

TRANSPORT ROUTE PLANNING MODELS BASED ON FUZZY APPROACH

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ABSTRACT. Transport route planning is one of the most important and frequent activities in supply chain management. The design of information systems for route planning in real contexts faces two relevant challenges: the complexity of the planning and the lack of complete and precise information. The purpose of this paper is to find methods for the development of transport route planning in uncertainty decision making contexts. The paper uses an approximation that integrates a specific fuzzy-based methodology from Soft Computing. We present several fuzzy optimization models that address the imprecision and/or flexibility of some of its components. These models allow transport route planning problems to be solve with the help of metaheuristics in a concise way. A simple numerical example is shown to illustrate this approach.

1. Introduction

One of the missions and functions of commerce, logistics and transport is to bring together disperse or specific supply with an unequally distributed demand from a geographical perspective. A second mission is to organize the transfer of goods during a chain of transactions that reaches different destinations in an optimal way. Logistics operations include all planning and management activities that are carried out in the distribution channels which allow products to arrive under specific conditions of quantity, time, quality and location to satisfy market demands. From the operational point of view routing problems are an important elements of logistics systems. One of the most frequently studied routing problems is the Vehicle Routing Problem (VRP).

Technological developments are providing important advances in the availability of information systems that are able to support planning and management. However, support systems for logistics and transport planning must deal with the increase of the complexity of their development in a dynamic environment of rapid changes and high levels of uncertainty. Most of the transport parameters and decisions are characterized by undetermined, incomplete or unavailable information; and subjectivity, ambiguity or imprecision in descriptions and interpretations of the

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decision makers. In practical real-world problems decision makers use subjective knowledge or linguistic information when making decisions, measuring parameters, objectives and constraints and even when modelling the problem [28].

Classic methods and models based on numerical analysis and exact or binary value logic are characterized by their precision and categorization. However this approach, included in hard computing, although exact, is not always the most practical way to solve route transport problems. On the other hand, techniques based on Intelligent Computing, include Soft Computing (e.g. fuzzy logic, artificial neural networks, metaheuristics, etc.), are useful in route transport planning because they are flexible enough to deal with complex systems, offer acceptable approximate solutions and therefore add value. The advantages of employing Soft Computing in real-world problems are its capability to tolerate imprecision, uncertainty, and lack of information to achieve tractability and robustness in decision making with low costs [1, 18, 33].

Traditionally, the uncertainty found in the nature of the data and their settings has been handled by means of probability theory [34]. When some elements of the problem are stochastic or random, researchers usually specify them as Stochastic Vehicle Routing Problems (SVRP), modelled either as a chance constrained program (CCP) or as a stochastic program with recourse (SPR) [13, 24, 29]. In many practical cases uncertainty cannot be considered random phenomena and therefore probability theory cannot be applied successfully. Bellman and Zadeh [3] described and named this type of imprecision, stochastic uncertainty, in contrast with the semantic meaning of imprecision which is appropriate to model judgments, preference and values and which cannot be used to estimate exact numeric values, since they are vague or fuzzy. Most VRP models with fuzzy components that have appeared in the literature assume vagueness in customer demand [20, 21, 22, 26], and fuzzy times: service time and travel time parameters [14, 15, 16, 17, 25, 35]. Only a few references also consider uncertainty in time windows for service [10, 19]. In addition, some authors argue that it is possible that randomness and fuzziness be present at the same time. In this situation, an integration of probability and possibility theories are used to deal with this type of hybrid uncertainty [6, 11, 27, 36].

VRP are a well known combinatorial optimization problems included in the category of NP-hard problems. VRP can be solved by employing exact mathematical methods that guarantee finding an optimal solution if it exists. However, the required computational complexity leads to exponential processing time when the problem size is large. In this case, heuristic and metaheuristics methods are more suitable. The imprecision in some of the formulation components we can express with fuzzy terms we are faced with fuzzy optimization problem. Fuzzy optimization models with metaheuristics include advantages of modelling real problems with uncertainty in information.

The main contribution of this paper is to provide techniques for processing information with uncertainty which improves decision making in transport route planning. This paper presents a fuzzy-based methodology to handle the uncertainty of available information resources and the evaluation criteria of decision makers in

these problems. It is our intention to show the validity of the approach that in later works apply to real world problems.

The paper is organized as follows. Section 2 offers a description of all the factors involved in transport route planning and specifically those that are found in a real world context where uncertainty is present. Section 3 formulates transport route problems and their variants as a combinatorial optimization problem. Section 4 presents the fuzzy optimization approach which allows imprecise data route problems to be solved. Section 5 helps to illustrate the proposed method with a numerical example. Conclusions are offered in the end of the paper.

2. Transport Route Planning in an Uncertain Environment

Distribution and transport processes require decisions to be made practically on a daily basis, if not in a completely dynamic sense. These decisions concern specific routes that the transport units need to follow to distribute materials and products between different points in the supply chain. The establishment of routes that follow these units for the delivery or collection of goods is the central part of operational decision making in logistics management for many businesses.

Problem characteristics and decision making in general and specifically in transport planning are conditioned by resources, objectives, constraints and the nature of the information; these components influence model formulation and solution methods [12]. Transport planning problems require knowledge (see Table 1) of the transport modes, the characteristics of the road network, the fleet of mobile media, customer or demand points and the distribution centres (warehouses, depots, platforms, transit centres, ports, airports and even other mobile media or platforms). These problems also need to determine the route(s) for each depot and the available fleet that satisfy the set of established constraints while trying to attain the proposed objectives.

Planning objectives are established by the decision maker, considering different criteria, such as:

- minimize fixed and variable costs,
- minimize fleet size,
- minimize total transport times and/or total distance travelled,
- minimize waiting times,
- maximize operating profits,
- maximize the load capacity of each vehicle,
- maximize the utility function and customer satisfaction.

Several different sources of uncertainty in transport are present in real world contexts. In general users and decision makers diligently establish measurements based on observations and perceptions which determine the problem parameters and in the same way affect the evaluation of objectives and obtained solutions.

The relative uncertainty found in measuring time and distance is one of the important phenomena related to transport planning. In many situations exact time and distance between two nodes is not known or expressed with precision. Examples are seen with the travel time between two nodes, which can be influenced

Items	Information
Modes and networks	Air Maritime Land - highways Rail
Mobile means, vehicles	Heterogeneous or homogeneous Number of means Types of products to transport Load capacity in weight and volume Speed Related costs
Customers or demand points	Number of clients Location Demand quantity, size or volume Type of product of service Frequency Time windows
Distribution centres	One or several centres Related fleet Maximum capacity of service Opening and closing times
Routes	Beginning and ending in the same point One vehicle per route Several vehicles per route One vehicle for several routes Only deliveries One by one deliveries Delivery and pickup (simultaneous or not) Periodic or not Several routes Timetable, frequency and capacity constraints

TABLE 1. Required Knowledge

by traffic congestion or road conditions and usually distance information in map or cartography available is not sufficiently precise or updated. It can also be difficult to specifically express customer service time, unload times, down times and waiting times. It is possible that customers want to establish their own service time limits, and these are normally expressed with preferences that are ambiguous, flexible, or imprecise.

Another source of uncertainty in transport activities is customer demand. A common dilemma facing customers is to indicate a specific demand with sufficient advance time, especially when stating the exact demand or expressing proper and understandable units with standardized measures. This situation creates an environment of uncertainty regarding demand for the decision makers.

In addition it is difficult to establish with precision vehicle loads and capacities

in relation to different types of loads that can be transported and the ways that they are measured. In this respect the standardization of packing and packaging is not sufficient, given the wide variety of product types and containers found in the vehicles.

On the other hand, there are problems where the decision maker needs to define a complex cost function which has information regarding transport mode, used vehicles and specified route parameters, many of which are imprecise or incomplete. Finally, we point out that there are also problems where all of the objectives and constraints are not necessarily strict or need to be exactly satisfied with the same precision. In other words it is possible to establish problems where the objectives are not strictly attainable and/or where constraints are not completely satisfied but with the same degree of precision.

We assume uncertainty is an important part of decision making, intrinsic for most real world routing planning problems that cannot be ignored. In this paper we use a fuzzy approach to deal with a type of imprecision associated with the vague and imprecise nature of linguistic terms that are used in the problem. In addition, this approach permits greater tolerance in the evaluation of objectives and constraints. In both cases the specification of probabilities and the simulation are not possible or are very expensive.

3. Route Planning Formulation and Variants

Vehicle Routing are optimization problems where a number of vehicles must serve a specified set of dispersed demand points, satisfying a finite set of constraints and minimizing total operating costs [30]. The standard VRP, usually named Capacitated VRP (CVRP) is a problem whose objective is to determine the set of routes for a number of vehicles of a limited capacity that minimizes total path costs, usually given as a function of time, distance or number of routes such that:

- Customers are visited exactly once per route
- Each route begins and ends at a unique depot
- Total customer demand for each route does not exceed vehicle capacity.

The CVRP includes a known set of customers and demands, however it is not possible to pick up goods but instead only perform deliveries. All of the vehicles are homogeneous and there is only one depot where the vehicles leave from to carry out the deliveries, and only one constraint is introduced on the load capacity of the vehicles. This problem can be correctly formulated and modelled as linear programming problem. The general formulation of linear programming problems (LP) are characterized by coefficients in the technological matrix a_{ij} , resources b_i and costs c_j is as follows:

$$\max\{z = cx/Ax \leq b, x \geq 0\}, \quad (1)$$

where $A = [a_{ij}]$ is an $m \times n$ matrix of real numbers, $b = [b_i]$ and $c = [c_j]$ are real vectors with dimensions m and n , respectively.

The proposed formulation of the CVRP uses indices i and j which represent the localization of the customer, and starts from 0 to n_c , where n_c is the number of

customers, and 0 represents the depot, and index k is the number of vehicles where m_v is total available routes or vehicles. The variables used in the model are the binary decision variables x_{ij}^k that represent the routes given by

$$x_{ij}^k = \begin{cases} 1 & \text{if the vehicle } k \text{ goes from } i \text{ to } j \\ 0 & \text{otherwise} \end{cases}$$

and the auxiliary integer variables y_i^k which represent the order that node i is visited in route k given by $y_j^k = y_i^k + 1$ if $x_{ij}^k = 1$.

The LP model of the VRP is the following:

$$\min \sum_{k=1}^{m_v} \sum_{i=0}^{n_c} \sum_{j=0}^{n_c} c_{ij}^k x_{ij}^k \quad (2)$$

subject to:

$$\sum_{k=1}^{m_v} \sum_{i=0}^{n_c} x_{ij}^k = 1, \quad j = 1, 2, \dots, n_c \quad (3)$$

$$\sum_{k=1}^{m_v} \sum_{i=0}^{n_c} x_{ji}^k = 1, \quad j = 1, 2, \dots, n_c \quad (4)$$

$$\sum_{j=1}^{n_c} x_{0j}^k = 1, \quad k = 1, 2, \dots, m_v \quad (5)$$

$$\sum_{i=1}^{n_c} x_{i0}^k = 1, \quad k = 1, 2, \dots, m_v \quad (6)$$

$$\sum_{i=0}^{n_c} x_{ij}^k - \sum_{i=0}^n x_{ji}^k = 0, \quad j = 1, 2, \dots, n_c, k = 1, 2, \dots, m_v \quad (7)$$

$$\sum_{i=0}^{n_c} \sum_{j=0}^{n_c} d_j x_{ji}^k \leq Q_k, \quad k = 1, 2, \dots, m_v \quad (8)$$

$$y_i^k - y_j^k + n_c x_{ij}^k \leq n_c - 1, \quad i, j = 1, 2, \dots, n_c, k = 1, 2, \dots, m_v \quad (9)$$

$$x_{ij}^k \in \{0, 1\}, y_i^k \in \{1, 2, \dots, n_c\}, \quad i, j = 0, 1, \dots, n_c, k = 1, 2, \dots, m_v. \quad (10)$$

Equation (2) represents the objective function in terms of costs. Constraints (3) and (4) guarantee that only one route/vehicle enters and leaves from each node or that each customer is served exactly once. Constraints (5) and (6) ensure that each vehicle leaves the depot and returns to it, thereby limiting vehicle use to one trip. Constraint (7) establishes the conditions to maintain continuity of the route. Constraints (8) ensure that the sum of customer demand in any route does not exceed the vehicle capacity, where Q_k is the vehicle load capacity and d_i is the demand at node i . Finally (9) eliminates subtours and (10) establishes the conditions of the variables.

Vehicle route problems can take into account different types of constraints and objectives, depending on the problem characteristics and the decision making procedure [7], as seen in the previous section. These considerations can produce variants of the VRP, such as:

- VRP with mixed or heterogeneous fleets, Mix Fleet VRP (MFVRP), with vehicles having the same capacity.
- Multi-depot VRP (MDVRP) where several depots are used to serve demand.
- Periodic VRP (PVRP) where route planning occurs over several days.
- Split & Delivery VRP (SDVRP) that allows customers to be served by several vehicles.
- Pickup and Delivery VRP (PDVRP) where customers receive and return goods.
- VRP with backhauls (VRPB) where customers can order or return articles but all of the deliveries are carried out before the collections take place.
- VRP with Time Windows (VRPTW) where each customer has a time window to receive goods.
- Open VRP (OVRP) which does not require that vehicles return to the depot.

There are also other versions of the problems which are combinations of the previously mentioned variants or others that add specific characteristics, for instance, a dynamic version of the vehicle route problem, Dynamic VRP (DVRP).

4. Route Planning with Uncertainty Using Fuzzy Approach

Solutions to real distribution route planning problems require special approaches that are able to deal with problem uncertainty and complexity. These planning problems use vague or incomplete information and often have objectives and constraints, which cannot be described by exact numbers, conventional expressions and cost functions and also cannot be evaluated to search solutions by conventional methods.

Fuzzy sets and systems offer an appropriate methodology for problems with uncertainty, imprecision and subjectiveness in the information and decision making. It is especially useful with uncertainty that is not present because of a lack of knowledge, nor based on its random nature, but instead due to the imprecise nature of its expression.

Under settings where uncertainty and imprecision are present some of the components of the VRP can be expressed in fuzzy terms through the use of membership functions. In this scenario we are working with VRP modelled as a fuzzy optimization problem or a fuzzy linear programming problem. In Fuzzy Linear Programming (FLP) some of the coefficients in the objective function and the constraints can be expressed and evaluated as fuzzy numbers. It is in these cases where the variables x take on values expressed as fuzzy numbers, and addition and multiplication operations and inequalities are in terms of fuzzy numbers. The solutions to these problems can be considered as fuzzy solutions. Discussions concerning solutions do

not focus on their feasibility, if they are optimal solutions or not. We, in turn, have chosen to discuss the degree of feasibility and optimality of the solution.

The proposed classification in [32] identifies the following models of fuzzy linear programming problems:

- Models with fuzzy coefficients in the constraints.
- Models with fuzzy coefficients in the objective functions.
- Models with fuzzy constraints.
- Models with fuzzy objectives.

There are also mixed problems that belong to several of these problems.

Numerous models and methods that solve these problems can be found in the literature [2], the majority only assume that some of the elements described in the model are fuzzy.

A general FLP model is presented in [5] and [9], in which all of the elements are fuzzy. From this general model it is possible to derive each particular case of the FLP problem. This general fuzzy linear programming model is as follows:

$$\max\{z = c^f x / a_i^f x \leq_f b_i^f, i = 1, \dots, m, x \geq 0\}, \quad (11)$$

where the fuzzy elements are given by:

- For each cost $\exists \mu_j \in F(\mathfrak{R})$ such that $\mu_j : \mathfrak{R} \rightarrow [0, 1]$, $j = 1, 2, \dots, n$, which defines the fuzzy costs.
- For each row $\exists \mu_i \in F(\mathfrak{R})$ such that $\mu_i : \mathfrak{R} \rightarrow [0, 1]$, $i = 1, 2, \dots, m$, which defines the fuzzy number in the right-hand side of constraints.
- For each $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$, $\exists \mu_{ij} \in F(\mathfrak{R})$ such that $\mu_{ij} : \mathfrak{R} \rightarrow [0, 1]$, which defines the fuzzy number in the technological matrix.
- For each row $\exists \mu^i \in F(F(\mathfrak{R}))$ such that $\mu^i : F(\mathfrak{R}) \rightarrow [0, 1]$, $i = 1, 2, \dots, m$, which provides the degree of fulfilment for the fuzzy number $a_{i1}^f x_1 + a_{i2}^f x_2 + \dots + a_{in}^f x_n$, $i = 1, 2, \dots, m$, with respect to the i -th constraint, that is the adequacy between this fuzzy number and the corresponding b_i^f in relation to the i -th constraint.

Cadenas and Verdegay [4] define a solution for this general model which consists in substituting the constraints set of (11) by a convex fuzzy set through a ranking function as a comparison mechanism of fuzzy numbers.

Let $A, B \in F(\mathfrak{R})$. A simple method for ranking fuzzy numbers consists in defining a ranking function that maps each fuzzy number into the real line $g : \mathfrak{R} \rightarrow \mathfrak{R}$. If this function g is known, then

- $g(A) < g(B) \iff A$ is less than B
- $g(A) > g(B) \iff A$ is greater than B
- $g(A) = g(B) \iff A$ is equal to B

usually, g is called a linear ranking function if $\forall A, B \in F(\mathfrak{R}) : g(A + B) = g(A) + g(B)$ and $\forall r \in \mathfrak{R}, r > 0 : g(rA) = rg(A), \forall A \in F(\mathfrak{R})$.

To solve problem (11), let g be a fuzzy number linear ranking function and given the function $\psi : F(\mathfrak{R}) \times F(\mathfrak{R}) \rightarrow F(\mathfrak{R})$ so that:

$$\psi(a_i^f x, b_i^f) = \begin{cases} \tau_i^f & a_i^f x \leq_g b_i^f \\ \tau_i^f(-)a_i^f x(+)b_i^f & b_i^f \leq_g a_i^f x \leq_g b_i^f(+) \tau_i^f \\ 0 & a_i^f x \leq_g b_i^f(+) \tau_i^f \end{cases} \quad (12)$$

where $\tau_i^f \in F(\mathfrak{R})$ is a fuzzy number in such a way that its support is included in \mathfrak{R}^+ , and \leq_g is a relationship that measures that $A \leq_g B, \forall A, B \in F(\mathfrak{R})$, and $(-)$ and $(+)$ are the usual operation among fuzzy numbers.

Thus the membership function is also associated with the fuzzy constraint $a_i^f x \leq_f b_i^f$ with τ_i^f a fuzzy number giving the maximum violation of the i -th constraint as:

$$\mu^i : F(\mathfrak{R}) \rightarrow [0, 1] / \mu^i(a_i^f x, b_i^f) = \frac{g(\psi(a_i^f x, b_i^f))}{g(\tau_i^f)}$$

where g is a linear ranking function.

Given problem (11), \leq_f with membership function (12) and using the fuzzy number representation theorem, the following is obtained:

$$\begin{aligned} \mu^i(a_i^f x, b_i^f) \geq \alpha &\iff \frac{g(\psi(a_i^f x, b_i^f))}{g(\tau_i^f)} \geq \alpha \iff g(\tau_i^f(-)a_i^f x(+)b_i^f) \geq g(\tau_i^f)\alpha \\ &\iff g(\tau_i^f) - g(a_i^f x) + g(b_i^f) \geq g(\tau_i^f)\alpha \iff g(a_i^f) \leq g(b_i^f(+) \tau_i^f(1 - \alpha)) \\ &\iff a_i^f x \leq_g b_i^f + \tau_i^f(1 - \alpha) \end{aligned}$$

where \leq_g is the relationship corresponding to g .

Therefore, an equivalent auxiliary model to solve (11) is the following:

$$\max\{z = c^f x / a_i^f x \leq_g b_i^f + \tau_i^f(1 - \alpha), i = 1, \dots, m, x \geq 0, \alpha \in [0, 1]\}. \quad (13)$$

To solve (13), different fuzzy numbers ranking methods can be used in both the constraints and the objective function, or ranking methods can be used in the constraints and α -cuts in the objective, which will generate different traditional models that allow a fuzzy solution to be obtained.

This approach can also be used to model and solve traditional deterministic VRPs expanded to reflect situations where some features or terms are fuzzy, resulting in a Fuzzy VRP (FVRP). The linear programming model formulation of these FVRP can correspond to some of the previously proposed models. Each of these models can be solved using different metaheuristics to find solutions.

In real VRP, the increase of the number of parameters and variables creates complex and difficult problems to solve using exact methods. It is in this context that metaheuristics offer a practical approach. Thus it is common to adapt metaheuristics to different contexts in real world problems, with relatively few changes while adapting perfectly to the characteristics found in route planning problems[12]. The most popular metaheuristics that have been employed to handle various fuzzy aspects of the VRP and to solve them seem to be genetic and ant colony algorithms, see e.g. [14, 15, 16, 17, 21, 22, 25, 35, 36].

4.1. VRP with Fuzzy Demand and Load. Information regarding customer demand typically is not established with any level of precision or is not available to the decision maker. A similar situation occurs with truck loads. These loads depend on demand and truck capacity as well as the numbers of units that are used in the transport. These figures are vague and can be expressed in a different way. In this case the VRP is associated with the first proposed model, which contains constraints with fuzzy coefficients, fuzzy number in the technological matrix coefficient and right hand size coefficients. Specifically, the uncertainty in the VRP formulation affects the constraints (8), which are revised and formulated as follows:

$$\sum_{j=0}^{n-c} \sum_{i=0}^{n-c} \tilde{d}_j x_{ij}^k \leq_f \tilde{Q}_k, k = 1, \dots, m.v, \quad (14)$$

consider \tilde{d}_i and \tilde{Q}_k as fuzzy numbers.

A fuzzy solution is proposed by Delgado, Verdegay and Vila in [8] by solving a general model and it can be used for this type of FLP, expressed in the following terms:

$$\max\{z = cx/a_i^f, x \leq_f b_i^f, i = 1, \dots, m, x \geq 0\}, \quad (15)$$

where for each $i = 1, 2, \dots, m, j = 1, 2, \dots, n, a_i^f = (a_{i1}^f, a_{i2}^f, \dots, a_{in}^f)$ with $a_{ij}^f \in F(\mathfrak{R})$ and $b_i^f \in F(\mathfrak{R}), x \in X = \{x \in \mathfrak{R}^n / a_i^f x \leq_f b_i^f, i = 1, \dots, m, x \geq 0\}$ and $c \in \mathfrak{R}^n$.

In this approach violations in its constraint are admitted up to a maximum of tolerance level τ_i^f . Note that these values have to be a fuzzy number on account of the nature of the coefficients taking part in each constraint. The proposed approach substitutes the set of constraints of the model (15) by a convex fuzzy set, obtained through α -cuts, resulting in the following auxiliary model:

$$\max\{z = cx/a_i^f x \leq_g b_i^f + \tau_i^f(1 - \alpha), i = 1, \dots, m, x \geq 0, \alpha \in [0, 1]\}. \quad (16)$$

In order to compare the left hand side with the right hand a fuzzy comparison relation between fuzzy numbers g is to be considered. For this reason, the solution to the model is obtained by particularization of the different comparison methods of fuzzy numbers.

In our case, an auxiliary problem for the VRP with fuzzy demand and load can be solved by adding the following modified constraints:

$$\sum_{j=0}^{n-c} \sum_{i=0}^{n-c} \tilde{d}_j x_{ij}^k \leq_g \tilde{Q}_k + \tau_k^f(1 - \alpha), k = 1, \dots, m.v, \alpha \in [0, 1]. \quad (17)$$

4.2. VRP with Fuzzy Times. One possibility in route planning requires that the routes are completed at a minimum cost, and this cost is measured in the times that are needed to cover them. Traffic problems or other factors such as road conditions can create delays which are difficult to quantify. Situations like this and others confirm that precise estimations of time are one of the most common and difficult problems to solve. As such, the situation previously described reveals a linear programming model with fuzzy coefficients in the objective functions. The

VRP model with fuzzy times can be formulated with this objective function and follows:

$$\min \sum_{k=1}^{m.v} \sum_{j=0}^{n.c} \sum_{i=0}^{n.c} \tilde{t}_{ij} x_{ij}^k + \sum_{k=1}^{m.v} \sum_{j=0}^{n.c} \sum_{i=0}^{n.c} \tilde{u}_{ij} x_{ij}^k \quad (18)$$

The objective function is clearly fuzzy, where costs \tilde{t}_{ij} and \tilde{u}_{ij} , representing travel times from one node to another and unload times, respectively, are fuzzy numbers, and Σ refers to the sum of fuzzy numbers.

The FLP model with coefficients in the objective function that is defined by a fuzzy number can be written as

$$\max\{z = c^f x / Ax \leq b, x \geq 0\}, \quad (19)$$

with $c^f \in F^n(\mathfrak{R})$.

For each feasible solution, there is a fuzzy number which is obtained from the fuzzy objective function. In order to obtain the optimal solution and corresponding fuzzy value of the objective function, methods ranking the fuzzy numbers obtained from this function may be considered. The modelling and solution of this problem uses the approximation proposed in [8] which, by the use of a ranking function g of fuzzy numbers (relations of comparisons between fuzzy numbers, generally noted as \leq_g), transforms this fuzzy model into a simpler auxiliary model:

$$\max\{z = g(c^f x) / Ax \leq b, x \geq 0\}. \quad (20)$$

Therefore according to the ranking function g used, different auxiliary models that solve (19) can be obtained.

Using this approach the model can then be solved using some ranking function g , substituting (18) by:

$$\min \sum_{k=1}^{m.v} \sum_{j=0}^{n.c} \sum_{i=0}^{n.c} g(\tilde{t}_{ij}) x_{ij}^k + \sum_{k=1}^{m.v} \sum_{j=0}^{n.c} \sum_{i=0}^{n.c} g(\tilde{u}_{ij}) x_{ij}^k \quad (21)$$

There is a long list of methods for ranking fuzzy numbers that in many cases provide different rankings. The fuzzy nature of the problem and the existence of these different mechanisms allow for comparisons and optimal solutions to be obtained.

4.3. VRP with Fuzzy Demand Constraints. There are situations where constraints are not precise, that is, the decision maker will allow a certain degree of flexibility regarding their fulfilment. For example, when loads and demands are not fuzzy, there are cases which satisfy constraints with a certain degree of imprecision. In the fuzzy VRP formulation the constraints (8) are affected and can be expressed as follows:

$$\sum_{j=0}^{n.c} \sum_{i=0}^{n.c} d_j x_{ij}^k \leq_f Q_k, k = 1, \dots, m.v. \quad (22)$$

This is a model that contains fuzzy constraints in which the demand constraints are defined by a fuzzy set, with a membership function $\mu : \mathbb{R}^n \rightarrow [0, 1]$. In such a situation, for each constraint, a desirable quantity b is considered, but the possibility that it is greater is accepted until a maximum $b + \tau$ (τ is referred to as a violation tolerance level). This model is represented by:

$$\max\{z = cx/Ax \leq_f b, x \geq 0\}, \quad (23)$$

where the symbol \leq_f indicates the imprecision of the constraints and where each fuzzy constraint $a_i x \leq_f b_i$ is specified by a membership function in the form:

$$\mu_i(a_i x) = \begin{cases} 1 & a_i x \leq b_i \\ f_i(a_i x) & b_i \leq a_i x \leq b_i + \tau_i \\ 0 & a_i x \geq b_i + \tau_i \end{cases}$$

which means that, for each constraint i , given the level of tolerance τ_i , to each point (n -dimensional vector) x is associated with a number $\mu_i(x) \in [0, 1]$ known as the degree of fulfilment (or verification) of the constraint i . The functions f_i are assumed to be continuous and monotonous non-decreasing.

Specifically, Verdegay [31], using the representation theorem for fuzzy sets, proves that the solutions for the case of linear functions f_i can be obtained from the auxiliary model:

$$\max\{z = cx/Ax \leq b + \tau(1 - \alpha), x \geq 0, \alpha \in [0, 1]\}, \quad (24)$$

where $\tau = (\tau_1, \tau_2, \dots, \tau_m)$.

Constraint (22) from the VRP then takes on the following form, allowing feasible solutions to be found for each α and τ :

$$\sum_{j=0}^{n-c} \sum_{i=0}^{n-c} d_j x_{ij}^k \leq Q_k + \tau_k(1 - \alpha), k = 1, \dots, m-v, \alpha \in [0, 1]. \quad (25)$$

4.4. VRP that Minimizes Fuzzy Route Costs. Route planning problems generally try to minimize the cost for a fleet of vehicles in the designed delivery routes. Frequently these costs are quantified indistinctly in terms of time or distance. However there are other ways to quantify the fixed and variable costs of the fleet, for instance, through the amortization of investments, consumption, maintenance and costs from negative occurrences, especially those related to environment. The availability of this information and determining the optimal solution with certainty is not an easy task. In some cases the decision maker does not necessarily need the optimal solution, but a good solution is sufficient at a certain degree of precision, for example, if we want to accept the minimum, then we are willing to accept some slightly larger solutions. In these situations it is convenient to look for the optimal values as fuzzy solutions in the objective function in optimization problems. It is this fuzzy optimization model that we will refer to and it will be characterized by fuzzy objectives.

When dealing with the VRP the objective function can be formulated as follows:

$$\bar{\min} \sum_{k=1}^{m-v} \sum_{j=0}^{n-c} \sum_{i=0}^{n-c} c_{ij}^k x_{ij}^k \tag{26}$$

Considering that t_0 is the maximum quantity that the objective function can be lesser to the minimum objective c_0 , then each solution vector x is associated to a number $\mu_0(x)$, which represents the degree in which the decision maker considers that the objective will be reached. That is, the membership function:

$$\mu_0(x) = \begin{cases} 1 & cx < c_0 \\ f_0(cx) & c_0 \leq cx \leq c_0 + t_0 \\ 0 & cx < c_0 + t_0 \end{cases}$$

where f_0 is a continuous and monotone non-decreasing. We use the approach proposed by Zimmermann [37]. This approach present some difficulties which have been solved [23]. Using this method, we find a solution with an auxiliary model that transforms the objective function (26) and adds a new set of constraints, as follows:

$$\max \alpha \tag{27}$$

$$\sum_{k=1}^{m-v} \sum_{j=0}^{n-c} \sum_{i=0}^{n-c} c_{ij}^k x_{ij}^k \leq c_0 + t_0(1 - \alpha). \tag{28}$$

5. Illustrative Example

This section presents a representative application of the proposed models with a numerical example. Although these models are being studied to be applied in real practical applications, the main goal of this paper is not to present these real problems but the models and methods that finally will solve them, and therefore the main purpose of this simple numerical example is to illustrate the application of the proposed approach using one of the described models.

Consider the following instance represented in Figure 1. The problem consists in finding efficient routes for a fleet of two vehicles v_1 and v_2 , starting and ending at a depot represented by 0, to serve three customers labelled 1, 2 and 3. Assume that the travel distances between customers and between the depot and each customer are crisp numbers r_{ij} listed in Table 2.

	0	1	2
1	3		
2	3	3	
3	4	5	6

TABLE 2. Travel Distance Matrix

Let the demand amount of each customer be a triangular fuzzy number given by $d_1 = (2, 3, 4)$, $d_2 = (1, 3, 5)$ and $d_3 = (3, 4, 5)$ respectively for customer "0", "1" and "2". Let the limited vehicle capacities also be triangular fuzzy numbers given

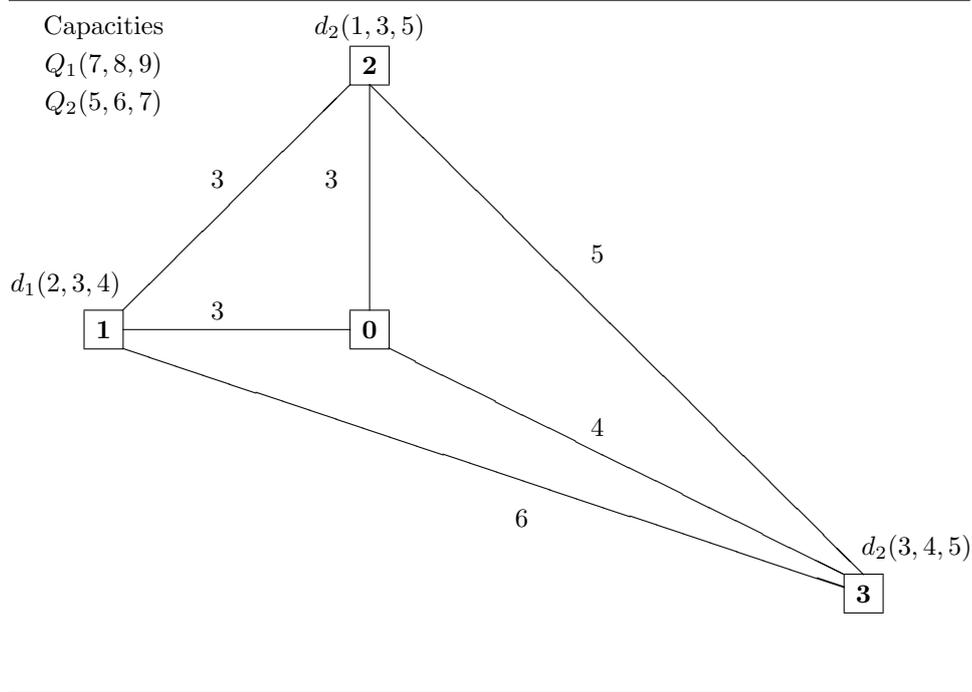


FIGURE 1. Illustrative Example

by $Q_1 = (7, 8, 9)$ and $Q_2(5, 6, 7)$. Let the operation cost (fuel and others) of each vehicle be calculated as a linear function of distance travelled. We assume that the cost per unit distance travelled is 2 for vehicle v_1 and 1 for vehicle v_2 ; i. e., $c_{ij}^1 = 2r_{ij}$ and $c_{ij}^2 = 1r_{ij}$.

Table 3 shows all the possible solutions; i. e., pairs of routes R_1 and R_2 for vehicles v_1 and v_2 . Table 3 also includes the loads and costs of the vehicles, the values of the objective functions and the feasibility conditions. The loads are calculated using arithmetic operations with triangular fuzzy numbers. The costs and objective values are calculated using usual arithmetic with crisp numbers. Let us show how we obtain the feasibility conditions.

We know that the routes shown in Table 3 satisfy all of the constraints of the model, except vehicle capacity, which is necessary to check. Since customer demand and capacity of vehicles are fuzzy numbers, the optimization model is a linear model with fuzzy coefficients in some constraints. According to the proposed approach we need to check the capacity constraint using an auxiliary model with the constraints (8). Therefore, the feasible solutions must satisfy the following constraints:

$$\sum_{j=0}^3 \sum_{i=0}^3 \tilde{d}_j x_{ij}^k \leq_g \tilde{Q}_k + \tau_k^f (1 - \alpha), k = 1, 2, \alpha \in [0, 1], \tag{29}$$

for some chosen fuzzy values of tolerance τ_k^f , $k = 1, 2$.

Solutions	Routes	Loads	Costs	Obj	Feasibility
Solution 1	$R_1 : 0 \rightarrow 1 \rightarrow 2 \rightarrow 0$	(3,6,9)	9	26	$\forall \alpha \in [0, 1]$
	$R_2 : 0 \rightarrow 3 \rightarrow 0$	(3,4,5)	8		
Solution 2	$R_1 : 0 \rightarrow 1 \rightarrow 3 \rightarrow 0$	(5,7,9)	12	30	$\forall \alpha \in [0, 1]$
	$R_2 : 0 \rightarrow 2 \rightarrow 0$	(1,3,5)	6		
Solution 3	$R_1 : 0 \rightarrow 2 \rightarrow 3 \rightarrow 0$	(4,7,10)	13	32	$\forall \alpha \in [0, 1]$
	$R_2 : 0 \rightarrow 1 \rightarrow 0$	(2,3,4)	6		
Solution 4	$R_2 : 0 \rightarrow 1 \rightarrow 2 \rightarrow 0$	(3,6,9)	9	25	$\forall \alpha \in [0, 1]$
	$R_1 : 0 \rightarrow 3 \rightarrow 0$	(3,4,5)	8		
Solution 5	$R_2 : 0 \rightarrow 1 \rightarrow 3 \rightarrow 0$	(5,7,9)	12	24	$\alpha = 0$
	$R_1 : 0 \rightarrow 2 \rightarrow 0$	(1,3,5)	6		$\forall \alpha \in [0, 0.6]$
Solution 6	$R_2 : 0 \rightarrow 2 \rightarrow 3 \rightarrow 0$	(4,7,10)	13	25	$\alpha = 0$
	$R_1 : 0 \rightarrow 1 \rightarrow 0$	(2,3,4)	6		$\forall \alpha \in [0, 0.6]$

TABLE 3. Demands and Costs of the Possible Routes

Using α -cut for values ranging from $\alpha = 0$ to $\alpha = 1$ allow us to determine route feasibility. We also need to determine the ordering function so that it can consistently compare triangular fuzzy numbers in these constraints. We use the third index of Yager and tolerance levels given by $\tau_1^f = \tau_2^f = (0, 1, 2)$.

Let us start with solution 1 to check feasibility. The routes are: $R_1 = [0 \rightarrow 1 \rightarrow 2 \rightarrow 0]$ and $R_2 = [0 \rightarrow 3 \rightarrow 0]$. We need to check whether $(3, 6, 9) \leq_g (7, 8, 9) + (0, 1, 2)(1 - \alpha)$ and $(3, 4, 5) \leq_g (5, 6, 7) + (0, 1, 2)(1 - \alpha)$. Using the third index of Yager, $(3 + 12 + 9) \leq (7 + 18 + 11) + (0 + 2 + 2)(1 - \alpha)$ and $(3 + 8 + 5) \leq (5 + 14 + 9) + (0 + 2 + 2)(1 - \alpha)$; i. e. $24 \leq 32 + 4(1 - \alpha)$ and $16 \leq 24 + 4(1 - \alpha)$, so we can see that both inequalities are true for any value of $\alpha \in [0, 1]$.

In the same way, for solution 2 we have $R_1 = [0 \rightarrow 1 \rightarrow 3 \rightarrow 0]$ and $R_2 = [0 \rightarrow 3 \rightarrow 0]$. We need to check whether $(5, 7, 9) \leq_g (7, 8, 9) + (0, 1, 2)(1 - \alpha)$ and $(1, 3, 5) \leq_g (5, 6, 7) + (0, 1, 2)(1 - \alpha)$. Again using the third index of Yager, $28 \leq 32 + 4(1 - \alpha)$ and $12 \leq 24 + 4(1 - \alpha)$ and that solution 2 is always feasible for any value of $\alpha \in [0, 1]$.

Following the same process we obtain the same results for solutions 3 and 4. Note that these four solutions are feasible for all tolerance fuzzy values τ_1^f and τ_2^f . However, the situation for solutions 5 and 6, is different.

Consider solution 5. The routes are $R_2 = [0 \rightarrow 1 \rightarrow 3 \rightarrow 0]$ and $R_1 = [0 \rightarrow 2 \rightarrow 0]$, we check whether $(5, 7, 9) \leq_g (5, 6, 7) + (0, 1, 2)(1 - \alpha)$ and $(1, 3, 5) \leq_g (7, 8, 9) + (0, 1, 2)(1 - \alpha)$. Applying the same procedure, $28 \leq 24 + 4(1 - \alpha)$ and $12 \leq 32 + 4(1 - \alpha)$. Therefore solution 5 is feasible only if $\alpha = 0$. Identical results occur for solution 6.

The corresponding column in Table 3 shows that solution 5 has the objective function value with minimum cost, 24, among the six solutions. Therefore we can determine that the optimal solution for $\alpha = 0$ is solution 5. However, for $\alpha > 0$ solutions 5 and 6 are not feasible. For these values, the first four solutions are feasible and the best one is solution 4 with objective value 25. Then, for $\alpha > 0$, the optimal solution is solution 4.

Now consider a higher level of tolerance for both vehicles. Let $\tau_1^f = \tau_2^f = (2, 2.5, 3)$. For solution 5 we have feasibility if $(5, 7, 9) \leq_g (5, 6, 7) + (2, 2.5, 3)(1 - \alpha)$ and $(1, 3, 5) \leq_g (7, 8, 9) + (2, 2.5, 3)(1 - \alpha)$. Now conditions are $28 \leq 24 + 10(1 - \alpha)$ and $12 \leq 32 + 10(1 - \alpha)$. Therefore route R_2 of solution 5 is not feasible for $\alpha > 0.6$. Similar results are true for solution 6.

As we can see for certain values of the tolerances and α -cut some routes are feasible and others not. We can also see that an increase in tolerance levels may lead to feasibility for some routes which were previously unfeasible.

6. Conclusions

In this paper a new and simple approach to deal with uncertainty in vehicle route planning is presented. A fuzzy optimization formulation is given to these problems. The proposed methodology is based on Soft Computing and allows techniques that use information with uncertainty to be integrated with intelligent computing strategies to find efficient and realistic solutions to routing problems. Built-in intelligent information systems can support experts in decision making, while fuzzy systems and metaheuristics provide techniques that can facilitate effective developments and implementations of the systems. The paper contributes to the literature on vehicle route planning in the following aspects: (a) it proposes a fuzzy version of VRP with imprecise data, (b) it offers new models and methods to solve fuzzy vehicle routing problems which are simpler, systematic and more integrated than others that have been proposed in the literature, and (c) it validates the solution approach using a simple numerical example. In future works authors will use these models and methods to find solutions to practical business problems.

REFERENCES

- [1] E. Avineri, *Soft computing applications in traffic and transport systems: a review*, Advances in Soft Computing, **1** (2005), 17-25.
- [2] A. Baykasoglu and T. Göcken, *Review and classification of fuzzy mathematical programs*, Journal of Intelligent & Fuzzy Systems, **19** (2008), 205-229.
- [3] R. E. Bellman and L. A. Zadeh, *Decision making in a fuzzy environment*, Management Science, **17(B)(4)** (1970), 141-164.
- [4] J. M. Cadenas and J. L. Verdegay, *Using fuzzy numbers in linear programming*, IEEE Transactions on Systems, Man and Cybernetics, **27(B)(6)** (1997), 1017-1022.
- [5] J. M. Cadenas and J. L. Verdegay, *A primer on fuzzy optimization models and methods*, Iranian Journal of Fuzzy Systems, **3(1)** (2006), 1-21.
- [6] R. Cheng and M. Gen, *Vehicle routing problem with fuzzy due-time using genetic algorithm*, Japanese Journal of Fuzzy Theory and Systems, **7(5)** (1995), 1050-1061.
- [7] J. F. Cordeau, G. Laporte, M. Savelsbergh and D. Vigo, *Vehicle Routing*, Handbook in OR & MS, **14(6)** (2007), 367-427.
- [8] M. Delgado, J. L. Verdegay and M. A. Vila, *Imprecise costs in mathematical programming problems*, Control and Cybernet, **16(2)** (1987), 113-121.
- [9] M. Delgado, J. L. Verdegay and M. A. Vila, *A general model for fuzzy linear programming*, Fuzzy Sets and Systems, **29** (1989), 21-29.
- [10] M. Djadane, G. Goncalves, T. Hsu and R. Dupas, *Dynamic vehicle routing problems under flexible time windows and fuzzy travel times*, Proceedings of 2006 International Conference on Service Systems and Service Management, **2** (2006), 1519-1524.

- [11] C. Erbao and L. Mingyong, *A hybrid differential evolution algorithm to vehicle routing problem with fuzzy demands*, Journal of Computational and Applied Mathematics, **231** (2009), 302-310
- [12] K. Ganesh, A. S. Nallathambi and T. T. Narendran, *Variants, solution approaches and applications for Vehicle Routing Problems in supply chain: agile framework and comprehensive review*, International Journal of Agile Systems and Management, **2(1)** (2007), 50-75.
- [13] M. Gendreau, G. Laporte and R. Seguin, *Stochastic vehicle routing*, European Journal of Operational Research, **88(1)** (1996), 312.
- [14] J. Y. Guo and J. Li, *A hybrid genetic algorithm to the vehicle routing problem with fuzzy traveling time*, Journal of Industrial Engineering Management, **19** (2006), 13-17.
- [15] L. Hong and M. Xu, *A model of MDVRPTW with fuzzy travel time and time-dependent and its solution*, Proceeding of Fifth International Conference on Fuzzy Systems and Knowledge Discovery, **3** (2008), 473-478.
- [16] L. Hong and M. Xu, *Real vehicle routing and dispatching with dynamic fuzzy travel times*, Proceeding of Second International Conference on Genetic and Evolutionary Computing, doi:10.1109/WGEC.2008.28, (2008), 32-37,
- [17] J. Jia, N. Liu and R. Wang, *Genetic algorithm for fuzzy logistics distribution vehicle routing problem*, Proceeding International Conference on Service Operations and Logistics, and Informatics, doi:10.1109/SOLI.2008.4686625, (2008), 1427-1432.
- [18] M. Ko, A. Tiwari and J. Mehnen, *A review of soft computing applications in supply chain management*, Applied Soft Computing, **10** (2010), 661-674.
- [19] R. J. Kuo, C. Y. Chiu and Y. J. Lin, *Integration of fuzzy theory and ant algorithm for vehicle routing problem with time window*, International Conference of the North American Fuzzy Information Processing Society, **23** (2004), 925-930.
- [20] C. S. Liu and M. Y. Lai, *The vehicle routing problem with uncertain demand at nodes*, Transportation Research Part E: Logistics and Transportation Review, **5(4)** (2009), 517-524.
- [21] P. Lucic and D. Teodorovic, *Vehicle routing problem with uncertain demand at nodes: the bee system and fuzzy logic approach*, In Fuzzy Sets Based Heuristics for Optimization, Springer Verlag, Berlin, (2003), 67-82.
- [22] P. Lucic and D. Teodorovic, *The fuzzy ant system for the vehicle routing problem when demand at nodes is uncertain*, Journal on Artificial Intelligence Tools (IJAIT), **16(5)** (2007), 751-770.
- [23] M. R. Safi, H. R. Maleki and E. Zaeimazad, *A note on the Zimmermann method for solving fuzzy linear programming problems*, Iranian Journal of Fuzzy Systems, **4(2)** (2007), 31-45.
- [24] W. R. Stewart Jr. and B. L. Golden, *Stochastic vehicle routing: a comprehensive approach*, European Journal of Operational Research, **14(4)** (1983), 371-385.
- [25] L. Tang, W. Cheng, Z. Xhang and B. Zhong, *Ant colony algorithm based on information entropy theory to fuzzy vehicle routing problem*, Proceedings ISKE, Series: Advances in Intelligent Systems Research, 2007.
- [26] D. Teodorovic and S. Kikuchi, *Application of fuzzy sets theory to the saving based vehicle routing algorithm*, Civil Engineering Systems, **8** (1991), 87-93.
- [27] D. Teodorovic and G. Pavkovic, *The fuzzy set theory approach to the vehicle routing problem when demand at nodes is uncertain*, Fuzzy Sets and Systems, **82** (1996), 307-317.
- [28] D. Teodorovi, *Fuzzy logic systems for transportation engineering: the state of the art*, Transportation Research, **33(A)** (1999), 337-364.
- [29] F. Tillman, *The multiple terminal problem with probabilistic demands*, Transportation Science, **3(3)** (2002), 192-204.
- [30] P. Toth and D. Vigo, *The vehicle routing problem*, Monographs on Discrete Mathematics and Applications, SIAM, **9** (2002).
- [31] J. L. Verdegay, *Fuzzy mathematical programming*, In: M. M. Gupta, E. Sanchez, eds., Fuzzy Information and Decision Processes, 1982.
- [32] J. L. Verdegay, *Fuzzy optimization: models, methods and perspectives*, Proceeding 6thIFSA-95 World Congress, (1995), 39-71.

- [33] J. L. Verdegay, R. R. Yager and P. Bonissone, *On heuristics as a fundamental constituent of Soft Computing*, Fuzzy Sets and Systems, **159** (2008), 846-855.
- [34] R. Viertl and D. Hareter, *Fuzzy information and stochastics*, Iranian Journal of Fuzzy Systems, **1(1)** (2004), 43-56.
- [35] J. Y. Zhang and J. Li, *Study on logistics distribution vehicle routing problem with fuzzy due-time*, International Conference on Management Science & Engineering, doi: 10.1109/ICMSE.2007.4421866, (2007), 311-317.
- [36] Y. S. Zheng and B. D. Liu, *Fuzzy vehicle routing model with credibility measure and its hybrid intelligent algorithm*, Applied Mathematics and Computation, **176** (2006), 673-683.
- [37] H. J. Zimmermann, *Description and optimization of fuzzy system*, International Journal of general System, **2** (1976), 209-216.

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