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A hybrid fuzzy modeling framework based on decomposed fuzzy sets and Z-numbers for risk prioritization in air traffic safety with a real case application

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Abstract

The rapid growth of global air traffic increases the complexity of airspace management, especially around risk management. To address related safety challenges, this study presents an integrated risk analysis model consists of Functional Hazard Analysis (FHA), Decomposed Fuzzy Sets (DFS), Z-numbers, and Fuzzy Inference System (FIS). The model systematically accounts for uncertainty in risk parameters and integrates confidences for experts' judgments. DFS assesses the consistency of experts' evaluations, while Z-numbers represent their reliability. Severity, probability, and detectability are evaluated within this fuzzy framework, and risks are classified by using IFS-based approach aligned with the ICAO risk matrix. The model is applied on 11 critical hazard scenarios from the Advanced Surface Movement Guidance and Control System (A-SMGCS) based on high traffic and low visibility. Results obtained confirm the model's ability to identify hazards and prioritize risks, offering a transparent, adaptable, and uncertainty-aware decision-support tool for aviation safety management.

Keywords: Functional hazard analysis, decomposed fuzzy sets, Z-numbers, fuzzy inference system, air traffic safety, uncertainty-based risk assessment, A-SMGCS.

1 Introduction

Air traffic management (ATM) ensures that aircraft operate safely, efficiently, and in an orderly manner. Air traffic controllers play a critical role in maintaining flight safety within controlled airspace by managing aircraft flow and preventing collisions. With increasing flight demand driven by globalization, air traffic density continues to grow, making ATM more critical than ever. Safety analyses are essential for minimizing the risk of incidents and improving system performance. Today's ATM system is still largely human-centered, relying on the coordination of controllers and pilots. However, rising traffic volumes have strained this model's limits [20]. Congestion now leads to increased delays, operational costs, emissions, and safety risks. To address these challenges, modern ATM requires advanced systems that support real-time human–machine collaboration. These systems integrate technologies such as radar, automatic, dependent, surveillance-broadcast (ADS-B), communication infrastructure, and decision-support software, enhancing situational awareness and decision-making. International standards, such as those set by International Civil Aviation Organisation (ICAO), emphasize the need for proactive risk identification through structured safety assessments [10]. Within this context, modeling uncertainty in expert-driven evaluations has become increasingly important. Especially in high-density airspaces such as Turkey's Ankara and Istanbul Flight Information Regions (FIRs), subjective, uncertain, and qualitative data often shape risk-related decisions.

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Traditional methods struggle to fully capture this uncertainty. Therefore, this study adopts a fuzzy logic–based framework enriched with Z-numbers, Decomposed Fuzzy Sets (DFS), and Fuzzy Inference System (FIS) to model expert assessments more effectively. Fuzzy logic enables the representation of linguistic terms (e.g., "high probability", "low probability", "high severity" etc.), while DFS enhances internal consistency by incorporating both positive and negative perspectives [2]. However, classical fuzzy models often overlook reliability for experts' evaluation. To overcome this problem, Z-numbers introduce a confidence measure, and the integration of a FIS allows for systematic reasoning based on predefined rules. As a result, the model delivers more robust and interpretable risk assessments under uncertainty [1]. Figure 1 shows the daily air traffic density in Turkish airspace, as recorded by live flight tracking systems.

Figure 1: Flight Traffic - Working Day (Turkey) [7]

The proposed model integrates DFS and Z-numbers into the Functional Hazard Assessment (FHA) framework, enhancing it with fuzzy logic and confidence-weighted expert input. To demonstrate its applicability, the model is applied to the Advanced Surface Movement Guidance and Control System (A-SMGCS) for evaluating surface movement hazards. The proposed model builds on the FHA approach defined in ICAO's Safety Management Manual (SMM). It is applied to the A-SMGCS to evaluate emerging hazards under high-traffic and low-visibility conditions. This integrated model supports multidimensional, uncertainty-aware risk analysis by incorporating both intuitive expert knowledge and the associated confidence level. Figure 2 illustrates the overall structure of the proposed model.

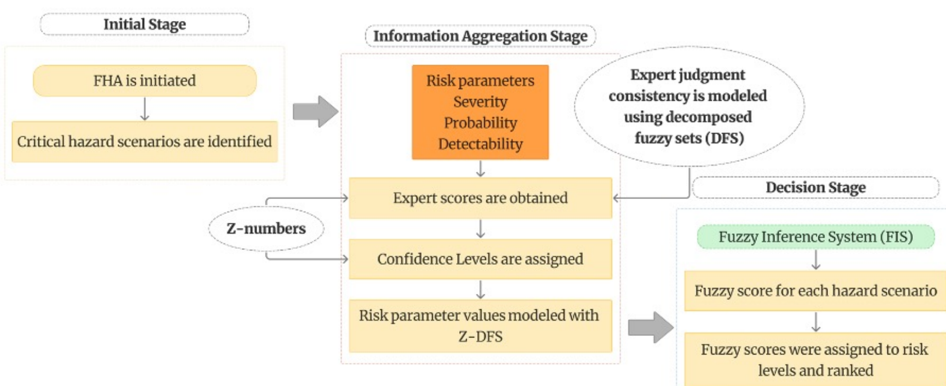

Figure 2: General structure of the proposed FHA method

There are four main different contributions based on our proposed model as follows:

i. The FHA method has been restructured to manage uncertainty of the process. The FHA method defined by ICAO has been extended using DFS and Z-numbers to more realistically model uncertainty, resulting in a multi-dimensional evaluation framework.

ii. Both the consistency and reliability of expert evaluations are considered. While DFS structures assess the consistency of expert judgments, Z-numbers represent their confidence levels, thereby enhancing the robustness of the decision-making process.

iii. An integrated risk analysis model combining quantitative and qualitative evaluation is proposed. Critical risk parameters severity, probability, and detectability are processed using a FIS, producing qualitative risk classifications in alignment with the ICAO risk matrix.

iv. The model's applicability is validated through real-world scenarios. It is tested on 11 critical hazard scenarios within the scope of A-SMGCS, demonstrating its capability to provide consistent and systematic risk prioritization under challenging conditions such as high traffic density and low visibility.

The structure of the paper is organized as follows: Section 2 provides a general evaluation of the current literature and existing studies. Section 3 discusses the evolving importance of air traffic management, ground movements, and the associated risks in terms of system safety. Section 4 presents an overview of system risk analysis, highlighting critical components that should be considered in the evaluation process. Section 5 introduces the proposed risk assessment methodology, which is based on DFS integration with FHA and enhanced through the integration of Z-numbers. Section 6 demonstrates the applicability of the model through a real case study conducted in Turkey, and the results are interpreted accordingly. Finally, the results and the conclusions are discussed in the last section.

2 A literature analysis on fuzzy methodologies in air traffic management systems

The steady increase in global flight volume has driven the need for more advanced and integrated ATM systems. Ensuring safe, orderly, and efficient air traffic has long been a core concern of academic research. The literature includes numerous completed and ongoing studies addressing various aspects of ATM, such as controller decision-making, system integration, human factors, and safety management systems. However, the continuous growth of global air travel further intensifies the complexity and urgency of effective ATM solutions [3]. This section presents a focused review of studies that use fuzzy methodologies in the design and enhancement of ATM systems.

For this aim, recent literature is systematically reviewed, with particular emphasis on air traffic safety, risk assessment, and uncertainty modeling. Special attention is given to the growing use of alternative techniques such as fuzzy logic, Functional Resonance Analysis Method (FRAM), Petri nets, decision support systems, and AI-based models in areas where conventional methods fall short. The selected studies include both theoretical frameworks and practical applications grounded in real-world scenarios. This dual perspective contributes to establishing an interdisciplinary basis for developing decision-support mechanisms in ATM. Table 1 summarizes the reviewed studies by publication year, methodology, and application domains.

Most existing studies on ATM focus on operational performance, addressing issues such as delay reduction, schedule optimization, and airspace capacity through mathematical and heuristic methods. However, these approaches often overlook the systematic assessment of safety risks under uncertainty particularly those related to weather, technical failures, and human factors. Additionally, the integration of ICAO's Safety Management System (SMS) into air traffic flow management remains limited in the literature. This study addresses these gaps by introducing a safety-oriented methodology applied to A-SMGCS. While previous research on A-SMGCS has largely emphasized system performance and sensor technologies, few studies have investigated risks stemming from human machine interaction and operational uncertainty especially under low visibility or high traffic density. The proposed model builds on the fuzzy based FHA approach, enhanced with Z-numbers and DFS, and employs FIS to perform rule-based risk evaluation. This integrated framework has an important capability to capture uncertainty and expert confidence, enabling early and systematic hazard identification. As a result, it offers a robust and interpretable decision-support tool for safety-critical ATM environments.

Table 1: Comparison of Related Studies Based on Methods and Application Areas

Related Study	**Method**	**Application Area**
C. Chen et al. [3]	Fermatean Fuzzy Z-Numbers, Muirhead mean operator	Failure Mode and Effect Analysis (FMEA), Risk Assessment
Lozano Tafur et al. [25]	Machine Learning, Deep Learning, Recurrent neural networks (RNN), Long Short-Term Memory (LSTM), Explainable AI (XAI)	Air Operations, Trajectory Prediction, Safety Analysis
Verma et al. [1]	Optimization Methods, Bibliometric Analysis, VOSviewer	Air Traffic Flow Management (ATFM), Delay Reduction, Capacity Planning
Y. Chen et al. [4]	Optimization, Bibliometric Analysis	ATFM, Capacity-Balance Problem, Traffic Forecasting
A. Kwasiborska and A. Stelmach [15]	Bow Tie, Event Tree Analysis (ETA), Event tree with fuzzy probabilities method (ETFP), Ishikawa Diagram, Risk Analysis Tool (RAT), Functional Resonance Analysis Method (FRAM), Human Factors Analysis and Classification System (HFACS)	Air Transport Safety, Unmanned aircraft (UAV) Operations, Ground Handling Risk Analysis
J. Tang et al. [26]	Game Theory, Cloud Matter Element Analysis	Air Traffic Control Systems, Safety Risk Assessment
E. Dudek and K. Krzykowska-Piotrowska [5]	FMEA, Risk Priority Number (RPN)	Free Route Airspace, Air Traffic Safety, Poland Flexible Airspace Management and Free Route (POLFRA)
P. Rutkowska et al. [23]	Functional Resonance Analysis Method (FRAM), Colored Petri Nets (CPN)	Aerodrome Ground Traffic, Incident Analysis, Safety Management
D. A. Pamplona and C. J. P. Alves [21]	Gamma-Poisson Distribution, RPN, Statistical Risk Analysis	Military Aviation, The United States Air Force (USAF), Fighter Aircraft Risk Assessment
W. Kaleta and J. Skorupski [13]	Hierarchical FIS, Expert System, Simulation	Localizer Performance with Vertical guidance (LPV-200) Approach Procedures, Small Airports, Controlled Flight into Terrain (CFIT) Risk Assessment
A. Volpe Lovato et al. [28]	Fuzzy Logic, Mamdani Model, Simulation	Air Traffic Conflict Management, Decision Support
J. Skorupski [24]	Fuzzy Risk Matrix, Petri Nets, Expert Opinion	Air Traffic Safety, Risk Assessment
M. Lower et al. [17]	Event Trees, Fuzzy Probabilities, Sub-Set Criteria	Air Traffic Incidents, Risk Analysis, Safety Assessment
A. Florowski and J. Skorupski [8]	Colored Stochastic Petri Nets, Simulation, Buffer Time Analysis	Airport Vicinity Traffic Flow Management, Landing Sequence Quality
M. Lower et al. [16]	Event Trees, Fuzzy Probability, Jaccard Similarity, Trapezoidal Membership Function	Air Traffic Incident Analysis, Human Factors, Risk Assessment
B. Kang et al. [14]	Z-number, Fuzzy Expectation, Fuzzy Set, Decision-making	Integration of Z-Number Systems with Classical Fuzzy Systems
L. Meyer et al. [18]	FHA, Euro Control Safety Assessment Methodology (SAM)	Virtual Control Towers, Visual Information Replacement

3 The role of airport ground movements in air traffic management

Ground movements covering aircraft maneuvers between aprons, taxiways, and runways are a critical component of airport operations. Their effective management directly contributes to flight safety by preventing collisions and ensuring rapid emergency response. From an efficiency standpoint, optimized taxi and sequencing reduce delays, fuel consumption, and operating costs. Given their importance, the safe and efficient coordination of ground movements forms a fundamental part of modern air traffic control systems. Challenges such as low visibility, traffic congestion, and operational errors complicate this process. To address these, A-SMGCS has been developed as an integrated solution. The system combines subsystems such as surface surveillance, routing, conflict detection, and decision support to enhance both safety and performance [9]. A-SMGCS is structured around four service modules: Surveillance, Airport Safety Support, Routing, and Guidance. These modules enable real-time aircraft tracking, conflict alerting, optimal taxi routing, and automated visual guidance, contributing significantly to situational awareness and controller effectiveness. The Controller Working Position (CWP), an ergonomically designed human–machine interface, allows controllers to manage movements using real-time data and electronic instruction entry tools, supporting both operational reliability and reduced human error [6]. As a safety-critical system, A-SMGCS must comply with international safety standards. According to the 2024 ICAO Safety Report, five high-risk incident categories have been prioritized, including Runway Incursions (RI) and Runway Excursions (RE), which A-SMGCS can directly or indirectly help prevent [11]. Malfunctions in the system may lead to severe incidents, highlighting the need for robust risk evaluation. Accordingly, this study aims to assess the effectiveness of A-SMGCS in mitigating operational risks on the airport surface. A fuzzy logic–based model is proposed to systematically analyze uncertainty in complex ground movement scenarios, offering an interpretable and safety-focused decision-support framework.

4 Assessment of system risk analysis

The safe and efficient management of airport ground movements depends on the performance of advanced systems such as A-SMGCS, which provides decision support to air traffic controllers. However, complexities arising from environmental conditions, technical limitations, and human factors introduce significant uncertainties. Therefore, evaluating A-SMGCS requires both technical performance assessment and systematic risk analysis. Hazard identification forms the basis of effective safety risk management. Misclassifying hazards or conflating them with risks may lead to serious consequences. In aviation, a hazard refers to any condition that may compromise the safety or functionality of aircraft, systems, or services. Given the high-risk nature of aviation operations, these hazards must be proactively addressed through well-defined mitigation strategies [12]. Conventional risk assessments typically consider the probability and severity of events, structured using risk matrices. This study introduces detectability as a third dimension, reflecting how easily a hazard can be identified before it occurs. Detectability is particularly critical in automated systems like A-SMGCS, where early detection of faults or human error is essential to safety. Unlike traditional models, this study explicitly incorporates uncertainty into the assessment process. In complex systems where many parameters cannot be precisely defined, a fuzzy logic–based approach enables the integration of expert judgment and qualitative data, improving both realism and adaptability. To extend the classical ICAO SMS standards, the proposed model introduces a comprehensive safety risk assessment framework that incorporates both detectability and uncertainty into the evaluation process. A dual-layered fuzzy approach is applied, combining DFS and Z-numbers, and utilizing a FIS for rule-based risk evaluation. DFS captures both optimistic and pessimistic dimensions of expert input, while Z-numbers reflect the associated confidence levels. The FIS processes these fuzzy and confidence-weighted inputs through predefined inference rules to generate consistent, interpretable risk scores. This integrated methodology enables a more flexible, credible, and operationally relevant framework for risk analysis in complex air traffic environments.

The fuzzy logic-based reinterpretation of severity values through decomposed Z-fuzzy sets is presented in Table 2. Each severity level is expressed using linguistic restriction terms enriched with optimistic and pessimistic evaluations, as well as corresponding reliability parameters. Figures 3 and 4 provide a graphical representation of these severity levels, illustrating the gradual transition and overlap among categories in accordance with fuzzy set theory.

Table 2: Membership and Non-Membership Values of Severity in DFS Framework

Linguistic Term	μ **(Optimistic)**	ϑ **(Optimistic)**	μ **(Pessimistic)**	ϑ **(Pessimistic)**	**Reliability**
Absolutely Low (AL)	(0.1, 0.2, 0.3)	(0.5, 0.55, 0.6)	(0.2, 0.3, 0.4)	(0.4, 0.5, 0.6)	(0.45, 0.50, 0.55)
Very Low (VL)	(0.2, 0.3, 0.4)	(0.4, 0.5, 0.6)	(0.3, 0.4, 0.5)	(0.3, 0.4, 0.5)	(0.55, 0.60, 0.65)
Low (L)	(0.3, 0.4, 0.5)	(0.3, 0.4, 0.5)	(0.4, 0.5, 0.6)	(0.2, 0.3, 0.4)	(0.65, 0.70, 0.75)
Moderate (M)	(0.4, 0.5, 0.6)	(0.2, 0.3, 0.4)	(0.5, 0.6, 0.7)	(0.1, 0.2, 0.3)	(0.75, 0.80, 0.85)
High (H)	(0.5, 0.6, 0.7)	(0.1, 0.2, 0.3)	(0.6, 0.7, 0.8)	(0.0, 0.1, 0.2)	(0.85, 0.90, 0.95)
Very High (VH)	(0.6, 0.7, 0.8)	(0.0, 0.1, 0.2)	(0.7, 0.8, 0.9)	(0.0, 0.05, 0.1)	(0.95, 1.00, 1.00)
Catastrophic (C)	(0.7, 0.8, 0.9)	(0.0, 0.05, 0.1)	(0.8, 0.9, 0.95)	(0.0, 0.0, 0.05)	(0.95, 1.00, 1.00)

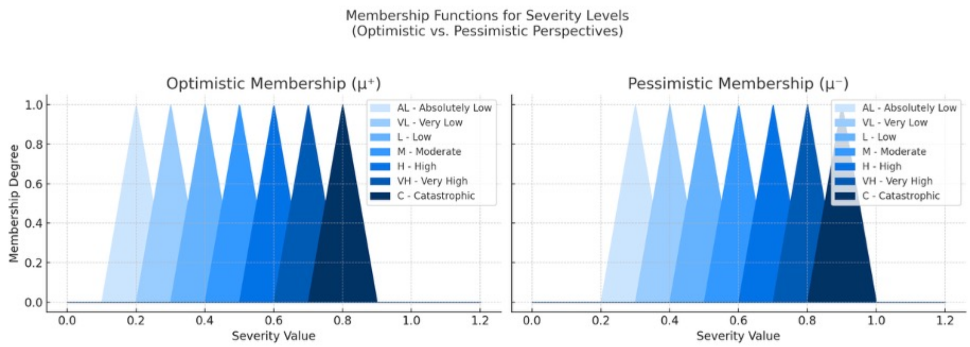

Figure 3: Membership Functions for the Severity Dimension Based on Decomposed Z-Fuzzy Modeling

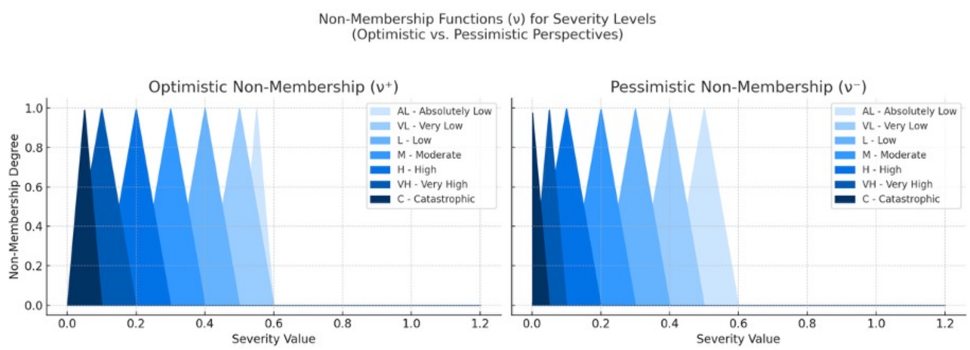

Figure 4: Non-Membership Functions for the Severity Dimension Based on Decomposed Z-Fuzzy Modeling

In this study, traditional probability classifications based on ICAO standards have been restructured by using the fuzzy set theory (FST). To flexibly and gradually evaluate the uncertainty associated with the likelihood of event occurrences, each probability level was modeled using triangular fuzzy numbers (TFNs). This method allows for a more realistic representation of transitions between probability levels and enables a more nuanced treatment of boundary uncertainties.

Table 3: Membership and Non-Membership Values of Probability in DFS Framework

Level	**Term**	μ **(Opt.)**	ϑ **(Opt.)**	μ **(Pes.)**	ϑ **(Pes.)**	**Reliability**
A	Very Frequent	(0.6, 0.7, 0.8)	(0.1, 0.15, 0.2)	(0.8, 0.9, 0.95)	(0.0, 0.0, 0.05)	(0.95, 1.00, 1.00)
B	Occasional	(0.5, 0.6, 0.7)	(0.15, 0.2, 0.25)	(0.7, 0.8, 0.9)	(0.05, 0.1, 0.10)	(0.85, 0.90, 0.95)
C	Rare	(0.3, 0.4, 0.5)	(0.25, 0.3, 0.35)	(0.5, 0.6, 0.7)	(0.1, 0.2, 0.3)	(0.75, 0.80, 0.85)
D	Very Rare	(0.2, 0.3, 0.4)	(0.3, 0.35, 0.4)	(0.4, 0.5, 0.6)	(0.2, 0.25, 0.3)	(0.65, 0.70, 0.75)
E	Almost Never	(0.1, 0.2, 0.3)	(0.4, 0.45, 0.5)	(0.3, 0.4, 0.5)	(0.3, 0.35, 0.4)	(0.45, 0.50, 0.55)

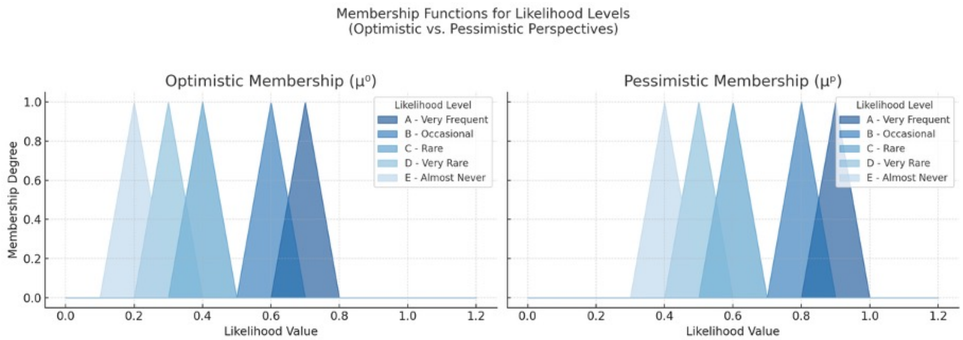

Figure 5: Optimistic and Pessimistic Membership Functions

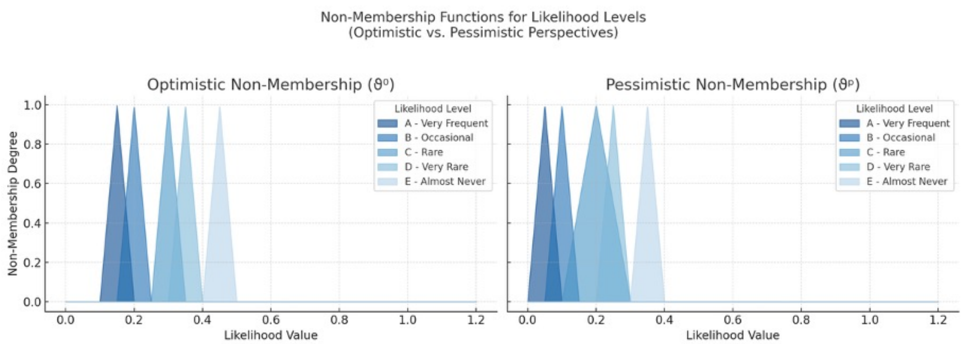

Figure 6: Optimistic and Pessimistic Non-Membership Functions

In this study, the detectability of each potential hazard is also modeled using TFNs and categorized into five linguistic levels, including "Difficult," "Medium," "Easy," and intermediate levels. This classification enables the integration of not only the presence of hazards but also the difficulty of their detection into the overall risk analysis process. These linguistic detectability levels are detailed in Table 4, while their graphical fuzzy representation is provided in Figure 7.

Table 4: Z-number Based Representation of Detectability Levels

Level	**μ (Opt.)**	**ϑ (Opt.)**	**μ (Pes.)**	**ϑ (Pes.)**	**Reliability**
Very Easy	(0.1, 0.2, 0.3)	(0.55, 0.60, 0.70)	(0.2, 0.3, 0.4)	(0.45, 0.50, 0.60)	(0.45, 0.50, 0.55)
Easy	(0.2, 0.3, 0.4)	(0.45, 0.50, 0.60)	(0.3, 0.4, 0.5)	(0.35, 0.45, 0.50)	(0.65, 0.70, 0.75)
Medium	(0.35, 0.5, 0.6)	(0.30, 0.35, 0.40)	(0.45, 0.6, 0.7)	(0.20, 0.25, 0.30)	(0.75, 0.80, 0.85)
Difficult	(0.5, 0.6, 0.7)	(0.20, 0.25, 0.30)	(0.6, 0.7, 0.8)	(0.10, 0.15, 0.20)	(0.85, 0.90, 0.95)
Very Difficult	(0.6, 0.7, 0.8)	(0.10, 0.15, 0.20)	(0.7, 0.8, 0.9)	(0.05, 0.08, 0.10)	(0.95, 1.00, 1.00)

In this study, the three fundamental components of risk; severity, probability, and detectability are modeled using TFNs within the DFS framework. This approach incorporates uncertainty and the reliability of expert judgment, offering a significant improvement over traditional risk assessment methods.

Unlike standard fuzzy models that use single membership functions, the DFS methodology decomposes each fuzzy number into optimistic and pessimistic perspectives, providing a more nuanced and cognitively aligned representation of subjective evaluations. By integrating Z-numbers, the model also reflects the confidence level of expert opinions not just what they believe, but how strongly they believe it.

A Fuzzy Inference System models the logical relationships between risk components through fuzzy rules, replacing the classical risk matrix. This rule-based structure enables flexible and interpretable risk evaluation without relying on fixed thresholds.

Overall, the integrated framework enhances both technical accuracy and practical relevance, offering a robust decision-support tool for safety-critical environments characterized by ambiguity and expert judgment.

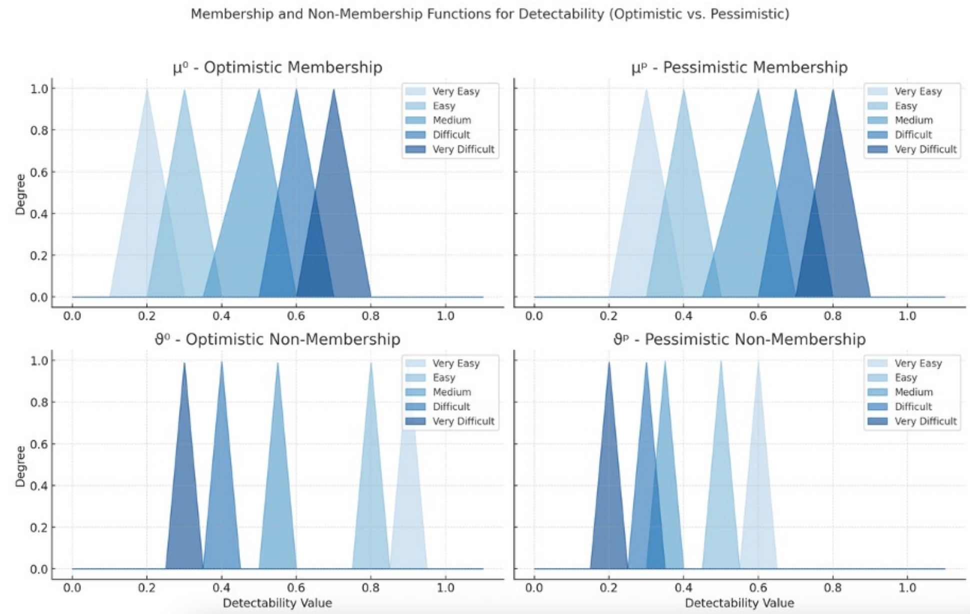

Figure 7: Membership and Non-Membership Functions for Detectability Based on Optimistic and Pessimistic Perspectives

5 Preliminaries

This section briefly outlines the core concepts of Z-fuzzy numbers and the DFS method, both used in the literature to support decision-making under uncertainty. Based on these, an integrated model is employed that combines optimistic and pessimistic membership functions with a reliability measure for experts' evaluations.

5.1 Ordinary Z-fuzzy numbers

Zadeh [29] introduced Z-fuzzy numbers as an extension of classical fuzzy numbers, allowing reliability information to be incorporated alongside uncertain linguistic evaluations. Z-fuzzy number is defined as an ordered pair of fuzzy numbers:

$$Z = (\widetilde{A}, \widetilde{R}),$$

where $\widetilde{A}$ represents the restriction part (i.e., the fuzzy assessment), and $\widetilde{R}$ represents the associated reliability of that assessment. This concept enables the modeling of expert judgments not only in terms of what is believed (restriction), but also in terms of how confidently it is believed (reliability), making it particularly valuable in uncertain environments such as risk assessment and decision analysis.

5.2 Decomposed fuzzy sets

In this section, we briefly present the basic definitions and conceptual structure of the DFS approach, which was originally proposed to enhance the expressive power of fuzzy models by incorporating both optimistic ($\mathcal{O}$) and pessimistic ($\mathcal{P}$) perspectives, along with the reliability of expert evaluations in the form of Z-numbers [2].

Definition 5.1 (DZFS [14, 19, 27])**.** *A DZFS is a structure of the form:*

$$\widetilde{A} = \left\{x, \left(\mathcal{O}\left(\mu^{\mathcal{O}}(x), \vartheta^{\mathcal{O}}(x)\right), \mathcal{P}\left(\mu^{\mathcal{P}}(x), \vartheta^{\mathcal{P}}(x)\right)\right)\right\}, \tag{1}$$

where the four components represent membership and non-membership degrees under each perspective. It must satisfy the following conditions:

$$0 < \mu^{\mathcal{O}}_{\widetilde{A}} + \vartheta^{\mathcal{O}}_{\widetilde{A}} \leq 1, \quad 0 < \mu^{\mathcal{P}}_{\widetilde{A}} + \vartheta^{\mathcal{P}}_{\widetilde{A}} \leq 1,$$

and the inconsistency in the judgment is given by:

$$I^{A} = 1 - \left(\mu^{\mathcal{O}}_{\widetilde{A}} + \vartheta^{\mathcal{O}}_{\widetilde{A}} + \mu^{\mathcal{P}}_{\widetilde{A}} + \vartheta^{\mathcal{P}}_{\widetilde{A}}\right),$$

where:

$$-1 \leq I^{A} \leq 1, \quad 0 \leq \mu_{\widetilde{A}}^{\mathcal{O}} + \vartheta_{\widetilde{A}}^{\mathcal{O}} + \mu_{\widetilde{A}}^{\mathcal{P}} + \vartheta_{\widetilde{A}}^{\mathcal{P}} \leq 2.$$

5.3 Decomposed Z-fuzzy numbers

Decomposed Z-fuzzy numbers are used in this study to represent expert evaluations by combining DFS-based restrictions with their corresponding decomposed fuzzy reliabilities. Expert opinions are collected through both functional (positive) and dysfunctional (negative) questions, creating a bipolar and reliability-sensitive evaluation structure.

This two-layer modeling approach addresses a common issue in fuzzy systems: inconsistent assignment of membership and non-membership values. The decomposed Z-fuzzy model allows uncertainty and confidence to be represented together, which is the key motivation for its use.

Although this method increases computational complexity, it offers richer semantic representation and improves interpretability compared to traditional fuzzy models. These advantages make it suitable for safety-critical analyses such as FHA.

Decomposed Z-fuzzy number $\widetilde{Z}_{DF} = (\widetilde{A}, \widetilde{R})$ is an ordered pair of fuzzy numbers, where both the fuzzy restriction and fuzzy reliability functions are defined by decomposed fuzzy numbers. Let the decomposed fuzzy restriction function be defined as [27]:

$$\widetilde{A} = \left(\mathcal{O}\left(\mu_{\widetilde{A}}^{\mathcal{O}}(x), \vartheta_{\widetilde{A}}^{\mathcal{O}}(x)\right), \mathcal{P}\left(\mu_{\widetilde{A}}^{\mathcal{P}}(x), \vartheta_{\widetilde{A}}^{\mathcal{P}}(x)\right)\right),$$

and the decomposed fuzzy reliability function as [27]:

$$\widetilde{R} = \left(\mathcal{O}\left(\mu_{\widetilde{R}}^{\mathcal{O}}(x), \vartheta_{\widetilde{R}}^{\mathcal{O}}(x)\right), \mathcal{P}\left(\mu_{\widetilde{R}}^{\mathcal{P}}(x), \vartheta_{\widetilde{R}}^{\mathcal{P}}(x)\right)\right),$$

where the membership and non-membership degrees under optimistic ($\mathcal{O}$) and pessimistic ($\mathcal{P}$) perspectives are represented by triangular fuzzy numbers. A general representation of the decomposed Z-fuzzy number is given in Eq. (2) [27].

Definition 5.2 (Decomposed Z-Fuzzy Number)**.** *A decomposed Z-fuzzy number $\widetilde{Z}_{DF}(\widetilde{A}, \widetilde{R})$ is defined as:*

$$\widetilde{Z}_{DF} = \Big(\big(\mathcal{O}(\mu_{\widetilde{A}}^{\mathcal{O}}(x), \vartheta_{\widetilde{A}}^{\mathcal{O}}(x)), \mathcal{P}(\mu_{\widetilde{A}}^{\mathcal{P}}(x), \vartheta_{\widetilde{A}}^{\mathcal{P}}(x))\big), \big(\mathcal{O}(\mu_{\widetilde{R}}^{\mathcal{O}}(x), \vartheta_{\widetilde{R}}^{\mathcal{O}}(x)), \mathcal{P}(\mu_{\widetilde{R}}^{\mathcal{P}}(x), \vartheta_{\widetilde{R}}^{\mathcal{P}}(x))\big)\Big), \tag{2}$$

where $\mu_{\widetilde{A}}^{\mathcal{O}} : X \to [0,1]$ and $\vartheta_{\widetilde{A}}^{\mathcal{O}} : X \to [0,1]$ are the membership and non-membership degrees of x with respect to the optimistic view of the restriction function, and similarly, $\mu_{\widetilde{A}}^{\mathcal{P}}, \vartheta_{\widetilde{A}}^{\mathcal{P}}$ represent the pessimistic view. For the reliability function, $\mu_{\widetilde{R}}^{\mathcal{O}}, \vartheta_{\widetilde{R}}^{\mathcal{O}}, \mu_{\widetilde{R}}^{\mathcal{P}}, \vartheta_{\widetilde{R}}^{\mathcal{P}}$ are defined in the same way. The following conditions must hold:

$$\begin{aligned} 0 < \mu_{\widetilde{A}}^{\mathcal{O}} + \vartheta_{\widetilde{A}}^{\mathcal{O}} \leq 1, \quad 0 < \mu_{\widetilde{A}}^{\mathcal{P}} + \vartheta_{\widetilde{A}}^{\mathcal{P}} \leq 1, \\ 0 < \mu_{\widetilde{R}}^{\mathcal{O}} + \vartheta_{\widetilde{R}}^{\mathcal{O}} \leq 1, \quad 0 < \mu_{\widetilde{R}}^{\mathcal{P}} + \vartheta_{\widetilde{R}}^{\mathcal{P}} \leq 1. \end{aligned}$$

Inconsistencies in the judgments are computed by:

$$\begin{aligned} I^{\widetilde{A}} &= 1 - \left(\mu_{\widetilde{A}}^{\mathcal{O}} + \vartheta_{\widetilde{A}}^{\mathcal{O}} + \mu_{\widetilde{A}}^{\mathcal{P}} + \vartheta_{\widetilde{A}}^{\mathcal{P}}\right), \\ I^{\widetilde{R}} &= 1 - \left(\mu_{\widetilde{R}}^{\mathcal{O}} + \vartheta_{\widetilde{R}}^{\mathcal{O}} + \mu_{\widetilde{R}}^{\mathcal{P}} + \vartheta_{\widetilde{R}}^{\mathcal{P}}\right), \end{aligned}$$

with the bounds:

$$\begin{aligned} -1 \leq I^{\widetilde{A}} \leq 1, \quad -1 \leq I^{\widetilde{R}} \leq 1, \\ 0 \leq \mu_{\widetilde{A}}^{\mathcal{O}} + \vartheta_{\widetilde{A}}^{\mathcal{O}} + \mu_{\widetilde{A}}^{\mathcal{P}} + \vartheta_{\widetilde{A}}^{\mathcal{P}} \leq 2, \\ 0 \leq \mu_{\widetilde{R}}^{\mathcal{O}} + \vartheta_{\widetilde{R}}^{\mathcal{O}} + \mu_{\widetilde{R}}^{\mathcal{P}} + \vartheta_{\widetilde{R}}^{\mathcal{P}} \leq 2. \end{aligned}$$

A decomposed triangular Z-fuzzy number is represented in Figure 8.

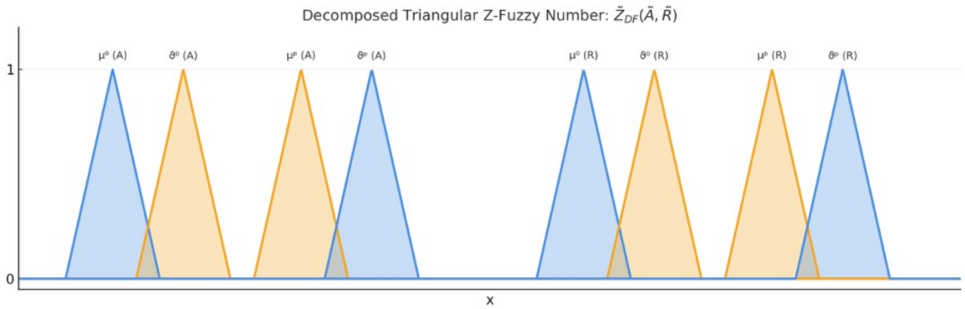

Figure 8: Graphical Representation of a Decomposed Triangular Z-Fuzzy Number

5.4 Consistency and scoring of decomposed Z-fuzzy numbers

The Consistency Index (CI) is employed to evaluate the reliability of expert-provided fuzzy inputs, while the Score Index (SI) is used to obtain crisp values for prioritization. These measures support the FIS by ensuring that only consistent and representative fuzzy information contributes to the final risk-based decision making.

$$CI\left(\widetilde{A}\right) = 1 - \sqrt{\frac{(a-d)^2 + (b-c)^2 + (1-a-b)^2 + (1-c-d)^2}{2}}.$$

For more reliable decisions, the value of CI should be close to 1 where 1 means perfectly consistent [22]. Once the consistency is ensured, the defuzzified score of the fuzzy number is calculated using the Score Index as follows:

$$SI\left(\widetilde{A}\right) = \begin{cases} (a+b+c+d) \cdot CI(A)^{2k}, & \text{if } CI(A) > 0 \\ 0, & \text{otherwise} \end{cases}$$

Here, k is a linguistic scaling factor (typically $k = 0.90$ or 0.95).

6 The proposed decomposed Z-numbers based fuzzy FHA methodology

In this study, we propose an FHA methodology enhanced with decomposed Z-fuzzy sets to better capture the uncertainty and subjectivity inherent in expert evaluations. The process starts with identifying system functions and their associated functional hazards. Evaluation criteria such as severity, probability, and detectability are defined using linguistic terms. Expert judgments are expressed as Z-numbers, which encapsulate both the evaluation and the expert's confidence. These are then decomposed into optimistic and pessimistic fuzzy components, enabling more nuanced modeling. The resulting fuzzy values are defuzzified into crisp scores, which are fed into a Fuzzy Inference System to compute a risk index for each hazard. By integrating both the content and the reliability of expert input, the proposed framework offers a robust and flexible tool for early-stage safety analysis in complex systems. A flowchart of the method is provided in Figure 9.

7 A case study

Within the scope of this study, the A-SMGCS; designed to ensure the safe and efficient management of surface movements and to optimize airport operations is selected as the application domain. A-SMGCS is a safety-critical system that plays a central role in surface movement control within air traffic management and may lead to serious incidents in the event of a malfunction. Therefore, comprehensive safety evaluations and risk analyses are essential prior to its deployment.

To address these needs, the study presents an application of an integrated risk assessment model aimed at managing uncertainty in air traffic systems and supporting air traffic controllers in making timely and accurate decisions under high traffic load and time pressure. According to ICAO Doc 9830, the functional scope of A-SMGCS is defined under five main categories, as summarized in Table 5.

To contribute to the risk analysis of the A-SMGCS system, potential hazards related to its five core functions are systematically identified. The scope of the analysis is based on the functional structure outlined in the A-SMGCS

Table 5: Main Functional Components of A-SMGCS [9]

Code	Function
F1	Surveillance Service
F2	Airport Safety Support Service
F3	Routing Service
F4	Guidance Service
F5	Controller Working Position

Manual [9], published by ICAO. The hazard identification process incorporates both system-level and operational considerations, ensuring a comprehensive integration of theoretical approaches with real-world practice.

Table 6: Functions and Associated Hazards

Function	Hazard Code	Hazard Description
F1	F1.1	Inaccurate or incomplete surveillance of moving objects
F1	F1.2	Incorrect identification of targets
F2	F2.1	Late or missing alerts from Runway Monitoring and Conflict Alert (RMCA)
F2	F2.2	Failure to detect conflicts using Conflicting ATC Clearances (CATC) function
F2	F2.3	Incorrect compliance detection by Conformance Monitoring for ATC Clearances (CMAC)
F3	F3.1	Incorrect or suboptimal route calculation
F4	F4.1	Incorrect guidance by taxiway centerline lighting system
F4	F4.2	Incorrect activation or deactivation of stop bars
F4	F4.3	Malfunction or misguidance by Advanced Visual Docking Guidance System (A-VDGS)
F5	F5.1	Incorrect or misleading information on Human-Machine Interface (HMI)
F5	F5.2	Incorrect or failed input via Electronic Clearance Input (ECI)

Figure 9: Flowchart of the proposed decomposed Z-fuzzy FHA methodology

To assess the risk levels associated with the identified hazards in the A-SMGCS system, expert evaluations are collected for each hazard based on three parameters: Severity, Occurrence, and Detectability. These evaluations are expressed using predefined linguistic terms, each accompanied by a confidence level to enable modeling through Z-number-based fuzzy sets. Tables 7, 8, and 9 present the expert inputs in terms of linguistic assessments and their associated confidence degrees, forming the basis for the subsequent Z-number-based decomposed fuzzy risk analysis.

Table 7: Expert Severity Assessments (Optimistic and Pessimistic) with Reliability Levels

Hazard Code	$\mathcal{O}$	**Reliability**	$\mathcal{P}$	**Reliability**
F1.1	High	High	Catastrophic	High
F1.2	High	High	Catastrophic	Moderate
F2.1	Moderate	High	Catastrophic	High
F2.2	Absolutely Low	High	Moderate	High
F2.3	Moderate	High	Catastrophic	Moderate
F3.1	High	High	Catastrophic	High
F4.1	High	Moderate	Catastrophic	Moderate
F4.2	Moderate	High	Catastrophic	High
F4.3	Moderate	High	Very High	High
F5.1	Moderate	High	Very High	High
F5.2	High	High	Very High	High

Table 8: Expert Probability Assessments (Optimistic and Pessimistic) with Reliability Levels

Hazard Code	$\mathcal{O}$	**Reliability**	$\mathcal{P}$	**Reliability**
F1.1	Almost Never	High	Very Rare	High
F1.2	Very Rare	High	Rare	High
F2.1	Almost Never	Moderate	Rare	Moderate
F2.2	Almost Never	High	Rare	High
F2.3	Almost Never	High	Rare	High
F3.1	Almost Never	High	Rare	Moderate
F4.1	Almost Never	High	Rare	High
F4.2	Almost Never	High	Rare	Moderate
F4.3	Almost Never	High	Rare	Moderate
F5.1	Almost Never	High	Rare	Moderate
F5.2	Almost Never	High	Rare	High

Table 9: Expert Detectability Assessments (Optimistic and Pessimistic) with Reliability Levels

Hazard Code	$\mathcal{O}$	**Reliability**	$\mathcal{P}$	**Reliability**
F1.1	Easy	Moderate	Very Difficult	High
F1.2	Easy	High	Very Difficult	Moderate
F2.1	Easy	High	Difficult	Moderate
F2.2	Easy	High	Easy	High
F2.3	Medium	Moderate	Difficult	Moderate
F3.1	Medium	Moderate	Medium	High
F4.1	Medium	High	Medium	High
F4.2	Easy	High	Medium	High
F4.3	Easy	High	Difficult	High
F5.1	Easy	High	Very Difficult	High
F5.2	Easy	High	Difficult	High

In the initial assessment stage, expert evaluations for each hazard concerning the Severity criterion are collected using linguistic terms (e.g., High, Moderate, Catastrophic), separately from both optimistic ($\mathcal{O}$) and pessimistic ($\mathcal{P}$) perspectives. Each linguistic evaluation is accompanied by an expert-defined reliability level that reflects the confidence in the corresponding judgment. These linguistic terms are then mapped to predefined triangular fuzzy numbers (TFNs), resulting in a decomposed fuzzy representation for each input.

For each hazard, the membership (μ) and non-membership (ϑ) functions are numerically quantified under both perspectives. Additionally, the optimistic and pessimistic reliability values ($\alpha_{\mathcal{O}}$ and $\alpha_{\mathcal{P}}$) are incorporated into the model to reflect confidence within the inference system.

Table 10 presents the fuzzy representations of the Severity criterion for all hazards, including the decomposed TFNs and their associated reliability levels. This transformation enables a systematic shift from linguistic assessments to computational reasoning within the FIS. The same procedure is applied to the Probability and Detectability criteria to ensure consistency across all input dimensions.

Table 10: Decomposed Z-Fuzzy Representation of Severity Assessments

Hazard Code	$\mathcal{O}$	$\mathcal{P}$	O-Reliability	P-Reliability
F1.1	$((0.5, 0.6, 0.7), (0.1, 0.2, 0.3))$	$((0.8, 0.9, 1.0), (0.0, 0.0, 0.05))$	$(0.85, 0.90, 0.95)$	$(0.85, 0.90, 0.95)$
F1.2	$((0.5, 0.6, 0.7), (0.1, 0.2, 0.3))$	$((0.8, 0.9, 1.0), (0.0, 0.0, 0.05))$	$(0.85, 0.90, 0.95)$	$(0.75, 0.80, 0.85)$
F2.1	$((0.4, 0.5, 0.6), (0.2, 0.3, 0.4))$	$((0.8, 0.9, 1.0), (0.0, 0.0, 0.05))$	$(0.85, 0.90, 0.95)$	$(0.85, 0.90, 0.95)$
F2.2	$((0.1, 0.2, 0.3), (0.5, 0.55, 0.6))$	$((0.5, 0.6, 0.7), (0.1, 0.2, 0.3))$	$(0.85, 0.90, 0.95)$	$(0.85, 0.90, 0.95)$
F2.3	$((0.4, 0.5, 0.6), (0.2, 0.3, 0.4))$	$((0.8, 0.9, 1.0), (0.0, 0.0, 0.05))$	$(0.85, 0.90, 0.95)$	$(0.75, 0.80, 0.85)$
F3.1	$((0.5, 0.6, 0.7), (0.1, 0.2, 0.3))$	$((0.8, 0.9, 1.0), (0.0, 0.0, 0.05))$	$(0.85, 0.90, 0.95)$	$(0.85, 0.90, 0.95)$
F4.1	$((0.5, 0.6, 0.7), (0.1, 0.2, 0.3))$	$((0.8, 0.9, 1.0), (0.0, 0.0, 0.05))$	$(0.75, 0.80, 0.85)$	$(0.75, 0.80, 0.85)$
F4.2	$((0.4, 0.5, 0.6), (0.2, 0.3, 0.4))$	$((0.8, 0.9, 1.0), (0.0, 0.0, 0.05))$	$(0.85, 0.90, 0.95)$	$(0.85, 0.90, 0.95)$
F4.3	$((0.4, 0.5, 0.6), (0.2, 0.3, 0.4))$	$((0.7, 0.8, 0.9), (0.0, 0.05, 0.1))$	$(0.85, 0.90, 0.95)$	$(0.85, 0.90, 0.95)$
F5.1	$((0.4, 0.5, 0.6), (0.2, 0.3, 0.4))$	$((0.7, 0.8, 0.9), (0.0, 0.05, 0.1))$	$(0.85, 0.90, 0.95)$	$(0.85, 0.90, 0.95)$
F5.2	$((0.5, 0.6, 0.7), (0.1, 0.2, 0.3))$	$((0.7, 0.8, 0.9), (0.0, 0.05, 0.1))$	$(0.85, 0.90, 0.95)$	$(0.85, 0.90, 0.95)$

Table 11: Decomposed Z-Fuzzy Representation of Probability Assessments

Hazard Code	$\mathcal{O}$	$\mathcal{P}$	O-Reliability	P-Reliability
F1.1	$((0.1, 0.2, 0.3), (0.4, 0.45, 0.5))$	$((0.4, 0.5, 0.6), (0.2, 0.25, 0.3))$	$(0.85, 0.90, 0.95)$	$(0.85, 0.90, 0.95)$
F1.2	$((0.2, 0.3, 0.4), (0.3, 0.35, 0.4))$	$((0.5, 0.6, 0.7), (0.1, 0.2, 0.3))$	$(0.85, 0.90, 0.95)$	$(0.85, 0.90, 0.95)$
F2.1	$((0.1, 0.2, 0.3), (0.4, 0.45, 0.5))$	$((0.5, 0.6, 0.7), (0.1, 0.2, 0.3))$	$(0.75, 0.80, 0.85)$	$(0.75, 0.80, 0.85)$
F2.2	$((0.1, 0.2, 0.3), (0.4, 0.45, 0.5))$	$((0.5, 0.6, 0.7), (0.1, 0.2, 0.3))$	$(0.85, 0.90, 0.95)$	$(0.85, 0.90, 0.95)$
F2.3	$((0.1, 0.2, 0.3), (0.4, 0.45, 0.5))$	$((0.5, 0.6, 0.7), (0.1, 0.2, 0.3))$	$(0.85, 0.90, 0.95)$	$(0.85, 0.90, 0.95)$
F3.1	$((0.1, 0.2, 0.3), (0.4, 0.45, 0.5))$	$((0.5, 0.6, 0.7), (0.1, 0.2, 0.3))$	$(0.85, 0.90, 0.95)$	$(0.75, 0.80, 0.85)$
F4.1	$((0.1, 0.2, 0.3), (0.4, 0.45, 0.5))$	$((0.5, 0.6, 0.7), (0.1, 0.2, 0.3))$	$(0.85, 0.90, 0.95)$	$(0.85, 0.90, 0.95)$
F4.2	$((0.1, 0.2, 0.3), (0.4, 0.45, 0.5))$	$((0.5, 0.6, 0.7), (0.1, 0.2, 0.3))$	$(0.85, 0.90, 0.95)$	$(0.75, 0.80, 0.85)$
F4.3	$((0.1, 0.2, 0.3), (0.4, 0.45, 0.5))$	$((0.5, 0.6, 0.7), (0.1, 0.2, 0.3))$	$(0.85, 0.90, 0.95)$	$(0.75, 0.80, 0.85)$
F5.1	$((0.1, 0.2, 0.3), (0.4, 0.45, 0.5))$	$((0.5, 0.6, 0.7), (0.1, 0.2, 0.3))$	$(0.85, 0.90, 0.95)$	$(0.75, 0.80, 0.85)$
F5.2	$((0.1, 0.2, 0.3), (0.4, 0.45, 0.5))$	$((0.5, 0.6, 0.7), (0.1, 0.2, 0.3))$	$(0.85, 0.90, 0.95)$	$(0.85, 0.90, 0.95)$

Table 12: Decomposed Z-Fuzzy Representation of Detectability Assessments

Hazard Code	$\mathcal{O}$	$\mathcal{P}$	O-Reliability	P-Reliability
F1.1	$((0.2, 0.3, 0.4), (0.75, 0.8, 0.85))$	$((0.7, 0.8, 0.9), (0.15, 0.2, 0.25))$	$(0.75, 0.80, 0.85)$	$(0.85, 0.90, 0.95)$
F1.2	$((0.2, 0.3, 0.4), (0.75, 0.8, 0.85))$	$((0.7, 0.8, 0.9), (0.15, 0.2, 0.25))$	$(0.85, 0.90, 0.95)$	$(0.75, 0.80, 0.85)$
F2.1	$((0.2, 0.3, 0.4), (0.75, 0.8, 0.85))$	$((0.6, 0.7, 0.8), (0.25, 0.3, 0.35))$	$(0.85, 0.90, 0.95)$	$(0.75, 0.80, 0.85)$
F2.2	$((0.2, 0.3, 0.4), (0.75, 0.8, 0.85))$	$((0.3, 0.4, 0.5), (0.45, 0.5, 0.55))$	$(0.85, 0.90, 0.95)$	$(0.85, 0.90, 0.95)$
F2.3	$((0.35, 0.5, 0.6), (0.5, 0.55, 0.6))$	$((0.6, 0.7, 0.8), (0.25, 0.3, 0.35))$	$(0.75, 0.80, 0.85)$	$(0.75, 0.80, 0.85)$
F3.1	$((0.35, 0.5, 0.6), (0.5, 0.55, 0.6))$	$((0.45, 0.6, 0.7), (0.3, 0.35, 0.4))$	$(0.75, 0.80, 0.85)$	$(0.85, 0.90, 0.95)$
F4.1	$((0.35, 0.5, 0.6), (0.5, 0.55, 0.6))$	$((0.45, 0.6, 0.7), (0.3, 0.35, 0.4))$	$(0.85, 0.90, 0.95)$	$(0.85, 0.90, 0.95)$
F4.2	$((0.2, 0.3, 0.4), (0.75, 0.8, 0.85))$	$((0.45, 0.6, 0.7), (0.3, 0.35, 0.4))$	$(0.85, 0.90, 0.95)$	$(0.85, 0.90, 0.95)$
F4.3	$((0.2, 0.3, 0.4), (0.75, 0.8, 0.85))$	$((0.6, 0.7, 0.8), (0.25, 0.3, 0.35))$	$(0.85, 0.90, 0.95)$	$(0.85, 0.90, 0.95)$
F5.1	$((0.2, 0.3, 0.4), (0.75, 0.8, 0.85))$	$((0.7, 0.8, 0.9), (0.15, 0.2, 0.25))$	$(0.85, 0.90, 0.95)$	$(0.85, 0.90, 0.95)$
F5.2	$((0.2, 0.3, 0.4), (0.75, 0.8, 0.85))$	$((0.6, 0.7, 0.8), (0.25, 0.3, 0.35))$	$(0.85, 0.90, 0.95)$	$(0.85, 0.90, 0.95)$

Table 13 presents the defuzzified values of Severity, Probability, and Detectability for each hazard, calculated using the decomposed FIS framework. Based on these individual criterion scores, the overall RPN is computed through multiplicative aggregation. The final row shows the ranking of hazards in descending order of risk, where a higher RPN value corresponds to a higher priority for mitigation.

Table 13: Defuzzified Values of Severity, Probability, and Detectability with RPN Scores

Hazard Code	**Severity**	**Probability**	**Detectability**	**RPN**
F1.1	0.4092	0.3956	0.3103	0.0502
F1.2	0.4175	0.4083	0.3117	0.0531
F2.1	0.3703	0.3695	0.3471	0.0475
F2.2	0.3491	0.3695	0.4309	0.0556
F2.3	0.3782	0.3695	0.4223	0.0590
F3.1	0.4092	0.3682	0.4477	0.0675
F4.1	0.4118	0.3655	0.4478	0.0674
F4.2	0.3703	0.3682	0.3727	0.0508
F4.3	0.3943	0.3682	0.3452	0.0501
F5.1	0.3943	0.3682	0.3082	0.0447
F5.2	0.4332	0.3655	0.3452	0.0547

Risk Priority Order: F3.1 > F4.1 > F2.3 > F2.2 > F5.2 > F1.2 > F4.2 > F1.1 > F4.3 > F2.1 > F5.1

Once the input data for each hazard are transformed into decomposed fuzzy numbers along with their associated reliability values, the FIS is employed to determine the corresponding risk level. At this stage, the optimistic and pessimistic membership (μ) and non-membership (ϑ) values, weighted by the square roots of their respective reliability levels (α), are used to calculate a defuzzified crisp score for each criterion, as described in Algorithm 1.

These crisp values are then fuzzified again by mapping them onto predefined fuzzy linguistic terms (e.g., *Low*, *Moderate*, *High*) based on their membership functions. The fuzzified inputs are processed through an expert-defined fuzzy rule base using the Mamdani inference method. Each activated rule contributes to the aggregation of the fuzzy output, which is finally defuzzified to produce the overall risk score.

```
Algorithm 1. Decomposed Fuzzy Inference System for Risk Evaluation

Input:

  • Optimistic and pessimistic membership and non-membership functions for each
    input variable: μ_O, ϑ_O, μ_P, ϑ_P
  • Associated optimistic and pessimistic reliability values: α_O, α_P
    (for each criterion: Severity, Probability, Detectability)

Output:

  • Linguistic risk level (e.g., Low, High, Very High)
  • Optionally: defuzzified risk score (numeric value)

Begin

  1. For each input criterion (Severity, Probability, Detectability):
       a. Compute the square root of the associated reliability values.
       b. Multiply the optimistic membership (μ_O) and non-membership (ϑ_O)
     values by √α_O.
       c. Multiply the pessimistic membership (μ_P) and non-membership (ϑ_P)
     values by √α_P.
  2. Calculate a crisp risk score using the weighted values from Step 1, based
     on the following expression:
         Risk_score = 0.5 + √((μ′_O + ϑ′_P) / 8) − √((ϑ′_O + μ′_P) / 8)
  3. Fuzzify the crisp score of each input by evaluating its degree of membership
     in predefined fuzzy sets.
  4. Evaluate the fuzzy rule base:
       IF (Severity is X) AND (Probability is Y) AND (Detectability is Z)
       THEN Risk is R
  5. Aggregate the outputs of all activated rules using fuzzy OR (e.g., max-min
     composition).
  6. Defuzzify the aggregated fuzzy output set using a suitable method (e.g.,
     centroid)
       to obtain a final crisp risk score.
  7. Map the crisp score to a linguistic risk level (e.g., Low, Moderate, High).
  8. Return:
       - The linguistic risk level
       - Optionally, the defuzzified numerical score

End
```

The FIS described in the pseudocode above is utilized to qualitatively assess the risk level associated with each identified hazard. It processes linguistic inputs namely severity, probability, and detectability along with their corresponding reliability levels. Using a predefined set of fuzzy logic rules, the system infers an overall risk category (e.g., Low, Moderate, High). This methodology facilitates a more human-centered and interpretable form of risk assessment, particularly in situations where precise numerical evaluation is infeasible or unreliable. The outputs generated through this fuzzy reasoning framework are summarized in Table 14.

Table 14: Risk Categorization based on FIS

Hazard Code	**S (Value)**	**S (Level)**	**P (Value)**	**P (Level)**	**D (Value)**	**D (Level)**	**Risk Category**
F1.1	0.41	Moderate	0.40	Very Rare	0.31	Easy	Low
F1.2	0.42	Moderate	0.41	Rare	0.31	Easy	Moderate
F2.1	0.37	Low	0.37	Very Rare	0.35	Easy	Low
F2.2	0.35	Low	0.37	Very Rare	0.43	Medium	Moderate
F2.3	0.38	Low	0.37	Very Rare	0.42	Medium	Moderate
F3.1	0.41	Moderate	0.37	Very Rare	0.45	Medium	Moderate
F4.1	0.41	Moderate	0.37	Very Rare	0.45	Medium	Moderate
F4.2	0.37	Low	0.37	Very Rare	0.37	Easy	Low
F4.3	0.39	Low	0.37	Very Rare	0.35	Easy	Low
F5.1	0.39	Low	0.37	Very Rare	0.31	Easy	Low
F5.2	0.43	Moderate	0.37	Very Rare	0.35	Easy	Low

7.1 A comparative analysis

To assess the effectiveness of the proposed model, we conduct a comparative analysis with the conventional risk scoring approach. Traditional methods often assign identical or similar scores to different hazards, which limits their ability to distinguish between scenarios. In contrast, our model introduces finer granularity by incorporating Z-numbers, enabling simultaneous evaluation of both optimistic and pessimistic perspectives. Unlike conventional approaches, the proposed method embeds expert confidence directly into the fuzzy inference process. Each evaluation is defined not only by a fuzzy value but also by an associated reliability measure, allowing for more realistic handling of uncertain inputs and variation in expert trust levels. For example, hazards such as F2.1 and F5.1, which receive similar scores under traditional models, are ranked differently in our approach. Likewise, F4.3 consistently rated as moderate using classical scoring is more distinctly categorized through the fuzzy-Z framework.

These differences stem from the model's ability to synthesize opposing perspectives into a unified assessment, rather than evaluating them in isolation. As summarized in Table 15, the proposed model demonstrates stronger discriminatory power, particularly in medium- and low-risk ranges where conventional methods tend to lose resolution. This enhanced sensitivity supports more informed and robust decision-making under uncertainty, making the approach well-suited to complex domains such as air traffic risk management.

Table 15: Comparison Analysis Results for Hazard Risk Ranking

Hazard Code	**Traditional Score (Opt.)**	**Rank**	**Traditional Score (Pess.)**	**Rank**	**Proposed Method Score**	**Rank**
F1.1	6	3	50	3	0.0502	8
F1.2	12	1	75	1	0.0531	6
F2.1	4	4	60	2	0.0475	10
F2.2	2	5	12	6	0.0556	4
F2.3	6	3	60	2	0.0590	3
F3.1	9	2	45	5	0.0675	1
F4.1	9	2	45	5	0.0674	2
F4.2	4	4	45	5	0.0508	7
F4.3	4	4	48	4	0.0501	9
F5.1	4	4	60	2	0.0447	11
F5.2	6	3	48	4	0.0547	5

8 Conclusion and future research suggestions

This study proposes an advanced risk assessment approach by integrating DFS, FIS, and Z-numbers into the FHA framework to support safety-critical decision-making based on uncertainty in air traffic operations. This integration enables the model to represent linguistic vagueness, quantify experts' evaluations and reliability, and produce interpretable outcomes. As a result, the proposed model enhances flexibility, adaptation to real case problems, and reliability in risk evaluations, especially when precise quantitative data is unavailable.

Unlike traditional methods that often assign identical scores to distinct hazards, the proposed model effectively distinguishes between similar scenarios by incorporating the reliability of expert judgments. Furthermore, the simultaneous consideration of both optimistic and pessimistic perspectives improves the robustness and clarity of risk prioritization. The model is applied to a set of 11 critical hazards within the A-SMGCS operational environment and demonstrates its effectiveness in differentiating risk levels, particularly under challenging conditions such as high traffic density and low visibility.

Additionally, the model provides a structured yet flexible framework for expert-based risk analysis, allowing for a comprehensive examination of risk components and their interrelations. It offers a more detailed perspective to manage of uncertainty, making it suitable for complex and dynamic decision-making environments for risk analysis.

However, the study also has certain limitations. The accuracy of the results depends on the consistency and quality of expert input, and the application is currently limited to a specific domain. Moreover, the fuzzy rules and membership functions are predefined, which may reduce adaptability in real-case or data-driven contexts. The model also assumes static risk conditions and does not account for temporal or contextual shifts in hazard behavior.

Future research may focus on enhancing the adaptability and scalability of the model by integrating real-case data sources such as weather, traffic density, or sensor-based alerts. Machine learning techniques will also be employed to develop adaptive fuzzy rules, and the model could be validated through broader expert panels or high-fidelity simulation environments. These directions may further strengthen the applicability and generalizability of the proposed framework across diverse air traffic management scenarios.

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