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Evaluation of sustainable third-party reverse logistics providers using a Fermatean fuzzy rough number-based decision-making framework

M. Akram [1] and S. Jamil [2]

[1,2] *Institute of Mathematics, University of the Punjab, New Campus, Lahore 4590, Pakistan*

m.akram@pucit.edu.pk, shajeea.jamil28@gmail.com

Abstract

Evaluating third-party reverse logistics providers (3PRLPs) is a complex task, often challenged by cognitive biases and incomplete, uncertain data in group decision-making contexts. To overcome these challenges, this study proposes a novel decision-support framework that integrates Fermatean fuzzy rough numbers with an extended entropy weight method and a new ranking technique. Fermatean fuzzy rough numbers effectively capture uncertainty and subjectivity without relying on predefined parameters, addressing key limitations in provider evaluation. The extended entropy weight method objectively determines the importance of decision criteria across economic, environmental, social, and risk dimensions. The framework enhances decision accuracy by integrating three ranking techniques into an aggregated index, minimizing information loss while accommodating expert preferences. A case study on sustainable 3PRLP evaluation in the Malaysian food industry demonstrates the model's practicality, while sensitivity and comparative analyses validate its robustness and superiority over existing approaches.

Keywords: Third-party reverse logistics providers, extended entropy weight method, ranking technique, Fermatean fuzzy rough numbers.

1 Introduction

Global economic integration intensifies competition for access to markets and investment opportunities. An organization's main objective is to harness its resources and capabilities to build and sustain strong market leadership [8, 26]. Logistics is an essential factor in enhancing profitability for companies, playing a pivotal role in achieving business success [20]. Embracing sustainable investment strategies can empower businesses to enhance their competitive position, minimize costs, improve productivity, and optimize supply chain management. A robust supply chain framework, based on circular economy principles, environmentally responsible practices, and eco-friendly solutions, along with effective reverse logistics, can provide significant advantages to a company [9]. Reverse logistics has become a critical sustainability strategy that enhances organizational competitiveness while mitigating environmental impacts [29]. It involves the systematic management of product, material, and information flows from the end user back to the origin for returns, repairs, recycling, or appropriate disposal [15, 29]. As a multidimensional concept, reverse logistics contributes directly to the economic, environmental, and social pillars of sustainability [18]. It promotes cost reduction and resource efficiency, supports environmental conservation through remanufacturing and recycling, and fosters social responsibility by raising public awareness about environmental preservation [9, 10, 29]. In today's global economy, reverse logistics is indispensable for sustaining competitive advantage and aligning operations with circular economy principles [19].
Growing environmental concerns and rising sustainability expectations have prompted companies to adopt reverse logistics as a strategic means of developing greener and more efficient supply chains [26]. Despite its advantages, reverse logistics implementation presents significant challenges, including infrastructure requirements, technological complexity, and increased operational risks that may undermine economic stability [10, 16, 18, 20, 26]. To mitigate these barri-

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ers, organizations increasingly outsource reverse logistics operations to 3PRLPs, aiming to enhance cost-effectiveness and advance long-term sustainability objectives [5, 14, 22, 26]. The selection of an appropriate 3PRLP is therefore critical, requiring alignment with organizational goals and demonstrated capabilities in areas such as data integration, transportation, material handling, and warehousing [11]. Environmental performance has become a central selection criterion, underscoring the growing emphasis on sustainability in logistics decision-making [8, 28]. In this context, multi-criteria decision-making (MCDM) methods provide a structured and effective approach for analyzing trade-offs and identifying the most suitable 3PRLPs [16, 26, 29].

Outsourcing, defined as the delegation of internal business functions to external entities, is a strategic practice that requires the careful selection of competent service providers to support organizational objectives [13, 14]. A successful evaluation process relies on a robust decision-making framework and well-defined, context-specific criteria [28]. Since many of these criteria are expert-driven and inherently vague, flexible methods are essential to convert qualitative judgments into measurable data [16]. Moreover, appropriate weighting techniques are necessary to reflect the varying significance of each criterion. Ultimately, the selected criteria should align with the organization's operational capabilities and strategic context to ensure an effective outsourcing decision [11, 29]. Extensive research has advanced MCDM methods for evaluating sustainable 3PRLPs, aiming to ensure transparent and objective assessments through systematic criteria weighting. The MULTIMOORA method [6] supports robust rankings by integrating multiple evaluation strategies, while Shannon's entropy [24] objectively determines criteria weights based on information dispersion. To address uncertainty and complexity, various hybrid approaches have been proposed. Prakash and Barua [18] applied a fuzzy AHP-VIKOR model to assess provider capabilities, and Li et al. [14] incorporated cumulative prospect theory to reflect decision-makers' behavioral biases. Zarbakhshnia et al. [28] used fuzzy SWARA-COPRAS for risk-based prioritization, while Govindan et al. [10] evaluated logistics providers using fuzzy AHP-TOPSIS. Bai and Sarkis [5] integrated rough set theory with VIKOR-TOPSIS for sustainability-driven decisions. Liu et al. [15] combined BWM, interval Pythagorean hesitant fuzzy sets, and self-organizing maps for mobile recycling decisions. Pamucar et al. [17] proposed a BWM-WASPAS-MABAC model using interval rough numbers. Zarbakhshnia et al. [29] developed a method for analyzing the impact of selection criteria. Chen et al. [8] used hesitant fuzzy linguistic term sets for multi-perspective evaluation. Eydi and Rastgar [9] employed fuzzy DEA in hospital waste logistics, and Yang et al. [26] applied MULTIMOORA with linguistic Pythagorean hesitant fuzzy sets for evaluating providers in EV battery recycling. As sustainability and cost-efficiency become central priorities in modern supply chains, the effective selection of 3PRLPs has emerged as a critical decision-making challenge. Many existing methods remain limited by their emphasis on economic and operational factors, often neglecting sustainability and struggling to address uncertainty in expert judgments. To bridge these gaps, previous studies have introduced advanced tools such as fuzzy sets [27], Fermatean fuzzy sets [7, 21], rough numbers [30], and fuzzy rough numbers [31] to improve the reliability of subjective evaluations. Building on these foundations, this study develops an integrated framework that combines Fermatean fuzzy rough numbers, an extended entropy-based weighting method, and the MULTIMOORA technique. Unlike traditional approaches (e.g., AHP, ANP, SAW), the entropy method ensures more objective weight determination, while the proposed model surpasses ranking methods like TOPSIS and WASPAS by incorporating multiple evaluation strategies. This enhances decision accuracy, minimizes sensitivity to weight variations, and supports resilient, sustainability-focused supply chain decisions. Table 1 presents an overview of integrating modeling techniques used to evaluate the 3PRLPs across different industries.

1.1 Motivations

The main motivations of the proposed research are presented in the following points:

1. In real-world scenarios, expert linguistic information is often vague and imprecise. Fermatean fuzzy rough numbers address these uncertainties with precision by aggregating subjective evaluations through upper and lower limits. This approach minimizes biases, reducing reliance on pre-existing data and assumptions in complex environments.
2. The Fermatean fuzzy rough number-based decision-making approach enhances sustainable supply chain management by addressing uncertainty and subjectivity in professional evaluations of the industry agents. It integrates diverse perspectives and key factors, providing a robust and flexible strategy for managing uncertainties in 3PRLP evaluation problems and effectively handling imprecision through the Fermatean fuzzy rough framework.
3. The proposed hybrid model integrates the extended entropy weight method with the MULTIMOORA technique, objectively assigning weights based on information content. This reduces subjectivity, making it effective for determining the weights of sustainable and risk criteria, such as cost efficiency and sustainability, while incorporating both qualitative and quantitative factors to prioritize logistics providers.
4. Third-party providers manage reverse logistics functions such as returns and recycling. With increasing demand for sustainable supply chains, selecting the right 3PRLP is essential. The proposed method determines criteria and sub-criteria weights to identify the optimal provider, validated through a case study in the food industry.

Table 1: Recent literature on integrated modeling techniques in 3PRLPs

Author(s)	Year	Method(s)	Type of data	Application region	Description
Senthil et al. [22]	2014	Fuzzy AHP-TOPSIS	Fuzzy numbers	Plastics recycling plant	Third-party reverse logistics contractor evaluation and selection
Prakash and Barua [18]	2016	Fuzzy AHP-TOPSIS	Fuzzy numbers	Electronics industry	Evaluation and selection of sustainable 3PRL partner through a robust hybrid model
Prakash and Barua [19]	2016	Fuzzy AHP-VIKOR	Fuzzy numbers	Electronics industry	Evaluation and selection of sustainable 3PRL partner through a robust hybrid model
Govindan et al. [10]	2017	fuzzy AHP-TOPSIS, Goal programming approach	Fuzzy numbers	Household electronic manufacturer	Evaluation of sustainable forward and reverse logistics partners
Bai and Sarkis [5]	2019	Neighborhood rough set based TOPSIS and VIKOR	Crisp and fuzzy data	Machine engine industry	Development of a conceptual framework integrating business, ecological and societal dimensions to enhance reverse logistics management
Pamucar et al. [17]	2019	Interval rough number-based BWM-WASPAS-MABAC	Interval numbers	Electronic firm	Selection of third-party logistics service partner to enhance the efficiency of supply chain management
Zarbakhshnia et al. [29]	2020	Fuzzy AHP, Gray MOORA	Fuzzy numbers	Car manufacturing company	Evaluation and selection of 3PRLPs for outsourcing sustainable reverse logistics
Jayant et al. [13]	2021	SAWARA based MOORA and WASPAS	Fuzzy numbers	Automotive parts manufacturing company	Robust hybrid model for evaluation and selection of sustainable 3PRLPs.
Yang et al. [26]	2022	Linguistic Pythagorean hesitant fuzzy MULTIMOORA	Linguistics hesitant Pythagorean Fuzzy set	Electric vehicle power battery recycling	Selection of optimal sustainable third-party reverse logistics partner
Eydi and Rastgar [9]	2022	Dual role factors and Fuzzy states based on DEA models	Fuzzy numbers	Hospital	Evaluating the efficiency of a 3PRLP in managing the collection and disposal of medical waste
Proposed approach	2024	Fermatean fuzzy rough number-based entropy-MULTIMOORA	Triangular Fermatean fuzzy numbers	Food industry	Evaluation of 3PRLPs to improve the food supply chain management in Malaysia

1.2 Innovative contributions

The primary innovative contributions of this research are presented as follows:

1. This study contributes to the Fermatean fuzzy rough approach by introducing the entropy-MULTIMOORA methodology to manage uncertain data. Designed to address vagueness and reduce subjective judgment distortions in logistics evaluations, the method employs triangular Fermatean fuzzy numbers to model qualitative assessments. The approach effectively aggregates individual assessments into group data, improving both precision and flexibility in expert perspectives.

2. This research combines sustainability dimensions and risk factors to evaluate reverse logistics suppliers, identifying key criteria for evaluating 3PRLPs. The extended entropy weight method determines the relative weights of these criteria and evaluates their impact on achieving sustainability goals.

3. The Fermatean fuzzy rough-MULTIMOORA method is applied to enhance decision-making by integrating three ranking techniques into a single aggregated index. This index incorporates sustainability criteria and risk factors, enabling a flexible expression of expert preferences. Using the Improved Borda rule, the final aggregated index determines the evaluation rank order of 3PRLPs, minimizing distortion and information loss.

4. We apply our presented methodology to real-world data from Malaysia's food industry to evaluate the sustainable 3PRLPs. The developed technique is validated through a case study designed to determine the optimal choice of 3PRLPs in the food supply chain management.

5. The outperformance of the developed approach is illustrated through logical analysis of robust numerical validation supported by comparative and sensitivity analysis.

1.3 Structure of the paper

The remaining paper is structured as follows: Section 2 presents a detailed explanation of the core concepts and terminologies. Section 3 formulates the presented group decision-making approach for evaluating 3PRLPs. Section 4 provides a practical case study concerning the 3PRLP evaluation by a food products importer and trader. A detailed discussion of the results, along with comparative analysis with established benchmarks, is presented in Section 5. The concluding section summarizes the study and provides potential avenues for future improvements.

2 Preliminaries

Definition 2.1. *Let $\mathcal{W}$ be a universal set of objects, and let ζ be a Fermatean fuzzy equivalence relation defined on $\mathcal{W}$. Consider a Fermatean fuzzy set (FFS) $C \in FF(\mathcal{W})$ on the universal set $\mathcal{W}$, where FF($\mathcal{W}$) represents a Fermatean fuzzy power set of $\mathcal{W}$. Then the space $(\mathcal{W}, \zeta)$ forms a Fermatean fuzzy approximation space. For each $C \in FF(\mathcal{W})$, the upper and lower approximations are FFSs on $\mathcal{W}$, defined respectively as follows:*

$$\begin{aligned} \overline{\zeta}(C) &= \{\langle c, \mu_{\overline{\zeta}(C)}(c), \nu_{\overline{\zeta}(C)}(c)\rangle \mid c \in \mathcal{W}\}, \\ \underline{\zeta}(C) &= \{\langle c, \mu_{\underline{\zeta}(C)}(c), \nu_{\underline{\zeta}(C)}(c)\rangle \mid c \in \mathcal{W}\}, \end{aligned} \tag{1}$$

where

$$\begin{aligned} \mu_{\overline{\zeta}(C)}(c) &= \sup\{\min\{\mu_\zeta(c,d), \mu_c(d)\} \mid d \in \mathcal{W}\}, \\ \nu_{\overline{\zeta}(C)}(c) &= \inf\{\max\{\nu_\zeta(c,d), \nu_c(d)\} \mid d \in \mathcal{W}\}, \end{aligned} \tag{2}$$

and

$$\begin{aligned} \mu_{\underline{\zeta}(C)}(c) &= \inf\{\max\{\nu_\zeta(c,d), \mu_c(d)\} \mid d \in \mathcal{W}\}, \\ \nu_{\underline{\zeta}(C)}(c) &= \sup\{\min\{\mu_\zeta(c,d), \nu_c(d)\} \mid d \in \mathcal{W}\}, \end{aligned} \tag{3}$$

are the membership and non-membership degrees for the upper approximation ($\underline{\zeta}(C)$) *and lower approximation* ($\overline{\zeta}(C)$), *associated with an element* $c \in \mathcal{W}$, *respectively, such that*

$$\begin{aligned} 0 \leq (\mu_{\overline{\zeta}(C)}(c))^3 + (\nu_{\overline{\zeta}(C)}(c))^3 \leq 1, \\ 0 \leq (\mu_{\underline{\zeta}(C)}(c))^3 + (\nu_{\underline{\zeta}(C)}(c))^3 \leq 1. \end{aligned} \tag{4}$$

The pair $\zeta(C) = (\underline{\zeta}(C), \overline{\zeta}(C)) = \{\langle c, (\mu_{\underline{\zeta}(C)}(c), \nu_{\underline{\zeta}(C)}(c)), (\mu_{\overline{\zeta}(C)}(c), \nu_{\overline{\zeta}(C)}(c))\rangle \mid c \in C\}$ *is known to be a Fermatean fuzzy rough set of* C, *provided that* $\underline{\zeta}(C) \neq \overline{\zeta}(C)$.

Definition 2.2. *A triangular Fermatean fuzzy number (TFFN)* $\widetilde{T} = [(\underline{t}, t, \overline{t}); \rho_{\widetilde{T}}, \eta_{\widetilde{T}}]$ *is a FFS* $T = \{\langle r, P_{\widetilde{T}}(r), N_{\widetilde{T}}(r)\rangle : r \in \mathbb{R}\}$ *over* $\mathbb{R}$, *where positive* ($P_{\widetilde{T}}$) *and negative* ($N_{\widetilde{T}}$) *membership functions are, respectively, defined as:*

$$P_{\widetilde{T}}(r) = \begin{cases} \frac{(r-\underline{t})\rho_{\widetilde{T}}}{t-\underline{t}}, & \underline{t} \leq r < t; \\ \rho_{\widetilde{T}}, & r = t\ ; \\ \frac{(\overline{t}-r)\rho_{\widetilde{T}}}{\overline{t}-t}, & t < r \leq \overline{t}; \\ 0, & otherwise\ , \end{cases} \tag{5}$$

$$N_{\widetilde{T}}(r) = \begin{cases} \frac{t-r+(r-\underline{t})\eta_{\widetilde{T}}}{t-\underline{t}}, & \underline{t} \leq r < t; \\ \eta_{\widetilde{T}}, & r = t\ ; \\ \frac{r-t+(\overline{t}-r)\eta_{\widetilde{T}}}{\overline{t}-t}, & t < r \leq \overline{t}; \\ 1, & otherwise\ , \end{cases} \tag{6}$$

The parameters $\rho_{\widetilde{T}}$ *and* $\eta_{\widetilde{T}}$ *represent the expert's positive and negative assessments, setting the thresholds for* $P_{\widetilde{T}}$ *and* $N_{\widetilde{T}}$, *respectively. These parameters,* $\rho_{\widetilde{T}}, \eta_{\widetilde{T}} \in [0, 1]$, *must satisfy the following condition:*

$$0 \leq \rho_{\widetilde{T}}^3 + \eta_{\widetilde{T}}^3 \leq 1.$$

Definition 2.3. *Assume that the universal set* $\mathcal{W}$ *comprises triangular Fermatean fuzzy numbers obtained from experts' analyses of 3PRLPs related to reverse logistics outsourcing activities. Consider a cluster* $\mathbb{K} = \{\widetilde{T}_1, \widetilde{T}_2, \ldots, \widetilde{T}_s\}$ *consisting of* t *distinct classes derived from the professional assessments and organized in strictly ascending order as:* $\widetilde{T}_1 < \widetilde{T}_2 < \ldots < \widetilde{T}_s$, *where* $\widetilde{T}_\alpha = [({t_\alpha}^l, {t_\alpha}^m, {t_\alpha}^u); \rho_{\widetilde{T}_\alpha}, \eta_{\widetilde{T}_\alpha}]$ *represents a Fermatean fuzzy evaluation for* $1 \leq \alpha \leq s$. *Consider the sets* $\mathbb{K}^l = \{t_1^l, \ldots, t_s^l\}$, $\mathbb{K}^m = \{t_1^m, \ldots, t_s^m\}$, *and* $\mathbb{K}^u = \{t_1^u, \ldots, t_s^u\}$ *as the ordered lower, middle, and upper bounds of TFFNs, where* $t_1^l \leq \ldots \leq t_s^l$, $t_1^m \leq \ldots \leq t_s^m$, *and* $t_1^u \leq \ldots \leq t_s^u$. *Moreover, define* $\mathbb{K}^\rho = \{\rho_{\widetilde{T}_1}, \ldots, \rho_{\widetilde{T}_s}\}$ *and* $\mathbb{K}^\eta = \{\eta_{\widetilde{T}_1}, \ldots, \eta_{\widetilde{T}_s}\}$ *as the collections representing the highest positive membership and lowest negative membership values, respectively, for each class* $\widetilde{T}_\alpha$, $1 \leq \alpha \leq s$. *Then* $\forall\, \widetilde{T}_\alpha \in \mathbb{K}$, *the lower approximation* $\underline{Apr}(\widetilde{T}_\alpha)$ *of each Fermatean fuzzy assessment is determined by the lower approximation of its individual components, defined as:*

$$\underline{Apr}({t_\alpha}^l) = \cup\{{t_\beta}^l \in \mathbb{K}^l \mid {t_\beta}^l \leq {t_\alpha}^l\}, \tag{7}$$
$$\underline{Apr}({t_\alpha}^m) = \cup\{{t_\beta}^m \in \mathbb{K}^m \mid {t_\beta}^m \leq {t_\alpha}^m\}, \tag{8}$$
$$\underline{Apr}({t_\alpha}^u) = \cup\{{t_\beta}^u \in \mathbb{K}^u \mid {t_\beta}^u \leq {t_\alpha}^u\}, \tag{9}$$
$$\underline{Apr}(\rho_{\widetilde{T}_\alpha}) = \cup\{\rho_{\widetilde{T}_\beta} \in \mathbb{K}^\rho \mid \rho_{\widetilde{T}_\beta} \leq \rho_{\widetilde{T}_\alpha}\}. \tag{10}$$

Given the requirement that ${\eta_{\widetilde{T}_\alpha}}^L \geq {\eta_{\widetilde{T}_\alpha}}^U$, *the lower approximation of* $\eta_{\widetilde{T}_\alpha}$ *is determined as:*

$$\underline{Apr}(\eta_{\widetilde{T}_\alpha}) = \cup\{\eta_{\widetilde{T}_\beta} \in \mathbb{K}^\eta \mid \eta_{\widetilde{T}_\beta} \geq \eta_{\widetilde{T}_\alpha}\}. \tag{11}$$

Correspondingly, the upper approximation of $\widetilde{T}_\alpha$, $1 \leq \alpha \leq s$, *i.e.,* ($\overline{Apr}(\widetilde{T}_\alpha)$), *can be defined as:*

$$\overline{Apr}({t_\alpha}^l) = \cup\{{t_\beta}^l \in \mathbb{K}^l \mid {t_\beta}^l \geq {t_\alpha}^l\}, \tag{12}$$
$$\overline{Apr}({t_\alpha}^m) = \cup\{{t_\beta}^m \in \mathbb{K}^m \mid {t_\beta}^m \geq {t_\alpha}^m\}, \tag{13}$$
$$\overline{Apr}({t_\alpha}^u) = \cup\{{t_\beta}^u \in \mathbb{K}^u \mid {t_\beta}^u \geq {t_\alpha}^u\}, \tag{14}$$
$$\overline{Apr}(\rho_{\widetilde{T}_\alpha}) = \cup\{\rho_{\widetilde{T}_\beta} \in \mathbb{K}^\rho \mid \rho_{\widetilde{T}_\beta} \geq \rho_{\widetilde{T}_\alpha}\}, \tag{15}$$
$$\overline{Apr}(\eta_{\widetilde{T}_\alpha}) = \cup\{\eta_{\widetilde{T}_\beta} \in \mathbb{K}^\eta \mid \eta_{\widetilde{T}_\beta} \leq \eta_{\widetilde{T}_\alpha}\}, \tag{16}$$

where $\overline{Apr}(\eta_{\widetilde{T}_\beta})$ is established by ensuring the same condition as stated earlier, i.e., ${\eta_{\widetilde{T}_\alpha}}^U \leq {\eta_{\widetilde{T}_\beta}}^L$.
To aggregate experts' Fermatean fuzzy preference scores in $\mathbb{K}$, the lower limit ($\underline{Lim}(\widetilde{T}_\alpha)$) can be expressed as:

$$\underline{Lim}({t_\alpha}^l) = \frac{1}{\mid \underline{Apr}({t_\alpha}^l) \mid} \sum {t_\beta}^l \mid {t_\beta}^l \in \underline{Apr}({t_\alpha}^l), \tag{17}$$

$$\underline{Lim}({t_\alpha}^m) = \frac{1}{\mid \underline{Apr}({t_\alpha}^m) \mid} \sum {t_\beta}^m \mid {t_\beta}^m \in \underline{Apr}({t_\alpha}^m), \tag{18}$$

$$\underline{Lim}({t_\alpha}^u) = \frac{1}{\mid \underline{Apr}({t_\alpha}^u) \mid} \sum {t_\beta}^u \mid {t_\beta}^u \in \underline{Apr}({t_\alpha}^u), \tag{19}$$

$$\underline{Lim}(\rho_{\widetilde{T}_\alpha}) = \frac{1}{\mid \underline{Apr}(\rho_{\widetilde{T}_\alpha}) \mid} \sum \rho_{\widetilde{T}_\beta} \mid \rho_{\widetilde{T}_\beta} \in \underline{Apr}(\rho_{\widetilde{T}_\alpha}), \tag{20}$$

$$\underline{Lim}(\eta_{\widetilde{T}_\alpha}) = \frac{1}{\mid \underline{Apr}(\eta_{\widetilde{T}_\alpha}) \mid} \sum \eta_{\widetilde{T}_\beta} \mid \eta_{\widetilde{T}_\beta} \in \underline{Apr}(\eta_{\widetilde{T}_\alpha}), \tag{21}$$

where expressions $\mid \underline{Apr}({t_\alpha}^e) \mid$, $\mid \underline{Apr}(\rho_{\widetilde{T}_\alpha}) \mid$, and $\mid \underline{Apr}(\eta_{\widetilde{T}_\alpha}) \mid$ represent the cardinality of $\underline{Apr}({t_\alpha}^e)$ ($e = l,\ m.\ u$), $\underline{Apr}(\rho_{\widetilde{T}_\alpha})$ and $\underline{Apr}(\eta_{\widetilde{T}_\alpha})$, respectively.
Correspondingly, the upper limit ($\overline{Lim}(\widetilde{T}_\alpha)$) can be established as:

$$\overline{Lim}({t_\alpha}^l) = \frac{1}{\mid \overline{Apr}({t_\alpha}^l) \mid} \sum {t_\beta}^l \mid {t_\beta}^l \in \overline{Apr}({t_\alpha}^l), \tag{22}$$

$$\overline{Lim}({t_\alpha}^m) = \frac{1}{\mid \overline{Apr}({t_\alpha}^m) \mid} \sum {t_\beta}^m \mid {t_\beta}^m \in \overline{Apr}({t_\alpha}^m), \tag{23}$$

$$\overline{Lim}({t_\alpha}^u) = \frac{1}{\mid \overline{Apr}({t_\alpha}^u) \mid} \sum {t_\beta}^u \mid {t_\beta}^u \in \overline{Apr}({t_\alpha}^u), \tag{24}$$

$$\overline{Lim}(\rho_{\widetilde{T}_\alpha}) = \frac{1}{\mid \overline{Apr}(\rho_{\widetilde{T}_\alpha}) \mid} \sum \rho_{\widetilde{T}_\beta} \mid \rho_{\widetilde{T}_\beta} \in \overline{Apr}(\rho_{\widetilde{T}_\alpha}), \tag{25}$$

$$\overline{Lim}(\eta_{\widetilde{T}_\alpha}) = \frac{1}{\mid \overline{Apr}(\eta_{\widetilde{T}_\alpha}) \mid} \sum \eta_{\widetilde{T}_\beta} \mid \eta_{\widetilde{T}_\beta} \in \overline{Apr}(\eta_{\widetilde{T}_\alpha}), \tag{26}$$

where expressions $\mid \overline{Apr}({t_\alpha}^e) \mid$, $\mid \overline{Apr}(\rho_{\widetilde{T}_\alpha}) \mid$, and $\mid \overline{Apr}(\eta_{\widetilde{T}_\alpha}) \mid$ represent the cardinality of $\overline{Apr}({t_\alpha}^e)$ ($e = l,\ m.\ u$), $\overline{Apr}(\rho_{\widetilde{T}_\alpha})$ and $\overline{Apr}(\eta_{\widetilde{T}_\alpha})$, respectively.
A Fermatean fuzzy rough number (FFRN) of $\widetilde{T}_\alpha$, $1 \leq \alpha \leq s$, can be defined as:

$$\begin{aligned} FFRN(\widetilde{T}_\alpha) = {} & [(\lceil \underline{Lim}({t_\alpha}^l), \overline{Lim}({t_\alpha}^l) \rfloor, \lceil \underline{Lim}({t_\alpha}^m), \overline{Lim}({t_\alpha}^m) \rfloor, \lceil \underline{Lim}({t_\alpha}^u), \overline{Lim}({t_\alpha}^u) \rfloor); \lceil \underline{Lim}(\rho_{\widetilde{T}_\alpha}), \overline{Lim}(\rho_{\widetilde{T}_\alpha}) \rfloor, \\ & \lceil \overline{Lim}(\eta_{\widetilde{T}_\alpha}), \underline{Lim}(\eta_{\widetilde{T}_\alpha}) \rfloor]. \end{aligned} \tag{27}$$

For convenience, it can be written as:

$$FFRN(\widetilde{T}_\alpha) = [(\lceil {t_\alpha}^{lL}, {t_\alpha}^{lU} \rfloor, \lceil {t_\alpha}^{mL}, {t_\alpha}^{mU} \rfloor, \lceil {t_\alpha}^{uL}, {t_\alpha}^{uU} \rfloor); \lceil {\rho_{\widetilde{T}_\alpha}}^L, {\rho_{\widetilde{T}_\alpha}}^U \rfloor, \lceil {\eta_{\widetilde{T}_\alpha}}^U, \eta_{\widetilde{T}_\alpha}^L \rfloor], \tag{28}$$

such that

$$0 \leq ({\rho_{\widetilde{T}_\alpha}}^L)^3 + ({\eta_{\widetilde{T}_\alpha}}^L)^3 \leq 1,$$

and

$$0 \leq ({\rho_{\widetilde{T}_\alpha}}^U)^3 + ({\eta_{\widetilde{T}_\alpha}}^U)^3 \leq 1,$$

where $1 \leq \alpha \leq s$.
Furthermore, the FFRN($\mathbb{K}$) can be calculated as follows:

$$\begin{aligned} FFRN(\mathbb{K}) = {} & \{FFRN(\widetilde{T}_1), FFRN(\widetilde{T}_2), \ldots, FFRN(\widetilde{T}_s)\} \\ = {} & \Big[\Big(\Big\lceil \frac{1}{s}\sum_{\alpha=1}^{s} {t_\alpha}^{lL}, \frac{1}{s}\sum_{\alpha=1}^{s} {t_\alpha}^{lU} \Big\rfloor, \Big\lceil \frac{1}{s}\sum_{\alpha=1}^{s} {t_\alpha}^{mL}, \frac{1}{s}\sum_{\alpha=1}^{s} {t_\alpha}^{mU} \Big\rfloor, \Big\lceil \frac{1}{s}\sum_{\alpha=1}^{s} {t_\alpha}^{uL}, \frac{1}{s}\sum_{\alpha=1}^{s} {t_\alpha}^{uU} \Big\rfloor \Big); \Big\lceil \frac{1}{s}\sum_{\alpha=1}^{s} {\rho_{\widetilde{T}_\alpha}}^L, \\ & \frac{1}{s}\sum_{\alpha=1}^{s} {\rho_{\widetilde{T}_\alpha}}^U \Big\rfloor, \Big\lceil \frac{1}{s}\sum_{\alpha=1}^{s} {\eta_{\widetilde{T}_\alpha}}^U, \frac{1}{s}\sum_{\alpha=1}^{s} \eta_{\widetilde{T}_\alpha}^L \Big\rfloor\Big]. \end{aligned} \tag{29}$$

Definition 2.4. *For any FFRN,* $\widehat{T} = [(\lceil t^{lL}, t^{lU} \rfloor, \lceil t^{mL}, t^{mU} \rfloor, \lceil t^{uL}, t^{uU} \rfloor); \lceil \rho_{\widetilde{T}}{}^{L}, \rho_{\widetilde{T}}{}^{U} \rfloor, \lceil \eta_{\widetilde{T}}{}^{L}, \eta_{\widetilde{T}}{}^{U} \rfloor]$*, the score function* $Scor(\widehat{T})$ *is given by:*

$$Scor(\widehat{T}) = \frac{(t^{lL} + t^{mL} + t^{uL}) + (\rho_{\widetilde{T}}^{L\,3} - \eta_{\widetilde{T}}^{L\,3}) + (t^{lU} + t^{mU} + t^{uU}) + (\rho_{\widetilde{T}}^{U\,3} - \eta_{\widetilde{T}}^{U\,3})}{10}. \tag{30}$$

To optimize the performance of the score function, it is essential to establish an accuracy function.

Definition 2.5. *Let* $\widehat{T} = [(\lceil t^{lL}, t^{lU} \rfloor, \lceil t^{mL}, t^{mU} \rfloor, \lceil t^{uL}, s^{uU} \rfloor); \lceil \rho_{\widetilde{T}}{}^{L}, \rho_{\widetilde{T}}{}^{U} \rfloor, \lceil \eta_{\widetilde{T}}{}^{L}, \eta_{\widetilde{T}}{}^{U} \rfloor]$ *be a FFRN, the accuracy function* $Acc(\widehat{T})$ *is defined as:*

$$Acc(\widehat{T}) = \frac{(t^{lL} + t^{mL} + t^{uL}) + (\rho_{\widetilde{T}}^{L\,3} + \eta_{\widetilde{T}}^{L\,3}) + (t^{lU} + t^{mU} + t^{uU}) + (\rho_{\widetilde{T}}^{U\,3} + \eta_{\widetilde{T}}^{U\,3})}{10}. \tag{31}$$

Definition 2.6. *Let* $\widehat{S} = [(\lceil s^{lL}, s^{lU} \rfloor, \lceil s^{mL}, s^{mU} \rfloor, \lceil s^{uL}, s^{uU} \rfloor); \lceil \rho_{\widetilde{S}}{}^{L}, \rho_{\widetilde{S}}{}^{U} \rfloor, \lceil \eta_{\widetilde{S}}{}^{L}, \eta_{\widetilde{S}}{}^{U} \rfloor]$ *and* $\widehat{T} = [(\lceil t^{lL}, t^{lU} \rfloor, \lceil t^{mL}, t^{mU} \rfloor, \lceil t^{uL}, t^{uU} \rfloor); \lceil \rho_{\widetilde{T}}{}^{L}, \rho_{\widetilde{T}}{}^{U} \rfloor, \lceil \eta_{\widetilde{T}}{}^{L}, \eta_{\widetilde{T}}{}^{U} \rfloor]$ *be two FFRNs, then*

1. *If* $Scor(\widehat{S}) > Scor(\widehat{T})$*, then* $\widehat{S} \succ \widehat{T}$ *(*$\widehat{S}$ *is superior to* $\widehat{T}$*);*
2. *If* $Scor(\widehat{S}) < Scor(\widehat{T})$*, then* $\widehat{S} \prec \widehat{T}$ *(*$\widehat{S}$ *is inferior to* $\widehat{T}$*);*
3. *If* $Scor(\widehat{S}) = Scor(\widehat{T})$*, then*
   - *If* $Acc(\widehat{S}) > Acc(\widehat{T})$*, then* $\widehat{S} \succ \widehat{T}$ *(*$\widehat{S}$ *is superior to* $\widehat{T}$*);*
   - *If* $Acc(\widehat{S}) < Acc(\widehat{T})$*, then* $\widehat{S} \prec \widehat{T}$ *(*$\widehat{S}$ *is inferior to* $\widehat{T}$*);*
   - *If* $Acc(\widehat{S}) = Acc(\widehat{T})$*, then* $\widehat{S} \sim \widehat{T}$ *(*$\widehat{S}$ *is equivalent to* $\widehat{T}$*).*

Definition 2.7. *Let* $\widehat{S} = [(\lceil s^{lL}, s^{lU} \rfloor, \lceil s^{mL}, s^{mU} \rfloor, \lceil s^{uL}, s^{uU} \rfloor); \lceil \rho_{\widetilde{S}}{}^{L}, \rho_{\widetilde{S}}{}^{U} \rfloor, \lceil \eta_{\widetilde{S}}{}^{L}, \eta_{\widetilde{S}}{}^{U} \rfloor]$ *and* $\widehat{T} = [(\lceil t^{lL}, t^{lU} \rfloor, \lceil t^{mL}, t^{mU} \rfloor, \lceil t^{uL}, t^{uU} \rfloor); \lceil \rho_{\widetilde{T}}{}^{L}, \rho_{\widetilde{T}}{}^{U} \rfloor, \lceil \eta_{\widetilde{T}}{}^{L}, \eta_{\widetilde{T}}{}^{U} \rfloor]$ *be two FFRNs, then the Euclidean distance between* $\widehat{S}$ *and* $\widehat{T}$ *is defined as:*

$$\begin{aligned} d(\widehat{S}, \widehat{T}) \;=\; & (\frac{1}{6}[(s^{lL} - t^{lL})^2 + (s^{lU} - t^{lU})^2 + (s^{mL} - t^{mL})^2 + (s^{mU} - t^{mU})^2 + (s^{uL} - t^{uL})^2 + (s^{uU} - t^{uU})^2])^{\frac{1}{2}} + \\ & (\frac{1}{4}[(\rho_{\widehat{S}}^{L\,3} - \rho_{\widehat{T}}^{L\,3})^2 + (\rho_{\widehat{S}}^{U\,3} - \rho_{\widehat{T}}^{U\,3})^2 + (\eta_{\widehat{S}}^{L\,3} - \eta_{\widehat{T}}^{L\,3})^2 + (\eta_{\widehat{S}}^{U\,3} - \eta_{\widehat{T}}^{U\,3})^2 + (\rho_{\widehat{T}}^{L\,3} - \rho_{\widehat{S}}^{L\,3} + \eta_{\widehat{T}}^{L\,3} - (\eta_{\widehat{S}}^{L})^3)^2 + \\ & (\rho_{\widehat{T}}^{U\,3} - \rho_{\widehat{S}}^{U\,3} + \eta_{\widehat{T}}^{U\,3} - \eta_{\widehat{S}}^{U\,3})^2])^{\frac{1}{2}}. \end{aligned} \tag{32}$$

3 Fermatean fuzzy rough number-based group decision-making method

This section presents a Fermatean fuzzy rough number-based group decision-making framework designed to evaluate the performance of 3PRLPs in managing outsourced reverse logistics services. The presented approach assesses the impact of sustainable practices and risk factors, analyzing supplier evaluation outcomes to assist the organization in evaluating and identifying the most suitable 3PRLP. In the reverse logistics supply chain, a panel of s experts $\mathcal{E} = \{\mathcal{E}_1, \ldots, \mathcal{E}_s\}$ evaluates q 3PRLPs, $\mathcal{P} = \{\mathcal{P}_1, \ldots, \mathcal{P}_q\}$, based on r sustainability-based criteria $\mathcal{C} = \{\mathcal{C}_1, \ldots, \mathcal{C}_r\}$, identified through literature and expert input. Linguistic ratings are converted into TFFNs to quantify expert assessments. The steps of the proposed multi-objective optimization approach are outlined below.

3.1 Extended entropy weight method for criteria weighting

Shannon's entropy [24] measures the expected information content and highlights the relative importance of indicators. In this study, it is combined with FFRNs to quantify uncertainty and subjectivity, enabling the calculation of objective weights for criteria and sub-criteria. The algorithmic framework for deriving entropy-based criteria weights is methodically outlined in the following sequential steps:

Step 1. Construction of a Fermatean fuzzy decision matrix based on the linguistic assessments.
To address the limitations of human knowledge and handle the inherent uncertainties in decisional issues, a team of logistics and procurement experts evaluates the alternatives. Their assessments are transformed into TFFNs using the Fermatean fuzzy scale. The Fermatean fuzzy decision matrix from the i^{th} expert for each logistics supplier is then constructed as follows:

$$\widetilde{T}_{\mathcal{E}_i} = \begin{array}{c} \\ \mathcal{P}_1 \\ \mathcal{P}_2 \\ \vdots \\ \mathcal{P}_q \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{C}_1 & \mathcal{C}_2 & \cdots & \mathcal{C}_r \end{array} \\ \begin{pmatrix} \widetilde{t}^i_{11} & \widetilde{t}^i_{12} & \dots & \widetilde{t}^i_{1r} \\ \widetilde{t}^i_{21} & \widetilde{t}^i_{22} & \dots & \widetilde{t}^i_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{t}^i_{q1} & \widetilde{t}^i_{q2} & \dots & \widetilde{t}^i_{qr} \end{pmatrix} \end{array}, \tag{33}$$

where $\widetilde{t}^i_{\alpha\beta} = [(t^{il}_{\alpha\beta}, t^{im}_{\alpha\beta}, t^{iu}_{\alpha\beta}); \rho_{\widetilde{t}^i_{\alpha\beta}}, \eta_{\widetilde{t}^i_{\alpha\beta}}]$, $1 \le \alpha \le q$; $1 \le \beta \le r$; $1 \le i \le s$ is the triangular Fermatean fuzzy performance score of the α^{th} alternative w.r.t β^{th} criterion.

Step 2. Extended decision-matrix formation.
Aggregate the individual Fermatean fuzzy decision-matrices $\widetilde{T}_{\mathcal{E}_i}$ from each supply chain expert into a consolidated matrix that captures the collective insights of their assessments. The aggregated Fermatean fuzzy assessment score $\tilde{\widetilde{t}}_{\alpha\beta} = \{\widetilde{t}^1_{\alpha\beta}, \widetilde{t}^2_{\alpha\beta}, \dots, \widetilde{t}^q_{\alpha\beta}\}$ reflects a set of assessment ratings assigned by each expert for assessing a 3PRLP. According to the Definition 2.3, the collective evaluation $\widetilde{t}^i_{\alpha\beta}$ can be converted to a FFRN as follows:

$$FFRN(\widetilde{t}^i_{\alpha\beta}) = [(\lceil t^{ilL}_{\alpha\beta}, t^{ilU}_{\alpha\beta} \rfloor, \lceil t^{imL}_{\alpha\beta}, t^{imU}_{\alpha\beta} \rfloor, \lceil t^{iuL}_{\alpha\beta}, t^{iuU}_{\alpha\beta} \rfloor); \lceil \rho_{\widetilde{t}^{iL}_{\alpha\beta}}, \rho_{\widetilde{t}^{iU}_{\alpha\beta}} \rfloor, \lceil \eta_{\widetilde{t}^{iU}_{\alpha\beta}}, \eta_{\widetilde{t}^{iL}_{\alpha\beta}} \rfloor]. \tag{34}$$

Further, FFRN($\tilde{\widetilde{t}}_{\alpha\beta}$) can be computed to form an extended evaluation-matrix, as outlined below:

$$\begin{aligned} FFRN(\tilde{\widetilde{t}}_{\alpha\beta}) &= \{FFRN(\widetilde{t}^1_{\alpha\beta}), FFRN(\widetilde{\mathfrak{t}}^2_{\alpha\beta}), \dots, FFRN(\widetilde{\mathfrak{t}}^s_{\alpha\beta})\}, \\ FFRN(\widetilde{\widetilde{\mathfrak{t}}}_{\alpha\beta}) &= \{[(\lceil t^{1lL}_{\alpha\beta}), t^{1lU}_{\alpha\beta} \rfloor, \lceil t^{1mL}_{\alpha\beta}, t^{1mU}_{\alpha\beta} \rfloor, \lceil t^{1uL}_{\alpha\beta}, t^{1uU}_{\alpha\beta} \rfloor); \lceil \rho_{\widetilde{\mathfrak{t}}^{1L}_{\alpha\beta}}, \rho_{\widetilde{\mathfrak{t}}^{1U}_{\alpha\beta}} \rfloor, \lceil \eta_{\widetilde{\mathfrak{t}}^{1U}_{\alpha\beta}}, \eta_{\widetilde{\mathfrak{t}}^{1L}_{\alpha\beta}} \rfloor], \\ &\quad [(\lceil t^{2lL}_{\alpha\beta}, t^{2lU}_{\alpha\beta} \rfloor, \lceil t^{2mL}_{\alpha\beta}, t^{2mU}_{\alpha\beta} \rfloor, \lceil t^{2uL}_{\alpha\beta}, t^{2uU}_{\alpha\beta} \rfloor); \lceil \rho_{\widetilde{\mathfrak{t}}^{2L}_{\alpha\beta}}, \rho_{\widetilde{\mathfrak{t}}^{2U}_{\alpha\beta}} \rfloor, \lceil \eta_{\widetilde{\mathfrak{t}}^{2U}_{\alpha\beta}}, \eta_{\widetilde{\mathfrak{t}}^{2L}_{\alpha\beta}} \rfloor], \dots, \\ &\quad [(\lceil t^{slL}_{\alpha\beta}, t^{slU}_{\alpha\beta} \rfloor, \lceil t^{smL}_{\alpha\beta}, t^{smU}_{\alpha\beta} \rfloor, \lceil t^{suL}_{\alpha\beta}, t^{suU}_{\alpha\beta} \rfloor); \lceil \rho_{\widetilde{\mathfrak{t}}^{sL}_{\alpha\beta}}, \rho_{\widetilde{\mathfrak{t}}^{sU}_{\alpha\beta}} \rfloor, \lceil \eta_{\widetilde{\mathfrak{t}}^{sU}_{\alpha\beta}}, \eta_{\widetilde{\mathfrak{t}}^{sL}_{\alpha\beta}} \rfloor]\}. \end{aligned} \tag{35}$$

The extended decision matrix, based on the Fermatean fuzzy rough preference scores of each logistics provider, is presented as follows:

$$\widetilde{\tau} = \begin{array}{c} \\ \mathcal{P}_1 \\ \mathcal{P}_2 \\ \vdots \\ \mathcal{P}_q \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{C}_1 & & \cdots & \mathcal{C}_r \end{array} \\ \begin{pmatrix} [(\lceil \overline{\tau}^{lL}_{11}, \overline{\tau}^{lU}_{11} \rfloor, \lceil \overline{\tau}^{mL}_{11}, \overline{\tau}^{mU}_{11} \rfloor, \lceil \overline{\tau}^{lL}_{uL}, \overline{\tau}^{uU}_{11} \rfloor); \lceil \overline{\rho^{\sim}_{\widehat{t}}}^{L}_{11}, \overline{\rho^{\sim}_{\widehat{t}}}^{U}_{11} \rfloor, \lceil \overline{\eta^{\sim}_{\widehat{t}}}^{U}_{11}, \overline{\eta^{\sim}_{\widehat{t}}}^{L}_{11} \rfloor] & \dots & [(\lceil \overline{\tau}^{lL}_{1r}, \overline{\tau}^{lU}_{1r} \rfloor, \lceil \overline{\tau}^{mL}_{1r}, \overline{\tau}^{mU}_{1r} \rfloor, \lceil \overline{\tau}^{uL}_{1r}, \overline{\tau}^{uU}_{1r} \rfloor); \lceil \overline{\rho^{\sim}_{\widehat{t}}}^{L}_{1r}, \overline{\rho^{\sim}_{\widehat{t}}}^{U}_{1r} \rfloor, \lceil \overline{\eta^{\sim}_{\widehat{t}}}^{U}_{1r}, \overline{\eta^{\sim}_{\widehat{t}}}^{L}_{1r} \rfloor] \\ [(\lceil \overline{\tau}^{lL}_{21}, \overline{\tau}^{lU}_{21} \rfloor, \lceil \overline{\tau}^{mL}_{21}, \overline{\tau}^{mU}_{21} \rfloor, \lceil \overline{\tau}^{uL}_{21}, \overline{\tau}^{uU}_{21} \rfloor); \lceil \overline{\rho^{\sim}_{\widehat{t}}}^{L}_{21}, \overline{\rho^{\sim}_{\widehat{t}}}^{U}_{21} \rfloor, \lceil \overline{\eta^{\sim}_{\widehat{t}}}^{U}_{21}, \overline{\eta^{\sim}_{\widehat{t}}}^{L}_{21} \rfloor] & \dots & [(\lceil \overline{\tau}^{lL}_{2r}, \overline{\tau}^{lU}_{2r} \rfloor, \lceil \overline{\tau}^{mL}_{2r}, \overline{\tau}^{mU}_{2r} \rfloor, \lceil \overline{\tau}^{uL}_{2r}, \overline{\tau}^{uU}_{2r} \rfloor); \lceil \overline{\rho^{\sim}_{\widehat{t}}}^{L}_{2r}, \overline{\rho^{\sim}_{\widehat{t}}}^{U}_{2r} \rfloor, \lceil \overline{\eta^{\sim}_{\widehat{t}}}^{U}_{2r}, \overline{\eta^{\sim}_{\widehat{t}}}^{L}_{2r} \rfloor] \\ \vdots & \ddots & \vdots \\ [(\lceil \overline{\tau}^{lL}_{q1}, \overline{\tau}^{lU}_{q1} \rfloor, \lceil \overline{\tau}^{mL}_{q1}, \overline{\tau}^{mU}_{q1} \rfloor, \lceil \overline{\tau}^{uL}_{q1}, \overline{\tau}^{uU}_{q1} \rfloor); \lceil \overline{\rho^{\sim}_{\widehat{t}}}^{L}_{q1}, \overline{\rho^{\sim}_{\widehat{t}}}^{U}_{q1} \rfloor, \lceil \overline{\eta^{\sim}_{\widehat{t}}}^{U}_{q1}, \overline{\eta^{\sim}_{\widehat{t}}}^{L}_{q1} \rfloor] & \dots & [(\lceil \overline{\tau}^{lL}_{qr}, \overline{\tau}^{lU}_{qr} \rfloor, \lceil \overline{\tau}^{mL}_{qr}, \overline{\tau}^{mU}_{qr} \rfloor, \lceil \overline{\tau}^{uL}_{qr}, \overline{\tau}^{uU}_{qr} \rfloor); \lceil \overline{\rho^{\sim}_{\widehat{t}}}^{L}_{qr}, \overline{\rho^{\sim}_{\widehat{t}}}^{U}_{qr} \rfloor, \lceil \overline{\eta^{\sim}_{\widehat{t}}}^{U}_{qr}, \overline{\eta^{\sim}_{\widehat{t}}}^{L}_{qr} \rfloor] \end{pmatrix} \end{array}. \tag{36}$$

Step 3. Defuzzification of Fermatean fuzzy rough assessment data.
Convert the Fermatean fuzzy rough assessment data of all logistic providers into precise numerical values derived from the decision matrix data. Apply the defuzzification formula from Equation (30) throughout the extended evaluation matrix to enhance the scope for in-depth analysis in 3PRLP management.

$$\mathfrak{t}_{\alpha\beta} = \frac{(t^{lL}_{\alpha\beta} + t^{mL}_{\alpha\beta} + t^{uL}_{\alpha\beta}) + ({\rho^{\sim}_{\widetilde{t}_{\alpha\beta}}}^{L}{}^{3} - {\eta^{\sim}_{\widetilde{t}_{\alpha\beta}}}^{L}{}^{3}) + (t^{lU}_{\alpha\beta} + t^{mU}_{\alpha\beta} + t^{uU}_{\alpha\beta}) + ({\rho^{\sim}_{\widetilde{t}_{\alpha\beta}}}^{U}{}^{3} - {\eta^{\sim}_{\widetilde{t}_{\alpha\beta}}}^{U}{}^{3})}{10}, \tag{37}$$

where $1 \le \alpha \le q$; $1 \le \beta \le r$.

Step 4. Normalization of crisp decision-matrix.
Normalization is a technique that adjusts data to a common dimensional scale, ensuring better comparability, consistency, and improved decision-making. This method transforms the crisp evaluation matrix to generate the projected indices $p_{\alpha\beta}$, according to the formula presented in Equation (38).

$$\mathfrak{p}_{\alpha\beta} = \frac{\mathfrak{t}_{\alpha\beta}}{\sum_{\alpha=1}^{q} \mathfrak{t}_{\alpha\beta}}, \tag{38}$$

where $1 \leq \alpha \leq q$; $1 \leq \beta \leq r$.

Step 5. Entropy calculation.
Entropy quantifies the average uncertainty in performance evaluations using discrete probability distributions. Greater variance suggests more reliable insights, whereas lower variance indicates higher entropy and increased unpredictability.

Calculate the entropy measure for each index using the Equation (39):

$$\breve{\mathcal{E}}_{\beta} = -(\ln q)^{-1} \sum_{\alpha=1}^{q} \mathfrak{p}_{\alpha\beta} \ln(\mathfrak{p}_{\alpha\beta}), \tag{39}$$

where $1 \leq \beta \leq r$.

Step 6. Evaluation of divergence and normalized criteria weights .
The entropy-based divergence of the α^{th} criterion across sustainable logistics providers forms the basis for evaluating the relative significance of each criterion. As the divergence of the α^{th} criterion decreases, the decision-making information required to prioritize the logistics providers becomes insufficient, leading to a reduction in the weights assigned to the criterion.

The divergence and objective weights for each criterion are determined based on entropy measures as defined in Equations (40) and (41):

$$div_{\beta} = 1 - \breve{\mathcal{E}}_{\beta}. \tag{40}$$

$$W_{\beta} = \frac{1 - \breve{\mathcal{E}}_{\beta}}{r - \sum_{\beta=1}^{r} \breve{\mathcal{E}}_{\beta}}, \tag{41}$$

where $1 \leq \beta \leq r$; $0 \leq W_{\beta} \leq 1$ and $\sum_{\beta=1}^{r} W_{\beta} = 1$.

3.2 Ranking 3PRLPs by the Fermatean fuzzy rough-MULTIMOORA method

After determining the objective weights, our primary aim is to represent the preferences of sustainable 3PRLPs using an aggregated ranking index derived from the Fermatean fuzzy rough-MULTIMOORA method. The procedure for ranking preferred sustainable 3PRLPs is detailed in the following stepwise algorithm:

Step 1. Performance evaluation of logistics providers using triangular Fermatean fuzzy numbers.
Logistics experts evaluate providers using relevant criteria, with their linguistic preferences quantified via the Fermatean fuzzy scale to accurately capture uncertainty in TFFNs. The i^{th} expert's preferences are formally expressed in Equation (42).

$$\widetilde{F}_{\mathcal{E}_i} = \begin{array}{c} \\ \mathcal{P}_1 \\ \mathcal{P}_2 \\ \vdots \\ \mathcal{P}_q \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{C}_1 & \mathcal{C}_2 & \cdots & \mathcal{C}_r \end{array} \\ \begin{pmatrix} \widetilde{\mathfrak{f}}^i_{11} & \widetilde{\mathfrak{f}}^i_{12} & \cdots & \widetilde{\mathfrak{f}}^i_{1r} \\ \widetilde{\mathfrak{f}}^i_{21} & \widetilde{\mathfrak{f}}^i_{22} & \cdots & \widetilde{\mathfrak{f}}^i_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{\mathfrak{f}}^i_{q1} & \widetilde{\mathfrak{f}}^i_{q2} & \cdots & \widetilde{\mathfrak{f}}^i_{qr} \end{pmatrix} \end{array}, \tag{42}$$

where $\widetilde{\mathfrak{f}}^{i}_{\alpha\beta} = [(\mathfrak{f}^{il}_{\alpha\beta}, \mathfrak{f}^{im}_{\alpha\beta}, \mathfrak{f}^{iu}_{\alpha\beta}); \rho_{\widetilde{\mathfrak{f}}^{i}_{\alpha\beta}}, \eta_{\widetilde{\mathfrak{f}}^{i}_{\alpha\beta}}]$, $1 \leq \alpha \leq q$; $1 \leq \beta \leq r$; $1 \leq i \leq s$ denotes the preference score assigned by the i^{th} expert to the α^{th} logistics provider based on the β^{th} criterion.

Step 2. Aggregation of performance evaluations for each logistics provider.
To derive a universally accepted solution, the individual perspectives of logistics experts are aggregated. The FFR approach is applied to convert the aggregated evaluation rating into a FFRN using Equation (29), resulting in the formation of the following aggregated evaluation matrix for logistics 3PRLPs:

$$\widehat{\mathcal{T}} = \begin{array}{c} \\ \mathcal{P}_1 \\ \mathcal{P}_2 \\ \vdots \\ \mathcal{P}_q \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{C}_1 & & \cdots & \mathcal{C}_r \end{array} \\ \begin{pmatrix} [(\lceil\bar{\mathfrak{f}}^{lL}_{11}, \bar{\mathfrak{f}}^{lU}_{11}\rfloor, \lceil\bar{\mathfrak{f}}^{mL}_{11}, \bar{\mathfrak{f}}^{mU}_{11}\rfloor, \lceil\bar{\mathfrak{f}}^{uL}_{11}, \bar{\mathfrak{f}}^{uU}_{11}\rfloor); \lceil\overline{\rho}^{L}_{\widetilde{\mathfrak{f}}11}, \overline{\rho}^{U}_{\widetilde{\mathfrak{f}}11}\rfloor, \lceil\overline{\eta}^{U}_{\widetilde{\mathfrak{f}}11}, \overline{\eta}^{L}_{\widetilde{\mathfrak{f}}11}\rfloor] & \cdots & [(\lceil\bar{\mathfrak{f}}^{lL}_{1r}, \bar{\mathfrak{f}}^{lU}_{1r}\rfloor, \lceil\bar{\mathfrak{f}}^{mL}_{1r}, \bar{\mathfrak{f}}^{mU}_{1r}\rfloor, \lceil\bar{\mathfrak{f}}^{uL}_{1r}, \bar{\mathfrak{f}}^{uU}_{1r}\rfloor); \lceil\overline{\rho}^{L}_{\widetilde{\mathfrak{f}}1r}, \overline{\rho}^{U}_{\widetilde{\mathfrak{f}}1r}\rfloor, \lceil\overline{\eta}^{U}_{\widetilde{\mathfrak{f}}1r}, \overline{\rho}^{L}_{\widetilde{\mathfrak{f}}1r}\rfloor] \\ [(\lceil\bar{\mathfrak{f}}^{lL}_{21}, \bar{\mathfrak{f}}^{lU}_{21}\rfloor, \lceil\bar{\mathfrak{f}}^{mL}_{21}, \bar{\mathfrak{f}}^{mU}_{21}\rfloor, \lceil\bar{\mathfrak{f}}^{uL}_{21}, \bar{\mathfrak{f}}^{uU}_{21}\rfloor); \lceil\overline{\rho}^{L}_{\widetilde{\mathfrak{f}}21}, \overline{\rho}^{U}_{\widetilde{\mathfrak{f}}21}\rfloor, \lceil\overline{\eta}^{U}_{\widetilde{\mathfrak{f}}21}, \overline{\eta}^{L}_{\widetilde{\mathfrak{f}}21}\rfloor] & \cdots & [(\lceil\bar{\mathfrak{f}}^{lL}_{2r}, \bar{\mathfrak{f}}^{lU}_{2r}\rfloor, \lceil\bar{\mathfrak{f}}^{mL}_{2r}, \bar{\mathfrak{f}}^{mU}_{2r}\rfloor, \lceil\bar{\mathfrak{f}}^{uL}_{2r}, \bar{\mathfrak{f}}^{uU}_{2r}\rfloor); \lceil\overline{\rho}^{L}_{\widetilde{\mathfrak{f}}2r}, \overline{\rho}^{U}_{\widetilde{\mathfrak{f}}2r}\rfloor, \lceil\overline{\eta}^{U}_{\widetilde{\mathfrak{f}}2r}, \overline{\eta}^{L}_{\widetilde{\mathfrak{f}}2r}\rfloor] \\ \vdots & \ddots & \vdots \\ [(\lceil\bar{\mathfrak{f}}^{lL}_{q1}, \bar{\mathfrak{f}}^{lU}_{q1}\rfloor, \lceil\bar{\mathfrak{f}}^{mL}_{q1}, \bar{\mathfrak{f}}^{mU}_{q1}\rfloor, \lceil\bar{\mathfrak{f}}^{uL}_{q1}, \bar{\mathfrak{f}}^{uU}_{q1}\rfloor); \lceil\overline{\rho}^{L}_{\widetilde{\mathfrak{f}}q1}, \overline{\rho}^{U}_{\widetilde{\mathfrak{f}}q1}\rfloor, \lceil\overline{\eta}^{U}_{\widetilde{\mathfrak{f}}q1}, \overline{\eta}^{L}_{\widetilde{\mathfrak{f}}q1}\rfloor] & \cdots & [(\lceil\bar{\mathfrak{f}}^{lL}_{qr}, \bar{\mathfrak{f}}^{lU}_{qr}\rfloor, \lceil\bar{\mathfrak{f}}^{mL}_{qr}, \bar{\mathfrak{f}}^{mU}_{qr}\rfloor, \lceil\bar{\mathfrak{f}}^{uL}_{qr}, \bar{\mathfrak{f}}^{uU}_{qr}\rfloor); \lceil\overline{\rho}^{L}_{\widetilde{\mathfrak{f}}qr}, \overline{\rho}^{U}_{\widetilde{\mathfrak{f}}qr}\rfloor, \lceil\overline{\eta}^{U}_{\widetilde{\mathfrak{f}}qr}, \overline{\eta}^{L}_{\widetilde{\mathfrak{f}}qr}\rfloor] \end{pmatrix} \end{array}. \tag{43}$$

Effectively synthesizing information and aggregating large data sets makes the analysis process more efficient and reveals important insights.

Step 3. Normalization and weighted normalization of Fermatean fuzzy rough performance evaluations.
Considering the different scales and factors of the assessment criteria, it is crucial to normalize the collective decision matrix to ensure a consistent and reliable framework for analysis. The optimized performance evaluation through the normalization process can be computed using Equations (44) and (45).

(i) For "profit" type criterion (where higher values are desirable),

$$\widehat{u}_{\alpha\beta} = \left[\left(\left\lceil\frac{\mathfrak{f}^{lL}_{\alpha\beta}}{\mathfrak{f}^{*}_{\beta}}, \frac{\mathfrak{f}^{lU}_{\alpha\beta}}{\mathfrak{f}^{*}_{\beta}}\right\rfloor, \left\lceil\frac{\mathfrak{f}^{mL}_{\alpha\beta}}{\mathfrak{f}^{*}_{\beta}}, \frac{\mathfrak{f}^{mL}_{\alpha\beta}}{\mathfrak{f}^{*}_{\beta}}\right\rfloor, \left\lceil\frac{\mathfrak{f}^{uL}_{\alpha\beta}}{\mathfrak{f}^{*}_{\beta}}, \frac{\mathfrak{f}^{uU}_{\alpha\beta}}{\mathfrak{f}^{*}_{\beta}}\right\rfloor\right); \left\lceil\rho^{L}_{\widetilde{\mathcal{F}}_{\alpha\beta}}, \rho^{U}_{\widetilde{\mathcal{F}}_{\alpha\beta}}\right\rfloor, \left\lceil\eta^{L}_{\widetilde{\mathcal{F}}_{\alpha\beta}}, \eta^{U}_{\widetilde{\mathcal{F}}_{\alpha\beta}}\right\rfloor\right], \quad \beta \in \Gamma_B, \tag{44}$$

(ii) For "cost" type criterion (where lower values are desirable),

$$\widehat{u}_{\alpha\beta} = \left[\left(\left\lceil\frac{\mathfrak{f}^{-}_{\beta}}{\mathfrak{f}^{uU}_{\alpha\beta}}, \frac{\mathfrak{f}^{-}_{\beta}}{\mathfrak{f}^{uL}_{\alpha\beta}}\right\rfloor, \left\lceil\frac{\mathfrak{f}^{-}_{\beta}}{\mathfrak{f}^{mU}_{\alpha\beta}}, \frac{\mathfrak{f}^{-}_{\beta}}{\mathfrak{f}^{mL}_{\alpha\beta}}\right\rfloor, \left\lceil\frac{\mathfrak{f}^{-}_{\beta}}{\mathfrak{f}^{lU}_{\alpha\beta}}, \frac{\mathfrak{f}^{-}_{\beta}}{\mathfrak{f}^{lL}_{\alpha\beta}}\right\rfloor\right); \left\lceil\eta^{U}_{\widetilde{\mathcal{F}}_{\alpha\beta}}, \eta^{L}_{\widetilde{\mathcal{F}}_{\alpha\beta}}\right\rfloor, \left\lceil\rho^{U}_{\widetilde{\mathcal{F}}_{\alpha\beta}}, \rho^{L}_{\widetilde{\mathcal{F}}_{\alpha\beta}}\right\rfloor\right], \quad \beta \in \Gamma_C, \tag{45}$$

where Γ_B and Γ_C represent the set of profit and cost-related attributes, respectively.
Moreover, for every feasible value of β,

$$\begin{aligned} \mathfrak{f}^{*}_{\beta} &= \max_{\alpha} \mathfrak{f}^{uU}_{\alpha\beta}, \quad \beta \in \Gamma_B, \\ \mathfrak{f}^{-}_{\beta} &= \min_{\alpha} \mathfrak{f}^{lL}_{\alpha\beta}, \quad \beta \in \Gamma_C. \end{aligned} \tag{46}$$

As a result, the normalization process yields the normalized Fermatean fuzzy rough matrix $\widehat{\mathbb{U}} = [\widehat{u}_{\alpha\beta}]_{q\times r}$.
Further, $\widehat{\mathbb{U}}$ can be converted into the weighted normalized matrix $\widehat{\mathbb{V}} = [\widehat{v}_{\alpha\beta}]_{q\times r}$ using Equation (47):

$$\begin{aligned} \widehat{v}_{\alpha\beta} &= \widehat{u}_{\alpha\beta} \otimes W_\beta \\ \widehat{v}_{\alpha\beta} &= [(\lceil u^{lL}_{\alpha\beta} \cdot W_\beta, u^{lL}_{\alpha\beta} \cdot W_\beta\rfloor, \lceil u^{lL}_{\alpha\beta} \cdot W_\beta, u^{lL}_{\alpha\beta} \cdot W_\beta\rfloor, \lceil u^{lL}_{\alpha\beta} \cdot W_\beta, u^{lL}_{\alpha\beta} \cdot W_\beta\rfloor); \\ &\quad \lceil\{1-(1-(\rho^{L}_{\widehat{u}_{\alpha\beta}})^3)^{W_\beta}\}^{\frac{1}{3}}\rfloor, \{1-(1-(\rho^{U}_{\widehat{u}_{\alpha\beta}})^3)^{W_\beta}\}^{\frac{1}{3}}\rfloor, \lceil(\eta^{U}_{\widehat{u}_{\alpha\beta}})^{W_\beta}, (\eta^{L}_{\widehat{u}_{\alpha\beta}})^{W_\beta}\rfloor], \end{aligned} \tag{47}$$

where W_β $(\beta = 1, \ldots, r)$ represents the objective criteria weights derived through the extended entropy method discussed in Section 3.1.

Step 4. Ratio system approach based on Fermatean fuzzy rough numbers.
The preference ranking of 3PRLPs, based on ratio system can be determined using the following steps:

1. To calculate the values in the Fermatean fuzzy rough ratio system, the process involves summing the Fermatean fuzzy rough weighted normalized evaluations for profit criteria and subtracting them for cost criteria, as detailed below:

$$\widehat{y}_\alpha = \sum_{\beta=1}^{k} \widehat{v}_{\alpha\beta} - \sum_{\beta=k+1}^{r} \widehat{v}_{\alpha\beta}, \quad \alpha = 1, \ldots, q. \tag{48}$$

where k represents the number of profit-type criteria, and (r-k) denotes the cost-type criteria. Equations (44) and (45) ensure that all criteria are regarded as beneficial, thereby allowing the Fermatean fuzzy ratio system to be defined as follows:

$$\widehat{y}_\alpha = \sum_{\beta=1}^{r} \widehat{v}_{\alpha\beta}, \quad \alpha = 1, \ldots, q. \tag{49}$$

2. Defuzzification of Fermatean fuzzy rough evaluations.
To aid in prioritizing the alternatives in the ratio system, the performance evaluations are converted into crisp numerical values that can be computed using Equation (50).

$${y_\alpha}^* = \frac{(y_\alpha^{lL} + y_\alpha^{mL} + y_\alpha^{uL}) + ({\rho_{y_\alpha}^L}^3 - {\eta_{y_\alpha}^L}^3) + (y_\alpha^{lU} + y_\alpha^{mU} + y_\alpha^{uU}) + ({\rho_{y_\alpha}^U}^3 - {\eta_{y_\alpha}^U}^3)}{10}. \tag{50}$$

3. The optimal alternative according to the ratio system is the one with the highest utility y_α $(\alpha = 1, \ldots, q)$ and the alternatives are prioritize in decreasing sequence of their utility values, as follows:

$$A_{RS} = \left\{ P_{\alpha \mid \max\limits_{\alpha} y_\alpha} \succ \ldots \succ P_{\alpha \mid \min\limits_{\alpha} y_\alpha} \right\}. \tag{51}$$

Step 5. Reference point approach using the Fermatean fuzzy rough numbers.
The following steps outline the process of evaluating the priority order of 3PRLPs using the presented reference point approach:

1. Equations (44) and (45) convert all criteria into beneficial ones, thereby defining the maximal objective Fermatean fuzzy rough reference point vector for calculating the utility as follows:

$$\widehat{\mathcal{R}}^* = \{\widehat{x}_1^*, \widehat{x}_2^*, \ldots, \widehat{x}_r^*\}, \tag{52}$$

such that

$$\widehat{x}_\beta^* = [(\lceil 1, 1\rfloor, \lceil 1, 1\rfloor, \lceil 1, 1\rfloor); \lceil 1, 1\rfloor, \lceil 0, 0\rfloor], \quad \beta = 1, \ldots, r. \tag{53}$$

2. The separation between the weighted relevance multi-objective reference point vector $\widehat{\mathcal{R}}^*$ and the corresponding weighted normalized rating $\widehat{v}_{\alpha\beta}$ is calculated using Equation (32). Th final utility score for each alternative, based on the reference point approach, is then derived using the Equation (54):

$$z_\alpha^* = \max_{\beta} d(\widehat{x}_\beta^*, \widehat{v}_{\alpha\beta}), \quad \alpha = 1, \ldots, q. \tag{54}$$

3. The optimal alternative in the reference point approach is the one with the lowest utility z_α^* and the alternatives are prioritized in ascending sequence as follows:

$$A_{RPA} = \left\{ P_{\alpha \mid \min\limits_{\alpha} z_\alpha} \succ \ldots \succ P_{\alpha \mid \max\limits_{\alpha} z_\alpha} \right\}. \tag{55}$$

Step 6. Full multiplicative form based on Fermatean fuzzy rough numbers.
The logistics suppliers can be prioritized under the full multiplicative full through the steps outlined below:

1. Since all criteria become beneficial after normalization (using Equations (44) and (45)), the Fermatean fuzzy rough utility of each alternative can be evaluated using the Equation (56):

$$\widehat{x}_\alpha = \prod_{\beta=1}^{r} \widehat{v}_{\alpha\beta}, \quad \alpha = 1, \ldots, q. \tag{56}$$

2. Defuzzification of the overall utility score evaluations.
The Fermatean fuzzy rough utility assessments can be transformed into definitive numerical values through the application of the defuzzification formula provided in Equation (57).

$$x_\alpha{}^* = \frac{(x_\alpha^{lL} + x_\alpha^{mL} + x_\alpha^{uL}) + (\rho_{\widetilde{X}_\alpha}^{L\ 3} - \eta_{\widetilde{X}_\alpha}^{L\ 3}) + (x_\alpha^{lU} + x_\alpha^{mU} + x_\alpha^{uU}) + (\rho_{\widetilde{X}_\alpha}^{U\ 3} - \eta_{\widetilde{X}_\alpha}^{U\ 3})}{10}. \tag{57}$$

3. The alternatives with the highest utility, according to the full multiplicative form, are considered the optimal, and the priority order is defined in decreasing sequence as follows:

$$A_{FMF} = \left\{ P_{\alpha \mid \max\limits_{\alpha} x_\alpha} \succ \ldots \succ P_{\alpha \mid \min\limits_{\alpha} x_\alpha} \right\}. \tag{58}$$

Step 7. Ranking aggregation using the improved Borda Rule.
The improved Borda Rule, based on the Borda Count, combines utilities and rankings from MULTIMOORA method, providing an advantage over Dominance Theory. Utilities are normalized using vector normalization, and the resulting evaluation, IMB(P_α), for $\alpha = 1, \ldots, q$ is computed as follows:

$$IMB(P_\alpha) = y_\alpha{}^* \frac{q - r(y_\alpha) + 1}{q(q+1)/2} - z_\alpha{}^* \frac{r(z_\alpha)}{q(q+1)/2} + x_\alpha{}^* \frac{q - r(x_\alpha) + 1}{q(q+1)/2}, \tag{59}$$

where $r(y_\alpha)$, $r(z_\alpha)$, and $r(x_\alpha)$ represent the rankings of the ratio system, reference point approach, and full multiplicative form, respectively. The optimal alternative is identified by the highest value of IMB(P_α), $\alpha = 1, \ldots, q$.

The framework of the proposed Fermatean fuzzy rough number-based entropy-MULTIMOORA approach for evaluating 3PRLPs is presented in Figure 1.

4 Case Study: Evaluating sustainable third-party reverse logistics providers in Malaysia's food industry

This study demonstrates the practical applicability and effectiveness of the proposed approach through a case study (adapted from [16]) that evaluates and prioritizes 3PRLPs to optimize the outsourcing of reverse logistics services to external suppliers. The analysis investigates a leading company in Malaysia's food industry, addressing the complexities and challenges of implementing sustainable practices in reverse logistics to enhance efficiency and resilience in food supply chain management.

4.1 Case background

The food supply chain plays a vital role in environmental, social, and economic sustainability, requiring thorough evaluation of various factors. Effective management often involves outsourcing reverse logistics to third-party providers committed to sustainable practices. The perishability of food and the return flow of packaging materials add complexity, demanding efficient reverse logistics systems. However, sustainable reverse logistics in food supply chains remains insufficiently explored. Aligning with Malaysia's commitment to green initiatives, this study assesses sustainable 3PRLPs to address the challenges of managing food distribution networks effectively.

This case study examines TOP EAST ASIA Sdn Bhd (TOP Group), founded in 2000 in Kuala Lumpur, Malaysia (`http://www.topeastasia.com`). Over the past decade, TOP Group has established a robust supply chain and emerged as a leading benchmark in Malaysia's food sector. The company has expanded globally, operating branches in Iran, Tunisia, Canada, Australia, Indonesia, New Zealand, and beyond. Specializing in import-export, TOP delivers products

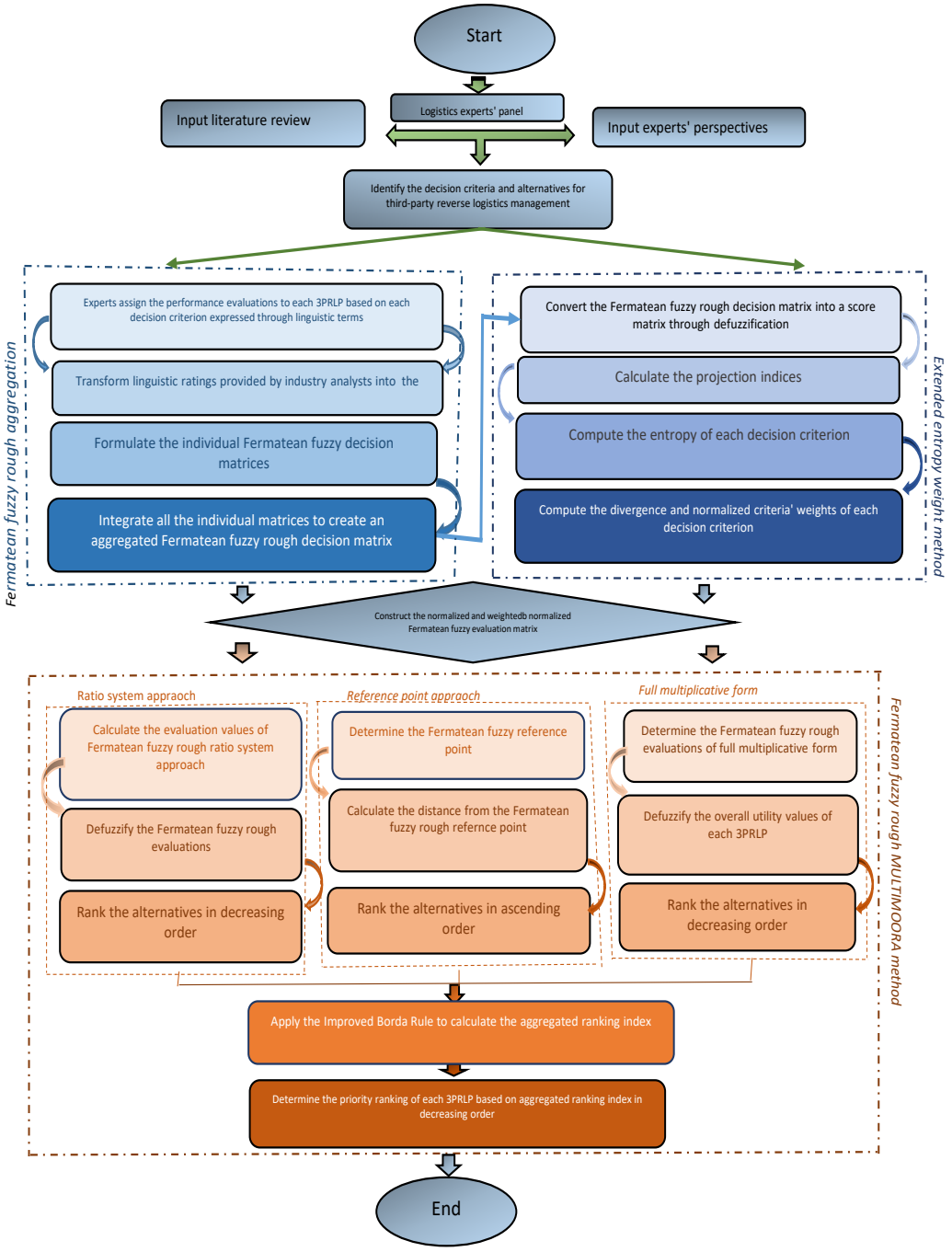


Figure 1: Flowchart of Fermatean fuzzy rough number-based method

reliably to international markets while maintaining high service standards. Table 2 details the annual trade capacities of each branch.

The TOP Group is a major importer and trader of food items, including dates, fresh and dried fruits, saffron, currants, and date-derived products. The TOP Group has collaborated with international suppliers from the USA, Palestine, Saudi Arabia, India, Iran, North Africa, and Egypt across various aspects of manufacturing and distribution. The company maintains a policy of evaluating only ISO-certified suppliers, who are subject to annual inspections to verify the adherence to guidelines and the consistent delivery of premium products.

Given the rising costs of raw materials and pressures from clients, government regulations, and sustainability policies, the company's leadership has resolved to incorporate reverse logistics operations into their supply chains, aligning them

Table 2: Annual trade capacities of TOP EAST ASIA branches

Import capacity		Export capacity	
Country	Capacity	Country	Capacity
Malaysia	2,500 tons	Malaysia	0 tons
Iran	0 tons	Iran	1,000 tons
Tunisia	0 tons	Tunisia	1,350 tons
Canada	300 tons	Canada	0 tons
Australia	150 tons	Australia	0 tons
Indonesia	300 tons	Indonesia	0 tons
New Zealand	100 tons	New Zealand	0 tons

with the enterprise's overall strategic objectives. Due to the complexities of managing reverse logistics operations, the company has opted to delegate these functions to a specialized 3PRLP. Considering the regulatory obligations and organizational rules, the senior management defined the following criteria for selecting a 3PRLP:

- The potential 3PRLP must possess the capability to deliver a comprehensive range of reverse logistics services and processes specifically designed for the food sector.
- The economic efficiency is a key consideration for the company, prompting managers to prioritize a 3PRLP that ensures both cost effectiveness and high profitability.
- The quality of products and operational flexibility are crucial factors for management in the selection of a 3PRLP.
- The 3PRLP must also guarantee the accurate and reliable communication of relevant information to clients.

Thus, the developed approach examines the influence of sustainable and risk factors, analyzing supplier evaluation outcomes to assist the business in assessing the chosen 3PRLPs and identifying the most optimal solutions.

4.2 Data gathering

This case study employed a structured survey to gather expert insights and define key criteria for prioritizing 3PRLPs. A decision-making panel of four logistics professionals ($\mathcal{E}_i$, $i = 1, \ldots, 4$)including a senior executive, manufacturing manager, sales director, and operations managerwas formed to ensure strategic alignment. Experts completed questionnaires through guided sessions to minimize misunderstandings. Through collaborative discussion and literature review, seventeen sustainable criteria (C_β, $\beta = 1, \ldots, 17$) and four candidate 3PRLPs were identified (see Table 3), supporting informed optimization of the food supply chain.

4.3 Performance evaluation of 3PRLPs

After identifying relevant sustainable criteria, logistics experts evaluate potential 3PRLPs using linguistic terms (Table 4). These assessments, gathered via a structured questionnaire, are summarized in Table 5. The decision-making process begins by converting the linguistic evaluations in Table 5 into TFFNs for further analysis.

4.4 The extended entropy method for evaluating the criteria weights

In the initial stage of our proposed approach, an extended entropy weighting method is applied to accurately ascertain the relative objective weights of sustainable decision criteria. The procedure for assigning objective weights to each criterion is elaborated in the following steps:

Step 1. Decision-makers in the food supply chain management evaluate the performance of each 3PRLP against the sustainable criterion C_β ($\beta = 1, \ldots, 17$), expressing their judgments using language-based variables, as defined in Table 4. The collective assessment data provided by every supply chain specialist $\mathcal{E}_i$ (as shown in Table 5) is represented using TFFNs based on the linguistic scale, which are then utilized to construct the corresponding Fermatean fuzzy decision matrices $T_{\mathcal{E}_i}$.

Table 3: Decision criteria for delegating reverse logistics services

Dimensions	Sub-Criteria	Preferred	Explanation
Economic (Ec)	Ec_1. Product cost	min	It represents the supplier's expertise in delivering products at a price that remains competitive in the market.
	Ec_2. Product profitability	max	It refers to earning a profit from the products provided by the supplier.
	Ec_3. Product quality	max	It involves various quality factors, including precision in order processing, complete fulfillment, personal service, effective evaluation, and merit assurance practices.
	Ec_4. Operational flexibility	max	It relates to qualities such as the supplier's adaptability, responsiveness to customer needs, readiness for future demands, and flexibility in meeting the requirements of target markets.
	Ec_5. Product delivery and service	max	It is related to aspects such as on-time shipments, a supplier's ability to meet delivery deadlines, the precision of transit time predictions, and the reliability of tracking systems.
	Ec_6. Lead time	min	It refers to the ability to deliver products within the short and competitive lead times.
	Ec_7. Location	max	It relates to the supplier's geographical attributes, including international regulations, market coverage, and global logistics networks.
Environment (En)	En_1. Environmental competencies	max	It involves the supplier's capacity to reduce environmental impact by using sustainable materials and technologies that reduce ecological harm in their operations.
	En_2. Green packaging and purchasing	max	It relates to the supplier's ability to consider environmental factors in the selection of packaging materials and making purchasing decisions.
	En_3. Reverse logistic practices	max	It relates to the supplier's capability to manage reverse logistics, covering processes such as recovery, recycling, sorting, processing, redistributions, storage, disposal, return policies, post-sales support, and recall management.
	En_4. Environmental management system	max	It refers to the supplier's strategies and operational frameworks designed to protect the environment.
Social (S)	S_1. Health and safety	max	It refers to standards ensuring the safety, health, and welfare of individuals within the supplier's organization.
	S_2. Information disclosure	max	It relates to the supplier's commitment to transparency by offering accurate information about raw materials and processes while enhancing the quality of customer interactions.
	S_3. Employee welfare	max	It involves the employee-related aspects, such as improving job satisfaction, increasing productivity, and ensuring long-term improvements in product quality.
Risk (R)	R_1. Operational risk	min	This represents the level of risk associated with suppliers concerning resource allocation, shipping production, and other operational factors.
	R_2. Financial risk	min	This indicates the level of risk associated with suppliers, including factors such as currency fluctuations, contractual obligations, backorders, rework, and investment risks.
	R_3. Organizational risk	min	It concerns to represent the level of risk related to a supplier's strategic decisions targeting new customers, markets, and other strategic ventures.

Table 4: Linguistic scale for the performance evaluation of 3PRLPs

Linguistic data	Triangular Fermatean fuzzy number
Very poor (Vp)	[(0,0,1);0.50,0.90]
Poor (P)	[(0,1,3);0.55,0.85]
Medium poor (Mp)	[(1,3,5);0.60,0.80]
Fair (F)	[(3,5,7);0.65,0.75]
Medium good (Mg)	[(5,7,9);0.70,0.70]
Good (G)	[(7,9,10); 0.75, 0.65]
Very good (Vg)	[(9,10,10);0.80,0.60]

Step 2. The Fermatean fuzzy data from the individual decision matrices is consolidated into a unified framework, with each element representing a combined evaluation assigned by the logistics experts.

Following the aggregation principles defined in Equations (34) and (35), transform each combined evaluation data (TFFNs) into the relevant Fermatean fuzzy rough evaluation information. As a result, an extended decision matrix was developed, reflecting the Fermatean fuzzy rough performance scores of each 3PRLP within the sustainable assessment criteria, as shown in Table 6.

Step 3. The extended entropy method is used to determine the weight of each criterion based on FFRNs, which quantify the entropy associated with each criterion. The Fermatean fuzzy rough assessment data is systematically

Table 5: Performance ratings of 3PRLPs for each decision criterion

3PRLPs	Experts	Criteria								
		$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_5$	$\mathcal{C}_6$	$\mathcal{C}_7$	$\mathcal{C}_8$	$\mathcal{C}_9$
$3PRLP_1$	$\mathcal{E}_1$	Mp	G	F	Vg	Mg	G	Mp	F	Mg
	$\mathcal{E}_2$	Mg	G	G	Vg	F	F	F	F	F
	$\mathcal{E}_3$	P	F	G	G	Mg	G	P	Mg	Mg
	$\mathcal{E}_4$	F	G	G	Vg	Mp	P	G	F	F
$3PRLP_2$	$\mathcal{E}_1$	Vp	F	Mg	P	Mp	P	P	F	F
	$\mathcal{E}_2$	P	F	F	G	P	P	F	F	F
	$\mathcal{E}_3$	G	G	F	P	Mp	F	P	Vp	Vp
	$\mathcal{E}_4$	Mg	F	F	Mp	Mp	Vp	G	F	F
$3PRLP_3$	$\mathcal{E}_1$	Mg	G	F	Vg	P	G	F	F	Mg
	$\mathcal{E}_2$	Mg	Mg	Mg	Mg	Mp	F	F	F	F
	$\mathcal{E}_3$	G	G	F	Mg	Mp	F	G	P	P
	$\mathcal{E}_4$	Mg	G	Mp	G	Mp	P	G	F	F
$3PRLP_4$	$\mathcal{E}_1$	Vp	Mp	Vg	Mp	G	F	F	Vg	G
	$\mathcal{E}_2$	Mp	F	Mg	Mp	Mg	F	F	F	F
	$\mathcal{E}_3$	Vp	Mg	Mg	Mg	Vg	Mp	G	G	G
	$\mathcal{E}_4$	P	Mp	G	F	G	Mg	G	F	F

3PRLPs	Experts	Criteria							
		$\mathcal{C}_{11}$	$\mathcal{C}_{11}$	$\mathcal{C}_{12}$	$\mathcal{C}_{13}$	$\mathcal{C}_{14}$	$\mathcal{C}_{15}$	$\mathcal{C}_{16}$	$\mathcal{C}_{17}$
$3PRLP_1$	$\mathcal{E}_1$	Mg	Mg	Vg	Vg	Vg	Mg	Vg	Vg
	$\mathcal{E}_2$	F	F	G	Vg	G	G	Vg	G
	$\mathcal{E}_3$	Mg	Mg	Mg	G	G	Mg	G	Vg
	$\mathcal{E}_4$	F	Mp	Mg	Vg	F	Mg	Vg	Vg
$3PRLP_2$	$\mathcal{E}_1$	F	F	F	Vp	P	P	F	Mp
	$\mathcal{E}_2$	F	F	G	P	P	G	Vg	Mp
	$\mathcal{E}_3$	Vp	F	F	P	P	P	P	Vp
	$\mathcal{E}_4$	F	Mp	F	F	F	F	Mg	F
$3PRLP_3$	$\mathcal{E}_1$	Mg	F	Vg	Vg	G	Mp	Vg	Vg
	$\mathcal{E}_2$	F	F	G	Vg	Mg	G	Vg	Mg
	$\mathcal{E}_3$	Mp	P	P	Mg	Mp	Mp	Mp	Mg
	$\mathcal{E}_4$	P	Mp	F	Mg	F	F	G	F
$3PRLP_4$	$\mathcal{E}_1$	Vg	Vg	Vg	G	Vg	Vg	Vg	Mp
	$\mathcal{E}_2$	F	Vg	Vg	Vg	Vg	G	Vg	G
	$\mathcal{E}_3$	Vg	Vg	Vg	F	G	G	G	G
	$\mathcal{E}_4$	F	G	Vg	Mg	F	G	G	Mg

Table 6: Aggregated Fermatean fuzzy rough decision matrix for potential 3PRLPs

$\widehat{T}$	$\mathcal{C}_1$	...	$\mathcal{C}_{17}$
$3PRLP_1$	$[(\lceil 1.02, 3.56\rfloor, \lceil 2.50, 5.50\rfloor, \lceil 4.50, 7.50\rfloor); \lceil 0.59, 0.66\rfloor, \lceil 0.74, 0.81\rfloor]$	...	$[(\lceil 8.13, 8.88\rfloor, \lceil 9.56, 9.94\rfloor, \lceil 10.0, 10.0\rfloor); \lceil 0.78, 0.80\rfloor, \lceil 0.60, 0.62\rfloor]$
$3PRLP_2$	$[(\lceil 1.17, 4.75\rfloor, \lceil 1.85, 6.73\rfloor, \lceil 3.27, 8.15\rfloor); \lceil 0.56, 0.69\rfloor, \lceil 0.71, 0.84\rfloor]$	...	$[(\lceil 0.65, 1.90\rfloor, \lceil 1.69, 3.77\rfloor, \lceil 3.21, 5.71\rfloor); \lceil 0.56, 0.62\rfloor, \lceil 0.78, 0.85\rfloor]$
$3PRLP_3$	$[(\lceil 3.13, 5.88\rfloor, \lceil 7.13, 7.88\rfloor, \lceil 9.06, 9.44\rfloor); \lceil 0.70, 0.72\rfloor, \lceil 0.68, 0.70\rfloor]$	...	$[(\lceil 4.29, 6.79\rfloor, \lceil 6.23, 8.31\rfloor, \lceil 8.10, 9.35\rfloor); \lceil 0.68, 0.75\rfloor, \lceil 0.66, 0.72\rfloor]$
$3PRLP_4$	$[(\lceil 0.06, 0.50\rfloor, \lceil 0.33, 1.75\rfloor, \lceil 1.54, 3.50\rfloor); \lceil 0.51, 0.56\rfloor, \lceil 0.84, 0.89\rfloor]$	...	$[(\lceil 3.50, 6.33\rfloor, \lceil 5.50, 8.33\rfloor, \lceil 7.25, 9.54\rfloor); \lceil 0.66, 0.73\rfloor, \lceil 0.67, 0.74\rfloor]$

converted into precise quantitative figures through the refined defuzzification technique given in Equation (37). This procedure results in a well-defined and consolidated performance assessment, as illustrated in Table 7.

Step 4. The normalization technique transforms the defuzzified performance scores of each 3PRLP into projected indices $\mathfrak{p}_{\alpha\beta}$, as governed by Equation (38), with the resulting matrix shown in Table 8.

Step 5. Using Equations (39), (40), and (41), the entropy quantification of each sustainable assessment criterion C_β ($\beta = 1, \ldots, 17$), the entropy-based divergence, and the objective weights obtained from data uncertainty, which emphasize their respective importance, are calculated, as shown in Table 9.

Table 7: Defuzzified performance ratings of Fermatean fuzzy rough evaluations

T	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_5$	$\mathcal{C}_6$	$\mathcal{C}_7$	$\mathcal{C}_8$	$\mathcal{C}_9$
$3PRLP_1$	2.414	4.665	4.665	5.702	3.265	3.511	2.714	3.278	3.585
$3PRLP_2$	2.547	3.535	3.278	2.186	1.483	1.139	2.498	2.248	2.248
$3PRLP_3$	4.257	4.972	2.971	4.870	1.483	2.971	4.100	2.406	2.692
$3PRLP_4$	0.672	2.726	4.870	2.726	5.121	2.971	4.100	4.263	4.100
	$\mathcal{C}_{10}$	$\mathcal{C}_{11}$	$\mathcal{C}_{12}$	$\mathcal{C}_{13}$	$\mathcal{C}_{14}$	$\mathcal{C}_{15}$	$\mathcal{C}_{16}$	$\mathcal{C}_{17}$	
$3PRLP_1$	3.585	3.265	4.870	5.702	4.789	4.257	5.702	5.702	
$3PRLP_2$	2.248	2.663	3.535	1.139	1.279	2.498	3.410	1.626	
$3PRLP_3$	2.971	2.092	3.781	5.029	3.527	3.016	4.576	4.316	
$3PRLP_4$	4.415	5.702	5.859	4.540	4.921	5.387	5.544	4.044	

Table 8: Projection indices of each 3PRLP

N	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_5$	$\mathcal{C}_6$	$\mathcal{C}_7$	$\mathcal{C}_8$	$\mathcal{C}_9$
$3PRLP_1$	0.244	0.293	0.296	0.368	0.288	0.331	0.202	0.269	0.284
$3PRLP_2$	0.258	0.222	0.208	0.141	0.131	0.108	0.186	0.184	0.178
$3PRLP_3$	0.430	0.313	0.188	0.315	0.131	0.280	0.306	0.197	0.213
$3PRLP_4$	0.068	0.171	0.309	0.176	0.451	0.280	0.306	0.350	0.325
	$\mathcal{C}_{10}$	$\mathcal{C}_{11}$	$\mathcal{C}_{12}$	$\mathcal{C}_{13}$	$\mathcal{C}_{14}$	$\mathcal{C}_{15}$	$\mathcal{C}_{16}$	$\mathcal{C}_{17}$	
$3PRLP_1$	0.271	0.238	0.270	0.347	0.330	0.281	0.296	0.363	
$3PRLP_2$	0.170	0.194	0.196	0.069	0.088	0.165	0.177	0.104	
$3PRLP_3$	0.225	0.152	0.210	0.306	0.243	0.199	0.238	0.275	
$3PRLP_4$	0.334	0.416	0.325	0.277	0.339	0.355	0.288	0.258	

Table 9: Entropy weights of each decision criterion

Weights' measures	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_5$	$\mathcal{C}_6$	$\mathcal{C}_7$	$\mathcal{C}_8$	$\mathcal{C}_9$
Entropy	0.894	0.981	0.984	0.948	0.902	0.952	0.982	0.975	0.981
Divergence	0.106	0.019	0.016	0.052	0.098	0.048	0.018	0.025	0.019
Normalized weights	0.142	0.025	0.021	0.070	0.131	0.064	0.024	0.034	0.025
	$\mathcal{C}_{10}$	$\mathcal{C}_{11}$	$\mathcal{C}_{12}$	$\mathcal{C}_{13}$	$\mathcal{C}_{14}$	$\mathcal{C}_{15}$	$\mathcal{C}_{16}$	$\mathcal{C}_{17}$	
Entropy	0.979	0.946	0.985	0.916	0.931	0.969	0.986	0.943	
Divergence	0.021	0.054	0.015	0.084	0.069	0.031	0.014	0.057	
Normalized weights	0.028	0.072	0.020	0.113	0.092	0.042	0.019	0.076	

4.5 Rank the potential 3PRLPs using the Fermatean fuzzy rough MULTIMOORA technique

Based on the objective criteria weights established in Section 4.4, prioritize the 3PRLPs to identify the most optimal one for optimizing sustainable food supply chain management that aligns with sustainability practices.

The initial steps of this approach use the same process as those employed in the earlier weighting technique.

Step 1. In accordance with the characteristics of each sustainable decision criterion, apply Equations (44) and (45) to normalize the defuzzified preference scores presented in Table 7. After normalizing the Fermatean fuzzy rough data, the matrix weighted normalization is derived using Equation (47), with the resulting normalized FFR decision matrix $\widehat{\mathbb{U}}$ and weighted normalized FFR decision matrix $\widehat{\mathbb{V}}$ detailed in Tables 10 and 11, respectively.

Table 10: Normalized Fermatean fuzzy rough decision matrix

$\widehat{\mathbb{U}}$	$\mathcal{C}_1$	...	$\mathcal{C}_{17}$
$3PRLP_1$	$[(\lceil 0.01, 0.01\rfloor, \lceil 0.01, 0.03\rfloor, \lceil 0.02, 0.06\rfloor); \lceil 0.74, 0.81\rfloor, \lceil 0.59, 0.66\rfloor]$	...	$[(\lceil 0.07, 0.07\rfloor, \lceil 0.07, 0.07\rfloor, \lceil 0.07, 0.08\rfloor); \lceil 0.60, 0.62\rfloor, \lceil 0.78, 0.80\rfloor]$
$3PRLP_2$	$[(\lceil 0.01, 0.02\rfloor, \lceil 0.02, 0.03\rfloor, \lceil 0.01, 0.05\rfloor); \lceil 0.71, 0.84\rfloor, \lceil 0.56, 0.69\rfloor]$	...	$[(\lceil 0.11, 0.20\rfloor, \lceil 0.17, 0.38\rfloor, \lceil 0.34, 1.00\rfloor); \lceil 0.78, 0.85\rfloor, \lceil 0.56, 0.62\rfloor]$
$3PRLP_3$	$[(\lceil 0.01, 0.01\rfloor, \lceil 0.01, 0.01\rfloor, \lceil 0.01, 0.02\rfloor); \lceil 0.68, 0.70\rfloor, \lceil 0.70, 0.72\rfloor]$	...	$[(\lceil 0.07, 0.08\rfloor, \lceil 0.08, 0.10\rfloor, \lceil 0.10, 0.15\rfloor); \lceil 0.66, 0.72\rfloor, \lceil 0.68, 0.75\rfloor]$
$3PRLP_4$	$[(\lceil 0.02, 0.04\rfloor, \lceil 0.04, 0.19\rfloor, \lceil 0.13, 1.00\rfloor); \lceil 0.84, 0.89\rfloor, \lceil 0.51, 0.56\rfloor]$	...	$[(\lceil 0.07, 0.09\rfloor, \lceil 0.08, 0.12\rfloor, \lceil 0.10, 0.19\rfloor); \lceil 0.67, 0.74\rfloor, \lceil 0.66, 0.73\rfloor]$

Table 11: Weighted Normalized fuzzy rough decision matrix

$\widehat{\mathbb{V}}$	$\mathcal{C}_1$	...	$\mathcal{C}_{17}$
$3PRLP_1$	$[(\lceil 0.01, 0.02\rfloor, \lceil 0.02, 0.04\rfloor, \lceil 0.03, 0.01\rfloor); \lceil 0.41, 0.47\rfloor, \lceil 0.93, 0.94\rfloor]$	...	$[(\lceil 0.01, 0.01\rfloor, \lceil 0.01, 0.01\rfloor, \lceil 0.01, 0.01\rfloor); \lceil 0.27, 0.28\rfloor, \lceil 0.06, 0.06\rfloor]$
$3PRLP_2$	$[(\lceil 0.01, 0.03\rfloor, \lceil 0.01, 0.01\rfloor, \lceil 0.02, 0.01\rfloor); \lceil 0.39, 0.50\rfloor, \lceil 0.92, 0.95\rfloor]$	...	$[(\lceil 0.01, 0.02\rfloor, \lceil 0.01, 0.03\rfloor, \lceil 0.03, 0.08\rfloor); \lceil 0.36, 0.41\rfloor, \lceil 0.04, 0.05\rfloor]$
$3PRLP_3$	$[(\lceil 0.01, 0.01\rfloor, \lceil 0.01, 0.01\rfloor, \lceil 0.02, 0.03\rfloor); \lceil 0.37, 0.39\rfloor, \lceil 0.95, 0.96\rfloor]$	...	$[(\lceil 0.01, 0.01\rfloor, \lceil 0.01, 0.01\rfloor, \lceil 0.01, 0.01\rfloor); \lceil 0.29, 0.33\rfloor, \lceil 0.05, 0.06\rfloor]$
$3PRLP_4$	$[(\lceil 0.03, 0.01\rfloor, \lceil 0.01, 0.03\rfloor, \lceil 0.02, 0.14\rfloor); \lceil 0.49, 0.54\rfloor, \lceil 0.91, 0.92\rfloor]$	...	$[(\lceil 0.01, 0.01\rfloor, \lceil 0.01, 0.01\rfloor, \lceil 0.01, 0.01\rfloor); \lceil 0.30, 0.34\rfloor, \lceil 0.05, 0.06\rfloor]$

Table 12: Normalized performance measures using different ranking approaches

3PRLPs	Ratio system approach		Reference point approach		Full multiplicative form		Improved Borda Rule	
	Evaluation value	Rank	Evaluation value	Rank	Evaluation value	Rank	Evaluation value	Rank
$3PRLP_1$	0.375	2	1.019	2	–0.164	3	–0.124	2
$3PRLP_2$	0.244	4	1.069	3	–0.163	2	–0.345	4
$3PRLP_3$	0.303	3	1.069	3	–0.173	4	–0.277	3
$3PRLP_4$	0.422	1	1.018	1	–0.154	1	0.005	1

4.6 Sensitivity analysis

To thoroughly assess the stability and performance of the framework, a detailed sensitivity analysis is conducted to evaluate how changes in industry stakeholders' preferences influence the final selection of sustainable 3PRLPs. This analysis involves a series of seventeen scenarios, S_β ($\beta = 1, \ldots, 17$) where two criterion weights are altered at a time while keeping all other weights fixed, in order to evaluate the performance of the developed approach for 3PRLP evaluation. The first scenario S_1 is based on the original weights $w_1, w_2, \ldots, w_{17}$ determined in Section 4.4, which are used as the reference for the analysis. In each scenario S_β, the weights of $\mathcal{C}_1$ and $\mathcal{C}_\beta$ are swapped (i.e., $w_1 \longleftrightarrow w_\beta$). This process continues, ultimately resulting in the updated decision criteria weights $w_{17}, w_2, \ldots, w_1$ for the final scenario S_{17}. The aggregated evaluation results, based on the Improved Borda Rule for each scenario, are illustrated in Figure 2.

4.7 Comparative analysis

To demonstrate the effectiveness and robustness of the developed group decision-making model for evaluating 3PRLPs, a comparative analysis is conducted against several existing techniques such as crisp entropy-MULTIMOORA [12], fuzzy entropy-MULTIMOORA [25], fuzzy rough entropy-REGIME [2], and Pythagorean fuzzy rough entropy-MARCOS [4]. This analysis highlights the superior capability of the proposed model in handling the complexities of multi-objective decision-making in 3PRLPs. The key outcomes of this comparison are summarized as follows:

1. The crisp MULTIMOORA method, developed by Brauers and Zavadskas [6], applies expert-assigned weights and converts linguistic assessments into deterministic values to rank 3PRLPs. While effective for straightforward evaluations, it fails to capture the ambiguity inherent in expert judgments and lacks a unified mechanism for aggregating diverse opinions. The proposed hybrid model addresses these gaps by integrating uncertainty modeling with a structured aggregation process. The resulting rankings are presented in Table 13.

2. Fuzzy approaches, such as fuzzy entropy MULTIMOORA [12], model uncertainty using membership functions

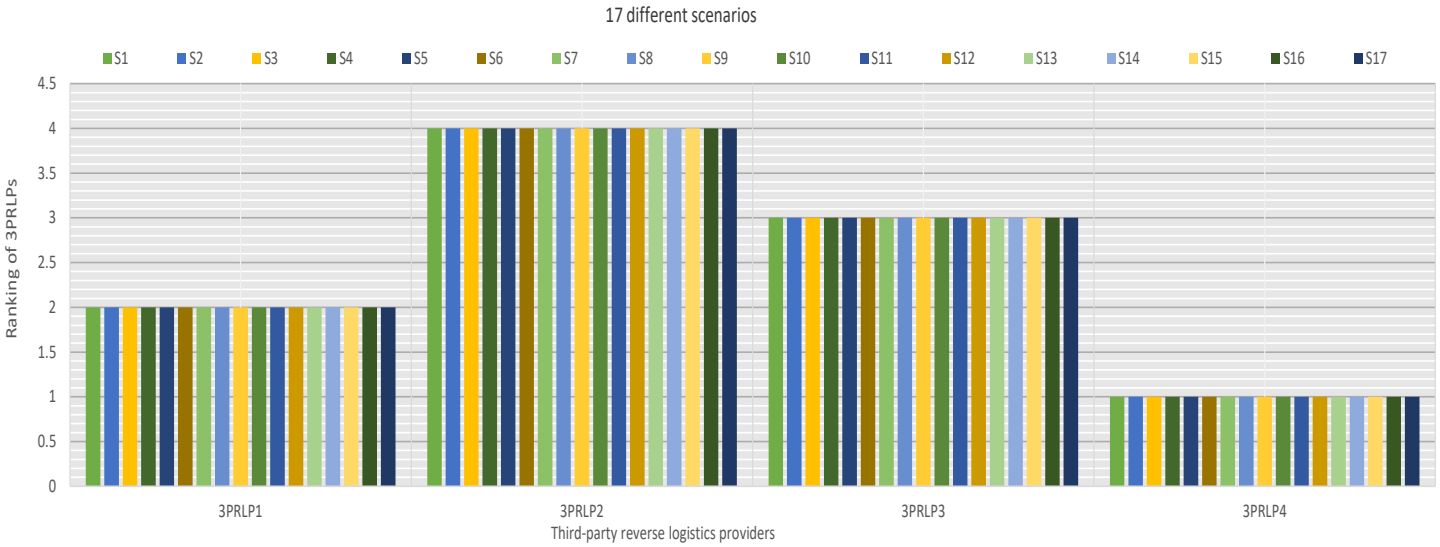


Figure 2: Sensitivity ranking results based on weights variations

derived from linguistic scales but often rely on arbitrary aggregation operators and involve interpretative complexity. The proposed model addresses these challenges by utilizing FFRNs, which represent uncertainty through well-defined upper and lower bounds without requiring additional assumptions. Unlike fuzzy MULTIMOORA [25], which applies expert-defined sustainability weights but encounters difficulties in preference aggregation, the proposed approach offers greater consistency, objectivity, and clarity in evaluations. The resulting rankings are presented in Table 13.

3. Fuzzy rough number (FRN) methods, such as fuzzy rough entropy-REGIME [2], use triangular fuzzy numbers derived from linguistic terms to represent imprecision through defined intervals. However, they offer limited flexibility in capturing varying levels of uncertainty. The proposed model addresses this by introducing rough boundary intervals for both positive and negative membership functions. Rankings based on aggregated performance weights and expert input are presented in Table 13.

4. A comparison is conducted with the entropy-MARCOS method using Pythagorean fuzzy rough numbers (PFRNs) [4], a widely adopted approach in sustainability-oriented decision-making. While PFRNs are more expressive than conventional fuzzy sets, they are less flexible than TFFNs in modeling complex uncertainty. TFFNs provide greater granularity and precision in representing vagueness, resulting in more reliable and interpretable outcomes. The final rankings, based on utility values from ideal and anti-ideal solutions, are shown in Table 13.

The overall ranking results of potential 3PRLPs evaluation for improving food supply chain management in the context of sustainability are displayed in Table 13 and Figure 3.

Table 13: Comparative analysis with existing techniques

3PRLPs	Crisp MULTIMOORA [12]	Fuzzy MULTIMOORA [25]	FR-REGIME [2]	PFR-MARCOS [4]	FFR-MULTIMOORA (proposed)
$3PRLP_1$	2	2	2	2	2
$3PRLP_2$	4	4	4	4	4
$3PRLP_3$	3	3	3	3	3
$3PRLP_4$	1	1	1	1	1

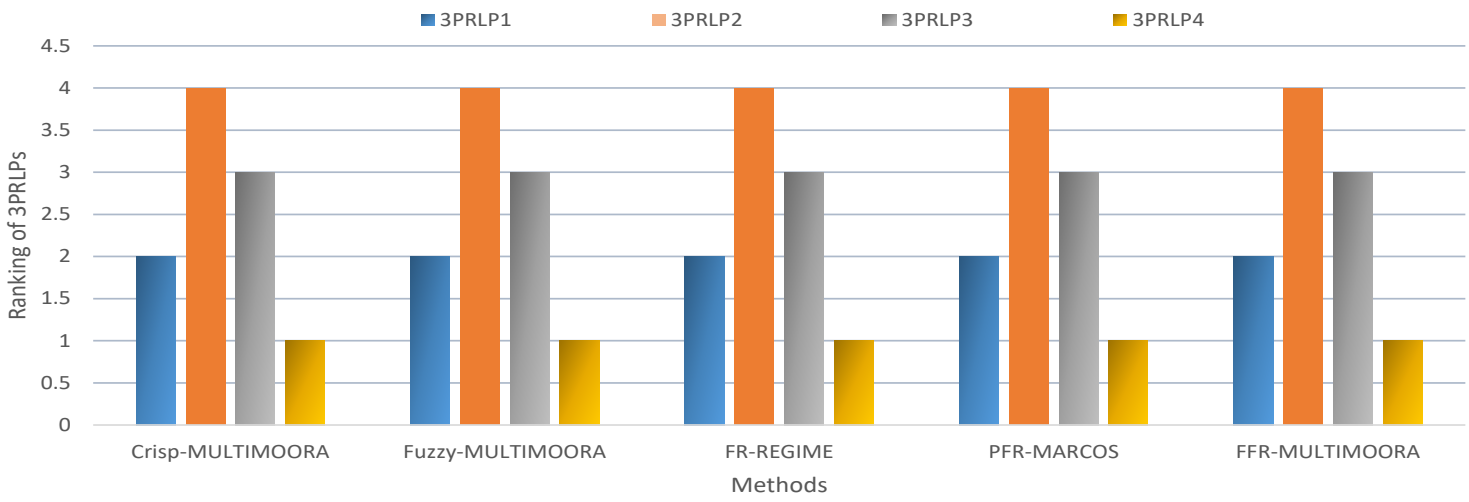


Figure 3: Ranking results obtained by comparative analysis with existing methodologies

5 Conclusions

Rising costs and sustainability demands make reverse logistics vital, prompting firms to rely on third-party providers due to limited internal capabilities. This study has emphasized the importance of evaluating 3PRLPs by addressing the limitations of existing methods in handling uncertainty and subjectivity in Fermatean fuzzy data for optimization modeling. The proposed approach integrates the entropy weight method with the MULTIMOORA technique under a Fermatean fuzzy rough number framework, thereby improving decision-making accuracy in uncertain contexts, reducing subjectivity, and operating without reliance on prior information. A case study on sustainable 3PRLPs in Malaysia's food industry validated the model's practicality, robustness, and superiority over traditional approaches. The findings indicate that the proposed method supports experts in assessing selection criteria more effectively, eliminating unsuitable providers, and identifying the most optimal 3PRLP. From a managerial perspective, this research offers valuable insights and greater flexibility for manufacturing decision-makers and third-party logistics firms. Future research may include stochastic uncertainty, larger datasets, social impact factors, alternative methods, and broader applications like transportation provider selection.

Declaration of competing interest

The authors declare no conflicts of interest.

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