{A}}) - g_j(\alpha_{\mathcal{C}})$	0.114	0.0811	0.0791	0.0472	−0.0157	0.0002	0.0386	0.0388	−0.0049	0.0346	0.0067	−0.0237	−0.0422	0.0369
$\mu_j(\alpha_{\mathcal{C}}, \alpha_{\mathcal{A}})$	0	0	0	0	1	1	0	0	1	0	0	1	1	0
$g_j(\alpha_{\mathcal{B}}) - g_j(\alpha_{\mathcal{C}})$	0.0286	0.0183	0.0205	0.0322	0.0015	−0.0029	0.0138	0.0155	0.0031	0.0156	−0.0011	−0.0188	−0.0253	0.0417
$\mu_j(\alpha_{\mathcal{C}}, \alpha_{\mathcal{B}})$	1	1	0.95	0	1	1	0	0	1	0.44	1	1	1	0

Table 12: Comprehensive concordance matrix

$\mathcal{C}$	$\alpha_{\mathcal{A}}$	$\alpha_{\mathcal{B}}$	$\alpha_{\mathcal{C}}$
$\alpha_{\mathcal{A}}$	–	1.1253	0.958
$\alpha_{\mathcal{B}}$	0.585	–	1.0094
$\alpha_{\mathcal{C}}$	0.4704	0.8762	–

Step 9. The discordance indices λ_j indicating the degree of inferiority of one action over the other with respect to each criterion ζ_j, are computed through Equation 3.24 and are arranged in Table 13.

Step 10. The credibility degree π for each pair of sustainable autonomous mining system $\alpha_{\mathcal{A}}$, $\alpha_{\mathcal{B}}$, and $\alpha_{\mathcal{C}}$, is calculated with the help of Equation 3.26 and is presented in Table 14.

Step 11. The concordance credibility index $\gamma_{\mathcal{C}}$, discordance credibility index $\gamma_{\mathcal{D}}$ and net credibility index γ of each alternative are determined by using Equations 3.28, 3.29, and 3.30, respectively, and are arranged in Table 15. The available sustainable autonomous mining systems are ranked in descending order of net credibility index, and shows that fully autonomous system $\alpha_{\mathcal{A}}$ is most suitable sustainable mining option. Figure 4 graphically illustrates the credibility analysis results, highlighting the key findings of the ranking procedure for the three sustainable mining systems under consideration.

Table 13: Discordance indices

$\lambda_j(\alpha_x, \alpha_y)$	Criteria													
	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6	ζ_7	ζ_8	ζ_9	ζ_{10}	ζ_{11}	ζ_{12}	ζ_{13}	ζ_{14}
$\lambda_j(\alpha_{\mathcal{A}}, \alpha_{\mathcal{B}})$	0	0	0	0	1	0	0	0	1	0	0	0	0	0
$\lambda_j(\alpha_{\mathcal{A}}, \alpha_{\mathcal{C}})$	0	0	0	0	1	0	0	0	0	0	0	1	1	0
$\lambda_j(\alpha_{\mathcal{B}}, \alpha_{\mathcal{A}})$	0.5133	1	1	0	0	1	0.74	1	0	0	1	0	0	0
$\lambda_j(\alpha_{\mathcal{B}}, \alpha_{\mathcal{C}})$	0	0	0	0	0	0.9	0	0	0	0	0	0.88	0	0
$\lambda_j(\alpha_{\mathcal{C}}, \alpha_{\mathcal{A}})$	1	1	1	1	0	0	1	1	0	1	0.85	0	0	0.845
$\lambda_j(\alpha_{\mathcal{C}}, \alpha_{\mathcal{B}})$	0	0	0	0.22	0	0	0.19	0.55	0	0	0	0	0	1

Table 14: Credibility indices

$\pi(\alpha_x, \alpha_y)$	$\alpha_{\mathcal{A}}$	$\alpha_{\mathcal{B}}$	$\alpha_{\mathcal{C}}$
$\alpha_{\mathcal{A}}$	–	1.1253	0
$\alpha_{\mathcal{B}}$	0	–	1.0094
$\alpha_{\mathcal{C}}$	0	0	–

Table 15: Ranking results

Alternatives	Concordance credibility	Discordance credibility	Net credibility	Ranking
	$\gamma_{\mathcal{C}}(\alpha_x)$	$\gamma_{\mathcal{D}}(\alpha_x)$	$\gamma(\alpha_x)$	
$\alpha_{\mathcal{A}}$	1.1253	0	1.1253	1
$\alpha_{\mathcal{B}}$	1.0094	1.1253	−0.1159	2
$\alpha_{\mathcal{C}}$	0	1.0094	−1.0094	3

5 Comparative analysis

This section presents a comparative analysis of the proposed interval rough-based methodology alongside other comparable methodologies found in the literature. On the basis of the idea used to address uncertainty, multi-criteria approaches were chosen for comparison. The following models were chosen for comparison based on predetermined criteria: interval rough HPA (Heronian and Power Averaging) [15], interval rough MAIRCA (Multi Attributive Ideal-Real Comparative Analysis) [17], and rough CoCoSo (Combined Compromise Solution) [29]. The MCDM problem concerning the evaluation of a sustainable mining system was addressed using the selected techniques, all under identical input data and conditions. The ranking results obtained through the applied decision-making methods are summarized in Table 16 and visually depicted in Figure 5.

5.1 Discussion

The key insights from the comparative analysis, which takes into account both computational processes and uncertainty handling models, are outlined in the following points:

- The interval rough-based proposed methodology, the interval rough MAIRCA method [17] and the interval rough HPA method [15] aim to handle uncertainty and imprecision in decision-making processes through IRNs, but they differ significantly in their approach and applications. The interval rough MAIRCA utilizes a compensatory model focused on calculating performance scores, offering a simpler approach to evaluating alternatives. In contrast, presented methodology employs the outstanding outranking mechanism of ELECTRE model that considers preference intensity, as well as indifference and preference thresholds, enabling a more refined comparison of alternatives. It further calculates two types of credibility indices referring to the amount of outperforming (concordance index) along whit its opposite aspect (that is, amount of outperformed (discordance index)) between each pair of alternatives and deduce a complete preference ordering through a detailed analysis of input data. The interval rough HPA method focuses on aggregating information using the Heronian mean and power averaging techniques, which are effective in capturing the overall performance of alternatives but may lack the detailed analysis required to distinguish between closely competing options. While the HPA method provides a simplified approach to aggregation, it may overlook subtle differences between alternatives when faced with highly conflicting criteria. The superiority of presented approach over interval rough MAIRCA and interval rough HPA is particularly evident when managing conflicting criteria and uncertainty. By building pairwise outranking relations, ELECTRE-III

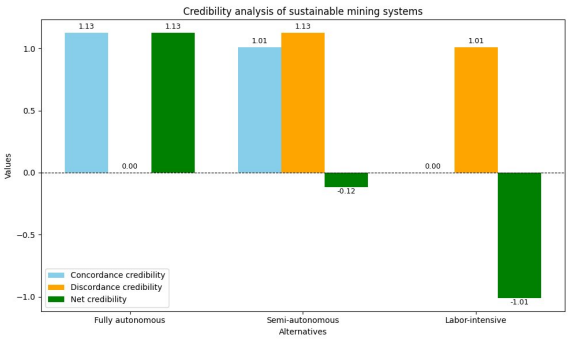

Figure 4: Graphical representation of ranking results

provides a more comprehensive and realistic reflection of real-world decision problems. This makes it more adept at handling imprecise data and delivering reliable results, particularly in contexts where ranking stability and sensitivity to criteria weights are critical.

- The interval rough ELECTRE-III method and the rough CoCoSo method [29] differ significantly in their approaches to handling MCDM under uncertainty. Firstly, while the rough model focuses on capturing the uncertainty and imprecision in decision-making by approximating vague concepts through lower and upper approximations, the interval rough model extends this by incorporating interval values to represent a range of possible assessments, thereby offering a more nuanced representation of uncertainty. This improves the applicability of the developed approach in decision-making scenarios involving interval-based data instead of precise values, thereby expanding its relevance to real-world problems. Moreover, the rough CoCoSo method follows a combined compromise solution approach, where the final rankings are derived through a weighted aggregation of criteria scores. While CoCoSo is efficient for straightforward decision scenarios, it may lack the ability to effectively deal with more complex situations involving significant uncertainty or conflicting preferences. In contrast, the presented MCGDM scheme is capable of performing a thorough inspection of the initial assessment information and capturing every minor detail corresponding to each conflicting criterion comprehensively.

Table 16: Comparison of decision results

Alternatives	Proposed method	Interval rough HPA [15]	Interval rough MAIRCA [17]	Rough CoCoSo [29]
	Ranking	Ranking	Ranking	Ranking
$\alpha_{\mathcal{A}}$	1	1	1	1
$\alpha_{\mathcal{B}}$	2	2	2	3
$\alpha_{\mathcal{C}}$	3	3	3	2

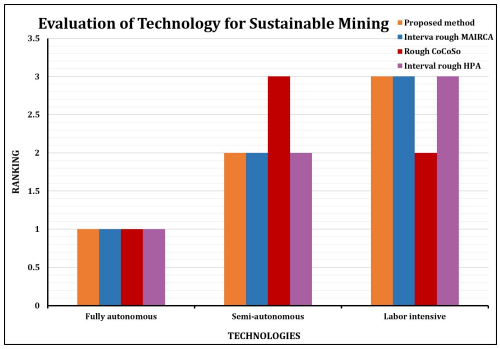

Figure 5: Comparative analysis results

6 Sensitivity analysis

Sensitivity analysis is a vital method used in decision-making, engineering, and finance to examine how changes in input parameters affect the outcomes of models. By adjusting these parameters, analysts assess the models sensitivity to different scenarios. This helps in understanding how outcomes vary when a single variable changes within a specified range. Additionally, research shows that small changes in the weighting of criteria can significantly alter the ranking of alternatives, highlighting the importance of these weights in decision-making processes. To simplify the calculations, we use an approach that treats the weight of the first criterion (ζ_1) as the baseline and examines how decision results vary by swapping ω_1 with the weights of other criteria progressively. In the first scenario (S_1), the original criteria weight vector $\mathcal{W}_1 = \{\omega_1, \omega_2, \ldots, \omega_{14}\}$ and its corresponding decision results serve as the reference. In second scenario (S_2), the weights of criteria ζ_1 and ζ_2 are swapped, and the calculations are carried out with the new weight vector $\mathcal{W}_2 = \{\omega_2, \omega_1, \ldots, \omega_{14}\}$. This process continues until 14th scenario (S_{14}), where the weights of criteria ζ_1 and ζ_{14} are swapped, resulting in the weight vector $\mathcal{W}_{14} = \{\omega_{14}, \omega_2, \ldots, \omega_1\}$. The comprehensive findings of our sensitivity analysis, summarized in Table 17, provide deeper insights into the analysis. Figures 6 and 7 visually illustrate the sensitivity of the net credibility index and the rankings of the three mining systems across 14 different scenarios. Across all phases, the autonomous mining system emerges as the optimal solution.

Table 17: Sensitivity analysis

Alternatives	S_1		S_2		S_3		S_4		S_5	
	Credibility index	Rank	Credibility index	Rank	Credibility index	Rank	Credibility index	Rank	Credibility index	Rank
Fully-autonomous ($\alpha_{\mathcal{A}}$)	1.1253	1	1.1253	1	1.1253	1	1.1253	1	1.1022	1
Semi-autonomous($\alpha_{\mathcal{B}}$)	−0.1159	2	−0.1159	2	−0.1159	2	−0.1159	2	−0.0928	2
Labor-intensive($\alpha_{\mathcal{C}}$)	−1.0094	3	−1.0094	3	−1.0094	3	−1.0094	3	−1.0094	3
Alternatives	S_6		S_7		S_8		S_9		S_{10}	
	Credibility index	Rank	Credibility index	Rank	Credibility index	Rank	Credibility index	Rank	Credibility index	Rank
Fully-autonomous ($\alpha_{\mathcal{A}}$)	1.1253	1	1.1253	1	1.1253	1	1.0315	1	1.1253	1
Semi-autonomous($\alpha_{\mathcal{B}}$)	−0.1159	2	−0.1159	2	−0.1159	2	−0.0221	2	−0.1159	2
Labor-intensive($\alpha_{\mathcal{C}}$)	−1.0094	3	−1.0094	3	−1.0094	3	−1.0094	3	−1.0094	3
Alternatives	S_{11}		S_{12}		S_{13}		S_{14}			
	Credibility index	Rank	Credibility index	Rank	Credibility index	Rank	Credibility index	Rank		
Fully-autonomous ($\alpha_{\mathcal{A}}$)	1.1253	1	1.1253	1	1.1253	1	1.091	1		
Semi-autonomous($\alpha_{\mathcal{B}}$)	−0.1298	2	−1.1253	3	0.0216	2	−0.017	2		
Labor-intensive($\alpha_{\mathcal{C}}$)	−0.9955	3	0	2	−1.1469	3	−1.074	3		

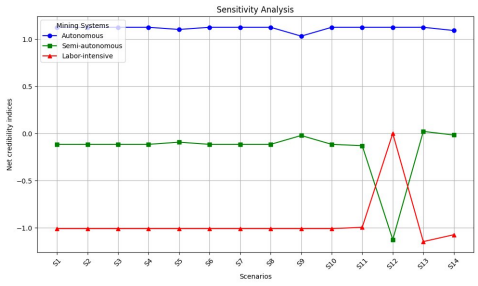

Figure 6: Sensitivity analysis: Credibility index of mining systems across scenarios

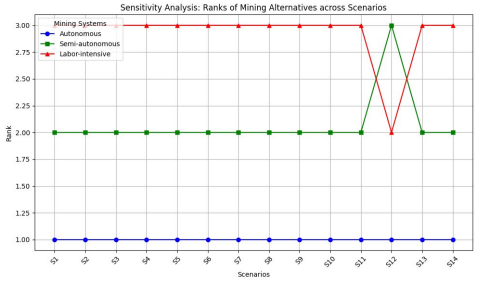

Figure 7: Sensitivity analysis of ranks

7 Limitations and future directions

Despite the numerous advantages of the proposed IRN-based ELECTRE III approach, it has certain limitations. Its reliance on predefined criteria weights restricts its applicability in scenarios where incomplete information about the significance levels of the criteria exists in the underlying MCDM problem. Additionally, the need for expert-defined threshold values introduces a degree of subjectivity into the decision-making process. To address these limitations, future research will focus on the following areas:

- The presented approach is not limited to the mining industry; we plan to expand its application to other complex decision-making scenarios across various fields, such as medicine, business, and construction management, in order to broaden its scope.
- Another future objective is to expand the family of interval rough-based outranking methods by developing hybrid approaches such as ELECTRE IV, ELECTRE TRI, PROMETHEE I, and PROMETHEE II.
- Additionally, we aim to explore the integration of interval-valued Pythagorean fuzzy rough numbers with other MCDM techniques for further enhancement.

8 Conclusions

This study introduces an innovative decision-making framework that integrates IRNs to address uncertainty in human opinions, effectively solving MCDM challenges through the ELECTRE-III outranking approach. By employing the LBWA technique, we systematically evaluate criteria weights, enhancing the robustness of the decision-making process. The mining industry, essential for providing raw materials to various sectors, particularly benefits from this framework in assessing sustainable mining technologies. The proposed technique effectively evaluates three sustainable mining technologies−autonomous, semi-autonomous, and labor-intensive mining systems−and identifies fully autonomous systems as the optimal solution for mining. Our case study demonstrates the effectiveness of the interval rough-based outranking MCGDM approach, showing improved precision and consistency in evaluation outcomes. This method serves as a reliable decision support tool not only for sustainable mining but also for other fields facing similar uncertainties. Overall, it marks a significant advancement in MCDM, equipping decision-makers with a powerful resource. Future research should explore further applications and refinements, particularly by integrating interval rough techniques with conventional MCDM models to better manage subjectivity and uncertainty. We aim to enrich the literature on interval rough-based decision-making by introducing IRN-PROMETHEE methods.
Conflict of Interest : The authors declare no conflict of interest.

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